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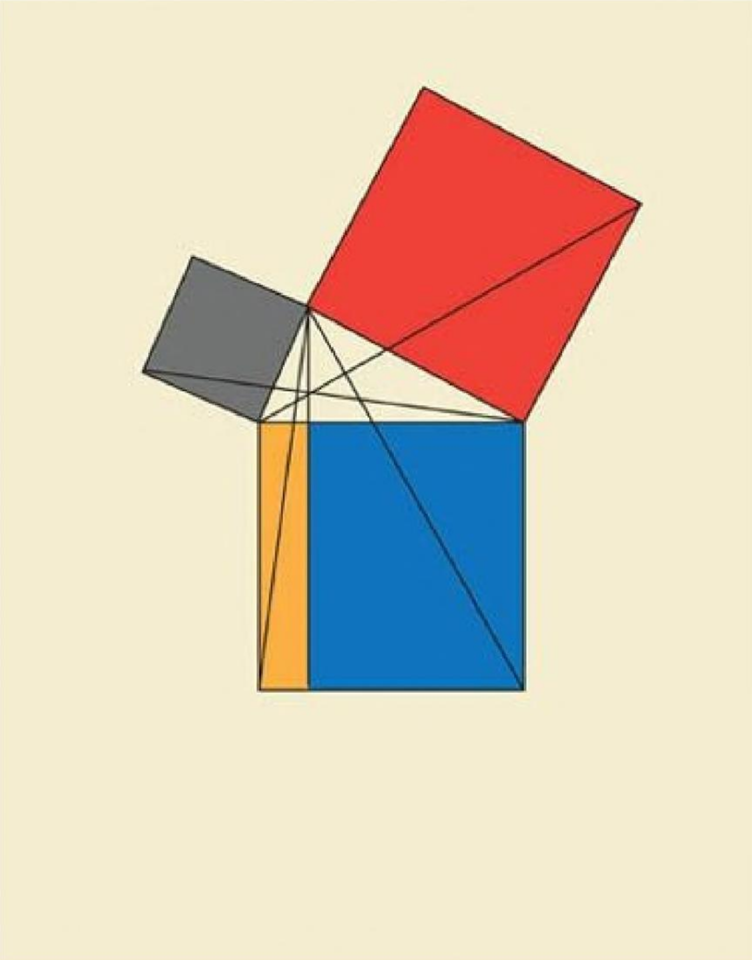
INTRODUCTION

Richard Brown



Elegant geometry

Mathematicians often 'see' mathematical objects like equations using geometry. This is a visual proof of the famous Pythagorean Theorem, $a^2 + b^2 = c^2$.

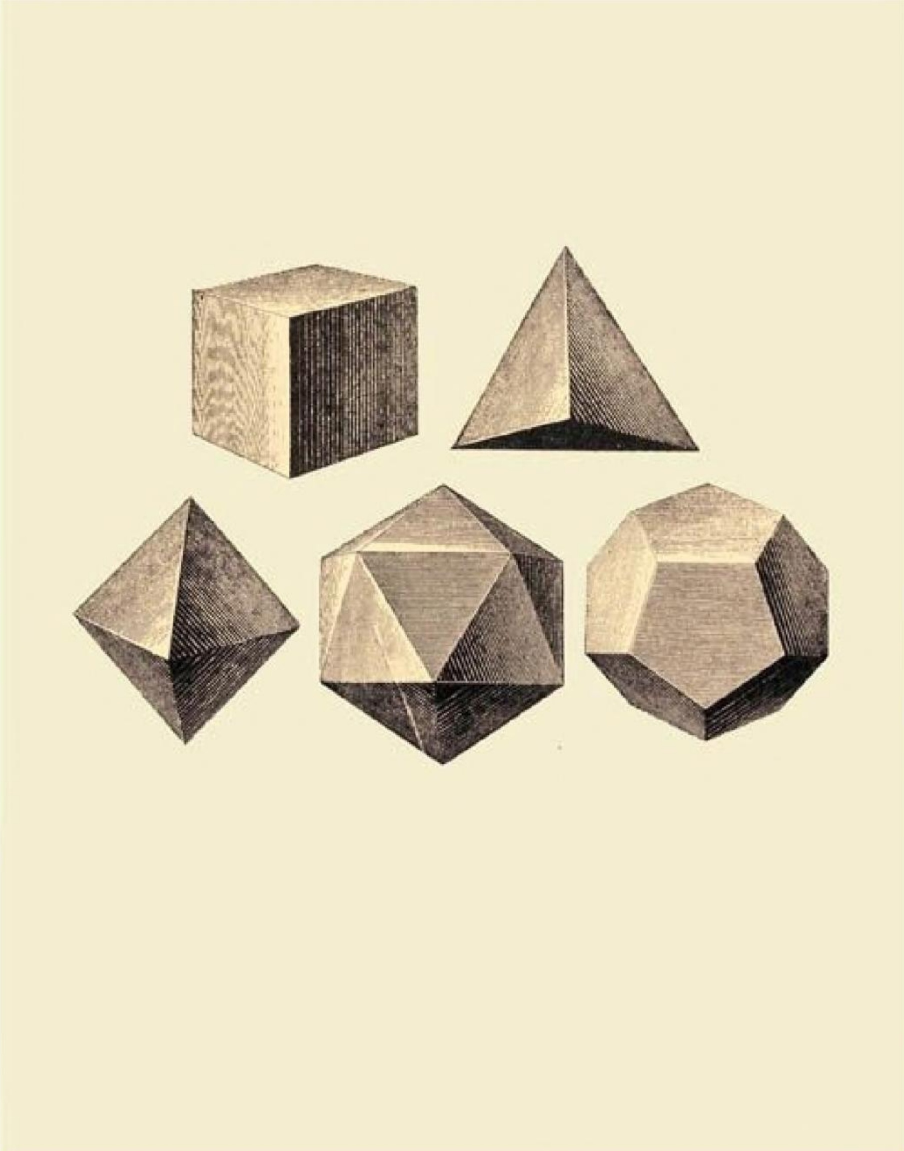
It is said that mathematics is the art of pure reason. It is the fundamental logical structure of all that exists, and all that doesn't exist, in this reality of ours. Far removed from the simple calculations that allow us to balance our accounts and calculate our everyday affairs, mathematics helps us to order and understand the very notion of everything we can imagine in life. Like music, art and language, the essential symbols and concepts of mathematics, many of which are defined and discussed in this book, allow us to express ourselves in amazingly intricate ways and to define unimaginably complex and beautiful structures. While the practical uses for mathematics are rife, what makes mathematics so magical is its elegance and beauty outside of any real application. We give the concepts in mathematics meaning only because they make sense and help us to order our existence. But outside of the meaning we give these elements of maths, they do not really exist at all except in our imagination.

The natural and social sciences use mathematics to describe their theories and provide structure to their models, and arithmetic and algebra allow us to conduct our business and learn how to think. But beyond these practical applications lies the true nature of the discipline; mathematics is the framework and provides the rules of engagement for the entire system of structured thought.

This text is a glimpse into the world a mathematician sees in everyday life. Herein lies a set of some of the more basic and fundamental elements in the field today, with definitions, a little history, and some insight into the nature of many basic mathematical concepts. This book contains 50 entries, each of which centres on an important topic in mathematics. They are ordered into seven categories, which roughly help to define their context. In **Numbers & Counting**, we explore the basic building blocks that allow us to enumerate our surroundings. We study some of the operations and structures on numbers in **Making Numbers Work**. These entries basically describe the arithmetic system that helps us to use mathematics in our everyday lives. In **Chance is a Fine Thing**, we detail some ideas and consequences when using mathematics to understand random events and chance happenings. Next, we lay out some of the deeper, more complex structures of numbers in **Algebra & Abstraction**. It is here that the path towards higher mathematics begins. In turn, we explore the more visual aspects of mathematical relationships in **Geometry & Shapes**. Since mathematical abstraction is one of pure imagination, we then explore what happens outside of our three dimensions in **Another Dimension**. And finally, in **Proofs & Theorems**, we discuss some of the more profound ideas and facts that our mathematical path has led us to.

Individually, each entry is a brief glimpse into one of the more beautiful and important ideas central to mathematics today. Each topic is presented in the same format, aimed at facilitating a proper introduction; the 3-second sum offers the briefest overview, the 30-second maths goes into further depth on the topic, and a 3-minute addition begins the process of pondering the deeper connections between the idea and its importance in the world. It is hoped that, taken together, these elements will help to open your eyes to a closer understanding of the nuts and bolts of what mathematics is really all about.

When used as a reference text, this book will provide the basics of some of the more profound ideas in mathematics. When read in full, this text may provide a glimpse into another world as rich and meaningful as the one you live in now: the world of mathematics.



Dimensional beauty

There are only five ways to construct a three-dimensional solid using regular polygons. It is not hard to see why. But does that make these objects special? Mathematicians think so.

NUMBERS & COUNTING

algebra One of the main branches of pure mathematics which studies operations and relations on numbers. Elementary algebra involves studying the rules of arithmetic on expressions involving variables. Advanced algebra involves studying these operations and relations on mathematical objects and constructions other than numbers.

algebraic number Any number that is a root of a non-zero polynomial that has integer coefficients. In other words algebraic numbers are solutions to polynomial equations (see here), such as $x^2 - 2 = 0$, where $x = \sqrt{2}$. All rational numbers are algebraic, but irrational numbers can be either algebraic or not. One of the best-known algebraic numbers is the golden ratio (1.6180339...), which is usually written ϕ.

binary (base 2) The counting system in which only the numbers 1 and 0 feature. Just as in our base 10 system there is a 1s column ($10^0 = 1$), 10s column (10^1) and 100s (10^2) column, and so on, in base 2 there is a 1s (2^0) column, a 2s column ($2^1 = 1$), a 4s column (2^2), and so on. For example, the binary version of 7 is written 111, as in $1 \times 1 + 1 \times 2 + 1 \times 4$.

coefficient A number that is used to multiply a variable; in the expression $4x = 8$, 4 is the coefficient, x is the variable. Although usually numbers, symbols such as a can be used to represent coefficients. Coefficients that have no variables are called constant coefficients or constant terms.

complex number Any number that comprises both real and imaginary number components, such as $a + bi$, in which a and b represent any real number and i represents $\sqrt{-1}$. See *imaginary number*.

factor One of two or more numbers that divides a third number exactly. For example 3 and 4 are factors of 12, as are 1, 2, 6 and 12.

figurate number Any number that can be represented as a regular geometric shape, such as a triangle, square or hexagon.

fractional number (fraction) Any number that represents part of a whole. The most common fractions are called common or vulgar fractions, in which the bottom number, the denominator, is a non-zero integer denoting how many parts make up the whole, whereas the top number, the numerator, represents the number of equal divisions of the whole. Proper fractions represent a value of less than 1, e.g., ⅔, whereas improper fractions represent a value greater than 1, e.g., 3/2, or 1⅓.

imaginary number A number that when squared provides a negative result. As no real number when squared provides a negative result, mathematicians developed the concept of the imaginary number unit i, so that $i \times i = -1$ or put another way $i = \sqrt{-1}$. Having an imaginary number unit that represents $\sqrt{-1}$ helps solve a number of otherwise unsolvable equations, and has practical applications in a number of fields.

integer Any natural number (the counting numbers 1, 2, 3, 4, 5 and so on), 0 or the negative natural numbers.

irrational number Any number that cannot be expressed as a ratio of the integers on a number line. The most commonly cited examples of irrational numbers are π and $\sqrt{2}$. A good way of identifying an irrational number is to check that its decimal expansion does not repeat. Most real numbers are irrational numbers.

number line The visual representation of all real numbers on a horizontal scale, with negative values running indefinitely to the left and positive to the right, divided by zero. Most number lines usually show the positive and negative integers spaced evenly apart.

polynomial An expression using numbers and variables, which only allows the operations of addition, multiplication and positive integer exponents, i.e., x^2. (See Polynomial Equations.)

rational number Any number that can be expressed as a ratio of the integers on a number line; or more simply any number that can be written as a fraction, including whole numbers. Rational numbers are also identified by finite or repeating decimals.

real number Any number that expresses a quantity along a number line or continuum. Real numbers include all of the rational and the irrational numbers.

transcendental number Any number that cannot be expressed as a root of a non-zero polynomial with integer coefficients; in other words non-algebraic numbers. π is the best-known transcendental number, and following the opening definition π therefore could not satisfy the equation $\pi^2 = 10$. Most real numbers are transcendental.

whole number Also known as a natural or counting number, a whole number is any positive integer on a number line or continuum. Opinion varies, however, on whether 0 is a whole number.

FRACTIONS & DECIMALS

the 30-second maths

The whole numbers, 0, 1, 2, 3..., are the bedrock of mathematics, and have been used by humans for millennia. But not everything can be measured using whole numbers. If 15 hectares of land are divided between 7 farmers, each will have 15/7 (or $2^1/_7$) hectares. The simplest non-whole numbers can be expressed in a fractional form like this. But for other numbers, such as π, this is awkward or impossible. With the development of science came the need to subdivide quantities ever more accurately. Enter the decimal system, an efficient column-based method using Hindu-Arabic numerals. Here, the number 725 has three columns, and stands for 7 hundreds, 2 tens and 5 units. By adding a decimal point after the units, and extra columns to its right, this approach easily extends to numbers smaller than a unit. So 725.43 stands for 7 hundreds, 2 tens, 5 units, 4 tenths (of a unit) and 3 hundredths. By incorporating ever more columns to the left or to the right, numbers both large and small can be written as precisely as needed. In fact every number in between the whole numbers can be expressed as a decimal (but not as a fraction), giving us the 'real' number system.

3-SECOND SUM

The starting point for mathematics is the system of whole numbers, 0, 1, 2, 3... But many things fall between the gaps, and there are two ways to measure them.

3-MINUTE ADDITION

Translating between fractions and decimals is not always straightforward. It is easy to recognize 0.25, 0.5 and 0.75 as ¼, ½ and ¾ respectively. But the decimal equivalent of ⅓, is 0.333333..., where the string of 3s never ends, and 1/7 is 0.142857142857142857..., also with a never-ending repeating pattern. It turns out all fractional numbers have repeating patterns in their decimal, while non-fractional numbers like π have decimals that do not repeat. These are the irrational real numbers.

RELATED THEORIES

RATIONAL & IRRATIONAL NUMBERS

COUNTING BASES

ZERO

3-SECOND BIOGRAPHIES

ABU 'ABDALLAH MUHAMMAD IBN MUSA AL-KHWARIZMI

c. 790–850

ABU'L HASAN AHMAD IBN IBRAHIM AL-UQLIDISI

c. 920–980

IBN YAHYA AL-MAGHRIBI AL-SAMAWAL

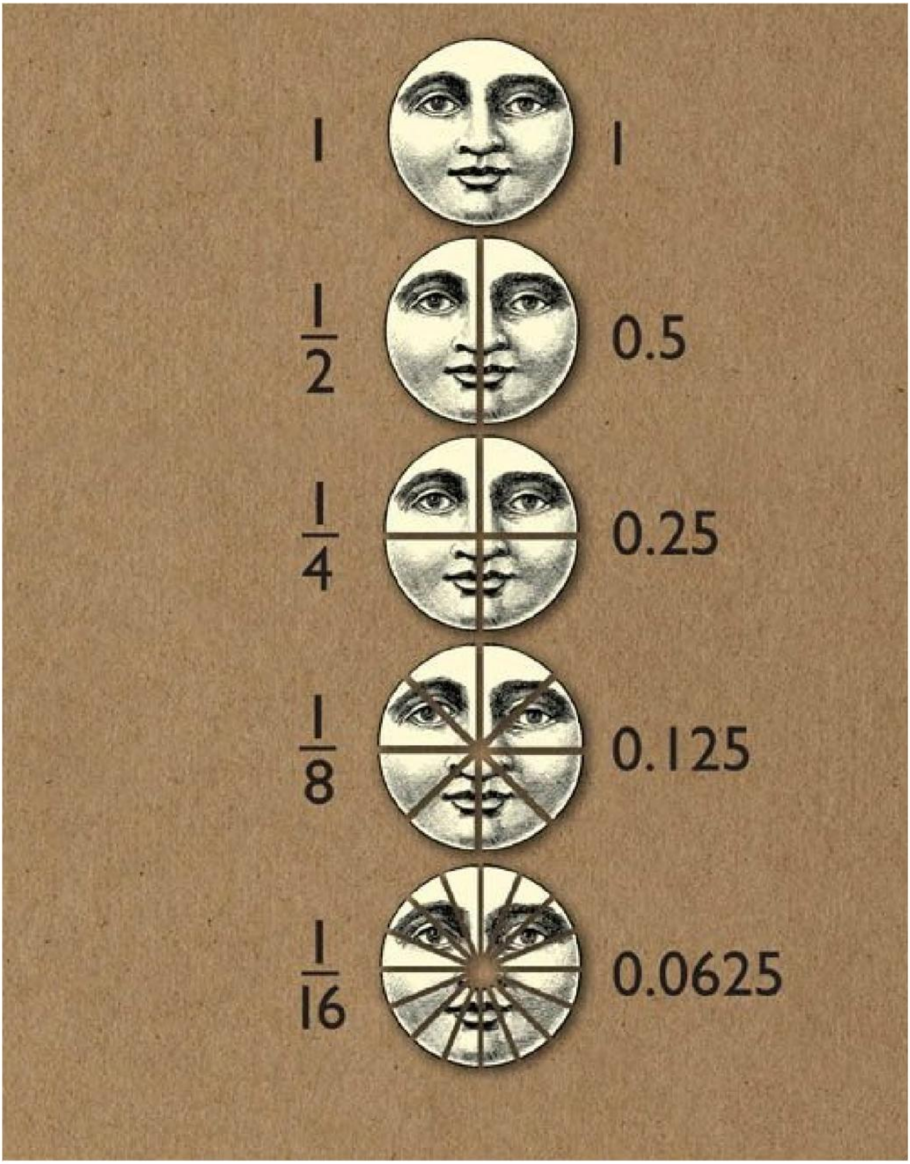
c. 1130–1180

LEONARDO PISANO (FIBONACCI)

c. 1170–1250

30-SECOND TEXT

Richard Elwes


1
1
1/2
0.5
1/4
0.25
1/8
0.125
1/16
0.0625

Whole numbers can be subdivided into fractions, and decimals express these divisions even more precisely.

RATIONAL & IRRATIONAL NUMBERS

the 30-second maths

Real numbers consist of positive numbers, negative numbers and 0, and these values can be categorized in several ways. The most fundamental way is to distinguish the real numbers that can be expressed as the fraction of two integers, such as ½ or –7/3 (called rational numbers), from those that cannot (called irrational numbers). The ancient Greeks believed all numbers were rational, until a follower of Pythagoras proved that √2 is not rational. You can tell if a number is rational or irrational by looking at its decimal expansion – if the digits ultimately repeat, the number is rational (think 3/11 = 0.272727...). Decimal expansions of irrational numbers (for example, π = 3.14159265...) have digits that do not repeat. But there's more. Rational numbers and many irrational numbers have something in common – they are algebraic, that is they are solutions to polynomial equations with integer coefficients. For example, √2 solves $x^2 - 2 = 0$ (see Polynomial Equations). But many more irrational numbers are not algebraic, and π is one example. Numbers that are not algebraic are called transcendental – only irrational numbers can be transcendental.

3-SECOND SUM

'Real' numbers — the numbers used to express quantities and representable via a decimal expansion — are either rational or irrational. But some irrationals are more unusual than others.

3-MINUTE ADDITION

The philosophy of the ancient Greeks held that all things measurable are, at worst, the ratio of whole numbers. Anecdotal history holds that the Pythagoreans were so distraught to discover that √2 is irrational that Hippasus of Metapontum was murdered to prevent revelation of this truth to the world. A number like π is perhaps more intuitively irrational, but it was only 250 years ago that this was proved true, and another century would pass before π was proved to be transcendental.

RELATED THEORIES

FRACTIONS & DECIMALS

EXPONENTIALS & LOGARITHMS

POLYNOMIAL EQUATIONS

PI – THE CIRCLE CONSTANT

PYTHAGORAS

3-SECOND BIOGRAPHIES

HIPPASUS OF METAPONTUM
active fifth century BCE

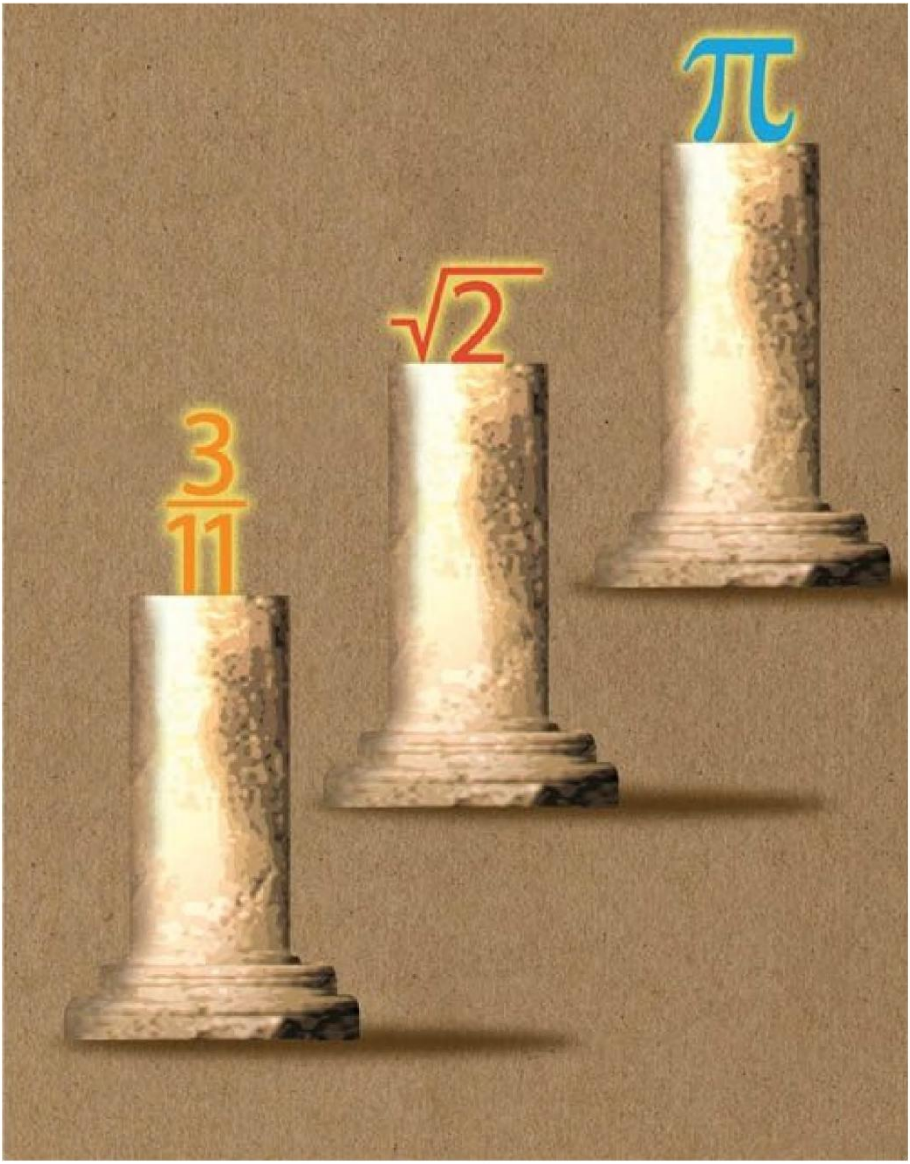
JOHANN LAMBERT
1728–1777

CHARLES HERMITE
1822–1901

FERDINAND VON LINDERMANN
1852–1939

30-SECOND TEXT

David Perry


π
√2
3/11

Be real – numbers are rational if they can be written as a fraction. Otherwise they are irrational.

IMAGINARY NUMBERS

the 30-second maths

Over the years, mathematicians have enlarged the number system several times. An early expansion was the inclusion of negative numbers. In business, for example, if +4 represents being in profit by 4 units, then −4 stands for being 4 units in debt. Negative arithmetic has a surprising property. Multiply a positive number by a negative, and you get a negative result: e.g., $-4 \times 3 = -12$. But multiply one negative number by another, and you get a positive result: $-4 \times -3 = 12$. So there was no number (positive or negative) which, when multiplied by itself, gives a negative answer. This meant that some simple equations, such as $x^2 = -1$, could never be solved, which was an obstacle to solving more sophisticated equations, even when solutions existed. This was corrected by a new 'imaginary' number i, defined as the square root of −1; that is to say $i \times i = -1$. This started off as a cheat to assist in calculations and was controversial early on; Descartes coined the term 'imaginary' as a derogatory term. Over time, however, it has become as accepted as all other types of number. Today, the number system that mathematicians prefer is termed 'complex numbers', comprising the likes of $2 + 3i$, or $\frac{1}{2} - \frac{1}{4}i$, or more generally $a + bi$, where a and b are any 'real' (that is to say decimal) numbers.

3-SECOND SUM

Today's mathematicians work in an expanded number system, which includes a new 'imaginary' number i, the square root of −1.

3-MINUTE ADDITION

The complex numbers allow for solutions to equations like $x \times x = -1$. One might ask next whether there are solutions to $x \times x = i$, for example, or whether we have to expand the system yet again. As it turns out, the complex numbers contain solutions to all possible polynomial equations, meaning that they are all we will ever need. This wonderful fact is known as the fundamental theorem of algebra.

RELATED THEORIES

FRACTIONS & DECIMALS

POLYNOMIAL EQUATIONS

RIEMANN'S HYPOTHESIS

3-SECOND BIOGRAPHIES

NICCOLÒ FONTANA ('TARTAGLIA')
1500–1557

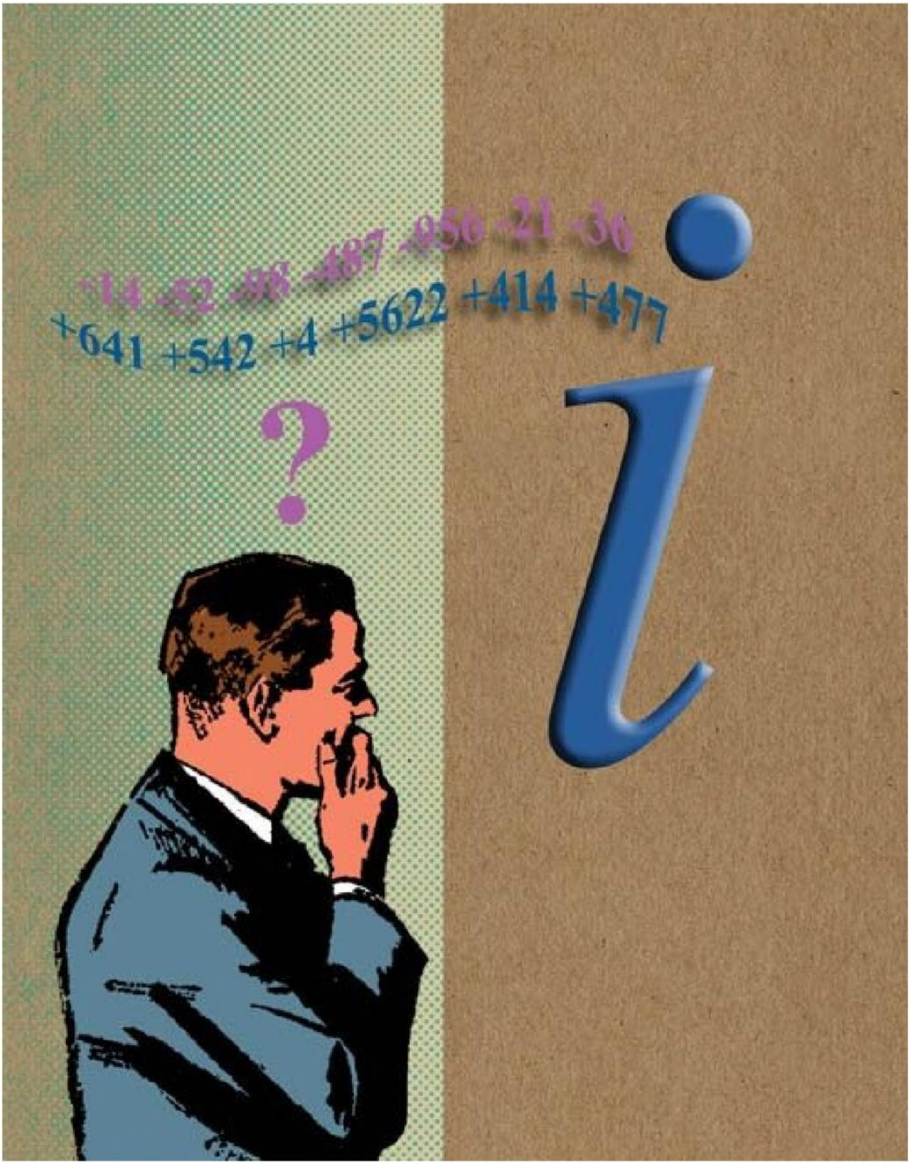
GIROLAMO CARDANO
1501–1576

RAFAEL BOMBELLI
1526–1572

CARL-FRIEDRICH GAUSS
1777–1855

AUGUSTIN-LOUIS CAUCHY
1789–1857

30-SECOND TEXT
Richard Elwes


-14 -52 -98 -487 -956 -21 -36
+641 +542 +4 +5622 +414 +477
?
i

Positive and negative integers weren't enough for some mathematicians – they needed imaginary numbers.

COUNTING BASES

the 30-second maths

When we count numbers beyond nine, we are used to putting a '1' in the next column and reusing the symbols. This is because we use the base 10 or decimal system. But base 10 has not always been the preferred system. Ancient Babylonians used base 60 (the sexagesimal system), for counting. Rather than stopping at nine and moving into the next column, they stopped at 59. Some reminders of this system include the continued use of 60 minutes in an hour, and 360° in a circle. References to base 12 counting, the duodecimal system, give us the concepts of dozen and gross (a dozen dozen). Base 20 counting, the vigesimal system, was common in early Europe (the 'score' in Abraham Lincoln's famous Gettysburg Address line, '4 score and 7 years ago', is 20). Modern computers use the base 2 or binary number system, where only 0 and 1 are used. Here it was easy to produce early systems for counting where only two mutually exclusive states are needed, like an open or closed electrical circuit. In any base, addition and multiplication are well-defined and one can do algebra. Try that the next time someone asks you for the value of 1 plus 1. It is obviously 10 (in binary arithmetic)!

3-SECOND SUM

A base refers to the number of unique digits that a counting system uses to represent numerical values.

3-MINUTE ADDITION

The Mayans of Central America also used base 20 for the 'long count' of their calendar, although they 'corrected' the third column from the normal 400 = 20 × 20 part to 18 × 20 = 360, maybe to reflect the approximate number of days in a year. If we prefer base 10 simply because our fingers are good calculators, did the Mayans see the value of their open-shoed toes in this endeavour?

RELATED THEORIES

ZERO

3-SECOND BIOGRAPHIES

GOTTFRIED LEIBNIZ

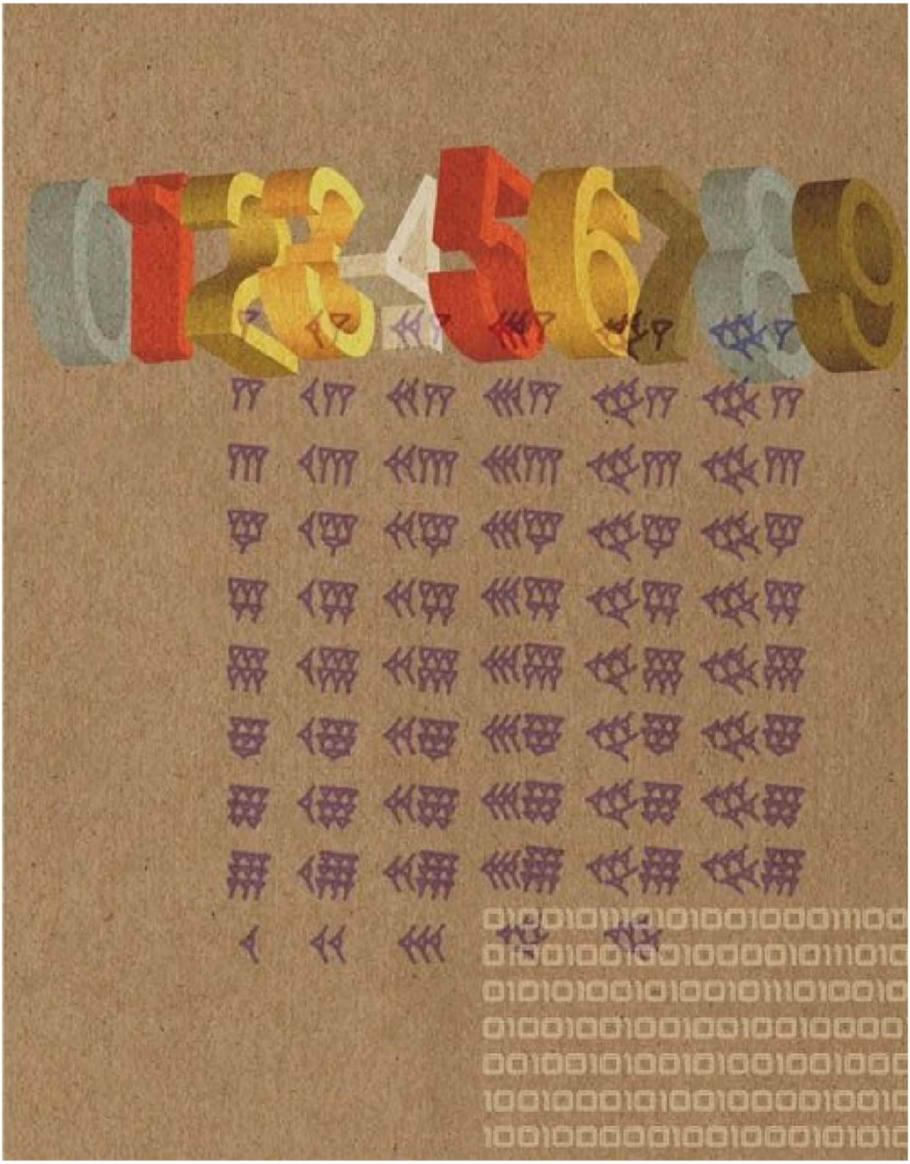
1646–1716

GEORGE BOOLE

1815–1864

30-SECOND TEXT

Richard Brown



The most commonly used counting system is base 10 – the Babylonians thought big with 60 unique digits. Computer code keeps it simple with a mere two digits.

PRIME NUMBERS

the 30-second maths

Most whole numbers will factor into smaller parts. For example, 100 = 4 × 25. It's also true that 100 = 20 × 5. If we take either of those and break the factors into still smaller factors, we ultimately come to the prime factorization of 100:100 = 2 × 2 × 5 × 5. We cannot break down the factors further – they are prime, divisible only by 1 and themselves. When mathematicians started listing the prime numbers, they searched for a pattern but did not see one. They raised the question of whether the list was finite or if one could find larger and larger primes. Euclid gave an elegant proof in his *Elements* that there are infinitely many primes. 17,463,991,229 is a large prime. How do we know it's prime? We could try dividing this integer by all smaller integers and find no factors other than 1, then declare it prime. This is slow, however, and there are better ways. The largest known primes have over 10,000,000 digits, and clever methods are required to establish them as such. Finding large primes might seem frivolous, but a revolutionary idea in the 1970s created a technique to effect secure communications by use of a system requiring the generation of large prime numbers. This technique pervades the internet, allowing us to shop online in safety.

3-SECOND SUM

A prime number is a positive integer that is divisible only by 1 and itself. Primes cannot be 'broken apart', and are to integers as the elements are to matter.

3-MINUTE ADDITION

When we take prime factorizations of numbers, it seems obvious that we will always get the same prime numbers at the end. The more one studies numbers, however, the less obvious this fact becomes. It's true, and is so important, that this fact bears the title of the fundamental theorem of arithmetic. Although no formula will generate each prime number in turn, the prime number theorem gives us an idea of what proportion of whole numbers are prime.

RELATED THEORIES

NUMBER THEORY

EUCLID'S ELEMENTS

3-SECOND BIOGRAPHIES

EUCLID
fl. 300 BCE

CARL FRIEDRICH GAUSS
1777–1855

JACQUES HADAMARD
1865–1963

CHARLES JEAN DE LA VALLÉE-POUSSIN
1866–1962

30-SECOND TEXT
David Perry

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Only divisible by 1 and themselves, prime numbers have fascinated mathematicians for centuries. The discovery of large primes has practical applications today.

FIBONACCI NUMBERS

the 30-second maths

In the Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, ... each term is the sum of the previous two terms. The resulting sequence, which plays a special role in number theory, possesses many curious numerical properties. If you add the terms in the Fibonacci sequence up to a certain point, the sum is always one less than a Fibonacci number; e.g., $1 + 1 + 2 + 3 + 5 + 8$ is one less than the Fibonacci 21. Adding the squares of these numbers produces a product of two Fibonacci numbers: $1 + 1 + 4 + 9 + 25 + 64 = 8 \times 13$. The ratios 1:1, 2:1, 3:2, 5:3, 8:5, ... approach the golden ratio $\phi \approx 1.618$. Geometrically, squares whose sides are Fibonacci numbers in length fit together nicely to form a golden spiral. Long before humans became fascinated with these patterns, plants had discovered the economy of Fibonacci numbers. The leaves or buds of many plants with a spiral structure – such as pineapples, sunflowers and artichokes – exhibit a pair of consecutive Fibonacci numbers. Examining a pineapple, you'll find 8 rows spiralling around in one direction and 13 in the other direction. In the animal kingdom, a honeybee has a Fibonacci number of ancestors in each generation.

3-SECOND SUM

A simple rule, adding the two previous terms to get the next term, produces one of Mother Nature's favourite sequences of numbers.

3-MINUTE ADDITION

In 1202, Leonardo Pisano, also known as Fibonacci, posed a riddle about breeding rabbits in his book *Liber Abaci* (*The Book of the Abacus*). Fibonacci posited, perhaps unrealistically, that after every month, each pair of adult rabbits produces one pair of baby rabbits, and baby rabbits take one month to become adults. If you start with a single pair of baby rabbits in January, you will have 144 pairs of rabbits by December!

Fibonacci numbers appear in the ancestral tree of a honeybee. Each male bee has only a female parent, while each female has two parents, one male and one female.

PASCAL'S TRIANGLE

the 30-second maths

What comes next in this sequence: (1 1), (1 2 1), (1 3 3 1), (1 4 6 4 1),...? This riddle is an important problem in algebra, known as 'expanding brackets'. Start with the expression $(1 + x)$ and multiply it by itself. This gives $(1 + x)^2 = 1 + 2x + 1x^2$. Multiplying three brackets gives $(1 + x)^3 = 1 + 3x + 3x^2 + 1x^3$. Four produces $(1 + x)^4 = 1 + 4x + 6x^2 + 4x^3 + 1x^4$. It is not the algebra which is difficult here, but the numbers. The next expression will look something like this: $(1 + x)^5 = 1 + ?x + ?x^2 + ?x^3 + ?x^4 + 1x^5$. But what are the right numbers to fill in here? Blaise Pascal wanted a way to find the answer quickly, and find it he did, in the rows of his famous triangle. It begins with a 1. Below that, there are two more 1s. Pascal's insight was that the process could be continued, with each number coming from the two above it, added together. (Earlier thinkers had come to similar conclusions, including the Indian thinker Pingala, over a thousand years earlier.) This process is simple to do: just a little addition and no complicated algebra. Each row then gives the answer to a bracket expanding problem. So to find $(1 + x)^5$, just read the numbers along the sixth row: 1, 5, 10, 10, 5, 1.

3-SECOND SUM

Blaise Pascal's celebrated triangle not only contains many fascinating numerical patterns, it is also an essential tool in algebra.

3-MINUTE ADDITION

Pascal's triangle contains many fascinating patterns. The first diagonal is a row of 1s, and the second counts: 1, 2, 3, 4, ... But the third comprises what are known as the triangular numbers: 1, 3, 6, 10, 15, ... If you want to arrange balls into a triangle (at the start of a game of pool, for example), these are the numbers that work. The Fibonacci numbers are also hiding in the triangle, as the totals of successive 'shallow diagonals' – see if you can find them!

RELATED THEORIES

FIBONACCI NUMBERS

THE VARIABLE PLACEHOLDER

POLYNOMIAL EQUATIONS

3-SECOND BIOGRAPHIES

PINGALA

c. 200 BCE

ABU BEKR IBN MUHAMMAD IBN AL-HUSAYN AL-KARAJI

953–1029

YANG HUI

1238–1298

BLAISE PASCAL

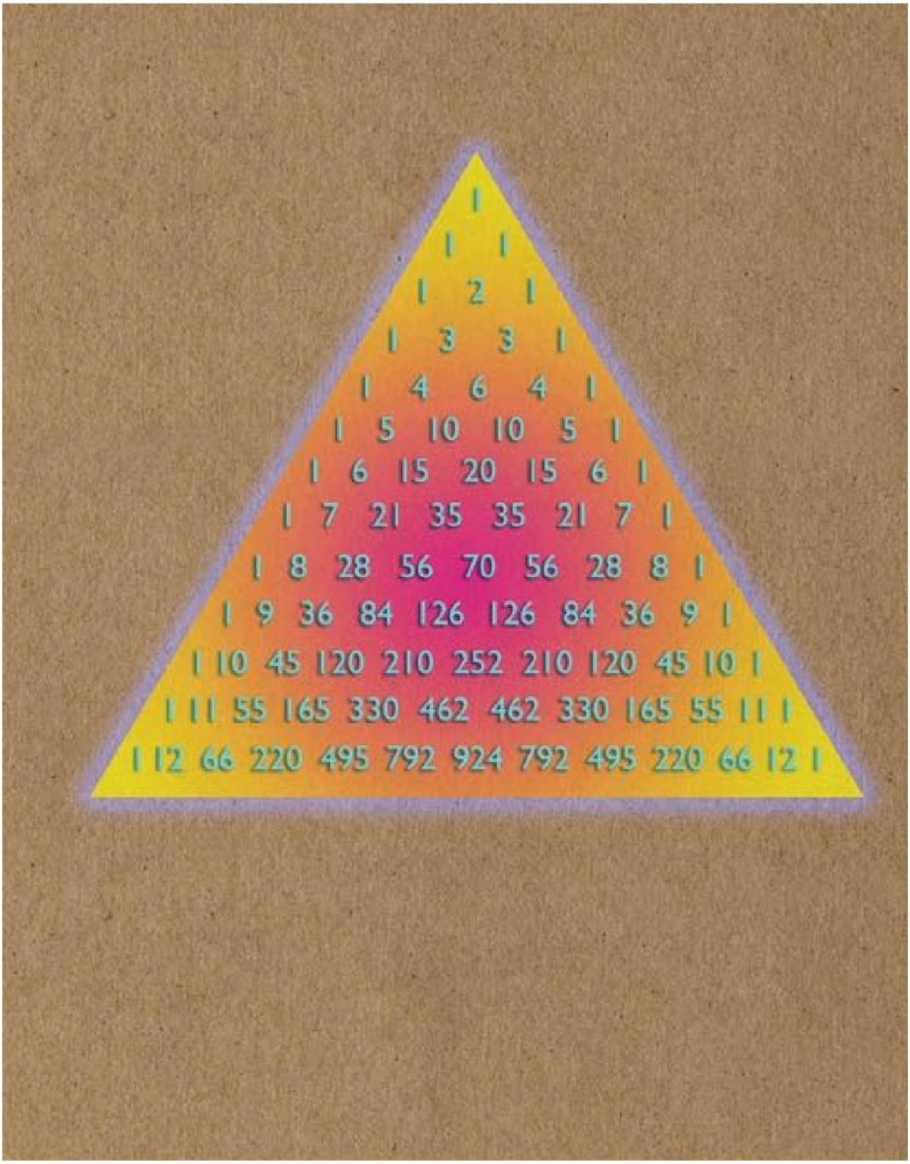
1623–1662

ISAAC NEWTON

1643–1727

30-SECOND TEXT

Richard Elwes


1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1

Pascal's triangle contains numerous mathematical patterns and provides a neat solution to some algebraic problems.

1658

Wrote *Essay on the Cycloids*

1668

Began work on *Pensées*, a collection of philosophical and theological notes

19 August, 1662

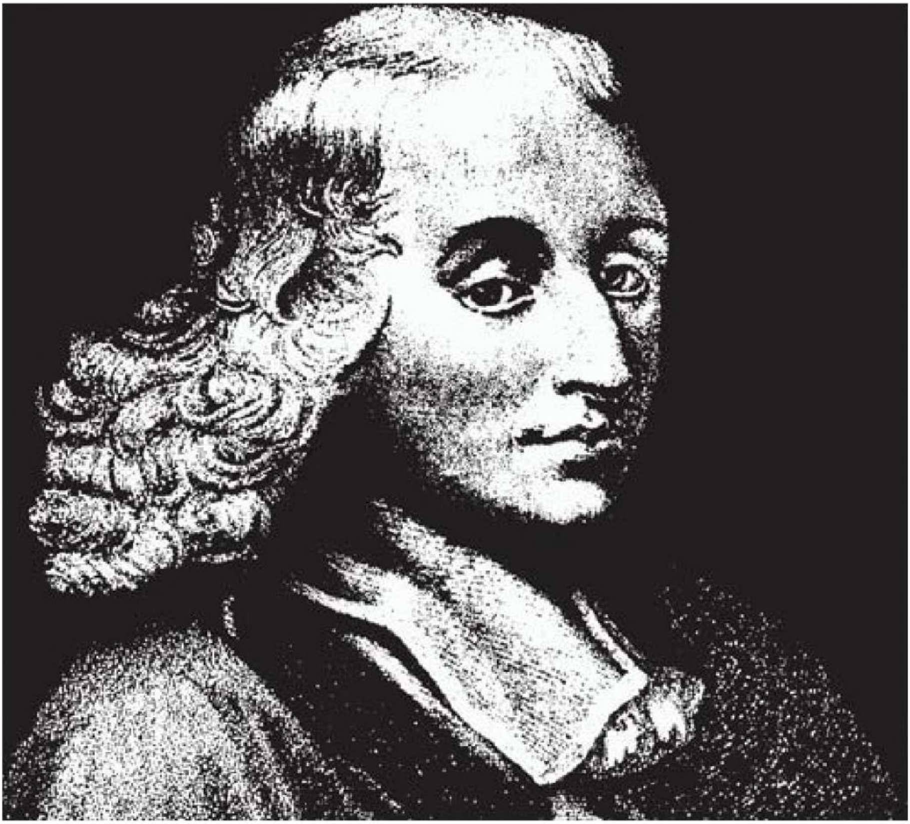
Died in Paris

1670

Pensées published posthumously

1779

Essay on Conics published



NUMBER THEORY

the 30-second maths

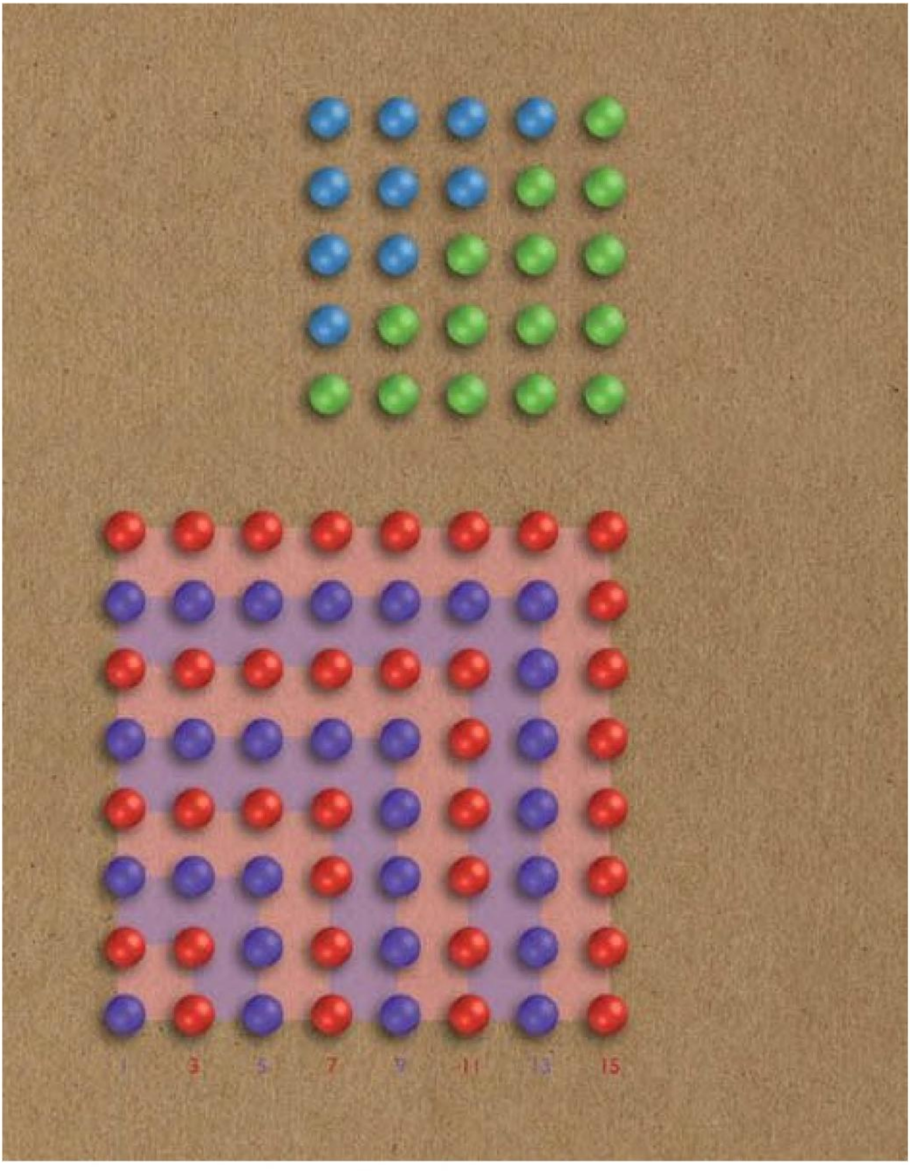
Number theory is the study of interesting properties that numbers possess. For example, choose any odd prime number and divide it by 4. The remainder will be either 1 or 3. It can be proven that if the remainder is 1, you can find two square numbers that add up to that prime. For example, dividing 73 by 4 gives 18 with a remainder of 1. After a short search, you can determine that $73 = 9 + 64 = 3^2 + 8^2$. On the other hand, a remainder of 3 means that no matter how hard you look, it is *impossible* to find two squares that add up to that prime (try 7 or 59). This begs the question: why? Mathematicians are never satisfied with discovering this kind of interesting behaviour – they want to prove that such properties are always true. Ancient Greek mathematicians began exploring properties of divisibility of integers, leading them to study prime numbers. They also enjoyed studying figurate numbers and their interrelationships. If you have a number of stones that can be arranged into an equilateral triangle, or a square, or a pentagon, and so forth, it is called figurate. Euclid even provided a formula for when any two squares add up to a third square. Pondering similar equations led Pierre de Fermat to conjecture what became his famous Last Theorem.

3-SECOND SUM

Number theory is the discipline devoted to the study of properties and the behaviour of various classes of numbers.

3-MINUTE ADDITION

Carl Friedrich Gauss declared that mathematics was the queen of the sciences and that number theory was the queen of mathematics. G. H. Hardy echoed this sentiment some 70 years ago, relishing an area of mathematics that is only studied for the surprising beauty of the discovered truths, an area unsullied by practical application. When number theory later began to show unanticipated application to cryptology, few thought the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MAKING NUMBERS WORK

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