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A Mathematician's Lament

PAUL LOCKHART

With a Foreword by Keith Devlin

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For Stanley, who asked me to write it.

If you want to build a ship, don't drum up people to collect wood and don't assign them tasks and work, but rather teach them to long for the endless immensity of the sea.

—ANTOINE DE SAINT EXUPÉRY

Foreword

IN LATE 2007, AN AUDIENCE MEMBER AT A TALK I gave handed me a 25-page typewritten document called *A Mathematician's Lament*, saying he thought I might like it. Written by a mathematics teacher called Paul Lockhart, the essay had been circling somewhat erratically through the mathematics education community since its author first wrote it in 2002, but it had never been published. The audience member's prediction turned out to be an understatement. I loved it, and felt that the words of this Paul Lockhart—whoever he was—deserved a much wider audience. And so I did something I have never done before, and probably never will again: after tracking down the essay's author—not entirely straightforward since the essay bore no contact information—and securing his permission, I devoted an entire issue of my monthly online column "Devlin's Angle" on the Mathematical Association of America's web-zine *MAA Online* (www.maa.org) to reproducing the entire essay in its original form. It was the quickest and most effective way I knew to get it in front of the mathematics and mathematics education communities.

When *A Mathematician's Lament* appeared in my March 2008 column, I introduced it with these words:

> *It is, quite frankly, one of the best critiques of current K-12 mathematics education I have ever seen.*

I was expecting a strong response. What ensued was a firestorm. Paul's words struck a very, very loud chord that resonated around the world. In addition to many emails expressing appreciation, requests flooded in—many to me, since by agreement I did not publish Paul's contact information—for reproduction and translation rights. (The volume you have in your hands arose in precisely this way.)

It wasn't that Paul was saying something that countless mathematicians and math teachers have not said before. Nor were the points he raised new to those in the sometimes divided world of mathematics education who wrote to disagree with much if not all of what he wrote. What was different was the eloquence of his

words and the obvious passion he injected into them. This was not just good writing; this was *great* writing, coming right from the heart.

Make no mistake about it, *A Mathematician's Lament*, and this greatly expanded book version, is an *opinion* piece. Paul has strong views on how mathematics should be taught, and he argues forcefully for his approach, and against much of the status quo in today's world of school mathematics education. What singles him out, besides his personal and captivating writing style, is that he brings to the thorny and much-debated issues of mathematics education a perspective that few others are able to draw upon. Paul is one of those very rare birds who began as an accomplished professional research mathematician, teaching students in universities, and then realized his true calling was in K-12 teaching, which is the career he has followed for many years now.

In my view, this book, like the original essay it came from, should be obligatory reading for anyone going into mathematics education, for every parent of a school-aged child, and for any school or government official with responsibilities toward mathematics teaching. You may not agree with everything Paul says. You may think his approach to teaching is not one that every teacher could successfully adopt. But you should read what he says and reflect on his words. *A Mathematician's Lament* is already a recognized landmark in the world of mathematics education that cannot and should not be ignored. I am not going to tell you how I think you should respond. As Paul himself would agree, that is for every individual reader to do. But I will tell you this. I would have *loved* to have had Paul Lockhart as my school mathematics teacher.

KEITH DEVLIN
Stanford University

PART I

Lamentation

A MUSICIAN WAKES FROM A TERRIBLE NIGHTMARE. In his dream he finds himself in a society where music education has been made mandatory. "We are helping our students become more competitive in an increasingly sound-filled world." Educators, school systems, and the state are put in charge of this vital project. Studies are commissioned, committees are formed, and decisions are made—all without the advice or participation of a single working musician or composer.

Since musicians are known to set down their ideas in the form of sheet music, these curious black dots and lines must constitute the "language of music." It is imperative that students become fluent in this language if they are to attain any degree of musical competence; indeed, it would be ludicrous to expect a child to sing a song or play an instrument without having a thorough grounding in music notation and theory. Playing and listening to music, let alone composing an original piece, are considered very advanced topics and are generally put off until college, and more often graduate school.

As for the primary and secondary schools, their mission is to train students to use this language—to jiggle symbols around according to a fixed set of rules: "Music class is where we take out our staff paper, our teacher puts some notes on the board, and we copy them or transpose them into a different key. We have to make sure to get the clefs and key signatures right, and our teacher is very picky about making sure we fill in our quarter-notes completely. One time we had a chromatic scale problem and I did it right, but the teacher gave me no credit because I had the stems pointing the wrong way."

In their wisdom, educators soon realize that even very young children can be given this kind of musical instruction. In fact it is considered quite shameful if one's third-grader hasn't completely memorized his circle of fifths. "I'll have to get my son a music tutor. He simply won't apply himself to his music homework. He says it's boring. He just sits there staring out the window, humming tunes to himself and making up silly songs."

In the higher grades the pressure is really on. After all, the students must be prepared for the standardized tests and college admissions exams. Students must take courses in scales and modes, meter, harmony, and counterpoint. "It's a lot for them to learn, but later in college when they finally get to hear all this stuff, they'll really appreciate all the work they did in high school." Of course, not many students actually go on to concentrate in music, so only a few will ever get to hear the sounds that the black dots represent. Nevertheless, it is important that every member of society be able to recognize a modulation or a fugal passage, regardless of the fact that they will never hear one. "To tell you the truth, most students just aren't very good at music. They are bored in class, their skills are terrible, and their homework is barely legible. Most of them couldn't care less about how

important music is in today's world; they just want to take the minimum number of music courses and be done with it. I guess there are just music people and non-music people. I had this one kid, though, man was she sensational! Her sheets were impeccable—every note in the right place, perfect calligraphy, sharps, flats, just beautiful. She's going to make one hell of a musician someday."

Waking up in a cold sweat, the musician realizes, gratefully, that it was all just a crazy dream. "Of course," he reassures himself, "no society would ever reduce such a beautiful and meaningful art form to something so mindless and trivial; no culture could be so cruel to its children as to deprive them of such a natural, satisfying means of human expression. How absurd!"

Meanwhile, on the other side of town, a painter has just awakened from a similar nightmare . . .

. . . I was surprised to find myself in a regular school classroom—no easels, no tubes of paint. "Oh we don't actually apply paint until high school," I was told by the students. "In seventh grade we mostly study colors and applicators." They showed me a worksheet. On one side were swatches of color with blank spaces next to them. They were told to write in the names. "I like painting," one of the students remarked. "They tell me what to do and I do it. It's easy!"

After class I spoke with the teacher. "So your students don't actually do any painting?" I asked. "Well, next year they take Pre-Paint-by-Numbers," the teacher replied. "That prepares them for the main Paint-by-Numbers sequence in high school. So they'll get to use what they've learned here and apply it to real-life painting situations—dipping the brush into paint, wiping it off, stuff like that. Of course we track our students by ability. The really excellent painters—the ones who know their colors and brushes backwards and forwards—they get to the actual painting a little sooner, and some of them even take the Advanced Placement classes for college credit. But mostly we're just trying to give these kids a good foundation in what painting is all about, so when they get out there in the real world and paint their kitchen they don't make a total mess of it."

"Um, these high school classes you mentioned . . ."

"You mean Paint-by-Numbers? We're seeing much higher enrollments lately. I think it's mostly coming from parents wanting to make sure their kid gets into a good college. Nothing looks better than Advanced Paint-by-Numbers on a high school transcript."

"Why do colleges care if you can fill in numbered regions with the corresponding color?"

"Oh, well, you know, it shows clear-headed logical thinking. And of course if a student is planning to major in one of the visual sciences, like fashion or interior decorating, then it's really a good idea to get your painting requirements out of the way in high school."

"I see. And when do students get to paint freely, on a blank canvas?"

"You sound like one of my professors! They were always going on about expressing yourself and your feelings and things like that—really way-out-there abstract stuff. I've got a degree in painting myself, but I've never really worked much with blank canvasses. I just use the Paint-by-Numbers kits supplied by the school board."

* * *

Sadly, our present system of mathematics education is precisely this kind of nightmare. In fact, if I had to design a mechanism for the express purpose of *destroying* a child's natural curiosity and love of pattern-making, I couldn't possibly do as good a job as is currently being done—I simply wouldn't have the imagination to come up with the kind of senseless, soul-crushing ideas that constitute contemporary mathematics education.

Everyone knows that something is wrong. The politicians say, "We need higher standards." The schools say, "We need more money and equipment." Educators say one thing, and teachers say another. They are all wrong. The only people who understand what is going on are the ones most often blamed and least often heard: the students. They say, "Math class is stupid and boring," and they are right.

Mathematics and Culture

THE FIRST THING TO UNDERSTAND IS THAT MATHEMATICS is an art. The difference between math and the other arts, such as music and painting, is that our culture does not recognize it as such. Everyone understands that poets, painters, and musicians create works of art, and are expressing themselves in word, image, and sound. In fact, our society is rather generous when it comes to creative expression; architects, chefs, and even television directors are considered to be working artists. So why not mathematicians?

Part of the problem is that nobody has the faintest idea what it is that mathematicians do. The common perception seems to be that mathematicians are somehow connected with science—perhaps they help the scientists with their formulas, or feed big numbers into computers for some reason or other. There is no question that if the world had to be divided into the "poetic dreamers" and the "rational thinkers" most people would place mathematicians in the latter category.

Nevertheless, the fact is that there is nothing as dreamy and poetic, nothing as radical, subversive, and psychedelic, as mathematics. It is every bit as mind-blowing as cosmology or physics (mathematicians *conceived* of black holes long before astronomers actually found any), and allows more freedom of expression than poetry, art, or music (which depend heavily on properties of the physical universe). Mathematics is the purest of the arts, as well as the most misunderstood.

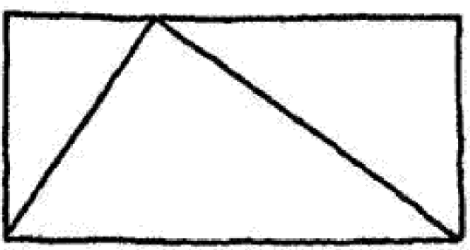
So let me try to explain what mathematics is, and what mathematicians do. I can hardly do better than to begin with G. H. Hardy's excellent description:

> A mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with *ideas*.

So mathematicians sit around making patterns of ideas. What sort of patterns? What sort of ideas? Ideas about the rhinoceros? No, those we leave to the

biologists. Ideas about language and culture? No, not usually. These things are all far too complicated for most mathematicians' taste. If there is anything like a unifying aesthetic principle in mathematics, it is this: *simple is beautiful*. Mathematicians enjoy thinking about the simplest possible things, and the simplest possible things are *imaginary*.

For example, if I'm in the mood to think about shapes—and I often am—I might imagine a triangle inside a rectangular box:



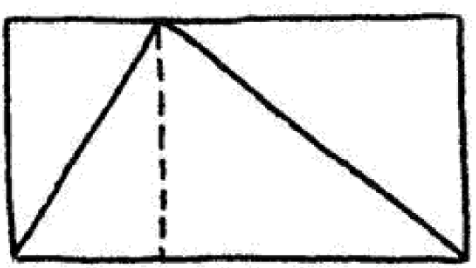
I wonder how much of the box the triangle takes up—two-thirds maybe? The important thing to understand is that I'm not talking about this *drawing* of a triangle in a box. Nor am I talking about some metal triangle forming part of a girder system for a bridge. There's no ulterior practical purpose here. I'm just *playing*. That's what math is—wondering, playing, amusing yourself with your imagination. For one thing, the question of how much of the box the triangle takes up doesn't even make any *sense* for real, physical objects. Even the most carefully made physical triangle is still a hopelessly complicated collection of jiggling atoms; it changes its size from one minute to the next. That is, unless you want to talk about some sort of *approximate* measurements. Well, that's where the aesthetic comes in. That's just not simple, and consequently it is an ugly question that depends on all sorts of real-world details. Let's leave that to the scientists. The *mathematical* question is about an imaginary triangle inside an imaginary box. The edges are perfect because I want them to be—that is the sort of object I prefer to think about. This is a major theme in mathematics: things are what you want them to be. You have endless choices; there is no reality to get in your way.

On the other hand, once you have made your choices (for example I might

choose to make my triangle symmetrical, or not) then your new creations do what they do, whether you like it or not. This is the amazing thing about making imaginary patterns: they talk back! The triangle takes up a certain amount of its box, and I don't have any control over what that amount is. There is a number out there, maybe it's two-thirds, maybe it isn't, but I don't get to say what it is. I have to *find out* what it is.

So we get to play and imagine whatever we want and make patterns and ask questions about them. But how do we answer these questions? It's not at all like science. There's no experiment I can do with test tubes and equipment and whatnot that will tell me the truth about a figment of my imagination. The only way to get at the truth about our imaginations is to use our imaginations, and that is hard work.

In the case of the triangle in its box, I do see something simple and pretty:



If I chop the rectangle into two pieces like this, I can see that each piece is cut diagonally in half by the sides of the triangle. So there is just as much space inside the triangle as outside. That means that the triangle must take up exactly half the box!

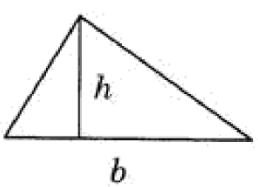
This is what a piece of mathematics looks and feels like. That little narrative is an example of the mathematician's art: asking simple and elegant questions about our imaginary creations, and crafting satisfying and beautiful explanations. There is really nothing else quite like this realm of pure idea; it's fascinating, it's fun, and it's free!

Now where did this idea of mine come from? How did I know to draw that line?

How does a painter know where to put his brush? Inspiration, experience, trial and error, dumb luck. That's the art of it, creating these beautiful little poems of thought, these sonnets of pure reason. There is something so wonderfully transformational about this art form. The relationship between the triangle and the rectangle was a mystery, and then that one little line made it obvious. I couldn't see, and then all of a sudden I could. Somehow, I was able to create a profound simple beauty out of nothing, and change myself in the process. Isn't that what art is all about?

This is why it is so heartbreaking to see what is being done to mathematics in school. This rich and fascinating adventure of the imagination has been reduced to a sterile set of facts to be memorized and procedures to be followed. In place of a simple and natural question about shapes, and a creative and rewarding process of invention and discovery, students are treated to this:

Triangle Area Formula: $A = \frac{1}{2}\, b\, h$



"The area of a triangle is equal to one-half its base times its height." Students are asked to memorize this formula and then "apply" it over and over in the "exercises." Gone is the thrill, the joy, even the pain and frustration of the creative act. There is not even a *problem* anymore. The question has been asked and answered at the same time—there is nothing left for the student to do.

Now let me be clear about what I'm objecting to. It's not about formulas, or memorizing interesting facts. That's fine in context, and has its place just as learning a vocabulary does—it helps you to create richer, more nuanced works of art. But it's not the *fact* that triangles take up half their box that matters. What matters is the beautiful *idea* of chopping it with the line, and how that might

inspire other beautiful ideas and lead to creative breakthroughs in other problems—something a mere statement of fact can never give you.

By removing the creative process and leaving only the results of that process, you virtually guarantee that no one will have any real engagement with the subject. It is like saying that Michelangelo created a beautiful sculpture, without letting me see it. How am I supposed to be inspired by that? (And of course it's actually much worse than this—at least it's understood that there *is* an art of sculpture that I am being prevented from appreciating).

By concentrating on *what*, and leaving out *why*, mathematics is reduced to an empty shell. The art is not in the "truth" but in the explanation, the argument. It is the argument itself that gives the truth its context, and determines what is really being said and meant. Mathematics is *the art of explanation*. If you deny students the opportunity to engage in this activity—to pose their own problems, to make their own conjectures and discoveries, to be wrong, to be creatively frustrated, to have an inspiration, and to cobble together their own explanations and proofs—you deny them mathematics itself. So no, I'm not complaining about the presence of facts and formulas in our mathematics classes, I'm complaining about the lack of *mathematics* in our mathematics classes.

If your art teacher were to tell you that painting is all about filling in numbered regions, you would know that something was wrong. The culture informs you — there are museums and galleries, as well as the art in your own home. Painting is well understood by society as a medium of human expression. Likewise, if your science teacher tried to convince you that astronomy is about predicting a person's future based on their date of birth, you would know she was crazy—science has seeped into the culture to such an extent that almost everyone knows about atoms and galaxies and laws of nature. But if your math teacher gives you the impression, either expressly or by default, that mathematics is about formulas and definitions and memorizing algorithms, who will set you straight?

The cultural problem is a self-perpetuating monster: students learn about math from their teachers, and teachers learn about it from their teachers, so this lack of understanding and appreciation for mathematics in our culture replicates itself indefinitely. Worse, the perpetuation of this "pseudo-mathematics," this emphasis

on the accurate yet mindless manipulation of symbols, creates its own culture and its own set of values. Those who have become adept at it derive a great deal of self-esteem from their success. The last thing they want to hear is that math is really about raw creativity and aesthetic sensitivity. Many a graduate student has come to grief when they discover, after a decade of being told they were "good at math," that in fact they have no real mathematical talent and are just very good at following directions. Math is not about following directions, it's about making new directions.

And I haven't even mentioned the lack of mathematical criticism in school. At no time are students let in on the secret that mathematics, like any literature, is created by human beings for their own amusement; that works of mathematics are subject to critical appraisal; that one can have and develop mathematical *taste*. A piece of mathematics is like a poem, and we can ask if it satisfies our aesthetic criteria: Is this argument sound? Does it make sense? Is it simple and elegant? Does it get me closer to the heart of the matter? Of course there's no criticism going on in school—there's no art being done to criticize!

Why don't we want our children to learn to do mathematics? Is it that we don't trust them, that we think it's too hard? We seem to feel that they are capable of making arguments and coming to their own conclusions about Napoleon. Why not about triangles? I think it's simply that we as a culture don't know what mathematics is. The impression we are given is of something very cold and highly technical, that no one could possibly understand—a self-fulfilling prophesy if there ever was one.

It would be bad enough if the culture were merely ignorant of mathematics, but what is far worse is that people actually think they *do* know what math is about—and are apparently under the gross misconception that mathematics is somehow useful to society! This is already a huge difference between mathematics and the other arts. Mathematics is viewed by the culture as some sort of tool for science and technology. Everyone knows that poetry and music are for pure enjoyment and for uplifting and ennobling the human spirit (hence their virtual elimination from the public school curriculum), but no, math is *important*.

> SIMPLICIO: Are you really trying to claim that mathematics offers no useful or practical applications to society?

SALVIATI: Of course not. I'm merely suggesting that just because something happens to have practical consequences doesn't mean that's what it is *about*. Music can lead armies into battle, but that's not why people write symphonies. Michelangelo decorated a ceiling, but I'm sure he had loftier things on his mind.

SIMPLICIO: But don't we need people to learn those useful consequences of math? Don't we need accountants and carpenters and such?

SALVIATI: How many people actually use any of this "practical math" they supposedly learn in school? Do you think carpenters are out there using trigonometry? How many adults remember how to divide fractions, or solve a quadratic equation? Obviously the current practical training program isn't working, and for good reason: it is excruciatingly boring, and nobody ever uses it anyway. So why do people think it's so important? I don't see how it's doing society any good to have its members walking around with vague memories of algebraic formulas and geometric diagrams, and clear memories of hating them. It might do some good, though, to show them something beautiful and give them an opportunity to enjoy being creative, flexible, open-minded thinkers—the kind of thing a *real* mathematical education might provide.

SIMPLICIO: But people need to be able to balance their checkbooks, don't they?

SALVIATI: I'm sure most people use a calculator for everyday arithmetic. And why not? It's certainly easier and more reliable. But my point is not just that the current system is so terribly bad, it's that what it's missing is so wonderfully good! Mathematics should be taught as art for art's sake. These mundane "useful" aspects would follow naturally as a trivial by-product. Beethoven could easily write an advertising jingle, but his motivation for learning music was to create something beautiful.

SIMPLICIO: But not everyone is cut out to be an artist. What about the kids who aren't "math people"? How would they fit into your scheme?

SALVIATI: If everyone were exposed to mathematics in its natural state, with all the challenging fun and surprises that that entails, I think we would see a dramatic change both in the attitude of students toward mathematics, and in our conception of what it means to be good at math. We are losing so many potentially gifted mathematicians—creative, intelligent people who rightly reject what appears to be a meaningless and sterile subject. They are simply too smart to waste their time on such piffle.

SIMPLICIO: But don't you think that if math class were made more like art class that a lot of kids just wouldn't learn anything?

SALVIATI: They're not learning anything now! Better to not have math classes at all than to do what is currently being done. At least some people might have a chance to discover something beautiful on their own.

SIMPLICIO: So you would remove mathematics from the school curriculum?

SALVIATI: The mathematics has already been removed! The only question is what to do with the vapid, hollow shell that remains. Of course I would prefer to replace it with an active and joyful engagement with mathematical ideas.

SIMPLICIO: But how many math teachers know enough about their subject to teach it that way?

SALVIATI: Very few. And that's just the tip of the iceberg . . .

Mathematics in School

THERE IS SURELY NO MORE RELIABLE WAY TO KILL enthusiasm and interest in a subject than to make it a mandatory part of the school curriculum. Include it as a major component of standardized testing and you virtually guarantee that the education establishment will suck the life out of it. School boards do not understand what math is; neither do educators, textbook authors, publishing companies, and, sadly, neither do most of our math teachers. The scope of the problem is so enormous I hardly know where to begin.

Let's start with the "math reform" debacle. For many years there has been a growing awareness that something is rotten in the state of mathematics education. Studies have been commissioned, conferences assembled, and countless committees of teachers, textbook publishers, and educators (whatever they are) have been formed to "fix the problem." Quite apart from the self-serving interest paid to reform by the textbook industry (which profits from any minute political fluctuation by offering up "new" editions of their unreadable monstrosities), the entire reform movement has always missed the point. The mathematics curriculum doesn't need to be reformed, it needs to be *scrapped*.

All this fussing and primping about which "topics" should be taught in what order, or the use of this notation instead of that notation, or which make and model of calculator to use, for god's sake —it's like rearranging the deck chairs on the *Titanic*! Mathematics is *the music of reason*. To do mathematics is to engage in an act of discovery and conjecture, intuition and inspiration; to be in a state of confusion—not because it makes no sense to you, but because you *gave* it sense and you still don't understand what your creation is up to; to have a breakthrough idea; to be frustrated as an artist; to be awed and overwhelmed by an almost painful beauty; to be *alive*, damn it. Remove this from mathematics and you can have all the conferences you like; it won't matter. Operate all you want, doctors: *your patient is already dead*.

The saddest part of all this "reform" are the attempts to "make math interesting" and "relevant to kids' lives." You don't need to *make* math interesting

—it's already more interesting than we can handle! And the glory of it is its complete *irrelevance* to our lives. That's why it's so fun!

Attempts to present mathematics as relevant to daily life inevitably appear forced and contrived: "You see, kids, if you know algebra then you can figure out how old Maria is if we know that she is two years older than twice her age seven years ago!" (As if anyone would ever have access to that ridiculous kind of information, and not her age.) Algebra is not about daily life, it's about numbers and symmetry—and this is a valid pursuit in and of itself:

> Suppose I am given the sum and difference of two numbers. How can I figure out what the numbers are themselves?

Here is a simple and elegant question, and it requires no effort to be made appealing. The ancient Babylonians enjoyed working on such problems, and so do our students. (And I hope you will enjoy thinking about it too!) We don't need to bend over backwards to give mathematics relevance. It has relevance in the same way that any art does: that of being a meaningful human experience.

In any case, do you really think kids even want something that is relevant to their daily lives? You think something practical like compound interest is going to get them excited? People enjoy fantasy, and that is just what mathematics can provide—a relief from daily life, an anodyne to the practical workaday world.

A similar problem occurs when teachers or textbooks succumb to cutesiness. This is where, in an attempt to combat so-called "math anxiety" (one of the panoply of diseases which are actually *caused* by school), math is made to seem "friendly." To help your students memorize formulas for the area and circumference of a circle, for example, you might invent a whole story about Mr. C, who drives around Mrs. A and tells her how nice his two pies are ($C = 2\pi r$) and how her pies are square ($A = \pi r^2$) or some such nonsense. But what about the *real* story? The one about mankind's struggle with the problem of measuring curves; about Eudoxus and Archimedes and the method of exhaustion; about the transcendence of pi? Which is more interesting—measuring the rough dimensions of a circular piece of graph paper, using a formula that someone handed you without explanation (and made you memorize and practice over and over), or hearing the story of one of the most beautiful, fascinating problems and one of the most brilliant and powerful ideas in human history? We're killing people's interest

in *circles* for god's sake!

Why aren't we giving our students a chance to even hear about these things, let alone giving them an opportunity to actually do some mathematics, and to come up with their own ideas, opinions, and reactions? What other subject is routinely taught without any mention of its history, philosophy, thematic development, aesthetic criteria, and current status? What other subject shuns its primary sources—beautiful works of art by some of the most creative minds in history—in favor of third-rate textbook bastardizations?

The main problem with school mathematics is that there are no *problems.* Oh, I know what *passes* for problems in math classes, these insipid "exercises." "Here is a type of problem. Here is how to solve it. Yes it will be on the test. Do exercises 1-35 odd for homework." What a sad way to learn mathematics: to be a trained chimpanzee.

But a problem, a genuine honest-to-goodness natural human *question*—that's another thing. How long is the diagonal of a cube? Do prime numbers keep going on forever? Is infinity a number? How many ways can I symmetrically tile a surface? The history of mathematics is the history of mankind's engagement with questions like these, not the mindless regurgitation of formulas and algorithms (together with contrived exercises designed to make use of them).

A good problem is something you don't know how to solve. That's what makes it a good puzzle, and a good opportunity. A good problem does not just sit there in isolation, but serves as a springboard to other interesting questions. A triangle takes up half its box. What about a pyramid inside its three-dimensional box? Can we handle this problem in a similar way?

I can understand the idea of training students to master certain techniques—I do that too. But not as an end in itself. Technique in mathematics, as in any art, should be learned in context. The great problems, their history, the creative process—that is the proper setting. Give your students a good problem, let them struggle and get frustrated. See what they come up with. Wait until they are dying for an idea, *then* give them some technique. But not too much.

So put away your lesson plans and your overhead projectors, your full-color