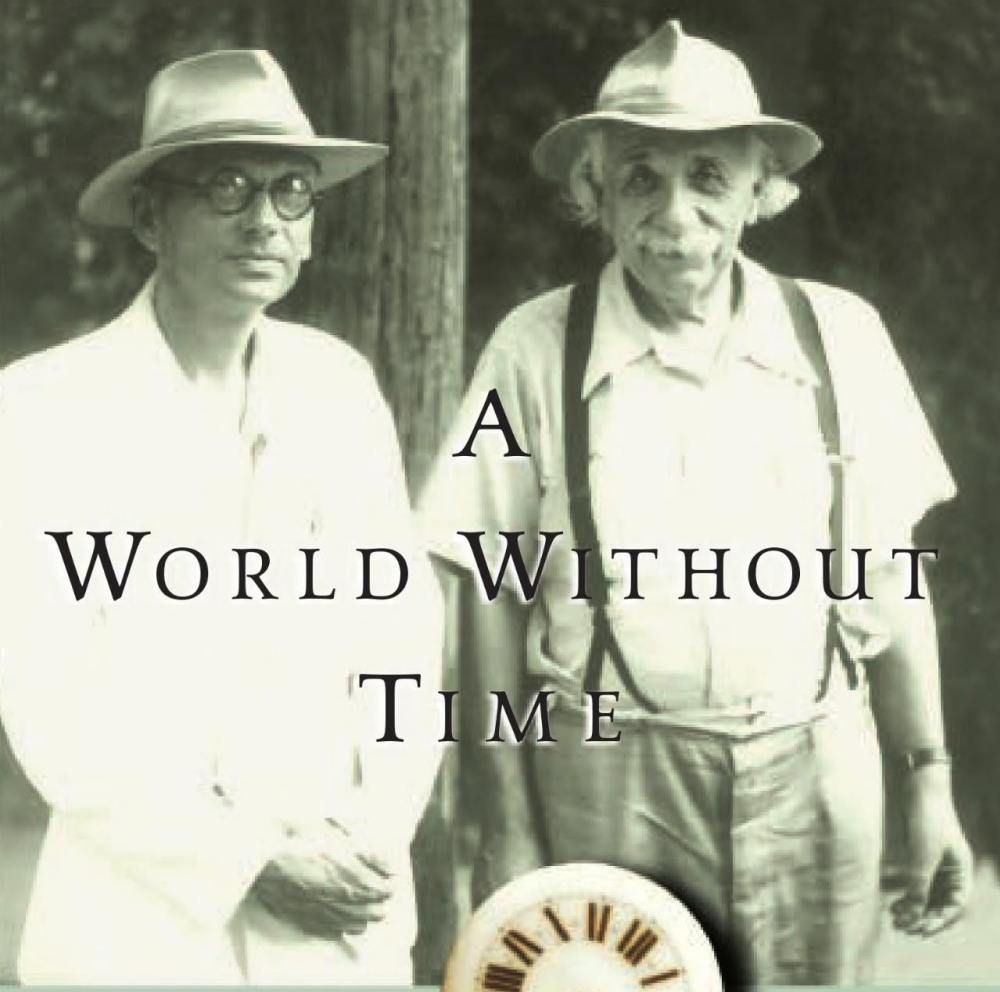

A World Without Time

A *DISCOVER MAGAZINE* BESTSELLER

The Forgotten Legacy of Gödel and Einstein

"Gödel's conclusion [about time] went almost unnoticed at the time, but it has since found a passionate champion in Palle Yourgrau."—*THE NEW YORKER*

Palle Yourgrau

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The Forgotten Legacy of Gödel and Einstein

Palle Yourgrau

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Acknowledgments

Having already written a book intended primarily for philosophers about Kurt Gödel's attempt to make sense of Einstein's theory of relativity, I was intrigued when William Frucht of Basic Books suggested I write another, this one accessible to normal readers. Such a book would focus on the sheer intellectual drama of the companionship of Gödel and Einstein—a relationship sorely neglected in the literature—and would place Gödel's and Einstein's epoch-making discoveries in the context of the great intellectual movements of the twentieth century, some of which, having helped to father, they tried, belatedly, to abandon. It was an offer too good to refuse, and I didn't. The task, however, turned out to be far from easy, and Frucht had to endure not only the late delivery of the final manuscript, but the drumbeat of my complaints about his editorial adventures at the expense of my beloved prose; it may not have been much, but it was all mine. I am grateful to Frucht both for the initial invitation and for (what turned out to be) his wise editorial advice, at every stage, on how to improve the manuscript.

I have greatly benefited from discussing the book with Mary Sullivan and Ben Callard. Sullivan, attentive as ever to the task of trying to keep me honest, read large parts of the book and offered acute, sobering advice, which I took to heart. Callard's critical remarks on every chapter were of great value and are reflected in the final draft. His strange affection for the work, moreover, kept my spirits up during the many dark moments when the project seemed to me ill-advised and

misbegotten. Mark van Atten also read the entire manuscript. His extraordinarily detailed remarks, on both substance and style, were a great boon. Robert Tragesser shared with me his deep understanding of Gödel's theorem, and Eli Hirsch helped make certain that my discussions of logic and mathematics were clear, accessible, and to the point. To each of these I owe a serious debt of gratitude, but especially to Callard and van Atten, for their extensive and thoughtful comments.

I would also like to express my appreciation for the conversations I had over a period of years with the late Hao Wang, of Rockefeller University, whom I came to know when I taught at Barnard College in New York. With Wang, who was one of Gödel's closest associates in his final years, I spent endless hours discussing Gödel's ideas, published and unpublished. The question of time was of particular interest to him. He confessed that, finding the topic uncongenial, he had resisted Gödel's efforts to discuss his thoughts on this theme, and regretted it. I regretted it, too. It did not diminish, however, the fruitfulness of our conversations, nor the enjoyment we took in contemplating what Gödel has said and written.

In addition to help from my friends, I have also depended on the kindness of strangers. My copy editor, David Kramer, in addition to improving the clarity and flow of the text, made a number of comments and suggestions of a more substantial nature on a wide variety of topics, from music to mathematics, which were an unexpected gift. AnnaLee Pauls, photoduplication coordinator at the Rare Books and Special Collections department of Princeton University's Firestone Library, provided much needed assistance in the delicate task of choosing and reproducing the right photos from the Gödel archives, housed in Firestone Library. I am grateful both for her assistance and for the spirit with which she provided it. I would also, finally, like to thank Arudra Burra, who is not a stranger, for his generous help, combined with fine judgment, in the search for the right photos to include in the book.

A World Without Time

1 | A Conspiracy of Silence

Gödel was . . . the only one of our colleagues who walked and talked on equal terms with Einstein.

FREEMAN DYSON

In the summer of 1942, while German U-boats roamed in wolf packs off the coast of Maine, residents in the small coastal town of Blue Hill were alarmed by the sight of a solitary figure, hands clasped behind his back, hunched over like a comma with his eyes fixed on the ground, making his way along the shore in a seemingly endless midnight stroll. Those who encountered the man were struck by his deep scowl and thick German accent. Speculation mounted that he was a German spy giving secret signals to enemy warships. The dark stranger, however, was no German spy. He was Kurt Gödel, the greatest logician of all time, a beacon in the intellectual landscape of the last thousand years, and the prey he sought was not American ships bound for Britain but rather the so-called continuum hypothesis, a conjecture made by the mathematician Georg Cantor about the number of points on a line. Gödel was spending the summer vacationing at the Blue Hill Inn with his wife, Adele, although fellow visitors at the inn rarely saw either of them. They materialized for dinner, but were never observed actually eating. To the locals, Gödel's scowl betrayed a dark disposition, but the innkeeper saw things differently. For her it was the expression of a man lost in thought. His last word to Blue Hill would not

decide the issue. He sent a letter accusing the innkeeper of stealing the key to his trunk.

The place Gödel would return to in the fall was a long way from Blue Hill—the prestigious Institute for Advanced Study in Princeton, New Jersey. There he would no longer have to walk alone, arousing the suspicions of neighbors. He had a walking companion, a colleague at the institute and his best friend. There was no danger that his reputation would intimidate his companion. For his friend, another German-speaking refugee with a mathematical bent, was the most famous scientist of all time, Albert Einstein, whose own meditative strolls already irritated the residents of Princeton.

"From a distance," a biographer wrote, "the [residents of Princeton] chuckled discreetly over [Einstein's] habit of licking an ice cream on Nassau Street on his way home from Fine Hall and were astonished by his utterly un-American long walks through the streets of Princeton." Indeed, toward the end of his career, when he was more or less retired, Einstein commented that his own work no longer meant much to him and that he now went to his office "just to have the privilege of walking home with Kurt Gödel." Ironically, it was not the scowling Gödel but his smiling companion who had once given indirect aid to the German U-boats, when, during World War I, although a courageous and committed pacifist, Einstein had helped improve the gyroscopes used by the German navy. Gödel's research would also, in the end, relate to gyroscopes, but these spun at the center of the universe, not in the dank bowels of submarines.

Washed up onto America's shores by the storm of Nazism that raged in Europe in the 1930s, the two men had awakened to find themselves stranded in the same hushed academic retreat, Princeton's Institute for Advanced Study, an exclusive intellectual club, whose members had only one assigned duty: to think. But Gödel and Einstein already belonged to an even more exclusive club. Together with another German-speaking theorist, Werner Heisenberg, they were the authors of the three most fundamental scientific results of the century. Each man's discovery, moreover, established a profound and disturbing *limitation*. Einstein's theory of relativity set a limit—the speed of light—to the flow of any

information-bearing signal. And by defining time in terms of its measurement with clocks, he set a limit to time itself. It was no longer absolute but henceforth limited or relative to a frame of measurement. Heisenberg's uncertainty principle in quantum mechanics set a limit on our simultaneous knowledge of the position and momentum of the fundamental particles of matter. This was not just a restriction on what we can know: for Heisenberg it signified a limit to reality. Finally, Gödel's incompleteness theorem—"the most significant mathematical truth of the century," as it would soon be described in a ceremony at Harvard University—set a permanent limit on our knowledge of the basic truths of mathematics: The complete set of mathematical truths will never be captured by any finite or recursive list of axioms that is fully formal. Thus, no mechanical device, no computer, will ever be able to exhaust the truths of mathematics. It follows immediately, as Gödel was quick to point out, that if we are able somehow to grasp the complete truth in this domain, then we, or our minds, are not machines or computers. (Enthusiasts of artificial intelligence were not amused.)

Einstein, Gödel, Heisenberg: three men whose fundamental scientific results opened up new horizons, paradoxically, by setting limits to thought or reality. Together they embodied the zeitgeist, the spirit of the age. Mysteriously, each had reached an ontological conclusion about *reality* through the employment of an epistemic principle concerning *knowledge*. The dance or dialectic of knowledge and reality—of limit and limitlessness—would become a dominant theme of the twentieth century. Yet Gödel's and Einstein's relations to their century were more uneasy than Heisenberg's.

The zeitgeist took root most famously in quantum mechanics. Here Gödel and Einstein would find themselves in lonely opposition to Heisenberg, who, on the wrong side in the war of nations, chose the winning team in the wars of physics. Heisenberg was a champion of the school of positivism, in quantum physics known as the Copenhagen interpretation in deference to Heisenberg's mentor, the Danish physicist Niels Bohr. What had been a mere heuristic principle in Einstein's special relativity—deducing the nature of reality from limitations on what can be known—became for Heisenberg a kind of religion, a

religion that Gödel and Einstein had no wish to join. Some, however, claimed to see in Gödel's theorem itself an echo of Heisenberg's uncertainty principle. That group did not include Gödel.

Einstein, himself one of the great pioneers of quantum mechanics, had known and inspired Heisenberg in Germany. In 1911, in Prague, years before Heisenberg came on the scene, Einstein once pointed out to his colleague Philipp Frank the insane asylum in the park below his study and remarked, "Here you see that portion of lunatics who do not concern themselves with quantum theory." By Einstein's lights, a bad situation had become even worse after Heisenberg. In an early encounter, Heisenberg, on the defensive against Einstein's harangue against quantum mechanics, fought back: "When I objected that in [my approach] I had merely been applying the type of philosophy that he, too, had made the basis of his special theory of relativity, [Einstein] answered simply, 'Perhaps I did use such philosophy earlier, and also wrote it, but it is nonsense all the same.'"

The two parted before the war, Einstein emigrating to the United States, Heisenberg remaining in Germany, to which he would remain loyal to the end. In Princeton, Einstein—pacifist, bohemian, socialist and Jew—was a man apart. To be sure, he found Gödel, but together they remained isolated and alone, not least because of their opposition to Heisenberg's positivist worldview, which ruled the intellectual scene even as Heisenberg's fatherland was attempting to dominate the world. Gödel and Einstein were not merely intellectual engineers, as so many of their brethren, inspired by positivism, had become, but philosopher-scientists. Ironically, while their stars had begun to wane, the sheer size of their reputations had made them unapproachable. Not to each other, however. "Gödel," wrote their colleague Freeman Dyson, "was the only one of our colleagues who walked and talked on equal terms with Einstein."

Their tastes, however, remained distinct. Einstein, a violinist, could never bring his friend to subject himself to the likes of Beethoven and Mozart. Gödel, in turn, had no more success, surely, in dragging Einstein to *Snow White and the Seven Dwarfs*, his favorite movie. History,

sadly, does not record which of the seven dwarfs was Gödel's favorite, but we do know why he favored fairy tales: "Only fables," he said, "present the world as it should be and as if it had meaning." That meaning, of course, may be dark. It is not known whether Alan Turing acquired an affection for *Snow White* from Gödel when he visited the institute in the 1930s, but some have heard an echo of the dark side of that tale in Turing's decision to end his life by eating a poisoned apple when, as a reward for his having broken the Enigma code of the German navy, the British government ordered him to receive hormone injections as a "cure" for his homosexuality.

Einstein, before fleeing Germany, had already become a refugee from mathematics. He later said that he could not find, in that garden of many paths, the one to what is fundamental. He turned to the more earthly domain of physics, where the way to the essential was, he thought, clearer. His disdain for mathematics earned him the nickname "lazy dog" from his teacher Hermann Minkowski (who would soon recast the lazy dog's special relativity into its characteristic four-dimensional form). "You know, once you start calculating," Einstein would quip, "you shit yourself up before you know it." Gödel's journey, in contrast, was in the opposite direction. Having befriended Gödel, Einstein commented that he knew now, at last, that in mathematics too one could find a path to the fundamental. In befriending Einstein, Gödel was reawakened to his early interest in physics. On their long walks home from the office, Einstein, forever cheerful, would attempt to raise the spirits of the gloomy and pessimistic Gödel by recounting his latest insights on general relativity. Sadly, however, pessimism blossomed into paranoia. The economist Oskar Morgenstern, calling one day on his good friend, was shocked to find the great Gödel hiding in the cellar behind the furnace.

From those long walks that Einstein and Gödel shared, from their endless discussions, something beautiful would soon be born. The scene was pregnant with possibility. Time, which has taunted thinkers from Plato to Saint Augustine to Kant, had finally met its match in Einstein. While the U-boats of his former fatherland were stalking the Allied fleet,

this most un-German of Germans was hunting a more elusive prey. He had amazed the world decades earlier when he alone succeeded in capturing time itself in the equations of relativity. "Every boy in the streets of Göttingen," his countryman David Hilbert wrote, "understands more about four-dimensional geometry than Einstein. Yet, in spite of that, Einstein did the work and not the mathematicians." Relativity had rendered time, the most elusive of beings, manageable and docile by transforming it into a fourth dimension of space, or rather, of relativistic space-time. Sharing with Gödel his latest thoughts on the four-dimensional universe of space-time that he himself had conjured into being, Einstein was sowing the seeds of relativity in the mind of a thinker who would later be described as a combination of Einstein and Kafka.

If Einstein had succeeded in transforming time into space, Gödel would perform a trick yet more magical: He would make time disappear. Having already rocked the mathematical world to its foundations with his incompleteness theorem, Gödel now took aim at Einstein and relativity. Wasting no time, he announced in short order his discovery of new and unsuspected cosmological solutions to the field equations of general relativity, solutions in which time would undergo a shocking transformation. The mathematics, the physics and the philosophy of Gödel's results were all new. In the possible worlds governed by these new cosmological solutions, the so-called rotating or Gödel universes, it turned out that the space-time structure is so greatly warped or curved by the distribution of matter that there exist timelike future-directed paths by which a spaceship, if it travels fast enough—and Gödel worked out the precise speed and fuel requirements, omitting only the lunch menu—can penetrate into any region of the past, present or future.

Gödel, the union of Einstein and Kafka, had for the first time in human history proved, from the equations of relativity, that time travel was not a philosopher's fantasy but a scientific possibility. Yet again he had somehow contrived, from within the very heart of mathematics, to drop a bomb into the laps of the philosophers. The fallout, however, from this mathematical bomb was even more perilous than that from

the incompleteness theorem. Gödel was quick to point out that if we can revisit the past, then it never really "passed." But a time that fails to pass is no time at all. Einstein saw at once that if Gödel was right, he had not merely domesticated time: he had killed it. Time, "that mysterious and seemingly self-contradictory being," as Gödel put it, "which, on the other hand, seems to form the basis of the world's and our own existence," turned out in the end to be the world's greatest illusion. In a word, if Einstein's relativity was real, time itself was merely ideal. The father of relativity was shocked. Though he praised Gödel for his great contribution to the theory of relativity, he was fully aware that time, that elusive prey, had once again slipped his net.

But now something truly amazing took place: nothing. Although in the immediate aftermath of Gödel's discoveries a few physicists bestirred themselves to refute him and, when this failed, tried to generalize and explore his results, this brief flurry of interest soon died down. Within a few years the deep footprints in intellectual history traced by Gödel and Einstein in their long walks home had disappeared, dispersed by the harsh winds of fashion and philosophical prejudice. A conspiracy of silence descended on the Einstein-Gödel friendship and its scientific consequences.

An association no less remarkable than the friendship between Michelangelo and Leonardo—if such had occurred—has simply vanished from sight. To this day, not only is the man on the street unaware of the intimate relationship between these two giants of the twentieth century, even the most exhaustive intellectual biographies of Einstein either omit all mention of this friendship or at best begrudge a sentence or two. Whereas a whole industry has grown up in search of Lieserl, the "love child" of Einstein's first marriage, the child of the imagination that was born of the friendship of Einstein and Gödel has been abandoned.

Only in the last few years has this child, the Gödel universe, received any glimmer of recognition. This comes from the redoubtable Stephen Hawking. Revisiting the rotating Gödel universe, Hawking was moved to deliver the highest of compliments. So threatening did

he find results like Gödel's demonstrating the consistency of time travel with the laws of relativity, that he put forward what amounts to an anti-Gödel postulate. If accepted, Hawking's famous "chronology protection conjecture" would precisely negate Gödel's contribution to relativity. So physically unacceptable did Hawking find conclusions like Gödel's that he felt compelled to propose what looks like an ad hoc modification of the laws of nature that would have the effect of ruling out the Gödel universe as a genuine physical possibility.

Hawking's attempt to neutralize the Gödel universe shows how dangerous it is to break the conspiracy of silence that has shrouded the Gödel-Einstein connection. Not only does this mysterious silence hide from the world one of the most moving and consequential friendships in the history of science, it also keeps the world from realizing the full implications of the Einstein revolution. It is one thing to overturn, as Einstein did, Newton's centuries-old conception of the absoluteness and independence of space and time. It is quite another to demonstrate that time is not just relative but ideal. Unlike Einstein, a classicist who forever sought continuity with the past, Gödel was at heart an ironist, a truly subversive thinker. With his incompleteness theorem he had shaken the foundations of mathematics, prompting the great mathematician David Hilbert to propose a new law of logic just to refute Gödel's results. The Gödel universe, correctly understood, shares with the incompleteness theorem an underlying methodology and purpose. It is a bomb, built from cosmology's most cherished materials, lobbed into the foundations of physics.

In the footsteps of Gödel and Einstein, then, can be heard an echo of the zeitgeist, a clue to the secret of the great and terrible twentieth century, a century that, like the seventeenth, will go down in history as one of genius. The residents of Blue Hill, preoccupied with war and the enemy out at sea, had failed to take the full measure of their man.

2 | A German Bias for Metaphysics

The German man of science was a philosopher.

J.T. MERZ

It is a remarkable fact . . . that at least in one point relativity theory has furnished a very striking confirmation of Kantian doctrines.

KURT GÖDEL

Physically they were opposites. Gödel, thin to the point of emaciation, hid his spectral body even in the heat of summer in overcoat and scarf. Gaunt, harrowed, and haunted, peering through thick glasses like an owl from another dimension, he could not fail to arouse suspicion. Early in life he had come to the conclusion that the less food one ate the better. This dubious insight he carried out with ruthless consistency, unencumbered by the excess baggage of common sense, a faculty he approached life without. His preconception, fueled by hypochondria that grew out of childhood rheumatic fever and by paranoia about the intentions of doctors, developed into a neurosis that would eventually take his life. During several periods of extreme stress he was confined to sanatoria, from one of which, by some accounts, he enlisted the services of his wife to escape. At his death he weighed a mere sixty-five pounds.

Einstein, in contrast, whose sanity was never in question, was as satisfied by a good sausage as by a good theorem. He had a taste for solid German cooking, which he consumed with relish, topped off by his omnipresent pipe. Friends and wives would be swept aside in the current of

his turbulent life, but his pipe never left him. Late in life he was the proud owner of a respectable professorial paunch. "I have firmly resolved," he wrote his wife Elsa, "to bite the dust, when my time comes, with the minimum of medical assistance, and until then to sin cheerfully . . . smoke like a chimney, work like a beaver, eat without thought or choice, and walk only in agreeable company, in other words, rarely."

With brown hair and blue eyes, Gödel measured barely five feet six. This number came as a surprise to his colleagues. His intellectual presence was so great that his modest height often went unnoticed. His frailty, however, was obvious. "Of course he has no children," the proprietor of the Blue Hill Inn said of Gödel; "he hasn't the strength to make babies." He did, however, have in his youth the strength to pursue women. "There is no doubt," wrote a college friend, Olga Taussky-Todd, "about the fact that Gödel had a liking for members of the opposite sex, and he made no secret about this fact." Gödel, she went on, was not beyond showing off his acquaintance with a pretty face. Taussky-Todd herself, to her dismay, was once enlisted to come to the mathematical aid of one such young woman who in turn was trying to make an impression on Gödel. Was this interest in women confined to Gödel's youth? Not if his wife, Adele, is to believed. Teasing her husband, she quipped that the Institute for Advanced Study—which she liked to call an *Altersversorgungsheim*, or home for elderly pensioners—was packed with pretty female students who lined up outside the office doors of the great professors. Einstein, who with well-knit limbs and hardy disposition measured five feet nine, did actually make babies, in and out of wedlock. Early and late, the constraints of marriage did not hamper him, even as his discoveries in physics were unconstrained by the conventions of classical physics. The event itself of entering into the institution of marriage bore the unmistakable stamp of unconventionality: Though Einstein wished to marry his cousin Elsa, he desired even more strongly to marry her twenty-year-old daughter, Ilsa. "Albert himself," wrote the flustered daughter to a friend, "is refusing to take any decision; he is prepared to marry either Mama or me."

Clothing too, like marriage, he considered a bourgeois affectation whose strictures he did his best to circumvent, spurning socks, tie, and belt whenever possible. Hair uncut and unkempt, he could embarrass a female guest when his robe, with nothing underneath, fell open, and then express surprise at her consternation. Bursting with the juices of life, he was an indefatigable optimist whose faith in common sense and human nature survived even the Holocaust.

Gödel, in contrast, was in the fullest sense of the phrase "buttoned up." Dressed severely even in the summer heat, he was the very model of dour reserve: gloomy, pessimistic, averse to all human contact except for the closest of friends and the direst of intellectual necessities. The institute still echoes with stories of Gödel's foolproof method for evading a rendezvous. He would carefully arrange a precise location in space and time for the projected meeting. With these coordinates in place, he confided to friends, he had achieved certainty as to where *not* to be when the appointed time arrived. Yet this method had its limitations. Finding himself trapped at an unavoidable institute tea, he negotiated the territory between guests, noted the mathematician Paul Halmos in his memoirs, with maximum attention to the goal of avoiding any possibility of physical contact.

Against every stereotype of the pure mathematician—and particularly one who, like Gödel, had studied and taught in Vienna—Gödel was all but allergic to the masters of classical music, preferring instead light classics and operettas, and was even more so to the abstractions of modern art. He was untouched by intellectual snobbery and made plain his love of fairy tales. His fondness for Walt Disney cartoons was no secret to his friends. Comedies, however, he disliked.

Einstein was consumed by his passion for the great Austrian-German classicists, Bach, Mozart and Beethoven, but especially Mozart. His friend and biographer, Philipp Frank, offered some shrewd observations about what made Mozart special. What passed for many as a sign of Einstein's cynicism was for Frank an expression rather of Einstein's urge "to make the serious things in the world tolerable by means of a playful guise." But this also characterizes much of Mozart's music,

"which might also be called 'cynical.' It does not take our tragic world very seriously." Einstein was always ready to perform Mozart at a moment's notice on his beloved violin, which he played, myths notwithstanding, very well. He "was an experienced sight reader," wrote the professional violinist Boris Schwartz, "with a steady rhythm, excellent intonation, a clear and pure tone, and a minimum of vibrato." Only his pipe was as familiar a companion. Violin and pipe: these will be forever the icons of the great scientist, together with his tousled hair.

Gödel, as is clear from photographs, was meticulously clean-shaven, every hair combed in place, whereas as every schoolchild knows, a small brush of a moustache floated above Einstein's full lips. Combs, moreover, in the Einstein household were verboten. With the visual signature comes the acoustic: When something or someone struck Einstein funny, a huge belly laugh welled up inside the scientist and erupted like a volcano that shook his entire body. More than a few times it shook up as well the surprised object of this laughter, who realized too late its full meaning. Gödel, in contrast, had a soft, high pitched chuckle, more a musing to himself on the ironies of the universe than a full-throated laugh. Raising the pitch of his voice at the end of each sentence and trailing off into silence, he left his audience with a feeling of detached query. (As a child of four he had been nicknamed *Herr Warum*, Mr. Why. "Why is your nose so large?" he asked an embarrassed guest.)

By age a generation apart, Einstein and Gödel shared an anniversary by one degree of separation. The year of Einstein's birth, 1879, was that of Gödel's mother, Marianne. (It was also the year that saw publication of Gottlob Frege's masterpiece, *Begriffsschrift,* and thus the birth of modern mathematical logic, a field Gödel would raise to unparalleled heights.) They were born into different religions, Einstein a Jew, Gödel baptized a Lutheran. Skeptical of the faith of his fathers in his youth, with the rise of Nazism Einstein rediscovered what he called his "tribal companions" and became a passionate, if thorny, Zionist. He never did, however, embrace the transcendent God of his people, accounting himself rather a "deeply religious unbeliever." His hero was not Moses but Spinoza, the pantheist and excommunicant, and he re-

flected this predilection throughout a scientific career in which such seemingly transcendent, untouchable things as space, time and light were revealed to be fully immanent and subject to physical causality.

Gödel was not a pantheist but rather a self-described theist, "following Leibniz," he said, "not Spinoza." Spinoza's God, he said, "is less than a person. Mine is more than a person. . . . He can play the role of a person." He noted the oft-neglected fact that the founders of modern science were not atheists. More radical than Einstein, he belonged to a rare breed of thinker: the true believers. Whereas "ninety per cent of philosophers these days," he would say, "consider it the business of philosophy to knock religion out of people's heads," he would exploit the machinery of modern logic to reconstruct Leibniz's famous "ontological argument" for the existence of God. Though not a Jew, he was nevertheless taken for one. In a Vienna teeming with Nazis, his wife once employed her umbrella to fend off a group of rowdies who were jostling Gödel, mistaking him for a Jew.

The misattribution was not confined to Nazis. While at the Institute for Advanced Study in Princeton, Gödel was for a time a member of an elite—a very elite—discussion group, consisting of himself, Einstein, the German physicist Wolfgang Pauli, and Bertrand Russell, one of the founders of modern "analytical" philosophy. Russell reacted badly to the discussions, finding them too philosophical in the "old-fashioned sense." (The failings of an entire century are crystallized in this fact.) In an unpleasant aside he vented his frustration: "All three of the others were Jews and exiles, and in intention, cosmopolitans," he wrote later, "[who shared] a German bias for metaphysics." "I am not a Jew," Gödel would respond later, "even though I don't think this question is of any importance." He admired the tenacity of the Jewish people. "Kurt had a friendly attitude toward people of the Jewish faith," said his friend Olga Taussky-Todd. "And once he said out of the blue that it was a miracle how, without a country, they were able to survive for thousands of years, almost like a nation, merely by their faith." Einstein, wishing to eliminate the Jewish need for miracles, pushed hard for most of his life for a homeland for the nation that had

survived so many years without a home. Never too concerned with consistency—unlike his logician companion—he was undisturbed by his earlier briefs against nationalism.

Seeing the handwriting on the wall, Einstein and Gödel abandoned comfortable university positions in Berlin and Vienna when the Nazis came to power in the 1930s. At the zenith of their powers, they were snatched up by the newly formed Institute for Advanced Study in Princeton, good European root stock for the vineyards of the new world. Together they wandered the narrow streets of a cloistered and provincial academic town, they who once strode the boulevards of the great capitals of Europe, centerpieces of a once great civilization crashing down in ruins. Strangers to each other in Europe, it was not until 1942 that they began the friendship that lasted until Einstein's death in 1955, a loss from which Gödel never recovered. Einstein, a German Jew in a nest of Wasps, felt out of place in Princeton. Gödel, already a recluse, resented less the isolation, although his wife, Adele, suffered. A café dancer in Vienna, Adele was out of her element in the elite college town. When an opportunity opened up to move to Harvard, she pleaded for the more cosmopolitan Cambridge, Massachusetts. But Gödel was not prepared to accept an offer where teaching was required.

What attraction could have drawn together such opposites as Einstein and Gödel? Certainly not scientific agreement. This was not a case of the strong force uniting like-charged protons in the atomic nucleus. The charges here were opposite. Gödel opined, in fact, that one of the reasons Einstein enjoyed his company was precisely because he made no attempt to hide his very different views, not just in politics and philosophy but in physics. "I frequently held an opinion," Gödel said, "counter to Einstein's and made no attempt to conceal my disagreement." Einstein's failed search, for example, for a unified field theory to unite the domain of quantum mechanics with general relativity, which occupied much of their discussions, was a particular target of Gödel's skepticism.

Indeed, Gödel was skeptical of the ultimate significance of natural science itself, despite its great success in enabling us (as he put it) to

build TVs and bombs. At a faculty dinner at the institute the young John Bahcall, having introduced himself as a new astrophysicist on the faculty, was taken aback when Gödel replied flatly that he didn't believe in natural science. By Gödel's lights, physics had taken the wrong turn centuries ago when it chose to follow the path laid by the naturalistically minded British empiricist Isaac Newton, rather than that of the German idealist Gottfried Leibniz. Gödel's fascination with Leibniz was boundless, prompting a mathematical colleague, Paul Erdös, to offer a rebuke: "You became a mathematician," he told Gödel, "so that people should study you, not that you should study Leibniz." Gödel even succeeded in transferring his own paranoia to Leibniz, arguing at length that some of his hero's crucial manuscripts had been secretly destroyed by "those who do not want man to become more intelligent." "You have a vicarious persecution complex," replied his friend Karl Menger, "on Leibniz's behalf." Menger, like most intellectuals a child of the Enlightenment, went on to ask why none of Voltaire's papers had been destroyed. "Who ever became more intelligent," Gödel answered, "by reading Voltaire?"

Further separating Einstein from Gödel was the fact that Einstein never fully resolved his native suspicion of mathematics. To the end, the great physicist favored his cherished physical intuitions. Even though it was precisely Minkowski's mathematical reworking of special relativity in terms of four-dimensional geometry (which Einstein resented at the time) that led to the mathematical abstractions of general relativity, the physicist remained forever wary of being led by the nose by mathematicians. He confessed once to being suspicious of a new move in general relativity that he said he could reach only mathematically (i.e., not intuitively). Gödel, in contrast, always felt most secure when he had formulated a problem in symbolic, mathematical terms. "If you had a particular problem in mind," wrote Taussky-Todd, "he would start by writing it down in symbols." Yet Gödel also believed, famously, that in mathematics too there are intuitions (a doctrine for which logicians still have not forgiven him). For Gödel the equations of mathematics, as opposed to the counsels of common

sense, would lead us into the promised land of new insights, whereas for Einstein, it was precisely common sense that was the final touchstone for assessing what the mathematicians had to offer.

Beneath these disagreements, however, or beyond them, there was much that united the two minds. Both had grown to maturity in the ancient capitals of Europe. They were heirs to the great Austrian-Germanic philosophical tradition, with "philosophy" understood here in its widest sense. Prejudice aside, Russell's comment on the "German bias for metaphysics" had not really missed its mark. Raised in this culture, the composer Gustav Mahler had kept, quite naturally, in his "composing hut," volumes by both Wolfgang Goethe and Immanuel Kant. It comes as no surprise, then, that Gödel and Einstein cut their philosophical teeth on the great works of Kant, whose fingerprints can be clearly discerned throughout the work of each. For Gödel, his writings on Einstein were as much an expression of his interest in Kant's and Leibniz's ideas of time as of his personal association with Einstein. He would characterize his own contributions to relativity theory—to Einstein's consternation—as showing that relativity had "verified" Kant's philosophical idealism.

Einstein's own reading of Kant, in turn, did much to free him from the excessive reliance on immediate sensory data to which many of his contemporaries, especially Ernst Mach, were susceptible. At the tender age of sixteen Einstein had reread Kant's weighty masterpiece, *The Critique of Pure Reason*—the same age at which Gödel too read Kant—and as a student at the Technical Institute in Zurich he had enrolled in a course on Kant. Still, he often made light of the tendency, especially strong in Germany, to venerate the German master. "Kant," he said, "is a sort of highway with lots and lots of milestones. Then all the little dogs come and each deposits his bit at the milestones."

"At the Institute in Princeton," Gerald Holton has noted, "[Einstein's] favorite topic of discussion with his friend Kurt Gödel was . . . Kant." Kant, deeply impressed by Newton—much of his *Critique*, indeed, was intended to provide a philosophical foundation for Newton and Euclid—had made famous the doctrine that science is fundamen-

tal and rigorous exactly to the degree to which it is mathematical. Einstein and Gödel, in turn, each in his own way, approached the world mathematically. For both, mathematics was a window onto ultimate reality, not, as for many of their scientific colleagues, a mere tool for intellectual bookkeeping.

Huddled over a desk in Fine Hall or walking home from the institute, they were a model of mathematical companionship. A chance photograph taken by a visiting mathematician finds the two friends together on the road, each sporting a white straw hat, Einstein beaming for the camera, his convex body bursting from rumpled, baggy pants held up by an ancient pair of suspenders, while the white linen of Gödel's fitted coat holds him closely, his eyes fixed in a cold stare. (Two gentlemen farmers from a Faulkner novel, commented one observer.) Each had found in the other a rare companion who could resist the charms of the "new physics" of Bohr and Heisenberg, according to whom mathematics could no longer provide for science a picture of the world as it actually is in itself—a worldview—but could serve only as a tool for calculation, a means for predicting the outcome of experiments. An impossible prescription to follow for "Mr. Why," and no less so for Einstein. For a signature of Einsteinian science is the Socratic search for "definitions," for what something "really is," in itself (a favorite expression of Plato's). Einstein, after all, was the man who had taught Kant what time "really was" (the fourth dimension of relativistic space-time), taught Newton what gravity "really was" (the curvature of four-dimensional space-time), and taught everyone what energy "really was" (as every schoolchild knows, $E = mc^2$).

As students of Kant, Einstein and Gödel were well aware that although space and time are the two fundamental forms of human experience—space, as Kant had it, the form of intuition of "outer sense," time the form of "inner sense"—it was space that was the natural object of scientific inquiry. And it was space that was first captured by the Greek mathematician Euclid, whose axiomatic-deductive system of geometry—the bane of every high school student—became the paradigm of science, a model from Newton to Einstein. Even in his new

physics of space, Einstein had simply generalized geometry from Euclid to the new non-Euclidean geometries, in which the angles of a triangle could sum to less, or more, than 180 degrees. (To the end of his life, Einstein could wax nostalgic about a boyhood gift that had turned his life around, his "holy geometry booklet.")

Yet as Einstein and Gödel well knew, it is not space but time that in the end poses the greatest challenge to science. The dynamic nature of time, the fact that it flows, is obviously its most striking feature. But it is another thing entirely to make sense of this seemingly obvious truth. After all, to flow is to flow in time. What sense can one attach, then, to the idea of the flow of time itself? Saint Augustine, in his *Confessions*, tied himself in knots over such conundrums. Western thought as such, one might say, is characterized by a kind of geometrical Midas touch. Whatever science touches becomes subject to geometry, the science of space. "Time," Kant himself had said, "is nothing but the form of inner sense, that is, of the intuition of ourselves and our inner states . . . and just because this inner intuition yields no [geometrical] shape, we endeavor to make up for this want by analogies." The analogy, for Kant, is to think of time, which is not space, as spatial! "We cannot," said Kant, "obtain for ourselves a representation of time which is not an object of outer intuition [i.e., of sensory experience] except under the [spatial] image of a line."

Thus when Einstein in 1905 captured time in special relativity, he once again transformed it into space, this time, into the fourth, temporal component of the geometrical structure of four-dimensional "space-time." Not for nothing did G.J. Whitrow write, "the primary object of Einstein's profound researches on the forces of nature has been well epitomized in the slogan, 'the geometrization of physics,' time being completely absorbed into the geometry of a hyper-space." The universe, however, not being empty of matter, is not governed by the matter-free idealization of special relativity but rather by Einstein's next brainchild, the general theory of relativity, the subject of Einstein's free tutorials with Gödel. Worse, the world of general relativity, much to Einstein's displeasure, was actually "expanding," that is, ex-

panding over time. (God, apparently, had for once failed to consult first with Einstein.)

But special relativity had taught the world that simultaneity, and thus time, is not, as Newton thought, worldwide and absolute, but rather local and relative. In what sense of time, then, could the universe itself be expanding, absolutely, over time? Time itself must have been smiling over the puzzle it had created. Appearances notwithstanding, Einstein had not after all succeeded in trapping this elusive prey in the net of general relativity. As Hubble showed, the universe really is expanding! The problem could not be avoided. But if even Einstein had run aground on these rocky shoals, who was left to take the lead? Whom could one compare with Einstein if not his traveling companion in general relativity, Kurt Gödel? But what made Gödel the logician, whose universe consisted of the timeless mathematical realm of sets and numbers, the right person to carry forward Einstein's torch into the uncertainties of the new space-time?

3 Vienna: Logical Circles

After one session in which Schlick, Hahn, Neurath and Waismann had talked about language, but in which neither Gödel nor I had spoken a word, I said on the way home, "Today we out-Wittgensteined these Wittgensteinians: we kept silent."

KARL MENGER

Born into the Austrian-German minority of Brno, a city now in the Czech Republic, the place where Mendel laid the foundations of the science of genetics, the Gödel brothers, Rudolf and Kurt, took it as a given that they would undertake their final academic studies at the storied University of Vienna. Vienna remained even after the Great War one of the premier intellectual centers of the world, distinguished in law, medicine (Rudolf would become a radiologist), physics, mathematics, social sciences, economics, philosophy, and theology. In those years there passed through the city many of the individuals who created the twentieth century, including Sigmund Freud, the founder of psychoanalysis; the composers Richard Strauss and Gustav Mahler as well as Arnold Schoenberg, the inventor of twelve-tone music; the painters Gustav Klimt and Oscar Kokoschka, as well as the revolutionary architect Adolf Loos, who presaged the famous Bauhaus school; the physicist-philosophers Ludwig Boltzmann and Ernst Mach; and the philosophers Karl Popper and Ludwig Wittgenstein. The list could be extended indefinitely. Wittgenstein, himself a kind of minimalist, was an admirer of the minimalism practiced by Loos, and

harbored architectural designs of his own. The attraction was mutual: "You are me!" said Loos to Wittgenstein when they met in 1914. The ratio of intellectual genius to square footage in Gödel's Vienna takes one's breath away.

Among those who were privileged to think the unthinkable, however, there is another name that belongs here. Adolf Hitler's path to Vienna began in Linz, the city of his birth, where in 1904 he attended the same realschule as Wittgenstein. Though the same age as young Ludwig, young Adolf was two years behind him at school. There exists a class photograph in which Wittgenstein appears to be placed near Hitler.

Both of the Gödel boys excelled in secondary school in Brno, but Kurt's gifts were clearly exceptional. He was a standout in all subjects, from science and mathematics to languages, and is said never to have made a single error in his Latin exercises. (It was in mathematics, ironically, that he received his only less than perfect grade.) Arriving in Vienna in 1924, Gödel decided at first to concentrate in physics, a choice that would serve him well. He also received a solid grounding in philosophy, especially the history of philosophy, with Heinrich Gomperz, and excelled in all his classes in mathematics, a subject in which he acquired by graduation a remarkable degree of depth as well as breadth, from geometry to number theory and mathematical logic. It would soon emerge that he was embarked on an intellectual journey in the direction of increased rigor and precision, from mathematical physics to mathematics, from there to mathematical logic, and finally from mathematical logic to mathematical philosophy.

As an undergraduate, Gödel was particularly impressed by the lectures on number theory, attended by hundreds of students, given by Philip Furtwängler, a cousin of the legendary orchestral conductor Wilhelm Furtwängler, whose fame in those years would turn to infamy when he declined to leave Germany during the next world war. Gödel claimed later that Philip Furtwängler, who was paralyzed from the neck down, gave the best lectures he had ever heard. It was Furtwängler whom Gödel credited with his turn to mathematics. The drama of

his lectures was heightened by the fact that Furtwängler lectured from his wheelchair, without notes, while an assistant wrote equations on the blackboard. The feeling of disembodiment this engendered fit the subject of the lectures perfectly. The natural numbers 0, 1, 2, 3, . . . seem to possess the kind of independent existence and "geometry" usually reserved for concrete physical objects. Unsurprisingly, therefore, this branch of mathematics is a breeding ground for Platonists, who like Plato believe in the objective, independent existence of ideal, disembodied "forms," of which the natural numbers are a paradigm. These are no more subject to the arbitrary manipulations of the human will than the distant stars, which we observe but cannot touch. As the minimalist mathematician Kronecker put it, "God made the natural numbers; all else is the work of man." For Gödel, all numbers are "the work of God."

Gödel's journey from physics to mathematical logic took place just as the new field was coming into its own as a well-established intellectual enterprise, although, truth be told, logic remains to this day in the eyes of many mathematicians a poor relation, not quite mathematics, not quite philosophy. Having for centuries been the province of rhetoricians and grammarians, logic emerged as a branch of mathematics at the turn of the century, due in large part to the work of the German philosopher-mathematician Gottlob Frege, an acquaintance of both Russell and Wittgenstein and a seminal influence on their thinking. Frege's early masterpiece, *Begriffsschrift* (*Concept Script*), published in 1879, succeeded in simultaneously axiomatizing logic and formalizing it, that is, formulating it in an artificially constructed, purely symbolic language, prefiguring today's computer programming languages. The rules of such a language are unambiguous and can be followed "mechanically," without the need to understand the meaning of the symbols. Not content with this, Frege employed this new mathematized logic—which for him was not a mere calculating device, but a proper science, with its own content and subject matter—as itself a foundation for mathematics, in particular for arithmetic, or number theory.