An Introduction To Logic And Scientific Method

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AN INTRODUCTION

TO LOGIC AND SCIENTIFIC METHOD

CHAPTER I

THE SUBJECT MATTER OF LOGIC

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§ 1. LOGIC AND THE WEIGHT OF EVIDENCE

Most of our daily activities are carried on without reflection, and it seldom occurs to us to question that which generally passes as true. We cannot, however, always remain in a state of unquestioned belief. For our habitual attitudes are frequently challenged by unexpected changes in our environment, if they are not challenged by our own curiosity or by the inquisitiveness of others.

Let us suppose the reader to be seated at his table some late afternoon. The gathering darkness is making his reading difficult. Ordinarily he would turn on the electric light near him and continue with his work. But on this occasion, we suppose, the Shade of Socrates suddenly appears to the busy reader, just as his hand is on the switch, and asks him to please tell what he is doing. The reader has stout nerves, and quickly recovering from his surprise, explains: "I wish to put on the light, and this is the switch. Since your day . . ." "Yes, yes," we can imagine the Shade to interrupt, "I know all about your modern methods and theories of lighting. You needn't take time to tell me about *that.* But I do wish you would tell me how you know that it is the electric switch you were just pointing to." The reader's temper may by this time have been thoroughly ruffled, and after an embarrassed silence, he may reply with pained surprise and some asperity: "Can't you see, Socrates?"—and turn on the light.

What is of interest to us in this imaginary dialogue is that a doubt, however slight, might be raised in the reader's mind about a proposition *This is the electric switch,* which had previously been accepted without question; and that the doubt might be resolved by claiming that any evidence besides *seeing* was superfluous. There are other propositions for which it would be difficult to find any evidence other than a direct seeing, hearing, touching, or smelling. *It is half-past eleven on my watch; My forehead is hot to the touch; This rose I am smelling has a fine fragrance; The shoes I have on are uncomfortable; That is a loud noise.* These are examples of propositions on account of which most of us would lose our tempers if we were pressed to give reasons why we believed them to be true.

Not all propositions, however, are regarded as so obvious. If the Shade should accost the reader entering the office of a life insurance company, and ask him what he is about, the reader might perhaps say: "I am going to buy a life insurance policy." Should the reader be pressed for his motives, a possible answer might be: "I shall die some day, and I wish to provide for my dependents." If Socrates should now demand why the reader believes in the truth of the proposition J *shall die some day,* the answer will no longer be, "Can't you see?" For we cannot literally see our own future death. But a little reflection may suggest the following reply: "All living creatures, O Socrates, must perish some day, and since I too am a living creature, I too shall die some day."

There are propositions, therefore, which we believe to be true because we can find some *other* propositions of whose truth we have no doubt and which we think will serve as *evidence* for the disputed proposition. *The sun is approximately ninety-three million miles away; Caesar crossed the Rubicon; There will be an eclipse of the sun next year in North America; The sum of the angles of a triangle is equal to two right angles.* These are a few propositions in whose truth we may believe because we think others, if not we ourselves, can find supporting propositions for them.

The distinction between propositions which are believed without grounds other than direct observation or apprehension and propositions which are believed because other propositions can be found to serve as evidence for them, cannot always be drawn very sharply. We sometimes believe a proposition to be true partly because we can make direct observations and partly because we can find supporting propositions. If we drop two rocks of unequal weight from the same height at the same time, we believe the proposition, *The two rocks strike the ground at the same time,* not only because we *see* that they do, but because we know a *reason* why they should do so. Moreover, many propositions whose truth seems very clear to us are in fact false. For we often see what we expect to see rather than that which actually happens. Many remarkable advances in knowledge have resulted from our questioning the truth of propositions which we previously regarded as "self-evident." And a critical study of human beliefs reveals how much "interpretation" is present in what at first sight seems like "immediate knowledge." But it is not necessary for our present purpose to settle the question as to what propositions, if any, can be known to be true "immediately."

All that we now require is the recognition of the general need of evidence for what we or others believe or question. In scientific or historic research, in courts of law, and in making up our minds as to all sorts of practical issues, we are constantly called upon to pass on diverse considerations offered in support of various propositions at issue. Sometimes we find such considerations to be irrelevant and to constitute no evidence at all, even though we have no doubt as to their truth, while other propositions we regard as conclusive or demonstrative proof of a point at issue. Between these two extremes we have situations in which there is some testimony or circumstance that points to a given conclusion but is not sufficient to exclude some alternative possibility. For most occasions we are satisfied with a preponderance of evidence, that is, if there is more evidence in favor of a proposition than against it; but in some cases, for example, when as jurymen we pass on the guilt of one accused of a crime, we are required to act affirmatively only if there is no reasonable doubt left, that is, no doubt which a "reasonably" prudent man would act on in the course of his affairs.

Logic may be said to be concerned with the question of the adequacy or probative value of different kinds of evidence. Traditionally, however, it has devoted itself in the main to the study of what constitutes proof, that is, complete or conclusive evidence. For, as we shall see, the latter is necessarily involved in determining the weight of partial evidence and in arriving at conclusions that are said to be more or less probable.

§ 2- CONCLUSIVE EVIDENCE OR PROOF

Let us consider the proposition *There are at least two persons in New York City who have the same number of hairs on their heads,* and let us symbolize it by *q.* How could its truth be established? An obvious way would be to find two individuals who actually do have the same number of hairs. But this would require an extremely laborious process of examining the scalps of perhaps six million people. It is not a feasible method practically. We may be able to show, however, that the proposition *q* follows from or is necessitated by other propositions whose truth can be established more easily. In that event, we could *argue* for the truth of the proposition *q,* in virtue of its being *implied* by the others, and in virtue of the established truth of the propositions offered as evidence. Let us try this method.

Suppose it were known by an actual count that there are five thousand barber shops in New York City. Would the proposition *There are five thousand barber shops in New York City* be satisfactory evidence for *q?* The reader will doubtless reply, "Nonsense! What has the number of barber shops to do with there being two persons with an identical number of scalp hairs?" In this way the reader expresses the judgment (based on previous knowledge) that the number of barber shops is no evidence at all for the equality in the number of hairs. Not all propositions are *relevant,* even if true, to the truth of a proposition in question.

Let us now consider the proposition *The number of inhabitants in New York City is greater than the number of hairs that any one of its inhabitants has on his head.* We shall denote this proposition by *p.* Is the truth of *p* sufficient to establish the truth of *q?* The reader might be inclined to dismiss *p,* just as he dismissed the information about the number of barber shops, as irrelevant. But this would be a mistake. We can show that if *p* is true, *q* must be true also. Thus suppose, taking small numbers for purposes of illustration, that the greatest number of hairs that any inhabitant of New York City has is fifty, and that there are fifty-one people living in New York City, no one of whom is completely bald. Let us assign a number to each inhabitant corresponding to the number of hairs that he has. Then the first person will have one hair, the second person two hairs, and so on, until we reach the fiftieth person, who will have, at most, fifty hairs. There is one inhabitant left and, since we have assumed that no person has more than fifty hairs, he will necessarily have a number of hairs that is the same as that possessed by one of the other fifty persons. The argument is perfectly general, as a little reflection shows, and does not depend on the number fifty we have selected as the maximum number of hairs. We may, therefore, conclude that our proposition *p, The number of inhabitants in New York City is greater than the number of hairs that any one of its inhabitants has on his head,* implies proposition *q. There are at least two persons in New York who have the same number of hairs on their heads.* The two propositions have been shown to be so related that it is impossible for the first (called the *evidence* or *premise)* to be true, and the second (called the *conclusion* or *that which is to be proved)* to be false.

Other instances of conclusive evidence can be multiplied indefinitely. Thus we can prove that a missing individual is dead by showing that he sailed on a boat destroyed at sea by an explosion that prevented anyone from being saved. So we can prove that our neighbor, Mr. Brown, has no right to vote by showing that he is not yet twenty-one years of age and that the law prohibits such individuals from voting.

Mathematics is, of course, a field in which proof is essential. A distinction, however, must be noted in this respect between applied and pure mathematics. In the former, as in the examples already mentioned, we assume that certain propositions, for example, the laws of mechanics, are *true;* and we prove the *truth* of other propositions by showing that they necessarily follow or are mathematically deducible from those assumed. In pure mathematics, on the other hand, we restrict ourselves to demonstrating that our primary assumptions necessarily imply or entail the theorems which are deduced from them, and ignore the question whether our conclusions as well as our axioms or postulates are in fact true.

It might be of some advantage to use the word "proof" for the former procedure (by which we conclude a proposition to be *true),* and to designate by "deduction" or "demonstration" the procedure which only establishes an *implication* or *necessary connection* between a premise and its conclusion irrespective of the truth or falsity of either. Such a terminology would permit us to say that a proposition is *proved* when, and only when, a premise *implies* that proposition and that premise is itself *true.* But so habitual is the usage which speaks of "proving" theorems in pure mathematics that it would be vain to try to abolish it. It is therefore safer to continue to speak of "proof" in pure mathematics, but to recognize that what we prove there are always implications, that is, that *if* certain propositions are true, certain others must be true. And this, after all, is the phase of all proof in which logic is primarily interested.

In all cases, then, of complete evidence or proof the conclusion is implied by the premises, and the reasoning or inference from the latter to the former is called *deductive.* We *infer* one proposition from another *validly* only if there is an objective relation of *implication* between the first proposition and the second. Hence, it is essential to distinguish *inference,* which is a temporal process, from *implication,* which is an objective relation between propositions. An implication may hold even if we do not know how to infer one proposition from another. Thus an inference to be valid requires that there be an implication between propositions. On the other hand, the being of an implication does not depend upon the occurrence of the psychological process of inferring.

§ 3. THE NATURE OF LOGICAL IMPLICATION

In every attempt at a complete proof of propositions of practical importance we thus find two questions involved:

1. Are the propositions offered as evidence true?
2. Are the conclusions so related to the evidence or premises that the former necessarily follow from and may thus be properly deduced from the latter?

The first question raises what is called a factual or material issue; and the answer to it cannot be assigned entirely to logic without making the latter include all the sciences and all common knowledge. Logic as a distinctive science is concerned only with the second question—with the relation of *implication* between propositions. Thus the specific task of logic is the study of the conditions under which one proposition necessarily follows and may therefore be deduced from one or more others, regardless of whether the latter are in fact true.

As any number of propositions can be combined into one, every instance of implication or logical sequence can be said to hold between two propositions, which might be most accurately designated as the *implicating* and the *implied,*[1] but are generally called *antecedent* and *consequent,* as well as *premise* and *conclusion.* We must, however, note that in using the terms "antecedent" and "consequent," or the expression, "It logically follows," we are referring to an abstract relation which, like that between whole and part, does not directly refer to any temporal succession. The logical consequences of. a proposition are not phenomena which follow it in time, but are rather parts of its meaning. While our apprehension of premises sometimes precedes that of their conclusion, it is also true that we often first think of the conclusion and then find premises which imply it.

Let us consider this relation of implication a little more closely.

Logical Implication Does Not Depend on the Truth of Our Premises.

The specific logical relation of implication may hold (1) between false propositions or (2) between a false and a true one, and (3) may fail to hold between true propositions.

1. Consider the argument *if Sparta was a democracy and no democracy has any kings, it follows that Sparta had no king.* The falsity of the proposition, *Sparta was a democracy*, does not prevent it from having certain implications nor from determining definite logical consequences.

No argument is more common in daily life than that which draws the logical implications of hypotheses contrary to fact. If there were no death there would be no cemeteries, funeral orations, and so on. All our regrets are based on drawing the consequences of propositions asserting what might have been but did not in fact happen.

"Had we never loved sae kindly,
Had we never loved sae blindly,
Never met or never parted,
We had ne'er been broken-hearted!"

It is a great error to suppose, as many have unthinkingly done, that in the reasoning we call scientific we proceed only from facts or propositions that are true. This view ignores the necessity for deduction from false hypotheses. In science as well as in practical choices, we are constantly confronted with alternative hypotheses which cannot all be true. Is the phenomenon of burning to be explained by the emission of a substance called phlogiston or by the combination with one called oxygen? Does magnetism act at a distance like gravitation, or does it, like sound, require a medium? We generally decide between such conflicting propositions by deducing the consequences of each and ruling out as false that hypothesis which leads to false conclusions, that is, to results which do not prevail in the field of observable fact. If false hypotheses had no logical consequences we should not thus be able to test their falsity.

That a proposition has definite logical consequences even if it is false follows also from the fact that these logical consequences or implications are part of its meaning. And we must know the meaning of a proposition before we can tell whether it is true. But in all cases (whether a proposition is true or false) the test as to whether there is a logical implication between one proposition and another is the impossibility of the former being true and the latter being false.

2. There is a widespread impression to the effect that false premises must logically lead to propositions that are false. This is a serious error, probably due to a thoughtless confusion with the true principle that if the consequences are false the premises must be false. But that true consequences may be implied by (or logically follow from) false

premises can be seen from the following simple examples:

If all Mexicans are citizens of the United States and all Virginians are Mexicans, it logically follows that all Virginians are citizens of the United States. If all porpoises are fishes and all fishes are aquatic vertebrates, it necessarily follows that porpoises are aquatic vertebrates. (The same conclusion follows if all porpoises are mollusks and all mollusks are aquatic vertebrates.) For again the relation between the antecedents and the consequents is such as to rule out the possibility of the former being true and the latter at the same time false.

Of course if a premise is false, the conclusion is *not proved* to be true even though the conclusion is implied by the premise. But it is of the utmost importance to realize that a proposition is not necessarily false, or proved to be so, if an argument in its favor is seen to rest on falsehood. A good cause may have bad reasons offered in its behalf.

3. We have already seen that the proposition *There are five thousand barber shops in New York City*, even if true, is irrelevant to and cannot prove or logically imply the proposition *There are at least two persons in New York City who have the same number of hairs on their heads.* Let us, however, take an instance in which the absence of logical connection or implication is perhaps not so obvious. Does the proposition *Perfect beings can live together without law and men are not perfect* imply *Men cannot live together without law?* Reflection shows that nothing in the premise rules out the *possibility* of there being men who, though not perfect, live together without law. We may be able, on other grounds, to prove that our conclusion is true, but the evidence here offered is not sufficient. There is no necessary connection shown between it and that which is to be proved.

Logical Implication Is Formal.

The fact that the logical implications of a proposition are the same whether it happens to be true or not, and that the validity of such implications is tested by the *impossibility* of the premise being true and its consequences false, is closely connected with what is called the formal nature of logic.

What do we mean by *formal?* The reader has doubtless had occasion to fill out some official blank, say an application for some position, a lease, a draft, or an income-tax return. In all these cases, the unfilled document is clearly not itself an application, lease, draft, or tax return; but every one of these when completed is characterized by conforming to the pattern and provisions of its appropriate blank form. For the latter embodies the character or fixed order which all such transactions must have if they are to be valid. A form is, in general, something in which a number of different objects or operations agree (though they differ in other respects), so that the objects may be varied and yet the form remain the same. Thus any social ceremony or act which diverse individuals must perform in the same way if they occupy a given position or office, is said to be formal. Similarly, logical implication is formal in the sense that it holds between all propositions, no matter how diverse, provided they stand to each other in certain relations. Consider any of the foregoing instances of proof, such as *Brown is a minor; all minors are ineligible to vote; therefore Brown is ineligible to vote*. The implication here does not depend on any peculiarity of Brown other than the fact that he is a minor. If any other person is substituted for Brown the argument will still be valid. We can indicate this truth by writing X *is a minor, all minors are ineligible to vote, therefore* X *is ineligible to vote,* where *X* stands for anyone of an indefinitely large class. Reflection shows that we can also replace the word "minor" with any other term, say, "felon," "foreigner," without invalidating the argument. Thus if X *is a* Y, *and all Y's are ineligible to vote, then* X *is ineligible to vote,* no matter what we substitute for *Y*. We can now take the third step and realize that the logical implication is not only independent of the specific character of the objects denoted by *X* and *Y*, but that the term "ineligible to vote" might be replaced by anything else *(provided it is the same in premise and conclusion).* Thus we get the formula: *If* X *is* Y *and all Y's are Z's, then* X *is a* Z as true in all cases no matter what *X, Y,* and *Z* denote. On the other hand it would be an error to assert that if *All Parisians are Europeans and all Frenchmen are Europeans,* it follows that *All Parisians are Frenchmen.* For if in the generalized form of this argument, *All X's are Y's, and all Z's are Y's, therefore all X's are Z's,* we substitute "Belgians" for "Parisians," we get an argument in which the premises are true but the conclusion false. Similarly we can assert the implication that

If Socrates is older than Democritus and Democritus is older than Protagoras, then Socrates is older than Protagoras. For this will hold no matter what persons are substituted for these three, provided we keep the form, X *is older than* Y *and* Y *is older than* Z *implies* X *is older than* Z. On the other hand, from the proposition: A *is to the right of* B, *and* B *is to the right of* C, it does not necessarily follow that A *is to the right of* C. For if three men are sitting in a circle *A* can be said to be to the left of C even though he is to the right of *B* and the latter to the right of C. It is the object of logical study to consider more detailed rules for distinguishing valid from invalid forms of argument. What we need to note at present is that the correctness of any assertion of implication between propositions depends upon their form or structure. If any form can be filled by premises which are true and conclusions which are false, that form is invalid, and the assertion of an implication in any such case is incorrect.

Two observations must be added to the foregoing:

1. The more general statement or formula is not a constraining force or imperative existing before any special instance of it. An argument is valid in virtue of the implication between premises and conclusion in any particular case, and not in virtue of the general rule, which is rather the form, in which we have abstracted or isolated what is essential for the validity of the argument. For the objects which enter into propositions are related in a certain way, and a form is an arrangement; hence an implication which holds for one arrangement of objects will not hold for another.

2. This formal character of implication (and thus of valid inference) does not mean that formal logic ignores the entire meaning of our propositions. For without the latter we can have only meaningless marks or sounds—not significant assertions or information having logical consequences. However, the fact that logic is concerned with necessary relations in the field of possibility makes it indifferent to any property of an object other than the function of the latter in a given argument. Formal properties must be common to all of a class.

Logical Implication as Determination

We have thus far considered the nature of logical implication from the point of view which regards it as an element in all proof or conclusive evidence. We may, however, also view it as entering into every situation or problem in which certain given conditions are sufficient to determine a definite result or situation. Take, for instance, the familiar problem: How long is the interval between the successive occasions when the hands of a clock are together? When the relative velocities of these hands are given, the value of the resulting interval is uniquely determined by the relation of logical implication—though if we are untrained in algebra, it may take us a long time before we see how to pass from the given conditions as premises to the conclusion or solution which they determine. The process of exploring the logical implications is thus a form of research and discovery. It should be noted, however, that it is not the business of logic to describe what happens in one's mind as one discovers rigorous or determinate solutions to a problem. That is a factual question of psychology. Logic is relevant at every step only in determining whether what *seems* an implication between one proposition and another is indeed such. Logic may, therefore, be also defined as the science of implication, or of valid inference (based on such implication). This may seem a narrower definition of logic than our previous one, that logic is the science of the weight of evidence. For implication as we have discussed it seems restricted to conclusive evidence. Reflection, however, shows that deductive inference, and hence the implication on which it ought to be based, enters into all determination of the weight of evidence.

§ 4. PARTIAL EVIDENCE OR PROBABLE INFERENCE

We have so far discussed the relation between premises and conclusion in case of rigorous proof. But complete or conclusive evidence is not always available, and we generally have to rely on partial or incomplete evidence. Suppose the issue is whether a certain individual, Baron X, was a militarist, and the fact that most aristocrats have been militarists is offered as evidence. As a rigorous proof this is obviously inadequate. It is clearly possible for the proposition *Baron X was a militarist* to be false even though the proposition offered as evidence is true. But it would also be absurd to assert that the fact that most aristocrats are militarists is altogether irrelevant as evidence for Baron X having been one. Obviously one who continues to make inferences of this type (Most X's are *Y's*, Z is an *X*, therefore Z is a *Y)* will in the long run be more often right than wrong. An inference of this type, which from true premises gives us conclusions which are true in most cases, is called *probable*. And the etymology of the word (Latin *probare)* indicates that such evidence is popularly felt to be a kind of proof even though not a conclusive one.

The reader will note that where the evidence in favor of a proposition is partial or incomplete, the probability of the inference may be increased by additional evidence. We shall in a later chapter consider with some detail when and how we can measure the degree of probability, and what precautions must be taken in order that our inferences shall have the maximum probability attainable. Here we can only briefly note that deductive inference enters as an element in such determination. To do this, let us consider first the case where a probable argument leads to a *generalization* or *induction*, and secondly, the case where such argument leads to what has been called a *presumption of fact*.

Generalization or Induction

Suppose we wish to know whether a certain substance, say, benzoate of soda, is generally deleterious. Obviously we cannot test this on everybody. We select a number of persons who are willing to take it with their food and whom we regard as typical or representative specimens of human beings generally. We then observe whether the ingestion of this substance produces any noticeable ill effect. If in fact all of them should show some positive ill effect we should regard that as evidence for the general proposition *Benzoate of soda is deleterious*. Such generalizations, however, frequently turn out to be false. For the individuals selected may not be typical or representative. They may have all been students, or unusually sensitive, or used to certain diets, or subject to a certain unnoticed condition which does not prevail in all cases. We try to overcome such doubt by using the inferred rule as a premise and deducing its consequences as to other individuals living under different conditions. Should the observed result in the new cases agree with the deduction from our assumed rule, the probability of the latter would be increased, though we cannot thus eliminate all doubt. On the other hand, should there be considerable disagreement between our general rule and what we find in the new cases, the rule would have to be modified in accordance with the general principle of deductive reasoning. Thus while generalizations from what we suppose to be typical cases sometimes lead to false conclusions, such generalizations enable us to arrive frequently at conclusions which are true in proportion to the care with which our generalization is formulated and tested.

Presumption of Fact

The second form of probable inference (which we have called presumption of fact) is that which leads us to deduce a fact not directly observable. Suppose that on coming home we find the lock on the door forced and a letter incriminating a prominent statesman missing. We believe in the general rule that violators of the law will not hesitate to violate it still further to save themselves from punishment. We then infer that the prominent statesman or his agents purloined the letter. This inference is obviously not a necessary one. Our evidence does not preclude the possibility that the letter was stolen by someone else. But our inference is clearly of the type that often leads to a right conclusion; and we increase this probability, when we show that if someone else than the one interested in the contents of the letter had stolen it, other valuables would have disappeared, and that this is not the fact.

Let us take another case. Suppose, for instance, we notice one morning that our instructor is irritable. We may know that headaches are accompanied by irritability. Consequently we may conclude that our instructor is suffering from a headache. If we examine this argument we find that the evidence for our conclusion consists of a proposition asserting the existence of a particular observable state of affairs (the instructor is irritable) and of another proposition asserting a rule or principle which may be formulated either as *All headaches are accompanied by irritability*, or *Many cases of irritability are due to headaches*. In neither case does our conclusion *The instructor has a headache* follow necessarily. His irritability may in fact be due to some other cause. But our inference is of the type that will lead to a true conclusion in a large number of cases, according to the extent to which irritability is connected with headaches. And we test the truth of the latter generalization (or induction) by deducing its consequences and seeing whether they hold in new situations.

This form of inference is so widespread, not only in practical affairs but even in advanced natural science, that illustration from the latter realm may be helpful. Various substances like oxygen, copper, chlorine, sulphur, hydrogen, when they combine chemically do so according to fixed ratios of their respective weights; and when the same amount of one substance, say chlorine, combines with different amounts of another substance, say oxygen, to form different compounds, it does so in ratios that are all small integral multiples of one. (This is the *observed event.)* We know also that if each of these substances were composed of similar atoms, or mechanically indivisible particles, the substances would combine in such integral ratios of their weights. (This is the *general rule.)* We conclude, then, that these substances are composed of atoms. (This is the *inferred fact.)* From the point of view of necessary implication the inference in this case is invalid. For it is possible that the observable facts may be due to an altogether different general cause than the assumed atomic constitution of matter. But our evidence has a very high degree of probability because we have used the general proposition (Matter has an atomic structure) as a premise from which to deduce-all sorts of consequences that have been found true by direct observation and experiment—consequences which have also been shown to be inconsistent with other known assumptions.

This is also the character of the evidence for such everyday generalizations as that bread will satisfy our hunger, that walking or taking some conveyance will get us to our destination. These generalizations are not universally true. Accidents unfortunately happen. Bread may disagree with us, and he who walks or rides home may land in the hospital or in the morgue. All of our life, in fact, we depend on using the most probable generalizations. If our friend should refuse to walk on wooden or concrete floors because it has not been absolutely proved that they might not suddenly disintegrate or explode, we should feel some concern about his sanity. Yet it is unassailably true that so long as we lack omniscience and do not know all of the future, all our generalizations are fallible or only probable. And the history of human error shows that a general consensus, or widespread and unquestioned feeling of certainty, does not preclude the possibility that the future may show us to be in error.

§ 5. IS LOGIC ABOUT WORDS, THOUGHTS, OR OBJECTS?

Logic and Linguistics

While it seems impossible that there should be any confusion between a physical object, our "idea" or image of it, and the word that denotes it, the distinction is not so clear when we come to complexes of these elements, such as the government or literature of a country. As the logical inquiry into the implication of propositions is not directly concerned with physical or historical fact, the view naturally arises that it is concerned exclusively with words. The etymology of the word "logic" (as in "logomachy") supports this view. The great English philosopher Hobbes speaks of logic and reason as "nothing but reckoning, that is, adding and subtracting, of the consequences of general names agreed upon."[2] We must not, however, confuse the fact that words or symbols of some sort are necessary for logic (as for all the advanced sciences) with the assertion that valid reasoning is *nothing but* a consequence of the act of naming. For we can change the names of things, as we do when we translate from one language into another, without affecting the logical connections between the objects of our reasoning or "reckoning." The validity of our reasoning depends on the consistency with which we use whatever language we have, and such consistency means that our words must faithfully follow the order and connection of the items denoted by them. Logic, like physics or any other science, starts from the common social fact usually recorded by lexicographers that certain words have certain meanings, that is, that they denote certain things, relations, or operations. But a knowledge of such usage in English, or in any other language, will not enable us to solve all questions as to the adequacy of evidence, for example, the validity of the proof by Hermite and Lindemann that π (the ratio between the circumference and the diameter of a circle) cannot be accurately expressed by rational numbers in finite form.

While the direct subject matter of logic cannot be restricted to words, or even to the meaning of words as distinguished from the meaning or implication of propositions, logic is closely connected with general grammar, and it is not always easy to draw a sharp line between the grammatical and the logical writings of philosophers like Aristotle, Duns Scotus, and C. S. Peirce. We have already mentioned that logic starts by taking the ordinary meaning of words for granted, and we shall later see how just discrimination in the meanings of words helps us to avoid logical fallacies. It must, however, be added that in the general study of the meaning of words (called semantics) we are dependent on logic. The information conveyed by words depends both logically and psychologically on propositions, or the information conveyed by sentences.

Perhaps the most significant distinction between logic and that part of linguistics called grammar can be put thus: The norm or correctness with which grammar is concerned is conformity to certain actual usages, while the norm or correctness of logic is based on the possibilities in the nature of things which are the objects of our discourse. Grammar is primarily a descriptive social science, describing in some systematic manner the way in which words are used among certain peoples. It is only incidentally normative, as the description of fashions in clothes is. It is assumed that certain styles, the King's English[3] or the usage of the "best people," is preferable. Many differences of linguistic form may not correspond to any difference of meaning, for example, the differences between the ablative and dative cases in Latin, or such differences as those between "proved" and "proven," "got" and "gotten" in English. But as language is sometimes used to convey significant information, as well as to express emotions, it is impossible for grammarians to ignore logical distinctions. As ordinary experience does not require great accuracy and subtle insight into the nature of things, ordinary language is not accurate. Logic is necessary to correct its vagueness and ambiguity.

In general, though words are among the important objects of human consideration, it is not true that all propositions are about words. Most propositions are about objects like the sun and the stars, the earth and its contents, our fellow-creatures and their affairs, and the like; and the implication between propositions, which is the subject matter of logic, has to do with the possible relations between all such objects. It is only as words are necessary instruments in our statement or expression of a proposition that logic must pay critical attention to them, in order to appreciate their exact function and to detect errors in inference.

Logic and Psychology

An old tradition defines logic as the science of the laws of thought. This goes back to a time when logic and psychology were not fully developed into separate sciences clearly distinguished from other branches of philosophy. But at present it is clear that any investigation into the laws or ways in which we actually think belongs to the field of psychology. The logical distinction between valid and invalid inference does not refer to the way we think—the process going on in someone's mind. The weight of evidence is not itself a temporal event, but a relation of implication between certain classes or types of propositions. Whether, for instance, it necessarily follows from Euclid's axioms and postulates that the area of no square can be exactly equal to that of a circle is a question of what is necessarily involved in what is asserted by our propositions; and how anyone actually thinks is irrelevant to it. Of course thought (and not mere sense perception) is necessary to apprehend such implications. But thought is likewise necessary to apprehend that the propositions of any science are true. That, however, does not make physics a branch of psychology—unless we deny that these sciences have different subject matters, in other words, unless we deny that physical objects and our apprehension of them are distinguishable and not identical. Similarly, our apprehension of the logical implication on which our inferences are based may be studied as a psychological event, but the relation directly apprehended is not itself a psychological event at all. It is a relation between the forms of propositions and indirectly one between the classes of possible objects asserted by them.

The realization that logic cannot be restricted to psychological phenomena will help us to discriminate between our science and rhetoric—conceiving the latter as the art of persuasion or of arguing so as to produce the feeling of certainty. People often confuse the two because the word “certainty” is sometimes used as a characteristic of what is demonstrated and sometimes as the feeling of unquestioning assurance. But such feeling of certainty may exist apart from all logic, and the factual persuasiveness of arguments is more often brought about by properly chosen words, which through association have powerful emotional influences, than through logically unassailable arguments. This is not to belittle the art of rhetoric or to accuse it of always using fallacious arguments. The art of persuasion, or getting others to agree with us, is one that almost all human beings wish to exercise more or less. Harmonious social relations depend on it. But strictly logical argumentation is only one of the ways, and not always the most effective way, of persuading those who differ from us. Our emotional dispositions make it very difficult for us to accept certain propositions, no matter how strong the evidence in their favor. And since all proof depends upon the acceptance of certain propositions as true, no proposition can be proved to be true to one who is sufficiently determined not to believe it. Hence the logical necessity revealed in implication, as in pure mathematics, is not a description of the way all people actually think, but indicates rather an impossibility of certain combinations of the objects asserted. The history of human error shows that the assertion, “I am absolutely certain,” or, “I cannot help believing,” in regard to any proposition is no adequate evidence as to its truth.

In general, the canons of logical validity do not depend upon any investigation in the empirical science of psychology. The latter, indeed, like all other sciences, can establish its results only by conformity to the rules of logical inference. But a study of psychology is of great help to logic, if for no other reason than that only a sound knowledge of psychology can help us to avoid unavowed but false psychological assumptions in logical theory.

Logic and Physics

If logic cannot be identified with linguistics or psychology, neither is it the same as physics or natural science. The propositions whose relations logic studies are not restricted to any special field, but may be about anything at all—art, business, fairy tales, theology, politics; and while the logical relation of implication is involved in physical science, it is not the primary object of the latter.

The fact that propositions may be about nonexistent objects does not militate against the objectivity of the relation of implication. This relation is objective in the sense that it does not depend upon our conventions of language or on any fiat of ours to think in a certain way. This may perhaps become more obvious if we consider the procedure of pure mathematics. In this field, as we have already indicated, we inquire only as to the implication of our initial propositions, without regard to their truth or to whether their subjects are existent or nonexistent, real or imaginary. And yet research in mathematics of the kind that has been going on for over two thousand years is as bound or determined by the nature of the material (logical implication) as is any geographical exploration of the earth or astronomical study of the movements of the stars.

No linguistic fiat or resolution to think differently can change the truths discovered or deduced in such fields as the theory of prime numbers. And this is true of all rigorous logical deduction.

Logic and the Metaphysics of Knowledge

The essential purpose of logic is attained if we can analyze the various forms of inference and arrive at a systematic way of discriminating the valid from the invalid forms. Writers on logic, however, have not generally been content to restrict themselves to this. Especially since the days of Locke they have engaged in a good deal of speculative discussion as to the general nature of knowledge and the operations by which the human mind attains truth as to the external world. We shall try to avoid all these issues—not because they are not interesting and important, but because they are not necessary for the determination of any strictly logical issues. Indeed, the answers to the questions of metaphysics, rational psychology, or epistemology (as they are variously called) are admittedly too uncertain or too questionable to serve as a basis for the science of all proof or demonstration. We wish, however, to dispose of one of these questions, which may trouble the reader: How can false propositions, or those about nonexistent objects, have implications that are objectively necessary?

This seeming paradox arises from a naïve assumption–that only actually existing things have determinate objective characters. It is rather easy to see that the world of science, the world about which there is true knowledge, cannot be restricted to objects actually existing but must include all their possible functions and arrangements. Consider such elementary propositions as *Carbon burns, Ice melts at 32° F., Metals conduct heat and electricity,* and the like. These all refer to classes or kinds of possibilities of the ideally continuous or recurrent existences we call carbon, ice, or metal. Now whatever actually exists is only one of an indefinite number of possibilities. The actual is a flying moment passing from the future which is not yet to the past which no longer is. Logic may be conceived as ruling out what is absolutely impossible, and thus determining the field of what in the absence of empirical knowledge is abstractly possible. History and the sciences of natural existence rule out certain possible propositions as false, for example: There are frictionless engines, free bodies, perfectly rigid levers, and so on. They are ruled out because they are incompatible with propositions which we believe to be true of the actual world. But the actual world at any one time is only one of a number of possible arrangements of things. A proposition proved false on one set of assumptions may be proved true on another. Thus while logical relations alone are not sufficient to determine which existence is actual, they enter into the determination of every arrangement of things that is at all possible. The essential properties which determine the value of $100 remain the same whether we do or do not possess that amount.

§ 6. THE USE AND APPLICATION OF LOGIC

Like any other science, logic aims at attaining truth in its own special field and is not primarily concerned with the values or uses to which these truths can be put. Bad men may be logically consistent. But correct inference is such a pervasive and essential part of the process of attaining truth (which process in its developed form we call *scientific method)* that a study of the way in which logic enters into the latter is a natural extension of our science, just as pure mathematics is extended and developed by its practical application. This will engage our attention in Book II of this volume. Even at this stage, however, we may note some of the ways in which formal, deductive logic aids us in arriving at true propositions.

1. It is obvious that it is often difficult, if not impossible, to determine the truth of a proposition directly, but relatively easy to establish the truth of another proposition from which the one at issue can be deduced. Thus we have observed how difficult it would be to show by actual count that there are at least two people in New York City who have exactly the same number of scalp hairs. But it is fairly easy to show that the number of inhabitants in New York City exceeds the maximum number of hairs on a human head. For the study of the physiology of hair follicles, as well as random samplings of human heads, enables us to establish that there are not more than five thousand hairs to a square centimeter. Anthropological measurements lead to the conclusion that the maximum area of the human scalp is much under one thousand square centimeters. We may conclude, therefore, that no human being has more than five million scalp hairs. And since the population of New York City is close to seven million, there must be more inhabitants in New York City than any human head has hairs. It follows, in virtue of our previous demonstration, that there must be at least two individuals in that city who are precisely alike in the number of hairs on their heads.

2. Many of our beliefs are formed to meet particular problems, and we are often shocked to find these beliefs inconsistent with one another. But they can be integrated, and their bearings on one another made clear, by exploring deductively their mutual relations. Thus it is deductive reasoning which enables us to discover the incompatibility between the following propositions: *Promise-breakers are untrustworthy; Wine-drinkers are very communicative; A man who keeps his promises is honest; No teetotalers are pawnbrokers; All communicative persons are trustworthy; Some pawnbrokers are dishonest.*

3. Deductive reasoning enables us to discover what it is to which we must, in consistency, commit ourselves if we accept certain propositions. Thus if we admit that

two straight lines cannot inclose an area, as well as some other familiar geometric propositions, we must also admit, as we soon discover, that the sum of the angles of any triangle cannot be greater than two right angles. The full meaning of what it is we believe is discovered by us when we examine deductively the connections between the diverse propositions which we consider. For the propositions which we may be inclined to accept almost without question may have implications altogether surprising to us and requiring us to modify our hasty acceptance of them as premises.

In pointing out these uses of deductive inference, it is not denied that men may and do successfully employ it without any previous theoretical study of logic, just as men learn to walk without studying physiology. But a study of physiology is certainly valuable in preparing plans for training runners. Any competent electrician can adjust our electric lights, but we think it necessary that an engineer who has to deal with new and complicated problems of electricity should be trained in theoretical physics. A theoretical science is the basis of any rational technique. In this way logic, as a theoretical study of the kinds and limitations of different inferences, enables us to formulate and partially mechanize the processes employed in successful inquiry. Actual attainment of truth depends, of course, upon individual skill and habit, but a careful study of logical principles helps us to form and perfect techniques for procuring and weighing evidence.

Logic cannot guarantee useful or even true propositions dealing with matters of fact, any more than the cutler will issue a guarantee with the surgeon's knife he manufactures that operations performed with it will be successful. However, in offering tribute to the great surgeon we must not fail to give proper due to the quality of the knife he wields. So a logical method which refines and perfects intellectual tools can never be a substitute for the great masters who wield them; none the less it is true that perfect tools are a part of the necessary conditions for mastery.[4]

[1] In grammar they are known as the *protasis* and *apodosis* of a subjunctive sentence.

[2] *Leviathan*, Pt. I, Chap. 5.

[3] The English kings from the eleventh to the fourteenth century were of course Frenchmen.

[4] More mature readers will do well to go over Appendix A carefully before undertaking Chapter II.

BOOK I

FORMAL LOGIC

CHAPTER II

THE ANALYSIS OF PROPOSITIONS

§ 1. WHAT IS A PROPOSITION?

In the last chapter logic was defined as dealing with the relation of implication between propositions, that is, with the relation between premises and conclusions in virtue of which the possible truth and falsity of one set limits the possible truth and falsity of the other. Both premises and conclusions are thus propositions; and for the purposes of logic, a *proposition* may be defined as anything which can be said to be true or false. But we shall understand this definition more clearly if we also indicate what a proposition is not.

1. A proposition is not the same thing as the sentence which states it. The three sentences, "I think, therefore, I am," "*Je pense, donc je suis,*" "*Cogito, ergo sum*" all state the same proposition. A sentence is a group of words, and words, like other symbols, are in themselves physical objects, distinct from that to which they refer or which they symbolize. Sentences when written are thus located on certain surfaces, and when spoken are sound waves passing from one organism to another. But the proposition of which a sentence is the verbal expression is distinct from the visual marks or sound waves of the expression. Sentences, therefore, have a physical existence. They may or may not conform to standards of usage or taste. But they are not true or false. Truth or falsity can be predicated only of the propositions they signify.

2. It should be noted, however, that while the proposition must not be confused with the symbols which state it, no proposition can be *expressed* or *conveyed* without symbols. The structure of the proposition must, therefore, be expressed and communicated by an appropriate structure of the symbols, so that not every combination of symbols can convey a proposition. "John rat blue Jones," "Walking sat eat very," are not symbols expressing propositions, but simply nonsense, unless indeed we are employing a code of some sort. Only certain arrangements of symbols can express a proposition. That is why the study of symbolism is of inestimable value in the correct analysis of the structure of propositions. And that is why, although grammatical analysis is not logical analysis, the grammar of a language will often clarify distinctions which are logical in nature.

3. A proposition, we have said, is something concerning which questions of truth and falsity are significant. Consequently when Hamlet declares, "Oh, from this time forth, My thoughts be bloody, or be nothing worth!" or when he asks, "Why wouldst thou be a breeder of sinners?" he is not asserting propositions *except implicitly*. For wishes, questions, or commands cannot as such be true or false. It should be noted, however, that the intelligibility of wishes, questions, and commands rests upon assumptions that certain states of affairs prevail. And such assumptions involve propositions. For consider the question, "Why wouldst thou be a breeder of sinners?" It obviously assumes, among other propositions, that the person addressed exists, is capable of breeding children, and that such children are certain to be sinners. Similarly, the exhortation, "My thoughts be bloody, or be nothing worth!" assumes that the speaker is capable of having ideas, that these ideas can be murderous, that they may have some kind of value, and so forth. Moreover, a command or wish may be put in the form of a declaration, which generally expresses a proposition, for example, *I wish you would come; I shall be pleased if you come; You will be sorry if you do not come.* To the extent that the declarations state something that may be true or false they are propositions.

4. Propositions are often confounded with the mental acts required to think them.

This confusion is fostered by calling propositions "judgments," for the latter is an ambiguous term, sometimes denoting the mental act of judging, and sometimes referring to that which is judged. But just as we have distinguished the proposition (as the objective meaning) from the sentence which states it, so we must distinguish it from the act of the mind or the judgment which thinks it.

5. Nor must propositions be identified with any concrete object, thing, or event. For propositions are at most only the abstract and selected relations between things. When we affirm or deny the proposition *The moon is nearer to the earth than the sun*, neither the moon alone, nor the earth, nor the sun, nor the spatial distance between them is the proposition. The proposition is the relation asserted to hold between them. The relations which are the objects of our thought are elements or aspects of actual, concrete situations. These aspects, while perhaps not spatially and temporally *separable* from other characters in the situation, are *distinguishable* in meaning. That is why sense experience never yields knowledge without a reflective analysis of what it is we are experiencing. For knowledge is *of* propositions. And propositions can be known only by discriminating within some situation relations between abstract features found therein.

6. While a proposition is defined as that which is true or false, it does not mean that we must *know* which of these alternatives is the case. *Cancer is preventable* is a proposition, though we do not know whether it is true.

A difficulty is nevertheless suggested: We may not be able to tell whether a given sentence does or does not express a proposition. Consider, for example, the expression, "Three feet make a yard." Are we raising questions of truth and falsity in asserting it? It must be acknowledged that the sentence has the appearance of expressing a proposition. But if we analyze what is usually meant by it, we find it expresses a *resolution* rather than something which is capable of being true. We *resolve* to use a unit of measure so that it will be made up of three feet. But the resolution as such cannot have truth or falsity predicated of it. Such resolutions, which often take the form of definitions, are expressed in ways analogous to the way propositions are expressed; but they must be distinguished from the latter.

The question whether the word "yard" is actually used in the sense defined is of course a factual one, and the answer may be true or false. But such propositions are about linguistic usages and not about the objects denoted by the words.

7. A related difficulty arises from the fact that we popularly speak of propositions as sometimes true and sometimes false, whereas our definition of propositions excludes this possibility and assumes that if a proposition is true it must always be true. Nothing is more common in discussion between candid people than their remark, "What you say is sometimes true, but not always so." This applies to such statements as "Religion preaches love of one's fellow men"; "It is difficult to resist temptation"; "A gentle answer turneth away wrath." We may, however, remove this difficulty by recognizing that if these propositions assert that something is *universally* the case, then the existence of an exception only proves that they are false. Such an assertion as "Religion sometimes preaches hate of one's neighbors" does not assert the absurdity that a universal proposition *Religion always preaches hate of one's neighbors is* sometimes true.

We can perhaps see this more clearly in the following case. *The present governor of Connecticut is Dr. Cross* seems to be a proposition true for certain years, but surely not always. This, however, is an inadequate analysis. For the phrase "the present governor" clearly presupposes a date; and as we complete our expression by including explicitly the date, we obtain expressions for different propositions, some of which are true and some false. In general, our everyday statements seldom contain all the qualifications necessary to determine whether what we say is true or false. Some of their qualifications we understand, others are not thought of. The incomplete expression is neither true nor false. And when we say that it is sometimes true and sometimes false, we can mean only that our expressions may be completed in some ways which express true propositions and in some ways which express false ones.

§ 2. THE TRADITIONAL ANALYSIS OF PROPOSITIONS

Terms: Their Intension and Extension

According to Aristotle, all propositions either assert or deny something of something else. That about which the assertion is made is called the *subject*, and that which is asserted about the subject is called the *predicate*. The subject and predicate are called the *terms* of the proposition; the proposition is the synthesis or unity of the terms by means of the *copula*, which is always some part of the verb "to be."

This analysis cannot readily be applied to very simple propositions such as *It is raining; There was a parade* and the like. The "It" and the "There" clearly do not denote subjects, of which "raining" and "a parade" are attributes. Nevertheless, there is a certain amount of truth in Aristotle's analysis, if we hold on to the distinction between *terms* and *propositions*, but drop the requirement that there must be just two terms. *It is raining* or *There was a parade* are properly said to be propositions, because they conform to his test, to wit, they are either true or false. "Raining" or "a parade" are not propositions, because they are not either true or false. When we hear the words "raining" or "a parade" we ask, "What about it?" Only when *assertions* are made about these objects can questions of truth or falsity be raised.

These objects, then, as *terms* enter as elements of propositions. A term may be viewed in two ways, either as a class of objects (which may have only one member), or as a set of attributes or characteristics which determine the objects. The first phase or aspect is called the *denotation* or *extension* of the term, while the second is called the *connotation* or *intension*. Thus the extension of the term "philosopher" is "Socrates," "Plato," "Thales," and the like; its intension is "lover of wisdom," "intelligent," and so on.

Although the intension and extension of a term are distinct aspects, they are inseparable. All words or symbols except pure demonstrative ones (those which serve to point out, like a gesture, or "this") signify some attributes in virtue of which they may be correctly applied to a delimited set of objects; and all general terms are capable of being applied to some object, even though at any given time no object may in fact possess the attributes necessary to include it in the extension of the term. *Why* a term is applied to a set of objects is indicated by its intension; the set of objects *to which* it is applicable constitutes its extension.

With many of the difficult problems of extension and intension we shall not concern ourselves. It will be convenient, however, to distinguish between several senses in which the term *intension* is frequently employed. It is necessary to do so if we wish to avoid elementary confusions.

1. The *intension of a term* is sometimes taken to mean the sum total of the attributes which are present to the mind of any person employing the term. Thus to one person the term "robber" signifies: "taking property not lawfully his, socially undesirable, violent," and so forth; to another person it may signify: "taking property with value greater than ten dollars not lawfully his, physically dangerous person, the result of bad disposition," and so on. The intension of a term so understood is called the *subjective intension*. The subjective intension varies from person to person, and is of psychological rather than of logical significance.

2. The *intension of a term* may signify the set of attributes which are essential to it. And by "essential" we mean the necessary and sufficient condition for regarding any object as an element of the term. This condition is generally selected by some convention, so that intension in this sense is called *conventional intension* or *connotation*. The conventional intension of a term, as we shall see later, constitutes its definition.

3. *The intension of a term* may signify *all* the attributes which the objects in the denotation of a term have in common, whether these attributes are known or not. This is called the *objective intension* or *comprehension*. Thus, if the conventional intension of "Euclidean triangle" is "a plane figure bounded by three straight lines," a part of the objective intension is: "a plane figure with three angles, a plane figure whose angle sum is two right angles," and so on.

It is the conventional intension of a term which is logically important. The denotation of a term clearly depends upon its connotation. Whether we may apply the word "ellipse" to some geometric figure is determined by the attributes included in the connotation of the term. From the point of view of knowledge already achieved, the understanding of the connotation of a term is prior to its denotative use: we must know the connotation of "amoeba" before we can apply it. In the order of the *development* of our knowledge, it is doubtful whether there is such a priority. Philosophers have been unable to resist the temptation of regarding either the intension or the extension of a term prior in every respect, and much ink has been shed over the question. It is reasonable to suppose, however, that the development of our knowledge concerning intension and extension, since they are inseparable aspects of the meaning of a term, go hand in hand. A group of objects, such as pieces of iron, bronze, tin, may be selected for certain special purposes in virtue of their possessing some prominent features in common, like hardness, opacity, fusibility, luster. Such objects may then be denoted by a common term, "metal." These striking features may then be taken as criteria for including other objects in the denotation of this term. But greater familiarity with such objects may lead us to note qualities more reliable as signs of the presence of other qualities. We may then group objects together in spite of superficial differences, or group objects differently in spite of superficial resemblances. The conventional intension of the term "metal" may thus become gradually modified. The assigning of the satisfactory conventional intension (or definition) to a term denoting objects with familiar common traits is a difficult task, and is a relatively late achievement of human thought.

Consider now the terms: "figure," "plane figure," "rectilinear plane figure," "quadrilateral," "parallelogram," "rectangle," "square." They are arranged in order of subordination, the term "figure" denoting a class which includes the denotation of "plane figure," and so on. Each class may be designated as the *genus* of its subclass, and the latter as a *species* of its genus. Thus the *denotation* of these terms *decreases:* the extension of "parallelogram" includes the extension of "rectangle," but not conversely, and so on. On the other hand, the *intension* of the term *increases:* the intension of. "rectangle" includes the intension of "parallelogram," but not conversely. Reflection upon such series of terms has led to the rule: *When a series of terms is arranged in order of subordination, the extension and intension vary inversely.*

But this formulation of the relation of extension to intension is not accurate. In the first place, "vary inversely" must not be understood in a strict numerical sense. For in some cases the addition of a single attribute to the intension of a term is accompanied by a greater change in its extension than in other cases. Thus the extension of "man" is reduced much more by the addition of the attribute "centenarian" than by the addition of the attribute "healthy." And in the second place, variation in the intension may be accompanied by no change in the extension. Thus, the extension of the term "university professor" is the same as the extension of "university professor older than five years." It is clear, moreover, that the relation of inverse variation must be taken between the *conventional* intension of a term, and its denotation in a specified universe of discourse. The law of inverse variation must, therefore, be stated as follows: *If a series of terms is arranged in order of increasing intension, the denotation of the terms will either remain the same or diminish.*

The Form of Categorical Propositions

According to the traditional doctrine all propositions can be analyzed into a subject and a predicate joined by a copula, either from the intensional or from the extensional point of view. *All cherries are luscious* may on the first view be interpreted to mean that the attribute of "being luscious" is part of the group of *attributes* which define the nature of cherries. On the second view, this proposition means that the objects called cherries are included in the denotation of the term "luscious." [1]

The traditional view recognizes other forms of propositions, called *conditional,* which it tries to reduce to the categorical form. We shall examine below these conditional forms, as well as other ways of analyzing propositions. Here we shall only note that on the traditional view all propositions are analyzable in just this way and only in this way.[2]

Propositions which do not obviously present a subject-predicate form must, then, be changed to exhibit that form. Thus *Germany lost the war* must be expressed as *Germany is the loser of the last war,* where "Germany" is the subject, "tire loser of the last war," the predicate, and "is," of course, the copula. The proposition *Ten is greater* *than five* must be analyzed into "ten" as subject, "a number greater than five" as predicate, and "is" as copula.

It is not difficult to exhibit any proposition as *verbally* conforming to the subject-predicate type, but such verbal identity often obscures fundamental logical distinctions. The chief criticism which modern logic has made of the traditional analysis is that the latter has lumped together (as categorical) propositions which have significant differences in form.

The reader may perhaps wonder what is the significance of this quarrel over the manner in which propositions should be analyzed. The answer is simple. The analysis of propositions is undertaken for the purpose of discovering what inferences may be validly drawn from them. Consequently, if there is a plurality of propositional forms, and such form or structure determines the validity of an inference, an increased refinement of analysis may help us to attain a more accurate view of the realm of possible inference.

Another reason for analyzing the structure of propositions is to devise some *standard* or *canonical* ways of representing what it is we wish to assert. We wish to find certain canonical formulations of propositions of a given type, in order that the process of inference shall be expedited. Thus in elementary algebra it is extremely convenient to write the quadratic equation $5x^2 = 3x - 5$ in the standard form $5x^2 - 3x + 5 = 0$. For if we do so, since we already know the roots of a general quadratic in the standard form $ax^2 + bx + c = 0$, it is very simple to find the numerical answers to our problems. Moreover, if we adopt a standard form for writing equations, it is much easier to compare different equations and note their resemblances. Similar considerations apply in logic. For if we can once establish criteria of validity for inferences upon propositions stated in a standard form, all subsequent testing of inferences becomes almost mechanical.

Much care must be taken, however, in reducing a proposition expressed in one verbal form to the standard form, lest some part of its original meaning be neglected. In reducing a line of Keats's poetry, for example, into a canonical form it is not easy to believe that every shade of meaning of the original has been retained.

Quantity

Categorical propositions have been classified on the basis of their *quantity* and their *quality*. In the proposition, *All steaks are juicy*, something is affirmed of *every* steak, while in the proposition, *Some steaks are tough*, information is supplied about an *indefinite* part of the class of steaks. Propositions which predicate something of *all* of a class are designated as *universal*, while those which predicate something of an *indefinite part* of a class are *particular*. The particles "all" and "some" are said to be *signs of quantity*, because they indicate of how large a part of the subject the predicate is affirmed. The distinction between them is more accurately stated if "all" is called the sign of a *definite* class, and "some" the sign of an *indefinite* part of a class. For in everyday usage, the signs of quantity are ambiguous. Thus in the proposition *Some professors are satirical* it would ordinarily be understood that a *part, but not the whole*, of the class of professors are satirical; here "some" means "some but not all." On the other hand, *Some readers of this book have no difficulty in understanding it* would generally be understood to mean that a portion of the readers, *not excluding the entire class*, had no difficulty with it; here "some" means "some and perhaps all." We shall obviate such ambiguity by agreeing that in logic "some" will be taken in the latter sense; that is, as not necessarily excluding "all."

A different sort of ambiguity occurs in the use of the word "all." Sometimes it denotes all of a finite and enumerated collection, as in the proposition *All the books on this shelf are on philosophy*. Sometimes, however, as in *All men are mortal* the "all" means "all possible," and cannot be taken, without distortion of intended meaning, to indicate merely an enumeration of the men who do in fact exist or have already existed. We shall find this distinction of paramount importance in the discussion of induction and deduction. Many mistaken views concerning the former are the outcome of ignoring it.

Is the proposition *Thaïs was a courtesan in Alexandria* universal or particular? The reader may be tempted to say it is the latter. But that would be an error, for he would then be using "particular" in a sense different from the one employed in classifying propositions. On the basis of the definition of universal propositions as those which affirm something of *all* of the subject, this proposition must be regarded as a universal. This would be even more obvious if, as we suggest, the terms *definite* and *indefinite* were used instead of *universal* and *particular*. However, since in such propositions we are affirming something of a single individual, traditional logic has sometimes designated them as *singular*. But singular propositions must be classified as universal on the traditional analysis. However, a more subtle analysis cannot be content with such a conclusion. Even untutored people feel dimly that there is a difference *in form* between *Dr. Smith is a reassuring person* and *All physicians are reassuring persons* although traditional logic regards both as universal. Modern logic corroborates this feeling by showing clearly that these propositions do in fact illustrate different logical forms. Nevertheless, for many purposes no harm results if, with traditional logic, we regard both as having the same structure.

Quality

A second classification of categorical propositions is concerned with their *quality*. In the proposition *All snakes are poisonous* the predicate is affirmed of the subject. The proposition is therefore said to be *affirmative*. In *No democracies are grateful* something is denied of the subject. The proposition is therefore said to be *negative*. If we think of a categorical proposition as a relation between classes of individuals, an affirmative proposition asserts the *inclusion* of one class or part of a class in another, while a negative proposition asserts the *exclusion* of one class or part of a class from another. It follows that the negative particle, the sign of quality, must be understood to characterize the copula, not the subject or the predicate.

How should we classify *All citizens are not patriots?* It seems to be a negative proposition, and the sign of quantity seems to indicate it as universal. But while it *may* be interpreted as asserting that *No citizens are patriots* it may also be understood as denying that *All citizens are patriots* or as asserting that *Some citizens are not patriots.* Expressions employing "all . . . not" as in the foregoing, or as in *All that glitters is not gold* are essentially ambiguous. In such cases, we must determine what is meant, and then state it in an unambiguous form.

On the basis of quantity and quality we may therefore distinguish four forms of categorical propositions. *All teetotalers are short-lived* is a universal affirmative, and is symbolized by the letter *A*. *No politicians are rancorous* is a universal negative, and is symbolized by *E*. *Some professors are soft-hearted* is a particular affirmative, and is symbolized by *I*. *Some pagans are not foolish* is a particular negative, and is symbolized by *O*. The letters *A* and *I* have been used traditionally for affirmative propositions: they are the first two vowels in *affirmo;* while *E* and *O* symbolize negative propositions: they are the vowels in *nego.*

Exclusive and Exceptive Propositions

In the propositions *The wicked alone are happy*, *Only the lazy are poor*, *None but savages are healthy,* something is predicated of something else in an exclusive fashion. They are therefore called *exclusive propositions.* Traditional logic reduces such propositions to the canonical form for categoricals. For example, *The wicked alone are happy* asserts the same thing as *All happy individuals are wicked. None but the brave deserve the fair* asserts the same as *All who deserve the fair are brave. None but Seniors are eligible* asserts the same as *All those eligible are Seniors.*

In the propositions *All students except freshmen may smoke, All but a handful were killed, No child may enter unless accompanied by a parent* the predicate is denied of some part of the denotation of the subject. They are therefore called *exceptive* propositions. They may also be stated in the standard form for categoricals. For exceptive propositions may always be expressed as exclusive ones. Thus *All students except freshmen may smoke* can be reduced to *Freshmen alone among students may not smoke*. Consequently, the above exceptive proposition can be stated as the following *A* proposition *All students who may not smoke are freshmen.*

Distribution of Terms

We shall now introduce a new technical term. We will say a term of a proposition is *distributed* when reference is made to *all* the individuals denoted by it; on the other hand, a term will be said to be *undistributed* when reference is made to an *indefinite part* of the individuals which it denotes.

Let us now determine which terms in each of four types of categoricals are distributed. It is evident that in universal propositions the subject term is always distributed, while in particular propositions the subject is undistributed. How about the predicate terms? In *All judges are fair-minded* is reference made to all the individuals denoted by "fair-minded"? Clearly not, because the proposition supplies no information whether *all* fair-minded individuals are judges or not. Hence the predicate in *A* propositions is undistributed. A similar conclusion is true for *I* propositions. We may therefore conclude that affirmative propositions do not distribute their predicates.

Does the same state of affairs obtain for negative propositions? Consider *No policemen are handsome.* This proposition asserts not only that every individual denoted by "policemen" is excluded from the class denoted by "handsome," but also that all individuals of the latter class are also excluded from the former. Consequently, the predicate in *E* propositions is distributed. A similar conclusion holds for *O* propositions. Thus in *Some of my books are not on this shelf* an indefinite part of the subject class is excluded from the *entire* class denoted by the predicate. The reader will see this clearly if he asks himself what part of the shelf indicated he would have to examine in order to assure himself of the truth of the proposition. Obviously it is not enough to examine only a *part* of the books on the shelf; he must examine *all* the books on the shelf. The predicate is therefore distributed.

We may summarize our inquiry by noting that universal propositions distribute the subject, while particular propositions do not distribute the subject. On the other hand, the predicate is distributed in negative propositions, but undistributed in affirmative ones.

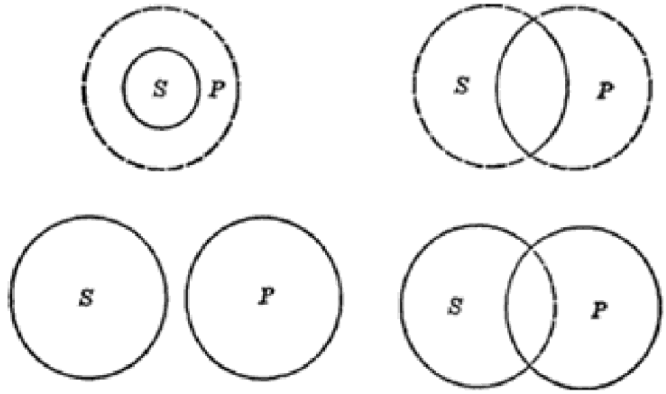
The concept of distribution of terms plays an important part in traditional logic, and is the fundamental idea in the theory of the syllogism. The reader is therefore advised to familiarize himself with it thoroughly. We may note in passing, however, that the subject-predicate analysis of propositions, together with the idea of distribution, sometimes leads to inelegant results. Thus on the traditional analysis *Socrates was snub-nosed* is a universal; its subject must be distributed, since snub-nosedness is predicated of the entire individual Socrates. Nevertheless, while in other universal propositions like *All children are greedy* a corresponding particular proposition may be obtained in which "children" is undistributed, no such corresponding proposition can be found for singular ones. For under no circumstances may the term "Socrates" be undistributed. We shall find other respects in which universal and singular propositions do not receive symmetrical development in traditional logic.

Diagrammatic Representation

The structure of the four types of categorical propositions can be exhibited in a more intuitive fashion if we adopt certain conventional diagrammatic representations. Many methods of doing this have been devised, some having different purposes in mind. The earliest is due to Euler, a Swiss mathematician of the eighteenth century. We shall first explain a slight modification of his method.

Let us agree to the following conventions. A circle drawn in solid line will indicate a distributed term; a circle drawn (in part or in whole) in dotted line will represent an undistributed term. A circle drawn inside another will indicate the inclusion of one class in another; two circles entirely outside each other will indicate the mutual exclusion of two classes; and two overlapping circles will represent either an indefinite partial inclusion or an indefinite partial exclusion.

The four relations between the classes "street-cleaners" and "poor individuals" which characterize the four categorical propositions (in which the former class is subject) may then be diagrammed as follows.

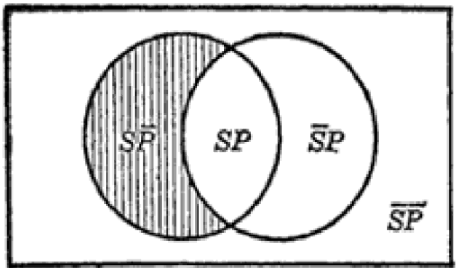


The *S* circle represents the class “street-cleaners” (the subject), and the *P* circle the class “poor individuals” (the predicate).

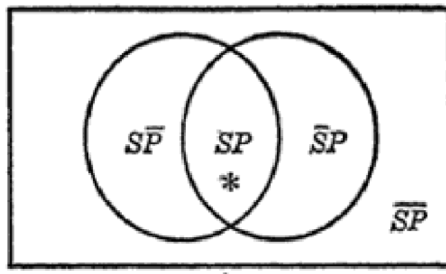
It is often useful to employ another method of representing the categorical propositions, which is due to the English logician John Venn. We first notice that every proposition tacitly refers to some context within which it is significant. Thus *Hamlet killed Polonius* refers to Shakespeare’s play. Let us call the domain of reference the *universe of discourse* and represent it diagrammatically by a rectangle. Now the reader will observe that two classes, together with their negatives, yield four and only four combinations. (By the negative of a class is understood everything in the universe of discourse excluded from that class.) For example, in the universe of discourse restricted to human beings there are the things which are both street-cleaners and poor_ (symbolized as *S P),* or which are street-cleaners and not poor (S $\bar{P}$), or not street-cleaners and poor (S *P)* or neither street-cleaners nor poor ($\bar{S}$ $\bar{P}$). The universe of discourse is thus divided into four possible compartments. However, in general not all these possible compartments will contain individuals as members. Which ones do and which ones do not depends upon what is *asserted* by propositions referring to that universe of discourse.

Let us therefore draw two overlapping circles within a rectangle, and so obtain automatically four distinct compartments, one for each of the four logical possibilities indicated. Now since the *A* proposition asserts that all street-cleaners are included in the class of the poor, the class of those who are street-cleaners and not poor cannot contain any members. To show this on the diagram, we shall agree to blot out by shading the corresponding compartment. The diagram for the *A* proposition will thus show that the compartment $S\bar{P}$ is missing. We may also indicate this explicitly by writing under the rectangle $S\bar{P} = 0$, where 0 means that the class in question contains no members. Hence the proposition *All street-cleaners are poor* declares that in its universe of discourse there are no individuals who are both street-cleaners and not poor.

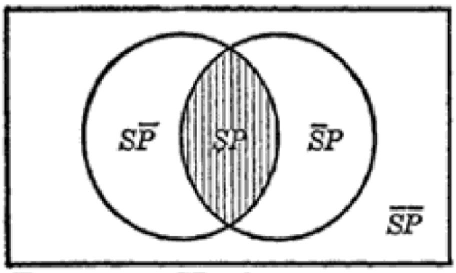
In the case of the *I* proposition the procedure of representing it is somewhat different. If we ask what *Some street-cleaners are poor* asserts, we find that it does *not* say that there *are* individuals who are street-cleaners and not poor (let the reader recall what we said above about the meaning of “some”). Neither does it say that *every* one of the four possible compartments has members. The minimum which the proposition requires for its truth is that the class of individuals who are street-cleaners and poor be *not* empty. We shall agree to designate this minimum by placing an *asterisk* into the *S P* compartment, to show that it cannot be empty. We thereby leave indeterminate whether the other compartments have members or not. We may indicate this further by writing S *P* ≠ 0 under the rectangle, which signifies that the left-hand member of this inequality is not devoid of members. The reader should study carefully the remaining diagrams. He will find that the analysis of the *E* and *O* propositions is similar to that of the *A* and *I* propositions respectively.



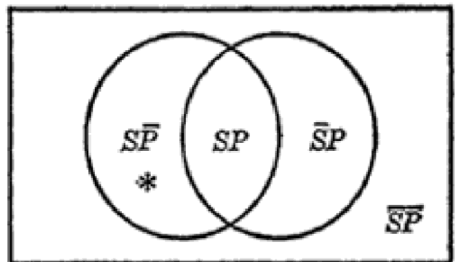
A: $S\bar{P} = 0$



I: $SP \neq 0$



E: $SP = 0$



O: $S\bar{P} \neq 0$

The Existential Import of Categoricals

If we now compare the diagrams, we discover that there is a remarkable difference between universal and particular propositions. Universals do not affirm the existence of any individuals, but simply *deny* the existence of certain kinds of individuals. Particulars do not deny the existence of anything, but simply *affirm* that certain classes do have members. Consequently, the universal *All street-cleaners are poor* means only this: *If any individual is a street-cleaner, why, then he is poor.* It does not say that there actually are individuals who are street-cleaners. On the other hand, the particular *Some street-cleaners are poor* means that there is at least one individual who is at the same time a street-cleaner and poor.

We will anticipate some later discussion by putting the matter as follows. The universal *All street-cleaners are poor* is to be interpreted to assert: *For all instances or values of* X, *if* X *is a street-cleaner, then* X *is poor.* The particular *Some street-cleaners are poor* is to be interpreted to assert: *There is an* X, *such that* X *is a street-cleaner and* X *is poor.* We are thus becoming prepared to understand why modern logic finds fault with classifying a proposition such as *Napoleon was a soldier* with propositions such as *All Frenchmen are soldiers.* The latter, we have seen, means when analyzed: *For all instances or values of* X, *if* X *is a Frenchman then* X *is a soldier.* The former, on the other hand, cannot in any way be interpreted in this manner. We shall return to this matter presently.

The conclusion we have reached, that universals do not imply the existence of any verifying instances, while particulars do imply it, will doubtless seem paradoxical to the reader. (Indeed, a fuller discussion than we can undertake is required to make clear how much of this conclusion is a matter of convention and how much is forced upon us by logical considerations.) The reader will perhaps cite propositions like *All dogs are faithful* and urge that they do imply the existence of dogs. Now it may well be that when the reader asserts *All dogs are faithful* he also intends to assert *There are dogs.* But he should note that then he makes two separate and distinct assertions. But the proposition *All those who are free from sin may cast stones* clearly does not imply that there actually are any individuals free from sin. The universal proposition may be simply a hypothesis concerning a class which we know cannot have any members.

Thus Newton's first law of motion states: All bodies free of impressed forces persevere in their state of rest or of uniform motion in a straight line forever. Will the reader affirm that this proposition asserts the existence of any body which is not under the influence of an impressed force? We need remind him only of the law of gravitation, according to which *all* bodies attract one another. What Newton's first law does assert is the hypothesis that *if* a body were free from impressed forces, it would persevere in its state of rest or in uniform motion in a straight line forever. In the same way, the principle of the lever states what would be the case if the lever were a perfectly rigid body; it does not assert that there is such a body. Indeed, reflection upon the principles of the sciences makes it quite clear that universal propositions in science always function as *hypotheses*, not as statements of fact asserting the existence of individuals which are instances of it. It is true, of course, that if there were no applications of the universal, it would be useless for the science which deals with matters of fact. It is also true that the *meaning* of universal propositions requires at least *possible* matters of fact. But we cannot identify abstract possibilities denoted by universals with actual existences in which these possibilities are annulled by or are combined with other possibilities. Thus, inertia is a phase of all mechanical action, though no instance of inertia by itself can be found in nature. The principle of the lever holds to the extent that bodies are actually rigid, even though no instances of pure rigidity exist in isolation from other properties of bodies.

We may view this matter from another angle. Hitherto we have been discussing propositions as asserting relations between classes of individuals. But we have seen (page 33) that propositions may be interpreted as asserting connections between attributes. When universal propositions are regarded as not implying the existence of any individuals, it is the interpretation in terms of constant relations between attributes which comes to the fore.

It should be noted, finally, that in raising questions of existential import we do not necessarily confine the reference of terms to the physical universe. When we ask, "Did Jupiter have a daughter?" or, "Was Hamlet really insane?" we are not raising questions of *physical* existence, but of the existence of individuals within a universe of discourse controlled by certain assumptions, such as the statements of Homer or Shakespeare. An individual who may thus be said to "exist" in one universe of discourse may not have an existence in another. The proposition *Samson is a pure myth* denies existence to Samson in the universe of authentic history, but obviously not in the field of biblical mythology.

When, therefore, it is said in formal logic that universal propositions do not imply, while particulars do imply, the existence of instances, the reader may find it helpful to interpret this (in part at least) on the basis of the different function each type of proposition plays in scientific inquiry. Just as we cannot validly infer the truth of a proposition concerning some matter of observation from premises which do not include a proposition obtained through observation, so we cannot infer the truth of a particular proposition from universal premises alone; unless indeed we tacitly take for granted the existence of members of the classes denoted by the terms of the universal proposition.

§ 3. COMPOUND, SIMPLE, AND GENERAL PROPOSITIONS

The analysis of propositions thus far has been restricted to those having the categorical form. But logical relations hold between more complicated forms of propositions. Consider the following:

1. The weight of *B* is equal to G.
2. The lines *AB* and *CD* are parallel.
3. If the angles *AFG*, *CGF*, are greater than two right angles, the remaining angles *BFG*, *DGF*, are less than two right angles.
4. The sum of the interior angles on the same side is equal to, greater than, or less than two right angles.

A comparison of the first two propositions with the last two shows that the second set contains propositions as components or elements, while the first set does not. Thus *The angles* AFG, CGF, *are greater than two right angles* and *The sum of the angles on the same side is equal to two right angles* are themselves propositions which are components in 3 and 4.

Let us apply the characterization *compound* to all propositions which contain other propositions as components. But the reader must be warned that the form of a sentence is not always indicative of the kind of proposition which it expresses. Let him recall the discussion at the end of the last section, where an analysis of categorical propositions was made.

Compound Propositions

Four types of *compound* propositions may be distinguished. Each type relates the component propositions in a characteristic manner.

1. Consider the proposition *If war is declared, then prices go up.* We have agreed to call the proposition introduced by "if" *(War is declared)* the *antecedent,* and the one introduced by "then" *(Prices go up)* the *consequent.* Sometimes the particle "then" is omitted, but it is tacitly understood in such cases. A compound proposition which connects two propositions by means of the relation expressed by "if . . . then" is called *hypothetical* or *implicative.*

When a hypothetical proposition is asserted as true, what do we mean? We clearly do not mean to assert the truth of the antecedent nor the truth of the consequent, although both may in fact be true. What we mean to affirm is that *if* the antecedent is true, the consequent is also true, or in other words, that the two are so connected that the antecedent cannot be true without the consequent's also being true. A hypothetical proposition is sometimes said to express doubt. But this is a misleading way of characterizing such a proposition. We may indeed doubt whether *War is declared,* but when we affirm the hypothetical proposition we do not doubt that *if war is declared, prices go up.*

The antecedent and consequent of a hypothetical proposition may themselves be compound propositions. The analysis of propositions with respect to their logical form will therefore be facilitated if we employ special symbols expressly devised to exhibit logical form. Let us agree that an accent placed over a parenthesis will signify the *denial* or *negative* of the proposition contained in it; hence *It is false that Charles I died in bed* may be written (Charles I died in bed)'. Also, let us replace the verbal symbols "if . . . then" by the ideographic symbol ⊃ The hypothetical *If Charles I did not die in bed, then he was beheaded* may then be written (Charles I died in bed)' ⊃ (he was beheaded).

2. Consider next the proposition *Either all men are selfish or they are ignorant of their own interests.* The relation connecting the compared propositions is expressed by "either . . . or." We shall call the component propositions *alternants,* and the compound proposition an *alternative* proposition.

What do we mean to affirm in asserting an alternative proposition? We do not intend to assert the truth or falsity of any of the alternants: we do not say that *All men are selfish* nor that *All men are ignorant of their own interests.* All that we do assert is that *at least one* of the alternants is true.

Do we mean to say, however, that *both* alternants *cannot* be true? Usage in everyday conversation in this respect varies. An editorial which sums up the economic situation in the alternative proposition *Either this country will adopt national economic planning, or a revolution cannot be avoided* would generally be taken to mean that one, but not both, of the alternatives must be true. On the other hand, when we say *Either he is a fool or he is a knave* we do not mean to exclude the possibility that both alternatives are true. In the interest of unambiguity, we shall adopt the interpretation according to which the minimum is asserted by such a proposition. Henceforth an alternative proposition will be understood to mean one in which *at least one* of the alternatives is true, and *perhaps both.*

It is convenient to introduce a special symbol for the relation expressed by "either . . . or." We shall use the symbol ∨ between the alternants to express this relation. *Either he is a fool or he is a knave* may then be written (He is a fool) ∨ (he is a knave).

3. Consider next the compound proposition *The moon is full, and Venus is a morning star.* The relation connecting the component propositions is the conjunctive "and." We shall call such com- pound propositions *conjunctives,* and their components *conjuncts.*

What does a conjunctive proposition assert? Obviously, it asserts not only the truth of *The moon is full* and the truth of *Venus “is a morning star* taken *singly;* it asserts the truth of the conjuncts *taken together.* Consequently, if either one of the conjuncts is false, the conjunctive proposition must itself be false. The conjunctive proposition must be regarded as a *single* proposition, and not as an *enumeration* of several propositions.

We shall employ a special symbol for the relation expressed by “and.” A dot (.) between propositions will hereafter represent a conjunction. Thus *The moon is full and Venus is a morning star* may be written (The moon is full). (Venus is a morning star).

4. The reader may wonder of what earthly use a conjunctive proposition can be in inference. A conjunctive proposition, he may claim, would never be adduced as evidence for any of its conjuncts. Thus if we are in doubt concerning the truth of *My watch keeps accurate time* would we offer in evidence the conjunctive *My watch keeps accurate time, and all spring-driven mechanisms are subject to climatic influences?* It is obviously more difficult to establish the truth of a conjunctive than to establish the truth of one of its conjuncts.

We may reply, however, that something may be inferred from a conjunctive which cannot be inferred from either of its conjuncts alone. Moreover, the *denial* of a conjunctive proposition yields a proposition that is extremely useful for inference. Indeed, the denial of a conjunctive proposition yields the fourth type of compound proposition. Thus, the denial of the proposition *My watch keeps accurate time, and all spring-driven mechanisms are subject to climatic influences* is *It is not the case that both my watch keeps accurate time and all spring-driven mechanisms are subject to climatic influences.* This means that at least one of the component propositions *My watch keeps accurate time* and *All spring-driven mechanisms are subject to climatic influence* must be false. We shall call the denial or negative of a conjunctive proposition a *disjunctive* proposition and its components *disjuncts.* Since a conjunctive asserts that *both* its conjuncts are *true,* its denial, which is a disjunctive proposition, asserts that *at least one* of the disjuncts is *false.* Both of two disjuncts cannot be true.

We have seen that in everyday conversation the alternants in an alternative proposition may be taken to be mutually exclusive. Thus in *Either he is single or he is married* the truth of one of the alternants excludes the truth of the other. Such alternative propositions *tacitly* assert a *disjunctive* proposition as well. As the usual meaning of *Either he is single or he is married* includes “He cannot be both,” it can be expressed as follows: [(He is single) ∨ (he is married)]. [(He is single). (he is married)]’.

Let us now employ the foregoing distinctions to exhibit the logical form of some compound propositions. Consider the following argument: If every distinct racial group is characterized by a distinct culture, then either all nations differ from one another culturally, or national distinctions do not coincide, in whole or in part, with racial ones. But neither is it the case that the different nations possess distinct cultures, nor is it true that national distinctions do not coincide at any point with racial distinctions. Hence it is not true that each race has a distinct culture.

Let us use the letters *p, q, r,* to designate the following component propositions in this argument.

$p \equiv$ Every distinct racial group is characterized by a distinct culture.

$q \equiv$ All nations differ from one another culturally.

$r \equiv$ National distinctions do not coincide, in whole or in part, with racial ones.

The premises and conclusion of the argument may therefore be represented as follows:

a. $p \supset (q \vee r)$
b. $q'.r'$
c. p'

Each of the propositions a, b, and c is of a logical form different from those of the other two, a difference which the symbolism helps to bring out. The validity of the argument depends on the structure or form of the proposition a, b, and c, since the conclusion follows from the premises only if the following is true:

d. $(a\ .\ b) \supset c$

The reader should note that an important distinction can be drawn between the relation which the antecedent of a hypothetical proposition has to its consequent (as in proposition *a*), and the relation which the premises of a valid argument have to the conclusion (as in *d*). For the former relation material (or factual) evidence must be offered; while for the relation between premises and conclusion such evidence is both irrelevant and impossible, since this latter relation holds only when one of the terms related is logically or analytically contained in the other. The two relations, however, have a common trait, namely, that neither holds when the antecedent or premise is true and the consequent or conclusion is false. It is this common trait which is denoted by "if . . . then" or $\supset$. The reader should be on guard against the fallacy of supposing either that two things in any way alike cannot be unlike in some other way, or that two things in some ways distinct cannot also be alike in other ways.

Simple Propositions

The analysis of compound propositions into propositional elements clearly belongs to logic. But the analysis of a sentence into its verbal elements is an affair of grammar. Logically propositions are prior to words, in the sense that the propositions are not produced by the union of words—but the meaning of the words can be derived only from some propositional context. Ultimately the meaning of words is determined by elementary propositions of the form *This is a truffle, This is the color magenta,* and the like, where the word "this" may be replaced by some gesture of pointing. But while propositions cannot be analyzed into verbal constituents, attention to the latter is often an aid in analyzing or classifying propositions for logical purposes. Consider the following propositions:

1. Archimedes was modest.
2. Archimedes was a mathematician.
3. Archimedes was a greater mathematician than Euclid.

According to the traditional doctrine, each of these is a categorical proposition and its components are a subject, a predicate, and a copula joining them; any proposition such as *Archimedes loved mathematics* or *Archimedes ran naked through the streets crying Eureka* is to be analyzed by transforming it into *Archimedes was one who loved mathematics* or *Archimedes was one who ran naked,* and so on. It may be questioned whether such transformation does not change the meaning somewhat. But in any case it is possible to analyze propositions in other ways than the traditional one. Thus, using proposition 2 given above as a model, we can regard every proposition as asserting that some object is a member of a class. Proposition 1 would then assert that Archimedes was a member of the class of beings called modest, and proposition 3 would assert his membership in the class of mathematicians greater than Euclid. This second mode of analysis is related to the first mode as the point of view of extension is to that of intension.

An entirely different mode of analyzing propositions is to resolve them into the assertion of some relation between at least two objects. Thus our first proposition asserts a relation between Archimedes and modesty (the substance-attribute relation), our second proposition asserts a relation which may be called that of class membership between Archimedes and the class of mathematicians. Such propositions as *Archimedes solved Hiero's problem* may thus be analyzed as *Archimedes stood in the relation of solver to Hiero's problem.*

Now it is quite clear that no one of these modes of analysis can claim to be the only one; nor are they mutually exclusive. Nevertheless, each of these modes of analysis fits some propositions better than others. It seems quite forced to say that in the proposition *The author of* Macbeth *is the author of* Hamlet, "the author of *Hamlet*" is an attribute of "the author of *Macbeth.*" It seems more appropriate to view it as the assertion of a relation of identity in denotation, despite a difference in intension or connotation.

Of more direct logical importance is it to note that if we fail to discriminate between class-membership propositions and those which typify some other relation, we miss something that bears on the nature of implication. Thus, while some relations are transitive, that of class membership is not. *Archimedes was a greater mathematician* *than Euclid and Euclid was a greater mathematician than Aristotle* implies that *Archimedes was a greater mathematician than Aristotle.* But *Archimedes was a member of the Syracusan city-state and the Syracusan city-state was a member of the Graeco-Carthaginian alliance* does not imply that *Archimedes was a member of the Graeco-Carthaginian alliance.*

In Chapter VI we shall undertake a more systematic study of the relation between classes and of the logical properties of relations in general.

General Propositions

Consider the proposition: *All mathematicians are skilled logicians.* This cannot be fittingly regarded as a proposition of the subject-predicate type, for it does not predicate a character or quality to some individual. Nor does it assert that an individual is a member of a class. Nor would it be accurate to say that it asserts a relation between one individual and one or more others. What it does assert is the specific relation of inclusion between *two classes.* Propositions which deal with the relations between classes, that is, with the total or partial inclusion (or exclusion) of one class in another, are called *general* propositions. We have already indicated what the proper analysis of such propositions should be, in discussing categorical propositions in a previous section. Let us now approach the same conclusion from a slightly different angle.

The series of propositions *Archimedes was a mathematician, Euclid was a mathematician, Ptolemy was a mathematician,* are all of the same form. They differ only in having different terms as subjects. Let us now examine the *expression* "*x* is a mathematician." This is not a proposition, because it is incapable of being true or false. But propositions may be obtained from it by substituting proper values for *x.* All the propositions so obtained will have the same form. An expression which contains one or more variables, and which expresses a proposition when values are given to the variables, is called a *propositional function.*

We may vary not only the subject but other terms of the proposition as well. By varying the relation in *Archimedes was killed by a Roman soldier,* we may obtain *Archimedes was praised by a Roman soldier, Archimedes was spoken to by a Roman soldier, Archimedes was a cousin of a Roman soldier,* and so on. If we represent the relation by a variable *R,* we get the propositional function: Archimedes *R* a Roman soldier. (It is to be read: Archimedes has the relation *R* to a Roman soldier.) In such a manner, by letting the terms and relations in a proposition vary, and by representing them by variables, we are able to exhibit logical form or structure in a precise fashion.

When we state that *All mathematicians are trained logicians* what we mean is that if *any* individual is a mathematician he is also a trained logician. We are really affirming an *implication* between being a mathematician and being a trained logician. This may be expressed as an implication between propositions obtained from propositional functions, as follows:

[For all values of *x,* (*x* is a mathematician) $\supset$ (*x* is a trained logician)], where the sign $\supset$ indicates as usual the "if . . . then" relation between the propositions, obtained from the propositional functions by giving values to *x.*

Propositions of this type, which assert the inclusion (or exclusion) of one class in (or from) another, are analogous in some ways to compound propositions. They should therefore not be confused with class-membership propositions. For the class-membership relation is not transitive, as we have seen, while the relation of class inclusion is transitive. Thus, if *All mathematicians are trained logicians* and *All trained logicians are college professors,* we may validly infer that *All mathematicians are college professors.*

Let us now express each of the four kinds of categorical propositions in this new notation.

1. *All students are independent thinkers* is equivalent to [For all *x*'s, (*x* is a student) $\supset$ (*x an independent thinker*)].

2. *No students are independent thinkers* is equivalent to [For all *x*'s, (*x* is a student) $\supset$(x is an independent thinker)'].

3. *Some students are independent thinkers* is equivalent to [There is an *x* such that (*x* is a student). (*x* is an independent thinker)].

4. *Some students are not independent thinkers* is equivalent to [There is an *x,* such that (*x* is a student). (*x* is an independent thinker)'].

The two universal propositions (1 and 2) evidently have a logical form quite distinct from the two particulars (3 and 4), while all four propositions are quite distinct in form from the subject-predicate form of propositions.

[1] The reader should note, however, that in the traditional analysis which we shall follow in this section the emphasis will be on the *extensional* interpretation.

[2] The view that all propositions are of the subject-predicate form has been associated historically with certain philosophical interpretations of the nature of things. The subject, on this view, is regarded as a *substance* in which various qualities inhere, and the task of all inquiry is to discover the inhering predicates in some concrete subject. According to Leibniz, for instance, there is an ultimate plurality of substances or monads, each of which is pregnant with an illimitable number of properties. These monads cannot be said to stand to one another in any relation, for if they did one monad would have to be a *predicate* of another, and therefore not self-subsistent. According to others, like Bradley, there is just *one* substance, so that all predication is the affirmation of something of *all reality* conceived as a single, unique individual. Neither of these extreme positions was adopted by Aristotle. He maintained that the ultimate subject of predication is some concrete, individual substance, and that there is an irreducible plurality of such, but that these substances arc systematically related.

CHAPTER III

THE RELATIONS BETWEEN PROPOSITIONS

§ 1. THE POSSIBLE LOGICAL RELATIONS BETWEEN PROPOSITIONS

The logician's interest in the structure of propositions arises from his desire to exhibit all possible propositional forms in virtue of which propositions imply one another. Propositions may be related in ways other than by implication. Thus *The exchange value of a commodity is proportional to the amount of labor required for its production* and *The supply of a commodity is proportional to the demand* are related propositions, since they are both about economics. And *Continuous eloquence wearies* and *Thought constitutes the greatness of man* are also related in virtue of the fact that Pascal believed both of them. This kind of relatedness, however, is not the logician's concern. The relations between propositions which are logically relevant are those in virtue of which the possible truth or falsity of one or more propositions limits the possible truth or falsity of others. Let us examine them.

The conclusion of Plato's dialogue *Protagoras* finds Socrates summarizing the discussion on the nature of virtue:

"My only object . . . in continuing the discussion, has been the desire to ascertain the nature and relations of virtue; for if this were clear, I am very sure that the other controversy which has been carried on at great length by both of us-you affirming and I denying that virtue can be taught—would also become clear. The result of our discussion appears to me to be singular. For if the argument had a human voice, that voice would be heard laughing at us and saying: 'Protagoras and Socrates, you are strange beings; there are you, Socrates, who were saying that virtue cannot be taught, contradicting yourself now by your attempt to prove that all things are knowledge, including justice, temperance, and courage—which tends to show that virtue can certainly be taught; for if virtue were other than knowledge, as Protagoras attempted to prove, then clearly virtue cannot be taught; but if virtue is entirely knowledge, as you are seeking to show, then I cannot but suppose that virtue is capable of being taught. Protagoras, on the other hand, who started by saying that it might be taught, is now eager to prove it to be anything rather than knowledge; and if this is true, it must be quite incapable of being taught.' Now I, Protagoras, perceiving this terrible confusion of our ideas, have a great desire that they should be cleared up."[1]

Let us examine the following propositions in this excerpt:

a. Virtue cannot be taught.
b. If virtue is not knowledge, then virtue cannot be taught.
c. If virtue is knowledge, then virtue can be taught.
d. Virtue can be taught.
e. Virtue is knowledge.
f. Virtue is not knowledge.

Very little reflection is required to show that propositions *a* and *d* are connected *logically*, and not only because both are about virtue. For there is clearly a limitation upon their possible truth or falsity. Both propositions cannot be true, since one affirms what the other denies; and both propositions cannot be false, for the same reason. Precisely the same relation holds between *e* and *f*. Such propositions are *contradictories* of one another.

How about *b* and *c?* The reader may be tempted to regard these as contradictories also. This, however, would be erroneous. Reflection shows that there is no contradiction in saying that virtue can be taught under certain contingencies (if virtue is knowledge) but not under others (if virtue is not knowledge). There is, indeed, no mutual limitation upon the possible truth or falsity of these two propositions. Such propositions, even though they deal with the same subject matter, are logically *independent.* The reader should determine for himself whether there are any other pairs of independent propositions in the set above.

Let us now consider *b* and *f* asserted *jointly,* thus forming a conjunctive proposition, and ask for the relation between this conjunctive and *a.* It is easy to see that if *both b and f* are true, *a* must also be true. But if *a* is true, does anything follow concerning the truth of *both b and f?* Evidently not, for *a* may be true on other grounds than those supplied by the conjunctive *b and f;* for instance, human obstinacy, bad habits, or the weakness of the flesh; and the conjunctive may be false even though *a* is true, for it may be true, for example, that virtue is knowledge and yet cannot be taught. Propositions so related that if the first is true the second is also true, but if the second is true the first is *undetermined* or *not thereby limited* in its truth-value, are said to be in the relation of *superaltern to subaltern.* The convenient name of *super implication* has also been devised for this relation. Can the reader find other combinations of propositions in this set which are in this relation?

We have thus far identified three types of relations between propositions: contradictoriness, independence, relation of superaltern to subaltern. Are these all the possible relations there are? We can obtain an exhaustive enumeration of such relations if we examine all the possible truth-values of a pair of propositions (where the "truth-value" of a proposition is either truth or falsity). Let *p* symbolize any proposition, and *q* any other. The following table contains all their possible truth-values. We must allow for the possibility that the truth-value of one or other of the propositions is not limited or determined by the other, by denoting such lack of determination as "Undetermined."

	p	*q*		*p*	*q*
1	True	True	4	False	True
2	True	False	5	False	False
3	True	Undetermined	6	False	Undetermined

Two propositions may be related to each other in any one of these six ways. But to impose only a *single* one of these six conditions upon a pair of propositions is not sufficient to determine their logical relation to one another *uniquely.* Thus the relation called contradictory, such as between *e* and *f,* needs two conditions to determine its properties, namely, the conditions 2 and 4. The relation of *superimplication* also requires two conditions, namely, 1 and 6. Reflection shows that the other possible logical relations similarly require two of these six conditions to determine them. By pairing each of the first three conditions with each of the last three, we then get nine possible relations between propositions, not all of which, however, are distinct.

1. If *p* is true, *q* is true.
   If *p* is false, *q* is true.

In this case, the truth-value of *q* is not limited by the truth-value of *p.* Two propositions so related are said to be *independent.*

2. If *p* is true, *q* is true.
   If *p* is false, *q* is false.

Two propositions so connected are said to be *equivalent.*

3. If *p* is true, *q* is true.
   If *p* is false, *q* is undetermined.

Propositions so related are said to be in the relation of *superaltern* (or *principal) to* *subaltern.* As noted before, we shall also use the designation *superimplication.*

4. If p is true, q is false.
   If p is false, q is true.

The reader will recognize this as the case of *contradictory* relation.

5. If p is true, q is false.
   If p is false, q is false.

Here the falsity of q does not depend on the truth or falsity of p, and the propositions are *independent.*

6. If p is true, q is false.
   If p is false, q is undetermined.

In this case, p and q are said to be *contraries:* both cannot be true, but both may be false.

7. If p is true, q is undetermined.
   If p is false, q is true.

In this case, both propositions cannot be false, but both may be true. Such propositions are called *subcontraries.*

8. If p is true, q is undetermined.
   If p is false, q is false.

In this case, the relation of p to q is the *converse* of the relation in 3, and p is said to stand in the relation of *subimplication* or that of *subaltern to principal* to q.

9. If p is true, q is undetermined.
   If p is false, q is undetermined.

In this case, p and q are also *independent,* since the truth-value of p does not determine the truth-value of q.

There are, therefore, seven distinct types of logical relations between one proposition or set of propositions and another proposition or set. (Note that 1, 5, and 9 are of the same type.) Propositions may be (1) equivalent, (2) related as principal to subaltern, (3) related as subaltern to principal, (4) independent, (5) subcontraries, (6) contraries, or (7) contradictories. These are all the fundamental types of logical relations between propositions, and *every discussion we shall enter into in the present book may be regarded as illustrating one of them.* A full understanding of these seven relations will give the reader an accurate synoptic view of the province of logic.[2]

§ 2. INDEPENDENT PROPOSITIONS

We have agreed to call two propositions independent, if the truth-value of one of them in no way determines or limits the truth-value of the other. Thus, if we were considering whether the proposition *Pericles had two sons* is true or false, we should not regard the truth or falsity of the proposition *Hertz discovered electric waves* as evidence either way. When one proposition is thus no evidence at all for the truth or falsity of the other, we also speak of the former as irrelevant to the latter. One of the duties of a court of law or of any other rational procedure is to rule out irrelevant testimony. This does not deny that propositions which we now do not know to be in any way related may later be discovered ta be indirectly connected with each other. No one before the middle of the 18th century could have seen any connection between propositions about thunder and lightning, about the color of mother of pearl and about the attractive power of a lodestone; and yet they are now all part of electromagnetic theory. But formal logic is not concerned with guaranteeing factual omniscience. The logical test of independence is simply whether it is possible for a given proposition to be a) true, or b) false, or c) have its truth undetermined, irrespective of whether another given proposition is true or false. Thus a) if the proposition, *The angle of reflection of a light ray is always equal to the angle of its incidence,* is true whether the hypothesis

Light consists of corpuscles is true or false, the former proposition is independent of the latter. Similarly, b) any proposition which can be demonstrated to be false, such as *The sum of two sides of a triangle is not greater than the third side*, will be independent of any proposition which may be considered as true or as false, e.g., *Through a point outside of a straight line only one parallel can be drawn*, c) A third instance of one proposition being independent of another can be seen in the pair, *The greatest contribution to physics in the 18th century was made by England* and *Sir Philip Sidney was the author of the Letters of Junius.*

In all these pairs of propositions, the truth-value of the first member of each pair is not limited or determined by whether the second member is true or false.[3]

§ 3.EQUIVALENT PROPOSITIONS

The realization that there are several ways of saying the same thing is very valuable in the search for truth. Futile controversies flourish not only because everyone prefers his own formulation of the beliefs he holds, but also because this preference makes very few of us willing to analyze apparently opposing expressions of belief in order to discover whether the obvious differences are verbal or whether they are material. In any case, the examination of what propositions are equivalent is an essential part of all rational inquiry.

Several forms of equivalent propositions have been studied by traditional logic. In turning to examine these, the reader will do well to employ either the diagrammatic representation of categorical propositions or the algebraic statements which express their import.

Conversion

Consider what the proposition *No agricultural country is tolerant on religious questions* asserts. Obviously, it contains the same information as *No countries tolerant on religious questions are agricultural*, for if agricultural countries are excluded from being tolerant, the latter must also be excluded from the former. These two propositions are equivalent—if one is true or false, the other is the same. They have the same terms as subject and predicate, but the subject of the first is the predicate of the second, and the predicate of the first is the subject of the second. The second proposition is said to be the *converse* of the first. The process by which we pass from a proposition to another that has the same truth-value and in which the *order* of subject and predicate is interchanged, is called *conversion.* An *E* proposition can therefore be converted.

Can each of the other categoricals be converted? May we validly infer from *All bald men are sensitive* the proposition *All sensitive men are bald?* We certainly may not. The reader will see this more clearly if he will note that in the first proposition the term “sensitive” is undistributed while in the converse the same term is distributed. This is not permissible, for it would be tantamount to asserting something of all of a class on the evidence of an assertion concerning only an indefinite part of that class. Indeed, we may state as a general principle: *In inferences from categorical propositions no term may be distributed in the conclusion which is undistributed in at least one of the premises.* Consequently, from *All bald men are sensitive* we may infer no more than *Some sensitive men are bald.* Thus, an *A* proposition can be converted only by *limitation* or *per accidens,* that is, its quantity must be changed. But as we saw before, no inference *per accidens* is valid without the assumption that the class denoted by the subject has members, in this case bald men.

The converse of the *I* proposition *Some Republicans are conservatives is Some conservatives are Republicans.* Hence the converse of an *A* proposition may be regarded as the converse of its subaltern.

What is the converse of *Some Italians are not dark-haired?* Is the reader tempted to say it is *Some dark-haired individuals are not Italians?* But this is clearly a fallacious inference. From the proposition *Some mortals are not men* it does not follow that

Some men are not mortals. Technically, this is expressed by saying that such an inference violates the principle concerning the distribution of terms. Indeed, the *O* proposition has no converse; but, as we shall see later, we can infer that *Some men not having dark hair are Italians.*

Obversion

We may obtain equivalent propositions in another way. If *All employees are welcome,* what may we infer about the relation of employees to those who are unwelcome? Evidently *No employees are unwelcome* is a valid conclusion. These two propositions are equivalent: the first declares that nobody is both an employee and unwelcome, and the second asserts the same thing. The inference is called *obversion* and each proposition is the *obverse* of the other. The subjects of the propositions are the same, but the predicate term of one is the negative or contradictory of the other; and the quality of the propositions is different. Care must be taken that the predicate of the obverse should be the contradictory of the predicate in the premise. Thus the obverse of *All leaves are green* is not *No leaves are blue,* for "green" and "blue" are not contradictory terms—they are merely contrary. Two terms are *contradictory* in a universe of discourse if they are exhaustive as well as exclusive; two terms are *contrary* if they are simply exclusive. The proper obverse of *All leaves are green* is *No leaves are non-green,* or in more colloquial English *No leaves are other in color than green.*

Each of the four types of categoricals may be obverted simply—that is, without limitation. The reader should verify that the obverse of *No Laplanders are educated* is *All Laplanders are uneducated;* of *Some college presidents are intelligent* is *Some college presidents are not unintelligent;* and of *Some gases are not poisonous* is *Some gases are nonpoisonous.*

Conversion and obversion are two forms of inference for passing from one proposition to another which is equivalent to it. Other types studied in traditional logic can be defined in terms of successive applications of these two.

Contraposition

If *All reasonable petitions are investigated,* what may be inferred concerning the relation of the uninvestigated things either to the reasonable petitions, or to the non-reasonable petitions? The reader may admit that one permissible conclusion is *No uninvestigated things are reasonable petitions* and that another is *All uninvestigated things are non-reasonable petitions.* But if the reader should not see that these conclusions necessarily follow, he will be able to do so if he performs the following series of obversions and conversions. We shall consider all four categorical propositions together.

1. All reasonable petions are investigated.	No reasonable petitions are investigated.	Some reasonable petitions are investigated.	Some reasonable petitions are not investigated.
2. No reasonable petitions are uninvestigated.	All reasonable petitions are uninvestigated.	Some reasonable petitions are not uninvestigated.	Some reasonable petitions are uninvestigated.
3. No uninvestigated things are reasonable petitions.	Some uninvestigated things are reasonable petitions.		Some uninvestigated things are reasonable petitions.
4. All uninvestigated things are non-reasonable petitions.	Some uninvestigated things are not non-reasonable petitions.		Some uninvestigated things are not non-reasonable petitions.

The first row contains the four categorical propositions. The second row contains the corresponding *obverses* of the propositions in the first row. The third row contains the *converses* of the propositions in the second row. And the fourth row contains the *obverses* of the propositions in the third row.

The propositions in the third row are called the *partial contrapositives* of the corresponding propositions in the first row. The partial contrapositive of a proposition is one in which the subject is the contradictory of the original predicate, while the predicate is the original subject; it also differs in quality from the original proposition. The *I* proposition has no partial contrapositive, and the *E* has one by limitation. The partial contrapositives of the *A* and *O* are equivalent to the original proposition.

The propositions in the fourth row are the *full contrapositives* of the corresponding propositions in the first row. The full contrapositive is a proposition in which the subject is the contradictory of the original predicate, and the predicate is the contradictory of the original subject. It has the same quality as the original proposition. As before, the *I* proposition has no full contrapositive, and the *E* has one only by limitation.

Obverted Converse

We have obtained equivalent propositions by performing a series of obversions and conversions, *in this order*, upon each of the four types of categoricals. A different set of equivalent propositions are obtained, however, if instead we first convert a given proposition, then obvert the result. The following table summarizes the outcome:

1.	All reasonable petitions are investigated.	No reasonable petitions are investigated.	Some reasonable petitions are investigated.	Some reasonable petitions are not investigated.
2.	Some investigated things are reasonable petitions.	No investigated things are reasonable petitions.	Some investigated things are reasonable petitions.	
3.	Some investigated things are not non-reasonable petitions.	All investigated things are non-reasonable petitions.	Some investigated things are not non-reasonable petitions.	

It will be noted that the *E* and *I* have an obverted converse without limitation, the *A* has a limited obverted converse, while the *O* has none at all.

Inversion

If *All physicists are mathematicians,* what may be inferred about the relation of the non-physicists to the mathematicians, or to the non-mathematicians? Let us discover what may be validly inferred by a successive application of conversions and obversions.

We may begin by first converting the proposition, then obverting, and so on, until we obtain a proposition which satisfies the problem; or we may begin by first obverting, then converting and so on. Let us develop the alternative methods in parallel columns, the first method in the left-hand column, the second in the right-hand one.

All physicists are mathematicians.	All physicists are mathematicians.
Some mathematicians are physicists.	No physicists are non-mathematicians.
Some mathematicians are not non-physicists.	No non-mathematicians are physicists.
	All non-mathematicians are non-physicists.
	Some non-physicists are non-mathematicians.
	Some non-physicists are not mathematicians.

Hence, if we first convert an *A* proposition we are soon brought to a halt, because an *O* proposition cannot be converted. If we first obvert, we get two propositions which are satisfactory. *Some non-physicists are not mathematicians* is called the *partial inverse* of the original proposition. Its subject is the contradictory of the original subject, its predicate is the original predicate. *Some non-physicists are non-mathematicians* is called the *full inverse.* Both its subject and its predicate are the contradictories of the original subject and predicate respectively.

Has each form of categorical proposition an inverse? If the reader will use the method we have indicated, he will discover that from *No professor is unkind* he can infer *Some non-professors are unkind* (the *partial inverse)* and *Some non-professors are not kind* (the *full inverse).* But no inverses can be obtained from either an *I* or an *O* proposition. Hence, only universals have inverses, and in each case inversion is by limitation.

The process of inversion may lead to absurd results as when, from *All honest men are mortal* we seem to get *Some dishonest men are immortal.* Where has the error crept in? The answer is: In our careless use of negatives. The true inverse of our proposition is *Some non-honest-men are non-mortal,* which is not at all an absurd result. For the class of all beings that are not *honest men* is wider than the class *dishonest men* and includes triangles and the like which are certainly non-mortal.

"But look here!" the reader may object. "The partial inverse of *All physicists are mathematicians* is *Some non-physicists are not mathematicians.* In the first proposition the predicate is undistributed, although the same term is distributed in the second. How then can you maintain that the second is a valid consequence of the first? Does it not violate the principle concerning distribution of terms?"

If the reader has understood the discussion of the existential import of propositions, he will have a ready reply. A universal proposition, he will say, asserts *nothing* about the existence or nonexistence of instances; particulars, on the other hand, clo have existential import. Consequently, a particular can never be validly inferred from a universal or combination of universals, *unless* the premises include a proposition asserting that the classes denoted by the terms of the universal contain at least one member. And specifically, the conversion of *A* is valid only if the predicate denotes such a class.

The source of the trouble in inversion is now apparent. To get the inverse of *All physicists are mathematicians* we are required to convert *All non-mathematicians are non-physicists.* This can be done only if we add the further premise *Some men are non-physicists* or what is the same thing *Some men are not physicists.* If this premise is supplied, the partial inverse does not violate the principle concerning distribution of terms.

If universals always did have an existential import, then not only would the terms of such propositions denote classes with members, but the contradictory terms would do so as well. Thus if *All men are mortal* required that there should be men and mortals, since we may validly infer *All immortals are non-men,* we would be compelled to affirm that there are immortals as well as non-men. That universals do not always have existential import, even in everyday conversation, can be seen from the following. Students of mathematics know that the ancient Greek problem, to construct a square equal in area to a circle with compass and ruler, is demonstrably impossible. We may therefore assert confidently that *No mathematicians are circle-squarers.* Its partial inverse is *Some non-mathematicians are circle-squarers.* But we assuredly did not intend to assert anything whose consequence is that there are some people who can in fact square the circle—for there is a proof that this cannot be done. Hence the original proposition could not have been intended to assert the existence of circle-squarers.

Inference by Converse Relation

If *Chicago is west of New York* we may validly infer that *New York is east of Chicago;* if *Socrates was a teacher of Plato* we may infer that *Plato was a pupil of Socrates;* if *Seven is greater than five* we may infer that *Five is less than seven.* In each of these pairs of propositions the two are equivalent. Such inferences are of the form: If *a* stands to *b* in a certain relation, *b* stands to *a* in the *converse relation.*

Equivalence of Compound Propositions

We must now examine what are the *equivalent* forms of compound propositions.

Consider the hypothetical *If a triangle is isosceles, its base angles are equal.* To assert it means, as we have seen, to assert that the truth of the antecedent involves the truth of the consequent, or that the antecedent could not be true and the consequent false. Hence the hypothetical simply asserts that the conjunctive proposition *A triangle is isosceles, and its base angles are unequal* is false, or, what is the same thing, that the disjunctive proposition *It is not the case that both a triangle is isosceles and its base angles are unequal* is true. It follows that from a hypothetical we may infer a disjunctive proposition.

Moreover, we may infer the hypothetical from the disjunctive as well. For if *It is not the case that both a triangle is isosceles and its base angles are unequal,* then the truth of one of the disjuncts is incompatible with the truth of the other: if one disjunct is true, the other must be false. From this disjunctive proposition we may therefore infer *If a triangle is isosceles, its base angles are equal.* Hence a disjunctive proposition can be found which is *equivalent* to a hypothetical.

If we employ our previous symbols, we may write this as follows:

[(A triangle is isosceles) ⊃(its base angles are equal)] ≡
[(A triangle is isosceles). (its base angles are equal)']'.

But this discussion also shows how we can infer an equivalent hypothetical from any hypothetical proposition. For if in the equivalent disjunctive the second disjunct is supposed true, the other disjunct must be false. We may therefore infer *If the base angles of a triangle are unequal, it is not isosceles.* Hence we may write:

[(A triangle is isosceles) ⊃ (its base angles are equal)] ≡
[(The base angles of a triangle are equal)' ⊃ (the triangle is isosceles)'].

These equivalent hypothetical are said to be the *contrapositives* of each other.

Consider next the alternative proposition *Either a triangle is not isosceles or its base angles are equal.* To assert it means to assert that *at least one* of the alternants is true. If, therefore, one of the alternants were false, the other would have to be true. Hence we may infer from the alternative above the following hypothetical *If a triangle is isosceles, its base angles are equal.* Moreover, the alternative may be inferred from this hypothetical. For the latter is equivalent to *It is not the case that both a triangle is isosceles and its base angles are unequal,* which asserts that *at least one* of the disjuncts must be false. From the disjunctive we may therefore infer *Either a triangle is not isosceles, or its base angles are equal.* We therefore may write the equivalence:

[(A triangle is isosceles)'∨(its base angles are equal)]≡
[(A triangle is isosceles) ⊃ (its base angles are equal)]

It follows that for every hypothetical there is an equivalent alternative, an equivalent disjunctive, and an equivalent hypothetical. A similar statement holds for every alternative and every disjunctive proposition. On the other hand, a conjunctive proposition is not equivalent to any one of the other three forms of compound propositions.

Let us now state the equivalents of *If he is happily married, he does not beat his wife.* They are *If he beats his wife, he is not happily married, Either he is not happily married or he does not beat his wife,* and *It is not the case that both he is happily married and he beats his wife.* In symbols they will read:

[(He is happily married) "⊃(He does not beat his wife)]≡
[(He does not beat his wife)' ⊃(He is happily married)']≡
[(He is happily married)'∨(He does not beat his wife)] ≡
[(He is happily married). (He does not beat his wife)']'

These equivalents may be stated more compactly, and the forms of the equivalent propositions exhibited more clearly, if we adopt further symbolic conventions. Let *p* represent the antecedent of a hypothetical proposition, and *q* its consequent. Any hypothetical may then be symbolized by $(p \supset q)$. The equivalences will then be written as follows:

$$(p \supset q) \equiv (q' \supset p') \equiv (p' \vee q) \equiv (p \,.\, q')'$$

We shall discuss in Chapter VII equivalences between *systems* of propositions. We may, however, offer at this point an example of two propositions which are equivalent in virtue of their place in a system. Let *p* ≡ *In Newtonian physics, light is reflected* *from a surface so that the angle of incidence is equal to the angle of reflection* and let *q* ≡ *In Newtonian physics light is reflected from a surface so that its path is a minimum: p* and *q* are equivalent.

§ 4. THE TRADITIONAL SQUARE OF OPPOSITION

Traditionally, the *opposition* between propositions has not been conceived in a manner as general as we have indicated. Since on the traditional view all propositions were analyzable into subject and predicate, only propositions in that form could be opposed. The opposition of compound propositions was not discussed, and the discussion of the opposition of singular propositions was most unsatisfactory.

In this section we shall examine the traditional account of opposition. According to it, two propositions are said to be opposed when they have the same subject and predicate, but differ in quantity or quality or both.

Consider, therefore, the four propositions:

A. All republics are ungrateful.
E. No republics are ungrateful.
I. Some republics are ungrateful.
O. Some republics are not ungrateful.

In discussing the existential import of propositions, we have seen that the universals do not require the existence of republics, while the particulars do. We cannot, therefore, without further assumptions, infer the truth of the *I* proposition from the truth of the *A* proposition. To do so, we require the assumption that there are republics. We shall make that assumption in the present section once for all, and explore the consequences of this hypothesis.

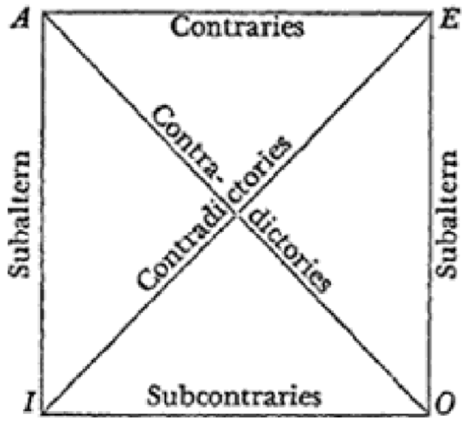
No two of the four propositions above are independent of one another, and no two are equivalent. We may, however, identify the other five relations of the possible nine as follows. The reader will find it helpful to make use of the diagrammatic representation of propositions.

1. *All republics are ungrateful* and *Some republics are not ungrateful* cannot both be true, and they cannot both be false. If one is true, we may validly infer the falsity of the other; and if one is false we may infer the truth of tire other. Hence the *A* and O propositions are *contradictories;* and the same holds for the *E* and *I* propositions.

2. *All republics are ungrateful* and *No republics are ungrateful* cannot both be true, so that if one were true, we could infer the falsity of the other. But if one were false, the truth-value of the other would be undetermined. Hence the *A* and *E* propositions are *contraries.*

3. Examining *All republics are ungrateful* and *Some republics are ungrateful,* we find that the truth of the second may be inferred from the truth of the first. But if the first is false, we can infer nothing about the truth-value of the second. Hence the *A* proposition is the principal or superaltern to the *I* proposition, which is the subaltern. The same relation holds for the *E* and *O* propositions. may infer the falsity of the *A* proposition. But from the truth of the / proposition we cannot infer the truth-value of the *A* proposition. Hence the *I* proposition stands to the *A* proposition as the subaltern to the principal. Similarly for the *O* and *E* propositions.

5. Finally, the truth of *Some republics are ungrateful* is compatible with the truth of *Some republics are not ungrateful,* although when we remember our convention that the word “some” is not to exclude “all” we see that we cannot infer the truth of one from the truth of the other. But if either of them is false, the other must be true. Hence the *I* and *O* propositions are sub contraries. This result also follows from the fact that the *A* and *E* propositions are contraries. For since the *O* and *I* propositions are the contradictories of the *A* and *E* propositions respectively, and since contraries cannot both be true, the *O* and *I* propositions cannot both be false; while since the *A* and *E* propositions may both be false, the *O* and *I* propositions may both be true.



These relations between the categorical propositions have been represented in the ancient square of opposition. We may also construct the following table of valid inferences from each of the four categoricals.

	A	*E*	*I*	*O*
If *A* is true		False	True	False
If *A* is false		Undetermined	Undetermined	True
If *E* is true	False		False	True
If *E* is false	Undetermined		True	Undetermined
If *I* is true	Undetermined	False		Undetermined
If *I* is false	False	True		True
If *O* is true	False	Undetermined	Undetermined	
If *O* is false	True	False	True	

In conclusion, we may point out that without the assumption concerning existence we have made at the beginning of this section, the *I* and *O* propositions cannot be inferred from the *A* and *E* proposition respectively. Moreover, without this assumption, the *A* and *E* propositions would not be contraries, since both might then be true. Thus *All immortal men are in this room* and *No immortal men are in this room* would be both true if there were no immortal men. For if it is false that there are immortal men, then (on the interpretation we have given to particular propositions) both the propositions *Some immortal men are in this room* and *Some immortal men are not in this room* are false. Hence the contradictories of these propositions must be true.

§ 5.THE OPPOSITION OF PROPOSITIONS IN GENERAL

One of the most fruitful sources of intellectual confusion is the too facile assumption that any two propositions which are not equivalent are mutually exclusive. Thus men have debated about the relation of mind to body, of heredity to environment, of selfishness to altruism, of art to nature, frequently without realizing that while the alternatives are not equivalent, it does not follow that they are mutually exclusive. It is well to note that two things related as part to whole are not identical, and yet surely are not contraries. The proposition *All Orthodox Greeks are true believers* is not the same as *All Christians are true believers* but the two are surely not incompatible.

The Contradictory Opposition of Compound Propositions

We have seen this relation illustrated in the traditional square of opposition by such simple cases as *All Mohammedans are true believers* and *Some Mohammedans are not true believers.* But it is a mistake to suppose that it is always easy to tell which propositions do have that relation to one another.

Suppose the reader in studying Rousseau’s *Social Contract* found himself in violent disagreement with the opening sentence of the first chapter: “Man is born free; and everywhere he is in chains.” How would he contradict this assertion? Or suppose he wished to deny a statement appearing later on: “The strongest is never strong enough to be always the master, unless he transforms strength into right, and obedience into duty.” With what proposition would he contradict this? Or, finally, what would he say is the contradictory of this: “Sovereignty, being the exercise of the general will, is the will either of the body of the people or of only a part of it”?

Let us begin with the conjunctive proposition *p: Man is born free; and everywhere he is in chains.* Since to assert this means to assert that both conjuncts are true, to deny it must mean that not both conjuncts are true, or what is the same thing, that at least one of them is false. Hence the alternative *q: Either some men are not born free or man is not in chains everywhere* is the proper contradictory of the original proposition. The reader should convince himself that these two propositions, *p* and *q,* cannot both be true and cannot both be false. Other forms of the contradictory can be obtained, since the alternative is equivalent to *If all men are born free, they are not in chains* as well as to // *man is in chains everywhere, some men are not born free.*

Consider next *The strongest is never strong enough to be always the master, unless he transforms strength into right, and obedience into duty.* This is really a hypothetical proposition, and may be expressed as *If the strongest does not transform strength into right and obedience into duty, he is never strong enough to be always master.* It is therefore equivalent to the disjunctive *It is not the case that both the strongest does not transform strength into right and obedience into duty, and he is sometimes strong enough to be always the master.* Hence its contradictory is *The strongest does not transform strength into right and obedience into duty, and he is sometimes strong enough to be always master.*

Finally, *Sovereignty, being the exercise of the general will, is the will either of the body of the people or of only a part of it* is an alternative proposition, which may be stated explicitly as *Either sovereignty, being the exercise of the general will, is the will of the body of the people, or it is the will of only a part of it.* Since this proposition is equivalent to a disjunctive one, its contradictory is the following conjunctive: *Sovereignty is not the will of the body of the people, and it is not the will of only a part of it.*

It follows that the contradictory of a hypothetical, disjunctive, or alternative proposition can always be stated in the form of a conjunctive proposition. On the other hand, the contradictory of a conjunctive is either a hypothetical, an alternative, or a disjunctive proposition. The precise relations between compound propositions and their contradictories may be compactly stated symbolically. Since

$$(p \supset q) \equiv (q' \supset p') \equiv (p' \vee q) \equiv (p \cdot q')',$$

the contradictory of any one of them will contradict any other. Hence

$$(p \supset q)' \equiv (q' \supset p')' \equiv (p' \vee q)' \equiv (p \cdot q'),$$

Or in words, the contradictory of “if p then *q*” is “p and 5’”; the contradictory of “either p or *q*” is “p’ and *q’*”*;* the contradictory of “not both p and 9” is *“p* and g.”

The reader should note the equivalence $(p' \vee q)' \equiv (p \,.\, q')$. This relation is perfectly general, and it does not matter what propositions we substitute for the symbols. Let us therefore replace p’ by r; then p will be replaced by *r’*. We then get

$$(r \vee q)' \equiv (r' \,.\, q'),$$

a relation known as De Morgan’s theorem. It asserts that the negative (or contradictory) of an alternative proposition is a conjunctive in which the conjuncts are the contradictories of the corresponding alternants. Another form of this theorem is given by

$$(p \cdot r)' \equiv (p' \vee r'),$$

which states that the negative of a conjunctive is an alternative proposition in which the alternants are the contradictories of the corresponding conjuncts.

We have found that specially devised symbols are a great help in exhibiting with great clarity the logical structure of propositions—a structure which is obscured by the unwieldiness of ordinary language. The reader will therefore doubtless agree that symbols are not an obstacle to understanding, but rather an aid. The generalizing power of modern logic, as of modern mathematics, is due in large measure to the adequacy of the symbolism it has adapted for the task.

As a test of his comprehension, we may ask the reader to give the contradictory of *Some men are poor but honest.* It must be understood that the force of the word "but" in this proposition is that the poor are generally dishonest although some also happen to be honest. Hence its explicit meaning is *Some men are poor and honest, and some men are poor and dishonest.* It follows that *Some men are not poor and honest* is not the contradictory of the original statement, and neither is *All men are not both poor and honest.* Application of the previously discussed principles shows that the contradictory is *Either all men are not both poor and honest or all men are not both poor and dishonest.* Similar considerations show that the contradictory of *John came home yesterday on a bicycle* is not *John did not come home yesterday on a bicycle* but rather *Either John did not come home or John did not come home yesterday or John did not come home on a bicycle.*

Contrary Opposition

This relation has also been illustrated in the traditional square of opposition. But clearly other examples of contrary propositions can be found. *I am seven feet tall* and *I am six feet tall. Socrates was the wisest of the Greeks* and *Plato was the wisest of the Greeks. Columbus was the first European to discover America* and *Leif Ericson was the first European to discover America.* These are examples of pairs of contraries which do not fall into the framework of the traditional scheme. It is evident that general propositions may have more than one contrary.

What is the contrary of *Either the book was stolen or I mislaid it?* One contrary is *The book was not stolen and I did not mislay it, and my brother did not borrow it;* these two compound propositions may be both false, for example, when *My brother borrowed it* is true. Symbolically, and in general, the following pair of compound propositions are contraries:

$$(p \vee q) \quad \text{and} \quad (p' \cdot q' \cdot r)$$

where *p, q,* and *r* are any propositions.

It should be clear from these examples that two propositions may be incompatible with each other even though neither may in fact be true. This elementary point seems to have been often overlooked in some of the great intellectual controversies in the history of human thought. The bitter fights between idealists and realists, between revolutionists and conservatives, between evolutionists and fundamentalists, between theists and deists, have been waged not only on the assumption that the respective points of view are exclusive of one another, but also on the supposition that if one of the parties were proved in the wrong, the other would be proved right. But such is the irony of history that these famous historical oppositions are not today generally acknowledged as dividing the truth between them.

Subcontrary Opposition

In addition to the propositions falling into the traditional schedule, the following pairs illustrate the relation of subcontrary opposition: *There is a page in this book containing misprints* and *There is a page in this book containing no misprints. Hydrogen is not the lightest element* and *Helium is not the lightest element San Marino is not the smallest country of Europe* and *Andorra is not the smallest country of Europe.* The two propositions in each pair cannot both be false, but both may be true.

Other examples of this important relation are: Under the prevailing organization of governments, the propositions: *Some European countries are monarchies* and *Some European countries are republics* are subcontraries. In a country which is planning to raise taxes, but where the budget must be balanced by increasing either the tariff or the income tax: *The income tax will be raised* and *The tariff will be increased* are again subcontraries. The relation of subcontrariness is thus a very simple one, and it may seem strange that serious errors should ever be made by failing to understand its nature or to recognize it in other examples. Yet the history of human thought shows a powerful tendency to conceive rival hypotheses as contrary or even contradictory when they are in fact subcontraries. Are we to obey the law or be free to change it? Are we to follow our elders or are we to try to improve on their ways? Libraries have been written on these and similar themes on the assumption that the alternatives were mutually exclusive. But it is only to an untutored mind that they appear so. Wisdom consists in seeing ways in which both may be true. Consider, for example, the controversy with respect to free trade and policies of protection. The issues are sometimes stated as if it were incompatible with the best interests of a country to adopt one policy at one time and the other at another time. But under certain conditions tariffs may be necessary for the economic development of a country, while under other conditions they may be detrimental. Hence, while the propositions *Tariffs are detrimental to a country* and *Tariffs are beneficial to a country* are in fact contraries when regarded as formulations of *universal* policies, the actual state of affairs makes it impossible to adopt such universal policies. Hence both propositions may be true if they are regarded as asserting that under *certain qualifying conditions* tariffs may be detrimental, and under others, beneficial. Meat can be both a food and a poison.

Superimplication

Every theory stands to its logical consequences in the relation of principal to subaltern—in the relation of superimplication. For this reason we shall study this and the converse relation in some detail in the sequel. Here we may note a single illustration of this ditional schedule allows. Let *p* represent the conjunction of postulates and axioms of Euclid, and *q* represent *The sum of the angles of a triangle is equal to two right angles:* then *p* is the principal or superimplicant to *q*.

In this connection, it will be useful to refer to a distinction, canonized by traditional logic, between immediate and mediate inference. An inference is said to be *immediate* when a proposition is inferred from a *single* other proposition; and inference is *mediate* when *at least two* propositions are required in the premises. But this distinction is not significant if any two propositions can be combined into one. We must also remember that some forms of so-called immediate inferences require special assumptions in order to be valid.

Logicians have sometimes been led into drawing a very sharp distinction between equivalent propositions, while on the other hand they have sometimes questioned whether the “immediate inferences” from one to another of an equivalent pair of propositions was a “genuine” case of inference at all. Reflection shows, however, that a part at least of this controversy arises because it is forgotten how arbitrary is the distinction between a proposition and what it implies. Two propositions so related that if the first is true the second is also true, and if the first is false the second is false also, are identical for certain purely logical purposes. For this reason it is not very significant whether we call the contrapositive of a proposition an immediate inference from the latter or whether we regard it as being its equivalent. None the less, although two equivalent propositions are identical with regard to their truth-value, the conventional meaning of the inferred proposition is often an elaboration of the meaning of the premise. It is true, however, that the dividing-line between equivalent propositions which have the same meaning and equivalent propositions which are not precisely alike in meaning is not a sharp one.

Two special cases of immediate inference which fall under the relation of subalternation should also be mentioned. These illustrate the logic of relations, which has been systematically studied only within recent times.

(a) Inference by Added Determinants

We may infer one proposition from another, if we *limit* the subject and predicate of the premise by the *same determinant.* Thus, from *Users of snuff are consumers of tobacco* we may infer *American users of snuff are American consumers of tobacco.* From *All popes are Italians* we may infer *All tall popes are tall Italians.* These propositions, however, are not equivalent. We cannot infer from *All American professors are American scholars* that *All professors are scholars.*

Care must be taken to qualify the subject and predicate with a determinant having the same meaning. Thus, if we argue that because *All husbands are wage-earners* therefore *All unsuccessful husbands are unsuccessful wage-earners* the determinant “unsuccessful” does not have the same meaning when applied to the subject as when it is applied to the predicate, although the same word is used. A husband is unsuccessful relative to his functions as husband; a wage-earner is unsuccessful relative to his wageearning functions. Hence determinants whose meaning involves a reference to different standards cannot be employed for inference by added determinants.

(b) Inference by Complex Conception

We may infer one proposition from another if we employ the subject and predicate as parts of a more complex conception Thus if *New York is the world's largest city* then *The center of New York is the center of the world's largest city.* If *A horse is an animal* then *The head of a horse is the head of an animal.* The inference consists in inferring that if one term stands to another in a certain relation, then whatever is related to the first term in some specific way stands to whatever is related to the second term in the same specific way in that same certain relation. Such inferences, however, do not yield equivalent propositions. We cannot infer from *The color of his nose is the color of a beet* that *His nose is a beet.*

We must observe the same caution as in the preceding type of inference. Thus it is fallacious to argue that because *All radicals are citizens* therefore *The wealthiest of radicals is the wealthiest of citizens.* For the person standing in the relation of "being the wealthiest of radicals" is measured by different standards of wealth than the person who stands in the relation of "being the wealthiest of citizens."

The Relation of Subimplication or Converse Subaltern

If *p* stands for *The angle sum of an isosceles triangle is equal to two right angles* and *q* for *The angle sum of any triangle is equal to two right angles, p* is the subimplicant or subaltern to *q*. For if *p* is true, nothing follows as to the truth of *q*, while if *p* is false, *q* must be false. As we shall see, the relation of a verifying instance of a theory to the theory is the relation of subaltern to principal.

It will become apparent later that no number of verifying propositions relating to a theory can demonstrate the theory, although strictly speaking only one contrary instance of a theory is needed to refute it. But great care must be exercised in making sure that what appear to be contrary instances are really so in fact.

[1] plato, *The Dialogues*, tr. by B. Jowett, 1892. 5 vols., Vol. I, pp. 186-87.

[2] There will be seemingly more relations if we introduce the question of symmetry or reversibility of the relations between the propositions *p* and *q*. Thus the relation between a hypothesis and its logical consequence will then be described by the tetrad: If *p* is true, *q* is true; if *p* is false, *q* is undetermined; if *q* is true, *p* is undetermined; if *q* is false, *p* is false. It is an interesting exercise for the student to try to determine how many tetrads of this kind are logically possible.

[3] Further consideration of the tests for, and proof of, the independence of propositions will be taken up in Chapter VII.

CHAPTER IV

THE CATEGORICAL SYLLOGISM

§ 1. THE DEFINITION OF CATEGORICAL SYLLOGISM

Consider the proposition *Tom Mooney is a danger to society*. What would constitute adequate evidence for this proposition? One may perhaps argue as follows: *All social radicals are a danger to society; and Tom Mooney is a social radical; it follows that Tom Mooney is a danger to society*. The reader would then have to admit that the first two propositions do imply the third, and *if* the first two were true, the third would be true necessarily. The conclusion is therefore a subaltern to the premises, since if the conclusion were true it would not follow that the premises are. It is also clear, however, that although the premises *if true* would be adequate evidence for the truth of the conclusion, the question *whether* the premises are true in fact is not determined by the logical relation in which they stand to the conclusion.

Arguments of this type are frequently employed. Some of them appear to be perfectly sound, although on reflection they are discovered to be faulty. An inference such as *All Parisians are Frenchmen, no Bostonians are Parisians, therefore no Bostonians are Frenchmen* or *All radicals are foreign-born, no patriotic citizen is a radical, therefore no patriotic citizen is foreign-born* is often regarded as sound by those who do not reflect. That neither argument is valid can be shown easily if we employ precisely the same type of inference, but about a different subject matter. Thus *All triangles are plane figures, no squares are triangles, therefore no squares are plane figures* is an argument of the same type as either of the preceding ones, but it will deceive practically no one.

Can we not discover some general rules, easy to apply, to which arguments of this type must conform to be valid? The matter was investigated by Aristotle, who laid the foundation for all subsequent logical inquiry, and his results have constituted the substance of logical doctrines for two thousand years. It is only in comparatively recent times that Aristotle's researches have received extension. The following discussion, however, makes little use of more modern logical techniques, although our procedure will follow the traditional analysis of the syllogism and not Aristotle's own. By departing in some ways from Aristotle's discussion of the categorical syllogism we shall at the same time exhibit the nature of a logical or mathematical system.

A *categorical syllogism* is defined as a form of argument consisting of three categorical propositions which contain between them three and only three terms. Two of the propositions are premises, the third is the conclusion. The premises *All football coaches are well paid* and *All baseball players are popular* cannot yield a syllogistic conclusion, since the premises alone contain four terms. The two propositions have no common term, while the premises of every syllogism have a common term. We may, in fact, interpret a syllogistic inference as a comparison of the relations between each of two terms and a third, in order to discover the relations of the two terms to each other. In the illustration with which we began this chapter, the common term is "social radical"; by examining the relations of the other two terms, "Tom Mooney" and "a danger to society," to this common term, the relation between "Tom Mooney" and "a danger to society" was found. For this reason, the syllogism is classified as a *mediate inference*. From the point of view of a generalized logic, however, the syllogism is a particular instance of an inference by *elimination* of one or more terms contained in the premises.

Is the following argument a categorical syllogism? A *is older than* B; B *is older than* C; *therefore* A *is older than* C. It certainly *looks* like one. But since every categorical proposition is to be analyzed into a subject term, a predicate term, and a copula which is some part of the verb "to be," it follows that this argument, although valid, is not a syllogism, since it contains *four* terms. The following argument, due to C. L. Dodgson (Lewis Carroll), is also not a syllogism as it stands, although it may be transformed into one: “A prudent man shuns hyaenas; no banker is imprudent; therefore no banker fails to shun hyaenas.”

The term which is contained in both premises is the *middle term;* the predicate of the conclusion is the *major term;* and the subject of the conclusion is the *minor term.* The premise which contains the major term is the *major premise,* and the premise containing the minor term is the *minor premise.* The order in which the premises are stated does not, therefore, determine which is the major premise. In the syllogism *All mystery tales are a danger to health, for all mystery tales cause mental agitation, and whatever is a cause of mental agitation is a danger to health* the conclusion is stated first, and the major premise last. It is usual, however, to state the major premise first.

§ 2. THE ENTHYMEME

Although syllogistic reasoning occurs frequently in daily discourse, often its presence is not noticed because the reasoning is incompletely stated. A syllogism that is incompletely stated, in which one of the premises or the conclusion is tacitly present but not expressed, is called an *enthymeme.*

The following are familiar illustrations of enthymemes: *This medicine cured my daughter’s cough; therefore this medicine will cure mine.* The inference is valid on the tacit admission of the major premise: *Whatever is a cure for my daughter’s cough is a cure for mine.* An enthymeme in which the major premise is unexpressed is of the *first order.*

All drunkards are short-lived; therefore John won’t live long. Here the missing premise is the minor: *John is a drunkard.* Enthymemes suppressing the minor premise are of the *second order.*

Usury is immoral, and this is usury. The conclusion *This is immoral* is here left unexpressed. Such an enthymeme is of the *third order.* The value of such enthymemes for purposes of innuendo is doubtless well known to the reader.

Although enthymemes do not introduce any new form of inference, their recognition is of very great importance practically. As we shall see, so-called inductive inferences are often believed to constitute a special mode of reasoning, when in fact they are simply enthymemes of the first order.

§ 3. THE RULES OR AXIOMS OF VALIDITY

We have, so far, merely defined what we shall understand by a categorical syllogism. We have not yet stated the conditions under which such an argument is valid. We shall do this by enumerating five propositions which jointly express the determining factors of any valid categorical syllogism. They are referred to as the *rules* or *axioms.* We shall state them without any attempt to prove them, since they will be our “first principles,” in terms of which we shall demonstrate other propositions. Nevertheless, although we make no attempt to prove the axioms, we assert them as expressing the conditions of valid syllogistic inference. Reflection on the syllogism as a form of inference in which a connection between two terms may be asserted because of their relations to a common third term, may enable us to “see” that these axioms do in fact express the conditions of validity. But such “seeing” must not be mistaken for *proof.* Since the axioms are principles of logic, we touch upon a fundamental characteristic of logical principles: not all logical principles can be demonstrated logically, since the demonstration must itself employ some principles of logic; and in particular, no proof of the principle of identity (If anything is *A* it is *A*) is possible, without assuming that the “anything that is *A*,” which occurs in one part of such an alleged proof is identical with the “anything that is *A*” occurring in another part.

The axioms of the categorical syllogism fall into two sets, those which deal with the quantity or distribution of terms, and those which deal with the quality of the propositions.

Axioms of Quantity

1. The middle term must be distributed at least once.
2. No term may be distributed in the conclusion which is not distributed in the premises.

Axioms of Quality

3. If both premises are negative, there is no conclusion.
4. If one premise is negative, the conclusion must be negative.
5. If neither premise is negative, the conclusion must be affirmative.

These axioms, together with the principles of hypothetical inference, are sufficient to develop the entire theory of the categorical syllogism. The axioms are not independent of each other, since we can derive some of them from the others. However, we shall take all of them as the axiomatic basis for our analysis.

§4. THE GENERAL THEOREMS OF THE SYLLOGISM

We shall now demonstrate four theorems.

Theorem I. The number of distributed terms in the conclusion must be *at least one less* than the total number of distributed terms in the premises.

Proof: The number of terms distributed in the conclusion cannot be greater than the total number distributed in the premises (Axiom 2).

The middle term, which must be distributed at least once in the premises (Axiom 1), does not appear in the conclusion *(definition of middle term)*.

Therefore the conclusion must contain at least one distributed term less than the premises.

Theorem II. If both premises are particular, there is no conclusion.

Proof: The two particular premises may be either (a) both negative, (b) both affirmative, or (c) one affirmative and one negative.

a. If both premises are negative, there is no conclusion (Axiom 3).

b. A particular affirmative proposition distributes no terms. If both premises are particular affirmative, the premises contain no distributed terms. Hence, there is no conclusion (Axiom 1).

c. An affirmative particular has no distributed terms, and a negative particular has only one. Hence the premises contain one and only one distributed term. Therefore, *if* there is a conclusion, it cannot contain *any* distributed terms (Theorem I). But since one premise is negative, the conclusion must be negative (Axiom 4). Therefore at least *one* term of the conclusion must be distributed. The assumption that there is a conclusion requires us to maintain that it contains at the same time *no* distributed terms and *at least one*. This is absurd. Hence there is no conclusion.

Theorem III. If one premise is particular, the conclusion must be particular.

Proof: The premises cannot both be particular (Theorem II). They must therefore differ in quantity, and may be either (a) both negative, (b) both affirmative, or (c) one affirmative and one negative.

a. If both premises are negative, there is no conclusion (Axiom 3).

b. One premise is an affirmative universal, the other an affirmative particular. The universal distributes only one term, the particular none. Hence the premises contain no more than one distributed term. Therefore the conclusion, if there is any, contains no distributed term (Theorem I). But a universal proposition contains at least one distributed term. Hence the conclusion, if there is any, is particular.

c. We may distinguish two cases: (α) the universal is negative, the particular is affirmative; (β) the universal is affirmative, the particular is negative.

α. The universal distributes two terms, the particular none. Hence the premises distribute two terms.

β. The universal distributes one terms, the particular also one. Hence the premises distribute two terms. In either case, therefore, the premises contain two and only two distributed terms. The conclusion, if there is any, cannot contain more than one distributed term (Theorem I). But the conclusion must be negative (Axiom 4), and its predicate must therefore be distributed. Hence its subject cannot be distributed, and it must be particular.

Theorem IV. If the major premise is an affirmative particular and the minor is a negative universal, there is no conclusion.

Proof: Since, by hypothesis, the minor is negative, the conclusion, if there is any, must be negative (Axiom 4), and its predicate, which is the major term, must be distributed. Hence the major term must be distributed in the major premise (Axiom 2). But the particular affirmative distributes none of its terms. There is therefore no conclusion.

The five axioms and these four theorems which we have demonstrated rigorously by their aid enable us to enumerate all possible valid syllogisms. The reader will do well to notice the nature of the demonstration: it has been shown that the theorems are necessary consequences of the axioms, so that if the axioms are accepted, the theorems must be accepted also, on pain of contradiction.

§5. THE FIGURES AND MOODS OF THE SYLLOGISM

But before we enumerate all the valid syllogistic forms, let us consider some syllogisms:

1. No musicians are Italians.
   All barbers are musicians.
   ∴ No barbers are Italians.
2. All gentlemen are polite.
   No gamblers are polite.
   ∴ No gamblers are gentlemen.
3. Some books are not edifying.
   All books are interesting.
   ∴ Some interesting things are not edifying.
4. All business men are self-confident.
   No self-confident men are religious.
   ∴ No religious men are business men.

Although these are all valid syllogisms, they differ from one another in two ways: (1) in the position of the middle term; and (2) in the quality and the quantity of the premises and the conclusion. In the first example, the middle term is the subject of the major and the predicate of the minor: in the second example, the middle term is the predicate of both premises; in the third, the middle term is the subject of both premises; and in the fourth, the middle term is the predicate of the major and the subject of the minor. The position of the middle term determines the *figure* of the syllogism, and on the basis of this distinction there are four possible figures. Letting *S, P, M,* denote the minor term, major term, and middle term respectively, we may symbolize the four figures as follows:

$M – P$
$S – M$
______

$\therefore S - P$

First Figure

$P - M$
$S - M$

$\therefore S - P$

Second Figure

$M - P$
$M - S$

$\therefore S - P$

Third Figure

$P - M$
$M - S$

$\therefore S - P$

Fourth Figure

Aristotle recognized only the first three figures. The introduction of the fourth is generally attributed to Galen, and is therefore called the Galenian figure. Logicians have disputed whether the fourth figure represents a type of reasoning distinct from the first three, and whether Aristotle was or was not mistaken in not recognizing it. If the distinction between figures is made on the basis of the *position* of the middle term, there can be no dispute that there are four distinct figures. But Aristotle did not distinguish the figures in this way. His principle of distinction was the *width* or extent of the middle term as compared with the other two. On this basis there are just three figures: the middle may be wider than one and narrower than the other, wider than either, and narrower than either.

The second way in which syllogisms may differ is with respect to the quantity and quality of the premises and the conclusion. This determines the *mood* of the syllogism. The first of the four syllogisms above is in the first figure, in mood *EAE*. The syllogism

5. All wholesome foods are cleanly made.
All doughnuts are wholesome food.
∴All doughnuts are cleanly made.

is a syllogism in the first figure, in mood *AAA*. Syllogisms may therefore differ in both figure and mood (for example, 1 and 3 above) or in figure alone (2 and 4) or in mood alone (1 and 5). However, not all moods are valid in every figure.

Let us count the total number of syllogistic forms, whether valid or not, taking account of differences in mood and figure. Since there are four types of categorical propositions, the major premise may be any one of the four types, and similarly for the minor premise and the conclusion. There are therefore 4 X 4 X 4 or 64 syllogistic moods in each figure, and 64 X 4 or 256 syllogistic forms in all four figures. Most of these, however, are invalid. But how shall we discover the valid forms? It would be an appalling task to examine each of 256 forms. Such a procedure is not necessary, however, since the invalid forms may be eliminated by applying the axioms and theorems.

Let us write down every possible combination of premises, where the first letter indicates the major premise, and the second letter the minor:

AA	*EA*	*IA*	*OA*
AE	*EE*	*IE*	*OE*
AI	*EI*	*IO*	*OI*
AO	*EO*	*II*	*OO*

But Axiom 3 shows that the combinations *EE, EO, OE,* and *OO* are impossible; Theorem II eliminates *II, IO, OI;* and Theorem IV eliminates *IE*. We are therefore left with the following eight combinations of premises, each of which will yield a valid syllogism in some or all figures: *AA, AE, AI, AO, EA, EI, IA, OA.* The eight combinations which have been eliminated yield no conclusion in *any* figure.

There now remains the task of discovering the valid moods in each figure. This may be done in either of the following ways:

1. Write premises in each of the figures having the quantity and quality indicated by each of the permissible combinations, and by inspection find those which yield a valid conclusion. This method has the disadvantage of being long.

2. Establish special theorems for each figure, and eliminate by their aid the invalid combinations of premises. This method is very elegant, and we shall employ it.

In what follows we shall assume once for all that the classes denoted by the terms of the propositions are not empty, and explore the consequences of this assumption. It will permit us to perform immediate inferences by limitation.

§ 6. THE SPECIAL THEOREMS AND VALID MOODS OF THE FIRST FIGURE

Since the form of the first figure is symbolized by

M – P
S – M
______
∴S – P

we prove:

Theorem I. The minor premise must be affirmative.

Suppose the minor is negative: then the conclusion must be negative (Axiom 4), and *P* must be distributed. Hence *P* must be distributed in the major premise (Axiom 2), so that the major must be negative. However, both premises cannot be negative (Axiom 3), and the minor must therefore be affirmative.

Theorem II. The major premise must be universal.

Since the minor premise must be affirmative, its predicate *M* cannot be distributed. Hence *M* must be distributed in the major (Axiom 1), making the latter universal.

By means of the special Theorem I, we may therefore eliminate the combinations *AE, AO,* and by means of the second theorem, the combinations *I A* and *OA*. Only the following four yield valid conclusions in the first figure: *AA, AI, EA* and *EI.* The six valid moods are therefore *AAA, (AAI), AII, EAE, (EAO), EIO.*

The moods we have encircled are called *subaltern* or *weakened moods,* because although the premises warrant a universal conclusion, the actual conclusion is only particular, and therefore "weaker" than it could be. Four of these six valid moods have been given special names, the vowels of which correspond to the quantity and quality of the premises and conclusion. Thus, *AAA* is *Barbara, AII* is *Darii, EAE* is *Celarent,* and *EIO* is *Ferio.* The names have in each of the figures may be recalled, and the moods in figures other than the first reduced to the first. We shall return to the problem of reduction.

§ 7. THE SPECIAL THEOREMS AND VALID MOODS OF THE SECOND FIGURE

The form of the second figure is symbolized by

$$\begin{array}{c} P - M \\ S - M \\ \hline \therefore S - P \end{array}$$

We prove:

Theorem I. The premises must differ in quality.

If both premises are affirmative, the middle term *M* is undistributed in each. Hence one of the premises must be negative (Axiom 1). But both premises cannot be negative (Axiom 3). Hence they differ in quality.

Theorem II. The major premise must be universal.

Since one of the premises is negative, the conclusion is negative (Axiom 4), and *P*, the major term, must be distributed. Hence *P* must be distributed in the major premise (Axiom 2), so that the latter is universal.

Theorem I eliminates the combinations *AA, AI,* and Theorem II eliminates *IA* and *OA.* We are left with four combinations in this figure: *AE, AO, EA,* and *EI,* from which we obtain six valid moods. *AEE (Camestres), (AEO), AOO (Baroco), EAE (Cesare), (EAO), and EIO (Festino).* The moods encircled are weakened syllogisms.

§ 8. THE SPECIAL THEOREMS AND VALID MOODS OF THE THIRD FIGURE

Employing the symbolic form of the third figure

$$\begin{array}{c} \mathrm{M - P} \\ \mathrm{M - S} \\ \hline \therefore \mathrm{S - P} \end{array}$$

we can prove:

Theorem I. The minor must be affirmative.

Suppose the minor negative: the conclusion would then be negative (Axiom 4) and *P*, its predicate, would be distributed. Hence *P* would be distributed in the major premise (Axiom 2), so that the latter would be negative. But this is impossible (Axiom 3), so that the minor cannot be negative.

Theorem II. The conclusion must be particular.

Since the minor must be affirmative, *S* cannot be distributed in the premises. Hence *S* cannot be distributed in the conclusion (Axiom 2), and the latter must be particular.

The first theorem eliminates the combinations *AE* and *AO,* and we are left with the six: *AA, AI, EA, EI, IA, OA.* Keeping in mind the second theorem, we obtain six valid moods: *(AAI) (Darapti), All (Daiisi), (EAO) (Felapton), EIO (Ferison), IAI (Disamis)* and *OAO (Bocardo).* In this figure there are no weakened moods. The two moods which we have encircled are called *strengthened syllogisms* because the same conclusion may be obtained even if we substitute for one of the premises its subaltern.

§ 9. THE SPECIAL THEOREMS AND VALID MOODS OF THE FOURTH

FIGURE

With the aid of the symbolic representation of the fourth figure

$$\begin{array}{c} P - M \\ M - S \\ \hline \therefore S - P \end{array}$$

we can prove:

Theorem I. If the major premise is affirmative, the minor is universal.

If the major is affirmative, its predicate *M* is not distributed. *M* must therefore be distributed in the minor premise (Axiom 1) and the latter is universal.

Theorem II. If either premise is negative, the major must be universal.

If either premise is negative, the conclusion is negative (Axiom tributed in the major premise (Axiom 2), which must, therefore, be universal.

Theorem III. If the minor is affirmative, the conclusion is particular.

If the minor is affirmative, its predicate *S* is not distributed. Hence *S* cannot be distributed in the conclusion (Axiom 2) and the latter must be particular.

The first theorem eliminates the combinations *AI, AO,* the second eliminates *OA.* We are left with the five combinations: *AA, AE, EA, IA* and *EI.* Remembering the third theorem, we obtain six valid moods: *AAI (Bramantip), AEE (Camenes), AEO IAI (Dimaris), EAO (Fesapo),* and *EIO (Fresison). AEO* is a weakened syllogism, while *AAI* and *EAO* are strengthened ones.

We thus find that there are just twenty-four valid syllogistic forms in the four figures, each figure containing six moods. The weakened and strengthened forms, however, are legitimate only on the assumption of existential import which we have explicitly made. Where such an assumption is not made, only fifteen valid moods can be obtained.

§ 10. THE REDUCTION OF SYLLOGISMS

We have discovered the valid moods by eliminating all forms incompatible with the axioms of validity and with the theorems derived from them. The only justification of the validity of the remaining forms we have given is that they are in conformity with the axioms. But a different approach to the justification of valid moods was taken by Aristotle, the original writer on the syllogism. According to him, the moods of the first figure were to be tested by applying to them directly a principle known since as the *dictum de omni et nullo.* This principle was, and frequently still is, believed to be "self-evident." It has been stated variously, one form of the *dictum* being: "Whatever is predicated, whether affirmatively or negatively, of a term distributed, may be predicated in like manner of everything contained under it." (Keynes) It is not difficult to show that the *dictum* is equivalent to the axioms and theorems relevant to the first figure. *It cannot, however, be applied directly to syllogisms in other figures.* The first figure was accordingly called the *perfect* figure, the others being *imperfect.*

Let us see how the *dictum* may test the validity of a syllogism in *Barbara: All Russians are Europeans; all communists are Russians; therefore all communists are Europeans.* "Europeans" is predicated affirmatively of the distributed term "Russians"; hence, according to the *dictum,* it may be predicated affirmatively of "communists," which is contained under "Russians." But the syllogism *All Parisians are Frenchmen; no Bostonians are Parisians; therefore no Bostonians are Frenchmen* does not conform to the *dictum.* “Frenchmen” is predicated affirmatively of the distributed term “Parisians”; it cannot, however, be predicated in any manner of “Bostonians,” since the latter term is not contained under “Parisians.”

Now if, with Aristotle, we regard the *dictum* as a “self-evident” principle, and if we believe with him that it is the *sole* “self-evident” principle which can test the validity of syllogistic forms, the only way in which moods in figures other than the first can be justified will be by showing that these imply valid moods in the *first* figure.

This process of exhibiting the connection of moods in other figures with moods in the first is called *reduction.* There are two varieties: (1) *direct reduction,* which is performed by conversion of propositions or transposition of premises; and (2) *indirect reduction,* which requires either obversion and contraposition of propositions, or a form of hypothetical inference known as *reductio ad absurdum.*

Although many logicians have regarded the process of reduction as unnecessary and even as invalid, there is no doubt that, given the basis upon which Aristotle develops the theory of the syllogism, reduction is an essential part of the theory. However, if the doctrine of the syllogism is developed on other bases, which do not assume that the first figure has any intrinsic superiority over the others, reduction cannot have the importance with which it has been regarded traditionally. Our own discussion has shown that the first figure need not be taken as central, and as we shall see later, the syllogism may be developed from a point of view even more general than the one we have taken. Moreover, it is possible to state *dicta* for each of the figures, all of which possess the same degree of “self-evidence” as the *dictum de omni.* Nevertheless, in spite of the diminution in the theoretical importance of reduction, it still serves as a valuable logical exercise. We shall now show how the process may be carried out by reducing to the first figure several moods in figures other than the first.

Direct Reduction

All almsgiving is socially dangerous.	↘ ↗	No socially dangerous institution is an educational institution.
No educational institution is socially dangerous.	converse	All almsgiving is socially dangerous.
∴ No educational institution indulges in almsgiving.		∴ No almsgiving is undertaken by an educational institution.
		∴ No educational institution indulges in almsgiving.

We have stated its equivalent syllogism on the right, which may be tested directly by means of the *dictum.* The reduction was effected by transposing the premises, converting the minor premise, and finally converting the conclusion. Therefore if we are not in doubt about the validity of the second syllogism, there can be no doubt concerning the validity of the original.

Consider next the *AII* mood in the third figure. It may be reduced to the valid syllogism in the first figure, stated at the right, as indicated:

All brokers are wealthy.	⟶	All brokers are wealthy.
Some brokers are intelligent.	converse ⟶	Some intelligent men are brokers.
∴ Some intelligent men are wealthy.		∴ Some intelligent men are wealthy.

Finally, the *IAI* in the fourth figure may be reduced as follows:

Some horses are spirited animals.	All spirited animals are difficult to manage.
All spirited animals are difficult to manage.	Some horses are spirited animals.
∴ Some creatures difficult to manage are horses.	∴ Some horses are difficult to manage.
	∴ Some creatures difficult to manage are horses.

Indirect Reduction

The reader will discover that the two syllogisms which contain a particular negative premise, *AOO* in the second figure and *OAO* in the third, cannot be reduced to the first by conversion and transposition of premises alone. If obversion is also permitted, however, the reader will have no difficulty in effecting the reduction.

But obversion was not included by Aristotle in the permissible means of effecting reduction. He discovered, however, a very important logical principle, which is in fact a generalization of the idea of contraposition of hypothetical propositions. Let us illustrate it before we state it.

Suppose that the premises in the following syllogism are true. We wish to demonstrate that the conclusion is necessarily true.

Some steel is not magnetic.
All steels are metals.
∴Some metals are not magnetic.

Now the conclusion is either true or false. If it is false, its contradictory: All metals are magnetic: is true. Combining this proposition with the minor of the syllogism, we get:

All metals are magnetic.
All steels are metals.
∴All steels are magnetic.

This, however, is a valid mood in the first figure. But since, by hypothesis, both premises of the original syllogism are true, the conclusion of this second syllogism cannot be true. For it contradicts the original major premise. Consequently, the major of this second syllogism cannot be true, or—what is the same thing—the conclusion of the first syllogism cannot be false. It must, therefore, be true.

The validity of the *OAO* syllogism in the third figure is thus demonstrated by means of a valid syllogism in the first figure and the principle known as the *reductio ad absurdum.* The validity of the *AOO* mood in the second figure may be shown in the same way. This method may be used for other moods as well.

Let us now exhibit the principle of the *reductio ad absurdum* in more abstract form. Let p represent *Some steel is not magnetic,* q represent *All steels are metals,* and r represent *Some metals are not magnetic.* And let p', q', r' symbolize the *contradictories* of each of these propositions respectively. Then the original syllogism asserts that p and q together imply r. Or symbolically, $(p \,.\, q) \supset r$. Now what we have shown was that the contradictory of r, together with q, implies the contradictory of p. Symbolically this may be stated: $(q \,.\, r') \supset p'$. And the reduction of the first syllogism depends on the *equivalence* between these two implications. This equivalence is an easy extension of the equivalence between a hypothetical proposition and its contrapositive, for we have shown that if a and b are any two propositions $(a \supset b) \equiv (b' \supset a')$. We now have:

$$[(p \,.\, q) \supset r] \equiv [(q \,.\, r') \supset p'] \equiv [(p \,.\, r') \supset q']$$

The principle of indirect reduction may therefore be analyzed as follows: *The syllogism is a form of inference in which two propositions* p *and* q *jointly imply a third* r, *where the three propositions contain three and only three terms.* If, however, we deny the implication $[(p \,.\, g) \supset r]$, we must also deny a second implication which is equivalent to it: $[(q \,.\, r') \supset p')$. But this second implication in our illustration above was a valid syllogism in *Barbara,* which cannot be denied. Therefore the first implication, which represents an *OAO* syllogism in the third figure *(Bocardo)* cannot be doubted either. For the denial of the validity of *Bocardo* commits us to the denial of the validity of *Barbara,* which is absurd.

If weakened and strengthened forms are not permitted (that is, if we do not assume existential import for universal propositions), reduction enables us to see that all syllogistic arguments can be reduced to two forms: one in which both premises are universal, and the other in which one premise is particular. The former is an argument in which both propositions may be pure hypotheses; the latter involves statements of fact ultimately dependent on observation.

§ 11. THE ANTILOGISM OR INCONSISTENT TRIAD

The principle involved in indirect reduction has been extended by Mrs. Christine Ladd Franklin in such a way as to provide a new and very powerful method for testing the validity of any syllogism. We shall, however, in discussing this method drop the assumption we have made concerning the existence of the classes denoted by the terms of the syllogism. As a consequence, the weakened and strengthened moods must be eliminated as invalid.

Consider the valid syllogism:

All musicians are proud.
All Scotchmen are musicians.
∴All Scotchmen are proud.

If we let *S, M,* and *P* symbolize the terms "Scotchmen," "musicians," and "proud individuals," and if we make use of the analysis we have given of what asserted is by categorical proposition in Chapter IV, this syllogism must be interpreted to assert the following:

$$M\bar{P}=0$$
$$S\bar{M}=0$$
$$\therefore S\bar{P}=0$$

Now if the premises *All musicians are proud* and *All Scotchmen are musicians* necessarily imply *All Scotchmen are proud,* it follows that these premises are incompatible with the *contradictory* of this conclusion. Hence the three propositions:

1. All musicians are proud.
2. All Scotchmen are musicians.
3. Some Scotchmen are not proud:

are *inconsistent* with one another. They cannot all three be true together. Symbolically stated,

$$M\bar{P}=0$$
$$S\bar{M}=0$$
$$S\bar{P}=0$$

are inconsistent. A triad of propositions two of which are the premises of a valid syllogism while the third is the contradictory of its conclusion, is called an *antilogism* or *inconsistent triad.*

An examination of the antilogism above reveals, however, that any two propositions of the triad necessarily imply the *contradictory* of the third. (This can be shown to be true in general, and is a further extension of the equivalence between a hypothetical proposition and its contrapositive.) Thus, if we take the first two of the triad as premises, we get:

$$M\bar{P}=0$$
$$S\bar{P}\neq 0$$
$$\therefore S\bar{M}\neq 0$$
$$SP=0$$

All musicians are proud.
All Scotchmen are musicians.
∴ All Scotchmen are proud:
which is the original syllogism from which the triad was obtained. If we take the first and third of the triad as premises, we get:

$$S\bar{M}=0$$
$$S\neq 0$$
$$\therefore M\bar{P}=0$$

All musicians are proud.
Some Scotchmen are not proud.
∴Some Scotchmen are not musicians:
which is a valid mood in the second figure. Finally, if we take the second and third of the triad as premises, we get:

$$S\bar{M}=0$$
$$S\bar{P}\neq 0$$
$$\therefore M\bar{P}\neq 0$$

All Scotchmen are musicians.
Some Scotchmen are not proud.
∴Some musicians are not proud:
which is a valid mood in the third figure.

The reader is advised to take a different valid mood of the syllogism, and obtain from it the inconsistent triad and the other two valid syllogisms to which it is equivalent.

The Structure of the Antilogism

Let us now examine the structure of the antilogism. The reader will note, in the first place, that it contains two universal propositions and one particular proposition. This is the same as saying that in the symbolic representation of the members of the triad, there are two *equations*, and one *inequation*, because a universal proposition is interpreted as denying existence, while a particular proposition asserts it. Confining his attention to the symbolic representation, the reader will find, in the second place, that the two universals have a common term, which is once positive and once negative. Finally, the particular proposition contains the other two terms. It can be shown without difficulty that these three conditions are present in every antilogism, and the reader should not hesitate to prove that this is so.

Now since every valid syllogism corresponds to an antilogism, we can employ the conditions we have discovered in every antilogism as a test for the validity of any syllogism. Hence it is possible to develop the theory of the categorical syllogism on the basis of the conditions for the antilogism. The single principle required is: *A syllogism is valid if it corresponds to an antilogism whose structure conforms to the three* *conditions above.*

The theory of the antilogism represents an attempt to discover a more general basis for the syllogism and other inferences studied in traditional logic. The reader will note the elegance and the power which result from the introduction of specially designed symbols. We shall indicate in the following chapter the close connection between advances in logical theory and improvement in symbolism. We will conclude this discussion, however, by indicating how the antilogism may be used to test syllogisms for their validity.

Is the following valid?

Some Orientals are polite.
All Orientals are shrewd.
∴Some shrewd people are polite.

Letting *S, P, O* stand for the minor, major, and middle terms respectively, the symbolic equivalent of this inference is: $OP \neq 0$, $O\bar{S} = 0$, '$SP \neq 0$. The equivalent antilogism is: $OP \neq 0$, $OS = 0$, $SP = 0$. *This contains two universals and one particular; the universal have a common term which is once positive and once negative; and the particular contains the other two terms. The syllogism is therefore valid.*

Is the following valid?

Some professors are not married.
All saints are married.
∴Some saints are not professors.

Letting *S, P, M* stand for the minor, major, and middle terms, this may be stated symbolically as: $P\bar{M} \neq 0$, $S\bar{M}$ —0, $S\bar{P} \neq 0$. The equivalent antilogism is: $P\bar{M} \neq 0$, $S\bar{M} = 0$, $SP = 0$. This contains two universals and one particular, but the common term in the former is not positive once and negative once. Hence the syllogism is invalid.

§ 12. THE SORITES

It sometimes happens that the evidence for a conclusion consists of more than two propositions. The inference is not a syllogism in such cases, and the examination of all possible ways in which more than two propositions may be combined to yield a conclusion requires a more general approach to logic than the traditional discussions make possible—or an elementary treatise permits. In certain special cases, however, the principles of the syllogism enable us to evaluate such more complex inferences. Thus, from the premises:

All dictatorships are undemocratic.
All undemocratic governments are unstable.
All unstable governments are cruel.
All cruel governments are objects of hate:

•we may infer the conclusion:

All dictatorships are objects of hate.

The inference may be tested by means of the syllogistic rules, for the argument is a *chain* of syllogisms in which the conclusion of one becomes a premise of another. In this illustration, however, the conclusions of all the syllogisms except the last remain unexpressed. A chain of syllogisms in which the conclusion of one is a premise in another, in which all the conclusions except the last one are unexpressed, and in which the premises are so arranged that any two successive ones contain a common term, is called a *sorites.*

The above illustration is an *Aristotelian sorites.* In it, the first premise contains the subject of the conclusion, and the common term of two successive propositions appears first as a predicate and next as a subject. A second form of sorites is the *Goclenian sorites.* The following illustrates it:

All sacred things are protected by the state.
All property is sacred.
All trade monopolies are property.
All steel industries are trade monopolies.
∴All steel industries are protected by the state.

Here the first premise contains the predicate of the conclusion, and the common term of two successive propositions appears first as subject and next as predicate.

Special rules for the sorites may be given. We shall state them and leave their proof as an exercise for the reader.

Special Rules for the Aristotelian Sorites.

1. No more than one premise may be negative; if a premise is negative, it must be the last.
2. No more than one premise may be particular; if a premise is particular, it must be the first.

Special Rules for the Goclenian Sorites.

1. No more than one premise may be negative; if a premise is negative, it must be the first.
2. No more than one premise may be particular; if a premise is particular, it must be the last.