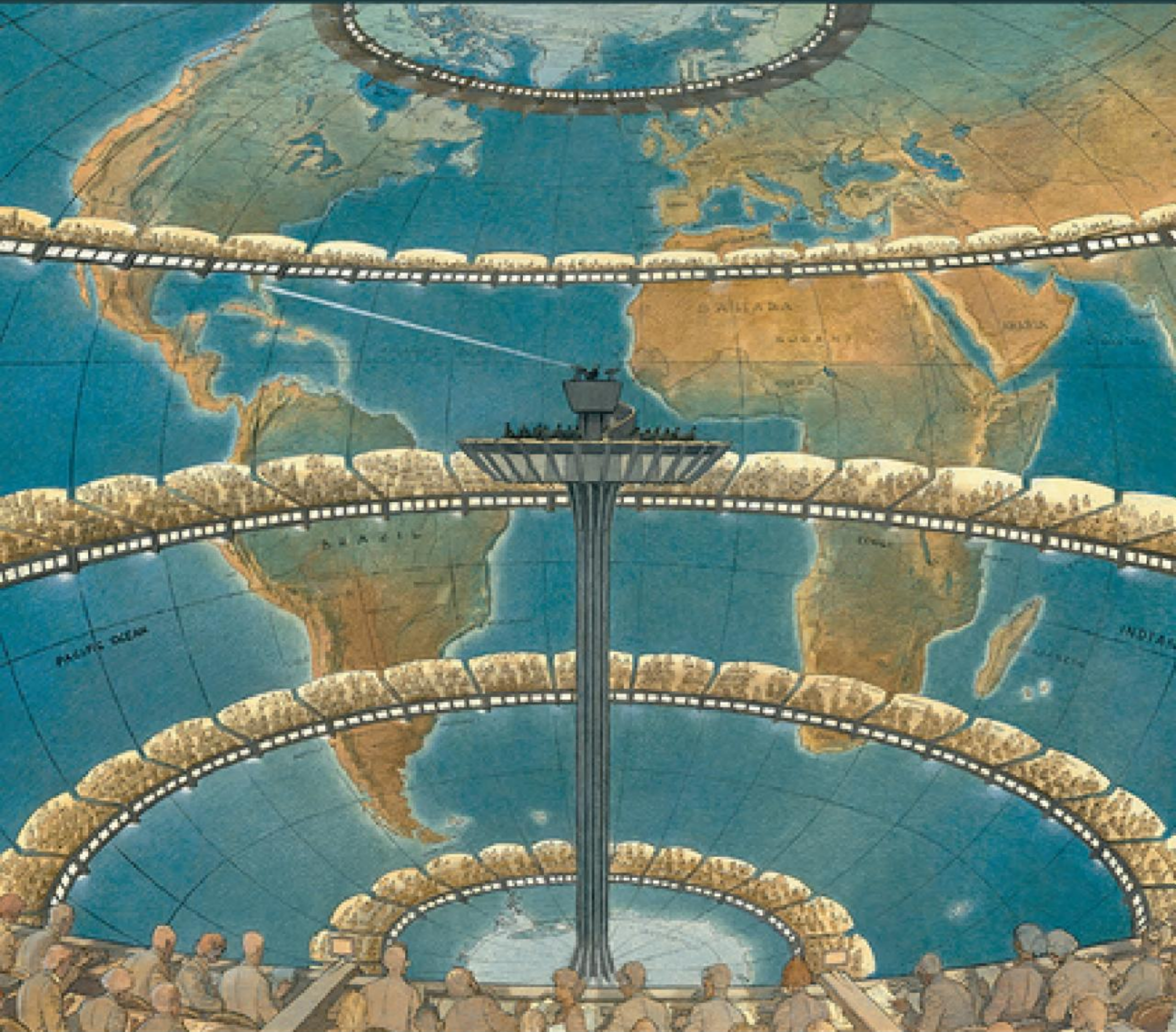
BECAUSE WITHOUT CAUSE



Non-Causal Explanations in Science and Mathematics

MARC LANGE

Because Without Cause

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PREFACE

WELCOME

Many explanations in science work by virtue of describing the world's network of causal relations. It is easy to find examples of "causal explanations." We explain the extinction of the dinosaurs by describing its various proximate causes, such as climate change, and we also explain it by describing its more distant causes, such as the terrestrial collision of one or more celestial bodies. According to classical physics, we explain the planets' motions by describing the gravitational influences causing those motions. In a more workaday example, we explain why a certain car fails to start by describing some respect in which its internal mechanism is malfunctioning, perhaps adding the causes of that malfunction. In other examples, we explain laws of nature. For instance, we explain why it is a law of nature that a gas's pressure climbs when it is compressed by a moveable piston under constant temperature. Our explanation describes the causal process underlying gas pressure: the collisions of gas molecules with the container's walls. Our explanation then gives certain of the laws governing that causal process, according to which molecules must collide more frequently with the container's walls as the gas is compressed under constant temperature.

This book is about some explanations that do *not* derive their explanatory power by virtue of describing the world's network of causal relations. Some of these explanations explain mathematical theorems; they are explanations *in mathematics*. Presumably, all explanations in mathematics are non-causal. The other non-causal explanations that this book will investigate are all *scientific* explanations; although many scientific explanations are causal, I contend that some are not. Non-causal explanations all involve "because without cause."[1]

Non-causal explanations in mathematics and science have generally been underappreciated in the vast recent philosophical literature about explanation. With regard to scientific explanation, causal explanation has received nearly all of the attention in recent decades.[2] Some philosophers have even declared that all scientific explanations are causal:

> [A]n explanation, I think, is an account of etiology: it tells us something about how an event was caused. Or it tells us something general about how some, or many, or all events of a certain kind are caused. Or it explains an existential fact by telling us something about how several events jointly make that fact true, and then perhaps something about how those truthmaker events were caused (Lewis 1986a, 73–74).

Even philosophers who officially leave room for non-causal scientific explanations, such as Woodward (2003, 221) and Strevens (2009, 5), devote scant attention to them. These philosophers take any non-causal explanations there may be as having to fit the models they have proposed of causal explanation—except that some sort of non-causal dependence must take the place of causal dependence. All told, then, non-causal scientific explanations have been largely neglected by philosophy of science. Likewise, explanation in mathematics has never been among the central topics in the philosophy of mathematics. Although mathematical explanation has recently begun to receive increased scrutiny, many philosophers still deny that there is any interesting sense in which certain mathematical proofs differ from other proofs of the same theorems in being able to explain why those theorems hold.

I will argue that non-causal explanations have long been recognized in both mathematics and science. I will offer many examples to persuade you not only that the task of giving explanations is an important part of mathematical practice, but also that non-causal scientific explanations play important roles that could not be played (even in principle) by causal explanations. I will not try to portray non-causal scientific explanations as working in roughly the same way as causal scientific explanations do (except that some variety of non-causal dependence appears in place of causal dependence). I will not even try to portray all non-causal scientific explanations as working in the same way as one another.

Once upon a time, many philosophers failed to acknowledge explanation over and above efficient description as a key aim of science.[3] Similarly, many philosophers today fail to see that some mathematics aims at explanation and that some important scientific explanations are non-causal. Fortunately, mathematicians and scientists never stopped looking for non-causal explanations just because some philosopher said that there is no such thing. In offering examples of non-causal explanations in mathematics and science, I will avoid examples that merely strike me (or some other philosopher) as explanatory. Rather, I will focus on examples that mathematicians and scientists themselves have proffered as explanatory. In many cases, I will look closely at their reasons for taking these examples to be explanatory. My aim throughout will be to account for these features of mathematical and scientific practice.

It would be of some value merely to have on hand a range of exemplary non-causal explanations (just as philosophy of science has settled upon some canonical examples of causal scientific explanations). However, I will not be content merely to offer some examples or even to classify them into natural kinds. Rather, I will try to elucidate how these various kinds of non-causal explanations work. If their explanatory power does not derive from their describing relevant features of the world's network of causal relations, then what does make them explanatory? To answer this question for various kinds of non-causal explanations in math and science is the principal aim of this book.

I will examine closely a great many fascinating and instructive explanations in math and science. For example, I will look at some "brute-force" mathematical proofs and contrast them with proofs of the same theorems that exploit symmetries to explain why those theorems hold. I will examine why combinatorial mathematicians regard simple bijective proofs of partition identities as explanatory, by contrast with non-explanatory proofs that appeal to generating functions out of which the simple partition identities seem to emerge miraculously from out of a welter of algebra. I will investigate proofs that explain theorems concerning the real numbers by placing them in the broader context of the complex numbers, and I will contrast these proofs with non-explanatory proofs of the same theorems that stick purely to the real numbers. I will present scattered theorems of Euclidean geometry that are explained by being unified under a single proof that uses properties from projective geometry, and I will investigate how these projective properties qualify as mathematically natural. I will present mathematical facts that have no explanations at all as well as mathematical explanations that are not proofs. I will identify mathematical explanations that reveal further, formerly invisible aspects of the theorems being explained, thereby provoking new questions about why those theorems hold. I will point out mathematical coincidences and specify the relation between being a mathematical coincidence and having a certain sort of mathematical explanation.

Turning to scientific explanations, I will describe how the minimum number of equilibrium points of any double pendulum is explained by the topology of any double pendulum's configuration space and how this explanation contrasts with a causal explanation of the same fact. This topological explanation and other "distinctively mathematical" scientific explanations work by showing how the fact being explained is inevitable considering the framework that any system *must* inhabit (regardless of whether and how it is caught up in causal relations)—where this variety of necessity transcends ordinary natural necessity. I will argue that an explanation that appeals to regression toward the mean works by depicting the phenomenon being explained as fallout from the mere fact that certain factors are statistically correlated, whatever this correlation's causal basis (if any). I will argue that explanations in evolutionary population biology that appeal to random drift are likewise "really statistical" explanations. On this view, natural selection and random drift are not different kinds of causal processes, but rather involve different kinds of explanations. In addition, I will distinguish several varieties of non-causal scientific explanation that work by showing how the fact being explained follows merely from the dimensional architecture. For instance, the reason why freely falling bodies obey Galileo's "odd number rule," rather than various alternatives to it that were proposed in Galileo's time, is that those alternatives are dimensionally impossible whereas Galileo's rule is dimensionally possible. Furthermore, I will maintain that an analogy between derivative laws of nature concerning physically dissimilar cases (such as cases in thermodynamics, electrostatics, and hydrodynamics) is physically coincidental insofar as these laws have no important common explainer among more fundamental laws. Yet this analogy between the various derivative laws is nonetheless mathematically no coincidence if their similarity arises from a similarity either in the cases' dimensional architecture or in the mathematical form of various, more fundamental laws. I will examine what would make it no coincidence that both gravitational and electric forces conserve energy, but instead for the law of energy conservation to explain this similarity between the two kinds of forces. I will also elucidate what it would take for a conservation law to be explained by a spacetime symmetry principle. I will argue that these explanations work by supplying (contextually relevant) information about the source of the especially strong necessity possessed by the fact being explained. I will use this account to understand several other non-causal scientific explanations, such as the way that the principle of relativity and other symmetry principles would explain the Lorentz transformations. These are among the topics that I will be investigating in this book.

■ WHAT THIS BOOK IS NOT ABOUT

Having sketched what this book will be about, I will now say a few words about what this book will *not* be about.

There are many non-causal explanations with which this book will not be concerned. For instance, a given base runner in a baseball game was out because he was beaten to first base by the thrown ball. Similarly, Carter can issue orders to Pyle because Carter is a sergeant and Pyle is a private. The rules of chess and the locations of various pieces on the chessboard explain non-causally why Fisher cannot move his king. The rules of a given language's grammar explain non-causally why a certain sentence is (or is not) grammatical in that language. Non-causal explanations can also be found in legal explanation. For instance, it is illegal in the United States to burn currency because a certain law is in force, and that law is a law, in turn, because certain legislative procedures were followed in its passage. Non-causal explanations can also be found in moral explanations. We might explain why a given character trait is a virtue (or a vice) or why a given action would be good (or courageous or mendacious). For example, the reason why it would be good for Alice to return the keys she borrowed is that she promised to return them and the rule of promise-keeping would maximize happiness/is a rule that no one could reasonably reject/is God's command/is (fill in your favorite moral theory here). Non-causal explanations can also be found in epistemology. Jones would be justified in believing that there are three other people joining him for dinner because of what he has been told and his background knowledge. In all of the explanations to which I have just referred, certain normative statuses are explained by other normative statuses or by certain non-normative facts together with certain norms. These diverse normative explanations are non-causal but are neither mathematical nor scientific explanations. They fall outside the scope of this book.

Likewise, some philosophers (e.g., Sturgeon 1985) have argued that a moral fact can explain a non-moral fact, such as when the rightness of Alice's returning the car keys she borrowed explains why Alice believes that it would be right for her to return them and ultimately why she returns them. I will not appeal to putative explanations of this kind in order to argue for the existence of non-causal scientific explanations. Likewise, suppose that Chris's experiencing pain is explained by his mental state's physical ground. In such a case, the instantiation of a supervening property is explained by the instantiation of a subvening property. Some philosophers (e.g., Kim 1998, 44; Gibbons 2006, 89) have characterized such an explanation as non-causal in that the subvening property's instantiation is not a *cause* of the supervening property's instantiation.

That the connection between these property instantiations is not causal, however, is insufficient to show that this scientific explanation works differently from standard examples of causal explanations. A "causal explanation" does not have to cite causes of whatever is being explained. Rather (as I will argue in chapter 1), an explanation that cites no causes of what it explains may deserve to be classified in the same category as an explanation that does cite causes of its explanatory target. They ought to be co-classified (as "causal explanations") because they work in the same way: each derives its power to explain by virtue of supplying (contextually relevant) information about the world's network of causal relations. This may well be precisely what happens in some cases where we explain a supervening property's instantiation by identifying the subvening property in which its instantiation consists. The explanation may work not merely by virtue of the supervenience relation (which could, unlike the explanatory relation, be symmetric), but also by virtue of describing an important aspect of the supervening property instantiation's place in the world's causal network. For example, by so locating the supervening property instantiation, the explanation may inform us that any cause of the supervening property instantiation operated ultimately by causing—or by supervening on events causing—the instantiation of the subvening property. For a similar reason, intentional explanations (e.g., by which an action is explained by the actor's beliefs and desires) may be causal explanations. Accordingly, I will not appeal to explanations of these kinds in order to argue for the existence of non-causal scientific explanations.

Furthermore, I will not make my case for non-causal scientific explanation by appealing to the spooky "passion at a distance" exhibited by quantum-mechanical systems. The specific peculiarities of quantum mechanics do not bear on the operation of the broad varieties of non-causal scientific explanations that I will study. Some philosophers believe that there are no causal relations in fundamental physics; if these philosophers are correct, then presumably any explanations supplied by fundamental physics are non-causal. But I will not use any argument along these lines to make the case for non-causal scientific explanations.

In addition, it has sometimes been argued that certain scientific explanations work by abstracting from, idealizing, mathematically massaging, or otherwise departing from the causal details of the case at hand and that these explanations therefore do not work by accurately describing the world's network of causal relations. I will not be relying on any such argument to show that there are non-causal scientific explanations. An explanation that works by giving an abstract description of the world's causal network still derives its power to explain by virtue of supplying information (of an abstract kind) about the world's causal network and so is (by my lights) a causal explanation. By contrast, I will look at many scientific explanations that abstract from the petty causal details but (as I will show) do *not* work by virtue of describing abstract features of the world's causal network. For instance, some non-causal scientific explanations work by identifying certain constraints to which the world must conform. These constraints (such as mathematical facts and symmetry principles) apply to causal processes, but not in virtue of their being causal processes. Rather, they apply in the same way to all aspects of the world, whether causal or not. Indeed, they would apply in the same way even if the world contained no causal network at all.

By the same token, "non-causal dependency relations" (Kim 1974, 41) were in play when Xanthippe became a widow as a result of Socrates's death, when the birth of my first child turned my father into a grandfather, and when my arrival in Paris gave the Eiffel Tower the property of being within 10 miles of me. In these cases, some *A*'s relational properties changed because of a change in some *B*'s properties (where *A* is not *B*). However, explanations of these "mere Cambridge changes" are not the kinds of non-causal explanations that I will be examining.

The same applies to many putative non-causal explanations involving metaphysical "grounding"—such as that the mereological sum of *A* and *B* exists because *A* exists and *B* exists, or that the disjunctive fact that it is raining or I am wearing a green shirt holds because it is raining, or that Jones and Smith have a common acquaintance because they both know Brown. I will look neither at "truthmaking" explanations (such as that *p* is true because *p*) nor at explanations of narrowly logical truths (perhaps it is the case that $(p \rightarrow p)$ or q because it is the case that $(p \rightarrow p)$). Similarly, I will not be invoking explanations according to which some relation holds between sentences (or propositions) by virtue of their logical forms, such as that "The meeting is likely to start at noon" is logically equivalent to "It is likely that the meeting will start at noon" because they have the same logical form.

Of course, some "in virtue of" explanations (as Rosen (2010) calls them) have tremendous scientific importance—such as that some fact about light holds in virtue of some fact about electromagnetic waves. But once again, I will not use such an explanation to make the case for non-causal scientific explanations because I suspect that this explanation works by describing features of the world's causal network—in particular, by tracing light's causal powers to the causal powers of electromagnetic fields. Likewise an explanation that reveals how a cube's water-solubility (or some other dispositional property instantiation) is grounded in the cube's molecular structure (or some other non-dispositional property instantiation) is a causal explanation (by the lights of chapter 1) even though the cube's molecular structure does not *cause* the cube to be water-soluble. The cube's molecular structure explains the cube's water-solubility by virtue of the fact that any manifestation of the cube's water-solubility (such as its dissolving when immersed in water) would have the cube's molecular structure as a cause. The explanation of the disposition in terms of its non-dispositional ground works by supplying information about the world's causal network. It is thus a causal explanation. Having given "causal explanation" such a broad scope, I find it especially interesting to discover that certain scientific explanations are non-causal.

There are scientific cases where facts of the form "There exists an *F*" are explained non-causally. But if an "in virtue of" (or "truthmaker") explanation of "There exists an *F*" must appeal to a fact of the form "*c* is *F*" (as in the explanation of Smith and Jones having a common acquaintance), then many non-causal scientific explanations of "There exists an *F*" are not "in virtue of" (or "truthmaker") explanations. For instance, Kelvin's account of why there is a lowest temperature ("absolute zero") is not that minus 273.15°C is it. (In chapter 1, I will present another example: the reason why there exists at this moment a pair of antipodal points on the Earth's equator having the same temperature.)

There are some scientifically important "in virtue of" explanations that may be non-causal but will not be examined in this book. A system may have a given electric charge in virtue of the charges of its parts. How this explanation works depends on why charges add. Although I will not look at this example, I will look carefully (in chapter 4) at how the "composition law" for forces (the "parallelogram law") is explained and what would make its explanation causal or non-causal.

Consider how a net force is explained by its component forces. This explanation might be termed a "constitutive explanation" in that the state of the whole is being explained by the states of its parts (together with a composition law: the parallelogram of forces). One might be tempted to characterize a constitutive explanation as "non-causal" in that the states of the parts do not *cause* the state of the whole; the whole's being in a certain state is not an event distinct from the parts' being in certain states. However, I will not appeal to "constitutive explanations" to argue that some scientific explanations are non-causal. At least some constitutive explanations work by supplying information about the world's causal network—as when we explain some capacity possessed by an intricate compound system (such as a machine or organism) by the diverse, simpler dispositions of its parts (together with the parts' interrelations). Such an explanation works by specifying an underlying causal mechanism. It works by telling us that (for example) any outcome of the given capacity of the system would be a manifestation of various dispositions of its parts—and thus by telling us about the world's causal network. Likewise, as I will show in chapter 4, the fact that the relation between component and net forces is a part/whole relation rather than a causal relation does not suffice to ensure that the explanation of the force composition law is non-causal. Although the net force is not distinct enough from its components to be an effect of them, an explanation of the force composition law will be a causal explanation if it works by tracing the individual effects of the component forces.

Many philosophical explanations are non-causal—including philosophical explanations of facts about the causal relation itself. For instance, a philosophical account of what causal relations consist in should explain why it is the case (if and insofar as it is indeed the case) that all token-level causal relations are transitive but some type-level causal relations are intransitive. Likewise, Kant proposed an account of why appearances stand in causal relations. Such philosophical explanations do not purport to be causal explanations; they do not work by virtue of supplying information about the particular lines of causal influence that exist, though they do purport to supply information about what causal relations are. Such philosophical explanations are not part of mathematics or science and so fall outside the purview of this book. Likewise, if God exists outside spacetime, then presumably God's properties do not cause whatever they explain, so such theistic explanation is non-causal. I do not examine theistic explanation.

There are some varieties of explanation in mathematics and some varieties of non-causal scientific explanation that I do not examine in this book (or that I merely mention briefly). For instance, that Samuel Clemens and Mark Twain are identical explains non-causally why they have the same height, weight, and birth dates. Of course, the fact that Clemens and Twain are identical conveys some information about the world's network of causal relations: it tells us, for instance, that any cause of Clemens's height is a cause of Twain's height. But that Clemens and Twain are identical does not owe its explanatory power to its supplying such information about the world's causal relations. Even if there were no causal relations at all, identities would explain in the same way. As Lycan (1981, 10) says, "What better way to explain a . . . correlation between A's and B's than by supposing . . . that in fact A's are just B's." This "identity explanation" (Achinstein 1983) of the correlation reveals that causal relations have nothing to do with why it holds.

That Clemens and Twain have the same height because Clemens is identical to Twain is thus unlike the explanation of visible light's having a certain speed in glass because visible light is identical to certain electromagnetic waves and those electromagnetic waves have that speed in glass. This explanation of light's speed is a causal explanation because it works by supplying the information that the factors causally responsible for light's speed are electromagnetic. Because this explanation is causal, it is asymmetric in the same way as causal relations are: the speed in glass of certain electromagnetic waves is not explained by the fact that those waves are identical to visible light and that visible light has the given speed in glass.

The Twain-Clemens non-causal explanation is like some purported explanations from science—which also purport to be non-causal explanations. For instance, when Wheeler proposed that an electron moving backward in time would be indistinguishable from a positron moving forward in time, so that a single electron might move both forward and backward in time, Feynman (1998, 163) reports Wheeler to have said, "I know why all electrons have the same charge and the same mass. . . . Because, they are all the same electron!"

In another kind of scientific explanation that I do not examine, we explain why p is the case by appealing to the fact that p is a law and so *had to* be the case; p was inevitable, unavoidable—necessary. For instance, the fact that every actual long, linear charge distribution with static uniform charge density λ has an electric field strength at a distance r equal (in Gaussian CGS units) to $2\lambda/r$ is explained by the fact that as a matter of natural law, every such charge distribution must have such an electric field. Although in chapter 2 I offer an account of what it would take for certain special kinds of laws (e.g., conservation laws) to possess certain special powers to explain, I do not in this book offer an account of what makes an ordinary law that p able to explain the fact that p. I have tried to offer such an account elsewhere, most recently in Lange (2009a), and I touch briefly upon that account in section 2.5. On my view, the power of p's lawhood to explain why p is the case arises from the connection between lawhood and necessity: if it is a law that p, then p holds because p must hold. In Lange (2009a), I offer an account of what this must-ness consists in. The law specifying the electric field of any static, uniform, long, linear charge distribution is explained, in turn, by Coulomb's law. This explanation is causal, on my view, because it works by describing the field's individual causes and how their separate effects compose to give the charge distribution's total electric field strength.

There are plenty of vexed questions about the role of non-causal explanation in science that I will not address. I will not study whether so-called teleological explanations in biology (e.g., mammals have hearts in order to circulate their blood) are causal explanations. I will not examine whether the apparently non-causal explanations given by extremal principles in physics (e.g., roughly speaking, that light takes a certain path from here to there in order to minimize its travel time in getting there) are actually parasitic on causal explanations (e.g., that light takes all possible paths from here to there, but the light waves taking paths near to the least-time path interfere constructively, the others canceling out). In short, I will not aim to give an exhaustive survey of non-causal explanations in science and mathematics. Rather, I will look at some varieties of non-causal explanation that have generally been overlooked by philosophers, that have significant connections with one another, and that have been especially important in science and mathematics.

■ COMING ATTRACTIONS

Although I have been describing the subject of this book as "non-causal explanation in math and science," this characterization is potentially misleading. My main concern will not be to label certain explanations as "causal" and others as "non-causal." There may be several illuminating distinctions that merit this terminology, and there may also be intermediate cases. My main concern will be to understand how various (interrelated, important, and relatively neglected) kinds of explanations in math and science work. That is, I want to understand where their power to explain comes from.[4]

The 11 chapters that follow divide into four parts.

Part I (chapters 1, 2, 3, and 4) focuses on a single prominent type of non-causal scientific explanation that I call "explanation by constraint." In chapter 1, I describe "distinctively mathematical" scientific explanations, offer an account of how they work, and identify an important respect in which they qualify as "non-causal." In chapter 2, I argue that other "constraints" can play the same role as mathematical facts do in "distinctively mathematical" explanations. Over the course of chapters 2 and 3, I offer an account of how "explanations by constraint" operate and what difference it would make that some law has such an explanation. In chapter 2, I focus on conservation laws and symmetry principles as possible "constraints," transcending the various force laws. In chapter 3, I look at the coordinate transformations (whether Galilean or Lorentz) as constituting possible constraints, and I identify what it would be for "top-down" explanations supplied by (what Einstein called) "theories of principle" to be autonomous from the "bottom-up" explanations supplied by "constructive theories." In chapter 4, I investigate what it would be for the parallelogram of forces (and other such composition laws) to transcend dynamics and how some scientists argue that the parallelogram law is explained by statics rather than dynamics.

Non-causal scientific explanations as a category are set apart merely by what they are *not*: causal explanations. Therefore, we should expect some non-causal scientific explanations to work differently from others. In part II (chapters 5 and 6), I examine several varieties of non-causal scientific explanation that are not "explanations by constraint." In chapter 5, I investigate explanations that reveal the fact being explained to be just a statistical "fact of life," such as explanations appealing to regression toward the mean. I call these "Really Statistical" (RS) explanations and argue that they are "non-causal" explanations in the sense elaborated in chapter 1. I apply my account of how "RS explanations" work to the case of explanations in population biology that appeal to random drift. In chapter 6, I look at several kinds of "dimensional explanation" in physics; I argue that they, too, are non-causal explanations by the lights of chapter 1. Some dimensional explanations are "explanations by constraint," whereas others are not. A causal explanation of a given derivative law can explain certain features of the law that a dimensional explanation cannot explain, but by the same token, a dimensional explanation can sometimes explain features of the derivative law that a causal explanation cannot explain. For instance, its dimensional explanation may reveal that one of its features results entirely from certain dimensional features of the more fundamental laws entailing it, so that other features of those more fundamental laws are not responsible for that feature of the derivative law.

Part III (chapters 7, 8, and 9) concerns explanation in mathematics rather than scientific explanations. In chapter 7, I give many examples (drawn from actual mathematical practice) of proofs that explain the theorems they prove, as well as proofs of those theorems that are not regarded in mathematical practice as explanatory. To capture these examples, I propose an account (the "big Lange theory") of what makes certain mathematical proofs not only prove their theorems, but also explain why those theorems hold. I also briefly identify some other kinds of explanation in mathematics, including explanations that do not explain theorems and explanations that do not consist of proofs. In chapter 8, I use my account to investigate explanations in mathematics that unify various mathematical facts. By virtue of having such an explanation, a given mathematical fact is no "mathematical coincidence." I also compare my account of explanation in mathematics to the accounts that have been proposed by Steiner and Kitcher. In chapter 9, I apply my account to the explanation of Desargues's theorem in Euclidean and projective geometry. I examine the status of mathematical properties such as the property of being a point in projective geometry—that is, being a Euclidean point *or* a "point at infinity." I propose an account of what makes such a property mathematically natural (that is, a genuine respect of mathematical similarity) rather than wildly disjunctive. I also investigate what makes Desargues's theorem in two-dimensional Euclidean geometry capable of being properly understood only in a broader environment: in three-dimensional projective geometry. The fact that projective geometry is where Desargues's theorem naturally belongs, the fact that the property of being a projective point is mathematically natural, and the fact that Desargues's theorem in projective geometry is no mathematical coincidence are all facts bound up with the mathematical explanation of Desargues's theorem. In this way, we can appreciate the significance of explanation in mathematics.

Finally, part IV (chapters 10 and 11) brings together ideas concerning both explanation in mathematics and non-causal explanation in science. In chapter 10, I examine scientific explanations that explain why certain physically unrelated laws of nature are so similar. These explanations work by revealing the laws' similarity to be no *mathematical* coincidence. Some of these explanations are dimensional explanations that account for dimensional similarities among otherwise unrelated laws of nature. To understand how these scientific explanations work, I will need to draw upon some of the ideas that I elaborated in connection with explanation in mathematics. Chapter 11 highlights some of the themes that ran through many of the earlier chapters, tying together different kinds of non-causal explanation and even causal explanation. I do not argue in this book that all explanations work in the same way; I do not give a single, general, abstract model of explanation that all kinds of mathematical and scientific explanations instantiate. However, it is also not the case that the explanations form an arbitrary, gerrymandered class having little or nothing in common besides our calling them "explanations." Despite their diversity, all of the different kinds of non-causal explanations deserve to be grouped with one another—and with the causal explanations—as species of the same genus. There are important respects in which they are all alike, especially in the ways that explanations can render similarities non-coincidental and properties natural. Non-causal explanations join causal explanations as all belonging to the same natural kind.

Scattered passages of this book draw on some of my previously published articles and are reproduced here with the kind permission of the publishers: "Dimensional Explanations," *Noûs* 43 (2009), 742–775; "A Tale of Two Vectors," *Dialectica* 63 (2009), 397–431; "What Are Mathematical Coincidences (and Why Does It Matter)?," *Mind* 119 (2010), 307–340; "Conservation Laws in Scientific Explanations: Constraints or Coincidences?," *Philosophy of Science* 78 (2011), 333–352; "Really Statistical Explanations and Genetic Drift," *Philosophy of Science* 80 (2013): 169–188; "What Makes a Scientific Explanation Distinctively Mathematical?," *British Journal for the Philosophy of Science* 64 (2013), 485–511; "Aspects of Mathematical Explanation: Symmetry, Unity, and Salience," *Philosophical Review* 123.4 (2014), 485–531; " 'There Sweep Great General Principles Which All the Laws Seem to Follow,' " in *Oxford Studies in Metaphysics*, vol. 7, edited by Karen Bennett and Dean Zimmerman (Oxford: Oxford University Press, 2012), 154–185; "How to Explain the Lorentz Transformations," in *Metaphysics and Science (Mind Association Occasional Series)*, edited by Stephen Mumford and Matthew Tugby (Oxford: Oxford University Press, 2013), 73–98; "How the Explanations of Natural Laws Make Some Reducible Physical Properties Natural and Explanatorily Potent," in *Laws of Nature: Metaphysics and Philosophy of Science*, edited by Walter Ott and Lydia Patton (Oxford: Oxford University Press, forthcoming); "Because Without Cause: Scientific Explanations by Constraint," in *Explanation beyond Causation*, edited by Juha Saatsi and Alexander Reutlinger (Oxford: Oxford University Press, forthcoming); "Explanation, Existence, and Natural Properties in Mathematics—A Case Study: Desargues' Theorem," *Dialectica* 69.4 (2015), 435–472.

PART I
Scientific Explanations by Constraint

1 What Makes a Scientific Explanation Distinctively Mathematical?

1.1 DISTINCTIVELY MATHEMATICAL EXPLANATIONS IN SCIENCE AS NON-CAUSAL SCIENTIFIC EXPLANATIONS

Mathematics figures in many scientific explanations. But some scientific explanations are *distinctively* mathematical: they are mathematical in a different way from ordinary scientific explanations that employ mathematics. In this chapter, I will argue that these "distinctively mathematical" scientific explanations are *non*-causal explanations, unlike many other scientific explanations employing mathematics. Because distinctively mathematical scientific explanations are non-causal in an especially dramatic way, I will use them to argue that we must recognize the existence of non-causal scientific explanations.

Of course, I must specify what it takes for a scientific explanation to qualify as "causal"—and thus what it would be for a scientific explanations *not* to be causal (i.e., to be non-causal). The distinction between "causal" and "non-causal" explanations (as I will use these terms) lies in how they work—that is, in what gives them explanatory power. A "non-causal" explanation may incidentally identify (or, at least, supply information about) causes of what is being explained. But it does not derive its explanatory power by virtue of doing so.

Having used distinctively mathematical scientific explanations as my point of entry into non-causal scientific explanations, I will devote the rest of parts I and II of this book to examining non-causal scientific explanations more closely. I will argue that different kinds of non-causal scientific explanations work in different ways. "Distinctively mathematical" scientific explanations are our first example of what I will call "explanations by constraint." In the subsequent three chapters (making up the rest of part I), I will give many more examples of explanations by constraint and I will offer an account of how these non-causal explanations work. I will argue that there are other "constraints" besides mathematical facts and so there are "explanations by constraint" that are not "distinctively mathematical" explanations. I will specify what it is to be a "constraint." In part II (chapters 5 and 6) and later in chapter 10, I will investigate several kinds of non-causal scientific explanations that are *not* explanations by constraint. Ultimately, I will try to understand how each of these kinds of non-causal scientific explanation works—that is, how explanations of these various kinds acquire their power to explain.

"Distinctively mathematical" explanations are *scientific* explanations, as distinct from explanations *in mathematics*—the subject of part III (chapters 7, 8, and 9). That is, part III is concerned with explanations in which the facts being explained (the "explananda") are theorems of mathematics, whereas the explanations with which I will be concerned in this chapter (and in the rest of parts I and II) take as their targets various facts about the natural, spatiotemporal world. However, occasionally in parts I and II it will be useful to cast an anticipatory glance at explanation in mathematics. For instance, the notion of a "coincidence" will arise in connection with both scientific explanations and explanations in mathematics. In addition, Steiner's (1978a, 1978c) account of (what I call) distinctively mathematical scientific explanations appeals to the notion of explanation in mathematics. In section 1.5, I will argue that explanations in mathematics are not connected to distinctively mathematical scientific explanations in the way that Steiner believes. I hope that these "spoiler alert" peeks at explanation in mathematics will encourage you to read part III!

In trying to characterize distinctively mathematical scientific explanations, I am not trying to explicate the meaning of the term "distinctively mathematical" so as to agree with some intuitions about its proper application. We may well have no pretheoretic notions at all regarding what a "distinctively mathematical" scientific explanation would be. Furthermore, my aim is not to explicate the meaning of "distinctively mathematical" so as to fit this term's use in scientific practice. No such term is commonly used in science. Nevertheless, the task of characterizing "distinctively mathematical" scientific explanations does aim to fit certain intuitions as well as certain features of scientific practice.

Shortly, I will present several examples of scientific explanations that are mathematical in a manner that intuitively differs profoundly from many familiar scientific explanations employing mathematics. One goal of this chapter is to explore this apparent difference in order to see whether it withstands careful scrutiny. I will use the term "distinctively mathematical" to mark this apparent difference. Ultimately, I will suggest that there is a fundamental difference between the explanations I will present and many familiar scientific explanations employing mathematics. My account of what makes certain scientific explanations but not others "distinctively mathematical" aims to accord with our intuitions about which scientific explanations are alike and which fundamentally differ.

An account of "distinctively mathematical" explanations aims to fit scientific practice by deeming to be explanatory only examples that would (if true) constitute genuine scientific explanations. But it also has a more ambitious aim: to reveal how "distinctively mathematical" explanations work.

The modern study of scientific explanation began with Hempel's and Oppenheim's 1948 proposal of the D-N model (Hempel 1965). Unfortunately, notorious counterexamples such as the flagpole, the eclipse, and the barometer (Salmon 1989, 46–50) demonstrated that the D-N model counts various non-explanations as explanatory. To avoid these problems for the D-N model, many philosophers have suggested that causal relations (which have no place in the D-N model) play a central role in scientific explanations. Indeed, many philosophers have gone on to suggest that *all* scientific explanations (or, at least, all scientific explanations of particular events or singular facts) are "causal explanations." For example, Salmon has written:

> To give scientific explanations is to show how events and statistical regularities fit into the causal structure of the world. (Salmon 1977, 162)
>
> Causal processes, causal interactions, and causal laws provide the mechanisms by which the world works; to understand why certain things happen, we need to see how they are produced by these mechanisms. (Salmon 1984, 132)[1]

The same note has been sounded by many other philosophers:

> Here is my main thesis: *to explain an event is to provide some information about its causal history.* (Lewis 1986b, 217; see Jackson and Pettit 1992, 12–13)[2]
>
> Causal explanation is the unique mode of explanation in physics. (Elster 1983, 18)
>
> The explanation of an event describes the "causal structure" in which it is embedded. (Sober 1984, 96)[3]
>
> An explanation is an adequate description of underlying causes helping to bring about the phenomenon to be explained. (Miller 1987, 60)

Recent accounts of scientific explanation (e.g., Woodward 2003; Strevens 2008) have continued to emphasize that scientific explanations work by describing causal connections.

I will argue that this view of scientific explanation cannot do justice to "distinctively mathematical" explanations. In arguing that these explanations are "non-causal," I am not appealing to some account of what makes an explanation "causal" that aims to fit either some pretheoretic intuitions about which explanations are "causal" or some scientific practice of labeling certain explanations "causal." Rather, I am trying to elaborate a notion of "causal" explanation that not only motivates many philosophers to contend that all scientific explanations are causal, but also helps us to understand how scientific explanations work. Distinctively mathematical explanations are "non-causal" because they do not work by supplying information about a given event's causal history or, more broadly, about the world's network of causal relations. A distinctively mathematical explanation works instead (I will argue) roughly by showing how the fact to be explained could not have been otherwise—indeed, was inevitable to a stronger degree than could result from the action of causal powers. If a fact has a distinctively mathematical explanation, then the modal strength of the connection between causes and effects is insufficient to account for that fact's inevitability. Accordingly, distinctively mathematical explanations do not qualify as "causal" even when the range of explanations that qualify as "causal" is broad enough to include every explanation that explains by virtue of describing the world's causal structure.

Thus, the importance of understanding how distinctively mathematical explanations work does not derive from the significance of any intuitions we may have regarding what makes an explanation "distinctively mathematical" or "causal." Rather, its importance lies in what it reveals about the kinds of scientific explanations there are and the ways they work—and, in particular, about the limits of philosophical accounts that place causal relations at the center of all scientific explanations.

Enough with the preliminaries! The best way to approach our topic is to give several examples of scientific explanations that intuitively are "distinctively mathematical." Here is a very simple example (inspired by Braine 1972, 144):

> The fact that 23 cannot be divided evenly by 3 explains why Mother fails every time she tries to distribute exactly 23 strawberries evenly among her 3 children without cutting any (strawberries—or children!).

The explanation seems no less distinctively mathematical when the "explanans" (i.e., the collection of explainers) includes certain contingent facts:

> That Mother has 3 children and 23 strawberries, and that 23 cannot be divided evenly by 3, explains why Mother failed when she tried a moment ago to distribute her strawberries evenly among her children without cutting any.

Notice that in the latter explanation, the explanandum (i.e., the fact being explained) concerns Mother's failure in a *particular* attempt, whereas in the former explanation, the explanandum is more general. In either case, the explanation is distinctively mathematical.

Here is another example along roughly similar lines. Suppose we select three jellybeans from a sample containing only red jellybeans and blue jellybeans. Why is it that of the three jellybeans we select, two are the same color? The answer is that with three objects but only two colors to distribute among them, two of the objects must share a color. This is an instance of what mathematicians call the "pigeonhole principle." Lipton (2009a, 46–47) gives a similar example: "I came to understand why my class had four students whose last birthdays fell on the same day of the week when it was explained to me that since there are only seven days in a week and twenty-two students in my class, there is no way to arrange the birthdays to avoid this result."

Here is an example given originally by Colyvan (1998, 321–322; 2001, 49–50; 2007, 120). Consider the fact that at every moment that Earth exists, on the equator (or on any other great circle) there exist two points having the same temperature that are located antipodally (i.e., exactly opposite each other in that the line between them passes through the Earth's center). Why is that? An explanation begins with the fact that temperature is a continuous function. That is, roughly speaking, as you move along the equator, temperature changes smoothly rather than jumping discontinuously. Now imagine placing your two index fingers on a globe at two antipodal points on the equator. Take the temperature on Earth at the location x that your left index finger is touching minus the temperature at the antipodal location that your right index finger is touching. This difference function $D(x)$ must change continuously as you move your two fingers eastward, keeping them at antipodal points on the equator (since a function is continuous if it is the difference between two continuous functions). Suppose without loss of generality that for the initial value of x, $D(x)$ is greater than zero (i.e., the left-finger location is warmer than the right). Then when you have moved your two fingers far enough around the equator that your left finger is where your right finger began (and vice versa), D must be less than zero. Hence, since D is continuous, there must have been a moment as you were moving your fingers when D went from positive to negative. That is, there must have been an x where $D(x) = 0$. (This step uses the intermediate value theorem: if f is a real-valued, continuous function on $[a,b]$ and u is a real number between $f(a)$ and $f(b)$, then there is a $c \in [a,b]$ such that $f(c) = u$.) Thus, there must be antipodal equatorial points at the same temperature.[4]

This explanation seems distinctively mathematical. Colyvan emphasizes that although it explains why there is always such a pair of points (rather than no such pair), it does not explain why two particular antipodal equatorial points are at the same temperature (rather than at different temperatures). To explain that fact, we would have to invoke the meteorological conditions in some neighborhoods of those points at some earlier moment. I will return to the contrast between these two explanations: one distinctively mathematical, the other not.

Pincock (2007) offers another example. Why has no one ever succeeded (or, in particular, why did a given person on a given occasion not succeed) in crossing all of the bridges of Königsberg exactly once (while remaining always on land or on a bridge rather than in a boat, for instance, and while crossing any bridge completely once having begun to cross it)? Here it is understood that the problem concerns the town's bridges as they were arranged when Euler considered this problem in 1735 (see fig. 1.1). A distinctively mathematical explanation is that in the bridge arrangement considered as a network, it is not the case

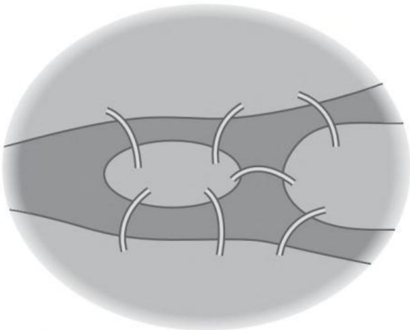

Figure 1.1 The bridges of Königsberg

that either every vertex or every vertex but two is touched by an even number of edges. (In fact, none is: one is touched by five edges, and each of the others is touched by three edges.) Any successful bridge-crosser would have to enter a given vertex exactly as many times as she leaves it unless the vertex is the start or the end of her trip. So among the vertices, either none (if the trip starts and ends at the same vertex) or two could touch an odd number of edges. Intuitively, this explanation is distinctively mathematical. It would not have been a distinctively mathematical explanation if it had instead been that no one ever tried turning left rather than right after crossing a given bridge, or that the bridges were made of a material that would immediately corrode anything in contact with them, or that someone was poised to shoot anyone who tried to cross a given bridge. By the same token, Lazare Carnot (1803, xxxvii) said: "If I propose to move a knight in the game of chess over all the squares on the chessboard, without passing twice over any given one, it doesn't concern me at all what is the mass of the knight and the force that I employ to move it." Carnot held that there is a separate science to cover such cases: "the theory of motion, considered in abstraction from the forces that produce or transmit it." This science supplies distinctively mathematical explanations.

As another example, consider why a particular attempt—or every past attempt, or every attempt ever—fails to unknot a given trefoil knot (see fig. 1.2) without cutting it. A distinctively mathematical explanation is that in three dimensions, the trefoil knot is distinct from the unknot. This explanation seems sharply different from an appeal to the fact that the knot was too tight, or that the rope was too hot to touch, or that all of those who tried to untie the knot gave up before they tried twisting the rope in a certain subtle way—each of which might explain why every attempt to disentangle some knot failed.[5]

Lipton gives the following example:

> Suppose that a bunch of sticks are thrown into the air with a lot of spin so that they twirl and tumble as they fall. We freeze the scene as the sticks are in free fall

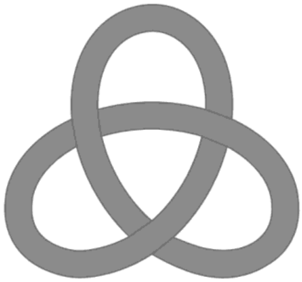

Figure 1.2 A trefoil knot

> and find that appreciably more of them are near the horizontal than near the vertical orientation. Why is this? The reason is that there are more ways for a stick to be near the horizontal than near the vertical. To see this, consider a single stick with a fixed midpoint position. There are many ways this stick could be horizontal (spin it around in the horizontal plane), but only two ways it could be vertical (up or down). This asymmetry remains for positions near horizontal and vertical, as you can see if you think about the full shell traced out by the stick as it takes all possible orientations. (Lipton 2004a, 9–10; see Lipton 2004b, 31–32; 2009b, 622)

Mancosu (2008, 134) says that the mathematical explanation of physical phenomena is "well illustrated" by Lipton's example. (Shortly I will demur.)

Perhaps these few examples suffice to suggest that, as Steiner (1978b, 18) says, "one senses a striking difference" between distinctively mathematical scientific explanations and ordinary scientific explanations that use mathematics. In the following sections, I will examine various proposals for capturing this difference and understanding how these explanations work. Ultimately, I will argue that roughly speaking, these explanations explain not by describing the world's causal structure, but rather by revealing that the explanandum is *necessary*—in particular, more necessary than ordinary laws of nature are. The Königsberg bridges as so arranged were never crossed because they *couldn't* be crossed. Mother's strawberries were not distributed evenly among her children because they *couldn't* be. The three jellybeans I selected were not all of different colors because they *couldn't* have been. Four students in Lipton's class had their last birthday on the same day of the week because this fact "could not have been otherwise" (Lipton 2009a, 47). A trefoil knot was never untied because it *couldn't* be. While Earth exists, there are always antipodal equatorial points at the same temperature because there *must* be. These necessities are stronger than the variety of necessity possessed by ordinary laws of nature, setting explanations like these apart from ordinary scientific explanations. Ultimately, I will suggest that distinctively mathematical scientific explanations work by appealing only to facts (including but not always limited to mathematical facts) that are modally stronger than ordinary laws of nature, together with contingent conditions that are contextually understood to be constitutive of the arrangement or task at issue in the why question.

In this way, distinctively mathematical explanations are examples of what I will dub "explanations by constraint," a kind of scientific explanation that has received scant attention from philosophers but that I will examine at length in the next three chapters. Explanations by constraint work not by describing the world's causal relations, but rather by describing how the explanandum arises from certain facts ("constraints") possessing some variety of necessity stronger than ordinary laws of nature possess. The mathematical facts figuring in distinctively mathematical explanations possess one such stronger variety of necessity: mathematical necessity. But (I will argue in subsequent chapters) science has taken seriously the idea that there are other varieties of necessity that are stronger than ordinary natural necessity and so that there are other "constraints" besides mathematical facts.

Although it has sometimes been suggested that distinctively mathematical scientific explanations are "non-causal," this idea requires careful elaboration (as I will show in the next section). Mancosu (2008, 135) tries to capture the distinction between distinctively mathematical and ordinary scientific explanations by saying that the former "is explanation in natural science that is carried out by essential appeal to mathematical facts."[6] But this criterion fails to exclude many ordinary scientific explanations. For example, to explain why the electric field strength at a distance r from a long, linear charge distribution with static uniform charge density λ is equal (in Gaussian CGS units) to $2\lambda/r$, we can integrate the contributions to the field (given by Coulomb's law) from all segments of the line charge (Purcell 1965, 28). When the integral is simplified, it becomes $(\lambda/r)\int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta$. The explanation then makes essential appeal to the mathematical fact that $\int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta = 2$. But intuitively this explanation is not distinctively mathematical.

An account of the distinction between distinctively mathematical explanations and ordinary scientific explanations that use mathematics should do justice to the conflicting ways in which we may find ourselves pulled in trying to classify a given explanation. For example, we may not entirely share Mancosu's confidence that Lipton's explanation of why more tossed sticks are nearly horizontal than nearly vertical is distinctively mathematical. Perhaps what is doing the explaining is a propensity of the stick-tossing mechanism (that it is equally likely to produce a tossed stick having any initial orientation) together with a propensity of the surrounding air molecules (that they are equally likely to push a tossed stick in any direction). After all, if the tossed sticks were instead all spinning uniformly about axes in the horizontal plane, they would be as likely at any moment to be vertical as horizontal, contrary to what we observe. If the explanans in Lipton's example includes the propensities of the stick-tossing mechanism and air molecules, then Lipton's example seems like the explanation of a fair coin's behavior in terms of the propensities of the chance setup (or the explanation of a gas's behavior in terms of the statistical-mechanical propensities of its molecules) rather than like the other examples I have given of distinctively mathematical explanations. An account of distinctively mathematical explanations should shed some light on this example. (I will return to it at the end of section 1.5.)

Most of the literature that I will cite concerning distinctively mathematical explanations has been motivated largely by "indispensability arguments" for the existence of mathematical entities. The basic thought behind these arguments is that if scientific theories must quantity over numbers, functions, sets, and other mathematical entities, then in accepting these theories, we are committed to the existence of these abstract entities just as we are committed to the existence of the concrete unobservable entities that these theories posit (such as electrons and the electromagnetic field). Some philosophers believe that this argument is strengthened by the fact that some scientific explanations are distinctively mathematical. Other philosophers believe that the mathematical entities figuring in these explanations are not doing the same kind of explanatory work as concrete unobservables do or that we do not become committed to the existence of abstract entities in accepting theories that quantify over them. In any case, this ontological debate in the philosophy of mathematics is irrelevant to my discussion. Philosophers engaged in this debate have generally paid relatively little attention to the questions I am pursuing: How do these "distinctively mathematical" scientific explanations differ from ordinary scientific explanations that use mathematics and how do they succeed in explaining?

Of course, we might say that a scientific explanation qualifies as "distinctively mathematical" exactly when it uses mathematics in the manner that indispensability arguments exploit—that is, exactly when the explanans must quantify over mathematical entities. However, on this way of using the term "distinctively mathematical," the ordinary scientific explanation of the fact that an infinite uniform line charge's electric field strength is inversely proportional to r counts as distinctively mathematical. This explanation quantifies over mathematical entities: the explanans includes the fact that there exists a function in which r appears solely as $(1/r)$ and that solves the integral generated by summing the contributions to the field from all segments of the line charge. Therefore, this way of using the term "distinctively mathematical" does not help to capture the intuitive difference between the various explanations that I have just given and ordinary scientific explanations that use mathematics.

In section 1.2, I will consider whether distinctively mathematical explanations are set apart by their failure to cite causes. I will argue that on the contrary, many ordinary scientific explanations fail to identify causes of the explanandum and at least some distinctively mathematical explanations do cite the explanandum's causes. Having adopted a broad notion of what makes an explanation "causal," I will argue in section 1.3 that distinctively mathematical explanations are non-causal. I will also argue that we must refine the explanatory target very carefully before we can agree with those philosophers who characterize certain explanations appealing to natural selection as distinctively mathematical. In sections 1.4 and 1.5, I will elaborate and defend my account of how distinctively mathematical explanations work. I will argue that even when such an explanation appeals to a contingent law of nature, it works by showing the explanandum to be necessary in a stronger sense than any causal explanation could.

■ 1.2 ARE DISTINCTIVELY MATHEMATICAL EXPLANATIONS SET APART BY THEIR FAILURE TO CITE CAUSES?

Mancosu (2008, 135) regards distinctively mathematical explanations as "counterexamples to the causal theory of explanation." Lipton (2004a, 9–10; 2009a, 47) and Kitcher (1989, 426) agree. Their remarks suggest that distinctively mathematical explanations are set apart from ordinary scientific explanations by their failure to specify the explanandum's causes. Writing about the tossed sticks, Lipton apparently thinks that an explanation is "causal" if and only if it cites causes of the explanandum: "The explanation why more sticks are near the horizontal than near the vertical is that there are two horizontal dimensions but only one vertical one. This is a lovely explanation, but apparently not a causal one, since geometrical facts cannot be causes" (Lipton 2004b, 31–32). However, there are two problems with the suggestion that distinctively mathematical explanations are non-causal by virtue of their failure to cite any causes of the explanandum: (1) many ordinary, causal scientific explanations fail to specify any causes of the explanandum, and (2) at least some distinctively mathematical explanations do cite the explanandum's causes. Although I agree with Mancosu, Lipton, and Kitcher that distinctively mathematical explanations are non-causal, we must be careful not to join Colyvan (1998, 324–325) in identifying an explanation as non-causal just when "it makes no appeal to causally active entities."[7]

For example, as Beebee (2004, 301–304) emphasizes, a typical explanation that appeals only to an omission or absence (along with laws of nature) is causal even though strictly speaking (according to many philosophers), an omission is not a cause since it is not even an event; it involves nothing happening. For example, Brandon (2006, 321) says that when we explain (according to classical physics) why a given body is moving uniformly (i.e., not accelerating) by citing the absence of any forces on it, we give a causal explanation that cites no causes since it cites no forces. Of course, on this view, the uniform motion of a body feeling no forces has no causes, strictly speaking.[8] Nevertheless, it has a causal explanation. The explanation works by citing a law that specifies how a body must move in the absence of any cause influencing its motion.

There are also causal explanations that cite no causes of the explanandum even though the explanandum has causes. For instance, many philosophers (e.g., Prior, Pargetter, and Jackson 1982) regard dispositions as causally impotent to bring about their manifestations. That is because the connection between a disposition (e.g., being water-soluble), its trigger (being immersed in water), and its manifestation (dissolving) is metaphysically necessary; the identities of these properties are enough to connect them in this way. In contrast, the link between causes and their effects is much weaker: mere natural necessity (that is, the kind of necessity possessed by ordinary laws of nature). Nevertheless, many philosophers (such as Lewis 1986b, 221; Jackson and Pettit 1992, 10) believe that a body's water-solubility, for example, causally explains why it dissolved when it was immersed in water. Though this explanation does not specify the molecular features of the body and water that caused the body to dissolve on being immersed in water, it is still informative regarding the causes of the body's dissolving. For instance, it rules out other possible causes of the body's dissolving, such as the body's being installed in a contraption that replaces the water with a different liquid—a powerful solvent—once the body is immersed in the water.

Instead of an explanation that uses a disposition to explain its manifestation, consider an explanation of the disposition itself. According to many philosophers, the possession of various categorical properties by various entities (together with some natural laws) explains why some entity possesses a given disposition. Such an explanation may be causal (I will argue) even though the disposition's categorical base does not *cause* the disposition. For instance, the categorical ground of a key's power to open a distant lock resides in the key's structure and the lock's structure, but the lock's structure is not a *cause* of the key's power (on pain of action at a distance on the cheap).

Thus, many explanations that fail to cite the explanandum's causes (even when it has causes) are nevertheless causal explanations. Accordingly, I will adopt a broad conception of what makes an explanation "causal": that it explains by virtue of describing contextually relevant features of the explanandum's causal history or, more broadly, of the world's network of causal relations. To do this, it does not need to specify any causes. When a body's water-solubility explains why it dissolved when it was immersed in water, the explanans explains without identifying the particular causally efficacious properties possessed by the immersed body and the water (having to do with various molecular forces and their causes). That the immersed body is water-soluble explains by supplying information about the explanandum's causal history. By the same token (as Jackson and Pettit 1990 argue, and Colyvan 1998, 324 also maintains), when the squareness of a rigid peg of side L explains its failure to fit through a round hole of diameter L (made through a rigid board), the peg's squareness is not itself a cause of the failure to fit. Rather, it ensures that there are such causes—namely, contact forces between the peg and the material surrounding the hole. Thus, the peg's squareness figures in a causal explanation despite not being causally efficacious.

The peg's squareness, though not itself a cause, ensures that the peg would still have failed to fit though the hole in the board even if the peg had been aligned differently with the hole so that the particular peg and board molecules that actually strongly repelled one another remained too far apart to interact. Other peg and board molecules would have strongly repelled one another instead. The peg's squareness explains not just by describing the actual causes of the outcome, but also by telling us that had different initial conditions prevailed, similar causes would have produced a failure to fit through the hole. As Jackson and Pettit (1992) emphasize, many causal explanations derive their explanatory power partly from describing what the world's network of causal relations would have been like under other conditions. Railton (1981, 251) nicely captures why such explanations are best understood as importantly like explanations that work by specifying the outcome's actual causes:

> this sort of causal process is such that its macroscopic outcomes are remarkably insensitive (in the limit) to wide variations in initial microstates. The stability of an outcome of a causal process in spite of significant variation in initial conditions can be informative about an ideal causal explanatory text in the same way that it is informative to learn, regarding a given causal explanation of the First World War, that a world war would have come about (according to this explanation) even if no bomb had exploded in Sarajevo. This sort of robustness or resilience of a process is important to grasp in coming to know explanations based upon it.

Similarly, the explanation of Mother's failure to distribute her 23 strawberries evenly (without cutting any) among her 3 children reveals that her failure is insensitive to her precise technique for distributing strawberries. However, this explanation does *not* work by describing the various causal relations that would have obtained if she had tried other ways of distributing the strawberries, and then showing that each of these causal processes would have failed to distribute the strawberries evenly. This explanation does not describe any technique of strawberry distribution. It derives its explanatory power neither by virtue of describing the actual causes nor by telling us about what the network of causal relations would have been like under other initial conditions. Rather (I will argue), this explanation shows the outcome to be inevitable to a stronger extent than facts merely about causal relations (actual and counterfactual) could make it. This explanation is distinctively mathematical.

Sober (1983; 1984, 139–142) has characterized "equilibrium explanations" that work by describing not only the actual causal relations, but also the causal relations that would have obtained under other initial conditions. Sober's chief example is R. A. Fisher's natural-selection explanation of the fact that the sex ratio at reproductive age is 1:1 in many species. The gist of Fisher's explanation is that if a population contains more males than females, then individuals who have a heritable tendency to produce more female than male offspring will tend to have more grandchildren than individuals without this tendency, and so this tendency will tend to spread and to restore the sex ratio to 1:1. If, on the other hand, a population contains more females than males, then the heritable tendency to produce more male than female offspring will be selectively advantageous, tending to restore the 1:1 ratio. Sober correctly emphasizes that such an equilibrium explanation of a population's current 1:1 ratio does not describe the particular causal path by which the population arrived at that ratio; it does not specify the varying sex ratios or selection pressures at earlier moments. Rather, the explanation works by showing how the tendency to return to a 1:1 ratio arises from various unchanging features of the population, including that the only selection pressure on the sex ratio is selection for individuals who have a tendency to overproduce the minority sex, that such a heritable tendency is present among some members of the population, and that male and female offspring require the same resources.

Sober (1983, 204) insists that a causal explanation of a population's current 1:1 ratio would differ from an equilibrium explanation in describing the actual history of the population's sex ratio and selection pressures. Sober concludes that an equilibrium explanation is not a causal explanation. But I see no reason to regard explanations that work by specifying the actual causes as fundamentally different from explanations that work by describing what the causes would have been like under certain conditions that extend significantly beyond the actual ones. Accordingly, unlike Sober in 1983, I take equilibrium explanations to be causal explanation.[9]

Both equilibrium explanations and distinctively mathematical explanations show the facts being explained to be inevitable. But equilibrium explanations (unlike distinctively mathematical explanations) work by describing the world's causal structure and so (I will argue) cannot show the fact being explained to be as inevitable as distinctively mathematical explanations do. Mother's success was *mathematically* impossible, not merely impossible by virtue of the world's causal structure. As Sober (1983, 207) remarks, an "equilibrium explanation situates [the sex ratio's] actual trajectory (whatever it may have been) in a more encompassing structure." That structure is a causal structure.[10]

Let's see another sort of causal explanation that specifies no actual causes of the fact being explained, but works by virtue of identifying the causal relations that would obtain under various conditions. Consider the explanation (given in section 1.1) of the derivative law concerning the strength of the electric field of a uniform infinite line charge. This explanation works by deducing the explanandum from Coulomb's law. This is a causal explanation even though "the explanation of a general law by deductive subsumption under theoretical principles is clearly not an explanation by causes" (Hempel 1965, 352) since laws are not causes. The explanandum is a law and so has no causes; its explainers are not causes. Nevertheless, this explanation works by describing part of the world's network of causal relations—in particular, the causes of the electric fields of uniform infinite line charges (namely, line-charge segments) and how the contributions made by the various causes of a given infinite line charge's field combine. A general regularity (whether a law, correlation, or statistical-relevance relation) is explained causally by relevant features of the causal mechanisms that produce the events figuring in the regularity. As another example, take the explanation of Kepler's "laws" by Newton's laws of motion and gravity (and some contingent features of the solar system). Harman (1986, 73) calls this explanation "non-causal" since it does not work by citing prior events that caused a given event; neither the explanans nor the explanandum specifies events. By contrast, I would emphasize that this explanation of Kepler's laws works by describing the way planetary motion is caused. Therefore, I deem it to be a causal explanation.[11]

Consider again the explanation of the derivative law concerning the strength of the electric field of a uniform infinite line charge. Perhaps there happens never to be a uniform infinite line charge. The explanation of the derivative law then cannot work by describing the causal histories of the fields of actual uniform infinite line charges. Nevertheless, the explanation remains causal. It works not by describing any actual event's causes, but rather by describing the world's network of causal relations—specifically, by describing what that network would have been like, had there been an infinite line charge.[12] In this respect, the explanation is like Sober's "equilibrium explanation": it works by describing what would cause what.

Likewise, to explain why neon is chemically inert, we could give the various causal mechanisms behind the chemical activity of other atoms and then show why atoms like neon (having filled outer electronic shells) are unable to form chemical bonds in any of these ways. This is a causal explanation of neon's inertness even though (as Kitcher 1985, 637 says) it does not describe any causal processes at work in neon. It explains by virtue of describing the world's network of causal relations (in particular, the causes of other atoms' chemical activity).

Similarly, when a disposition is explained by its categorical base (and natural laws), the categorical base does not cause the disposition (as I mentioned earlier). The explanation does not supply information about the disposition's causal history (since the disposition has none, strictly speaking). Lewis (1986b, 223–224) says that no event is being explained. But, as Lewis also says, the explanation is nevertheless causal in that it works by supplying relevant information about the world's network of causal relations. The categorical base explains the disposition by virtue of the base's role as what would cause any manifestations of the disposition.

By the same token, when we explain why some body is moving uniformly (rather than nonuniformly) by noting that the body is experiencing no forces, we are not giving the explanandum's causes (since it has none). But we are explaining by virtue of describing a relevant aspect of the world's network of causal relations. In particular, we are explaining by specifying the forces (the acceleration-causers) that are acting on the body, namely, none. That there are no forces acting on the body qualifies as explanatorily relevant by virtue of the fact that forces cause accelerations. Likewise, the explanation cites a law (Newton's first law of motion) that helps to explain by virtue of governing how bodies behave in the absence of any causes of accelerations (that is, forces). We have here a causal explanation because the facts that explain are explanatorily relevant by virtue of their significance regarding the world's network of causal relations.[13]

Likewise, we might explain why visible light has a given speed in a given medium by the fact that electromagnetic waves within a certain range of frequencies have that speed there together with the fact that visible light consists of electromagnetic waves in that frequency range. Light's speed is not *caused* by the speed of electromagnetic waves since visible light and electromagnetic waves of those frequencies are not distinct things; light is identical to electromagnetic waves in that frequency range. Nevertheless, an explanation of light's behavior that appeals to some properties of electromagnetic waves is a causal explanation because it works by supplying relevant information about the world's network of causal relations. For instance, an explanation of light's speed in a given medium that appeals to the speed of electromagnetic waves there works by telling us that light's speed in that medium is caused by whatever causes the speed of electromagnetic waves there—and that those factors cause light's speed by virtue of causing electromagnetic waves' speed. The reverse is not true; although light is an electromagnetic wave, light behaves as it does by virtue of being an electromagnetic wave, not the reverse. The laws of electromagnetism causally explain the laws of light (just as the Coulomb's law causally explains the laws giving the electric field of a long linear uniform charge distribution). Just as a machine's capacity to do something is explained causally by some of the capacities that its components possess (capacities that combine to form the capacity being explained), so likewise light's capacity to do something is explained causally by the capacities of the electromagnetic waves that constitute it. These are all causal explanations because they work by supplying information about the causes that some effects would have. For instance, light's color and intensity acquire their powers to affect the eye by virtue of being electromagnetic properties.[14]

In short, then, what makes an explanation "causal" is not that it cites the explanandum's causes, since the explanandum in a causal explanation need not have any causes, and even if it does, a causal explanation need not specify them—and (as I will show shortly) even a non-causal explanation may specify the explanandum's causes. Rather, what makes an explanation "causal" is how it works: that it derives its explanatory power by virtue of supplying relevant information about the explanandum's causes or, more broadly, about the world's network of causal relations. In other words, in a causal explanation, an explainer's explanatory credentials derive partly from the information it supplies regarding the world's network of causal relations. Any causes cited by a causal explanation explain by virtue of being causes and thereby supplying information about the network of causal relations—and since even non-causes can supply such information, they can figure in causal explanations, too. As I will show, when causes figure in non-causal explanations, the source of their explanatory power is not their status as causes.

Of course, this is not to say that in a causal explanation, any information whatever about the world's network of causal relations (or even about the explanandum's causes in particular) is explanatory. A principal aim of any account of causal explanation is to specify what makes some such information explanatorily relevant. The length of a building's shadow and the sun's angle of elevation in the sky supply some information about the cause of the building's height: that it caused a building tall enough to cast a shadow of that length when the sun is elevated to that angle. But the shadow's length and the sun's angle nevertheless do not help to explain the building's height because they do not supply the right sort of information about the world's network of causal relations. A causal explanation works by virtue of supplying the right sort of information about the world's causal nexus, and an account of causal explanation must say a good deal about what determines "the right sort" in various cases.

This conception of what it is for an explanation to be "causal" is hardly original. Lipton (2004b, 32), for example, writes that "causal explanations are explanatory *because* they are causal" (his emphasis). That is, what makes an explanation causal is not that it cites causes or that it supplies information about causes, but rather that it explains *by virtue of* doing so—that this is how the explanation manages to explain. I take this notion of "causal explanation" to be what many philosophers have had roughly in mind in holding that all scientific explanations are causal. Of course, one could use the term "causal explanation" more narrowly than I do, as when Sober (1983, 203) says "A causal explanation describes what the cause is" and concludes that equilibrium explanations are non-causal. As I mentioned earlier, I do not contend that my broader conception of "causal explanation" fits better with firm pretheoretic intuitions about the term's proper use (I doubt there are any) or with the term's widespread use in scientific practice (I doubt it has any). Rather, my conception of "causal explanation" fits the term's use by those philosophers who contend that all scientific explanation is causal and allows us to draw an important distinction between different ways in which scientific explanations work. As I have just shown, some explanations that do not "describe what the cause is" still work very much like explanations that do work by specifying the explanandum's cause. Therefore, it is illuminating to group these various explanations together—especially when we are trying to understand explanations that appear to work in a radically different way.[15] Although my conception of what it takes for an explanation to be "causal" is sufficiently broad to allow non-causes (such as laws) to supply causal explanations and even to admit causal explanations that cite no causes, it is not broad enough to encompass all scientific explanations. Not all scientific explanations derive their explanatory power from describing the world's network of causal relations.[16]

If (as I believe) Mancosu, Lipton, and Kitcher are correct in deeming distinctively mathematical scientific explanations to be non-causal, then those explanations cannot work by describing the world's network of causal relations. How, then, do they work? In section 1.4, I will propose that they work by constraining what there could be.

I have just argued that an explanation that fails to cite any causes nevertheless qualifies as causal if it explains by virtue of describing the world's network of causal relations. By the same token, some distinctively mathematical explanations, though non-causal, nevertheless happen to cite the explanandum's causes. Even so, they qualify as non-causal because they do not derive their explanatory power from their success in describing the world's network of causal relations specifically.

For instance, that Mother had 3 children and 23 strawberries were causes of her failure a moment ago when she tried to distribute her strawberries evenly among her children. That these were causes of her failure is the common verdict of many different accounts of causal relations. For instance, Lewis's counterfactual account says that C causes E exactly when there is a chain of stepwise "influence" from C to E, where C "influences" E exactly when "there is a substantial range C_1, C_2, . . . of different not-too-distant alterations of C (including the actual alteration of C) and there is a range E_1, E_2, . . . of alterations of E, at least some of which differ, such that if C_1 had occurred, E_1 would have occurred, and if C_2 had occurred, E_2 would have occurred, and so on" (Lewis 2007, 476). Such a pattern of counterfactual dependence obtains in the strawberry example: if Mother had had 24 strawberries (or 2 children and 22 strawberries), for instance, then she would not have failed. Alternatively, a manipulability account of causal relations (Gasking 1955; Woodward 2003) says roughly that C is a cause of E exactly when systematic changes in E can be brought about by suitable interventions on C. Clearly, manipulation of the numbers of strawberries or children would bring about corresponding changes in the outcome of Mother's attempt. Likewise, that there are 3 children and 23 strawberries raises the probability of the outcome from what it otherwise would be (in accordance with probabilistic accounts of causal relations), and there is a causal process of "maternal strawberry distribution" connecting the outcome to the initial conditions (in accordance with accounts inspired by Salmon 1984).

Nevertheless, I maintain that this explanation is non-causal because it does not work by virtue of describing the outcome's causes or, more broadly, the world's network of causal relations. The causal mechanism by which Mother distributed her strawberries does not enter into it. The numbers of children and strawberries do not figure in this explanation as causes of the outcome. A distinctively mathematical scientific explanation may happen to cite causes, but it does not appeal to them *as causes*. It does not work by exploiting their causal powers.[17]

That a distinctively mathematical explanation happens to cite facts about the explanandum's causes does not mean that it works by virtue of describing the explanandum's causes. In the distinctively mathematical explanation, Mother's having 3 children helps to explain her failure to distribute the strawberries evenly not by virtue of being a cause of her failure, but rather by virtue of helping to make her success mathematically impossible. By the same token, the fact that 23 cannot be divided evenly by 3 supplies information about the world's network of causal relations: it entails that there are no causal processes by which 23 things are distributed evenly (without being cut) into 3 groups. But in the distinctively mathematical explanation of Mother's failure, the fact that 23 cannot be divided evenly by 3 does not possess its power to explain by virtue of supplying this information about causal processes in particular. The distinctively mathematical explanation does not exploit what the world's *causal structure* is like as a matter of mathematical necessity. Rather, it exploits what *the world* is like as a matter of mathematical necessity: the fact that 23 things cannot mathematically possibly be divided evenly (while remaining uncut) into 3 groups explains why no collection of 23 things is in fact ever so divided. The mathematical fact entails that even a pseudoprocess rather than a causal process (and even a world without causal processes) cannot involve such a division of 23 things. The mathematical fact supplies information about the world's network of causal relations (just as any fact does: that the cat is on the mat tells us that the world's network of causal relations includes no events caused by the cat's being off the mat). But its supplying information about the world's *causal* network per se is not responsible for its explanatory power in the distinctively mathematical explanation. In contrast, in a causal explanation, a fact's supplying (the right sort of) information about the world's causal network per se is responsible for the fact's explanatory significance.

As I showed earlier, there is a non-causal explanation for the existence at a given moment of antipodal equatorial points having the same temperature. Perhaps this occurrence also has a causal explanation that appeals to whatever prior meteorological conditions (and natural laws) explain why those particular antipodal equatorial points have the same temperature (rather than different temperatures) at the given moment. Of course, the non-causal explanation shows that even if meteorological conditions had been different so that the temperatures at those particular antipodal points had been unequal, there would still have been a pair of antipodal equatorial points at the same temperature. Perhaps, then, prior meteorological conditions do not explain why a pair of antipodal equatorial points having the same temperature exists, even though the explanandum is entailed by a (contingent) fact that the meteorological conditions do explain, namely, that these particular antipodal equatorial points have the same temperature.[18] On the other hand, there are well-known examples where C causally explains E even though E would still have occurred, had C not occurred. (Standard examples include cases of preemption, as when Assassin kills Victim, but had Assassin not pulled the trigger, Backup would have done so.) Whether or not the existence of a pair of antipodal equatorial points having the same temperature has a causal explanation is a question about causal explanation and overdetermination that I will put aside.

However, even if there is such a causal explanation for the existence *at a given moment* of a pair of antipodal equatorial points having the same temperature, I contend that if we take this causal explanation and combine it with another causal explanation regarding another such pair of points *at another moment*, then we do *not* thereby explain why it is that *at both moments* there are antipodal equatorial points at the same temperature. That is because this pair of causal explanations inaccurately depicts this similarity between the two moments as utterly coincidental—as having no important common explainers—since the earlier meteorological conditions relevant to one moment are largely disjoint from those relevant to the other moment.

Let's linger momentarily on this notion of being a "coincidence." It is a coincidence that President Kennedy and President Lincoln both had vice presidents named Johnson. This fact is coincidental in virtue of its two components having no common cause—or, at least, none of any interest; in a typical context in which this Kennedy-Lincoln fact is entertained, the cosmological Big Bang (for example) is not of any interest. We can explain why Kennedy and Lincoln both had vice presidents named Johnson by causally explaining why Kennedy had a vice president named Johnson and similarly explaining why Lincoln did. These two causal explanations have nothing interesting in common, and the same goes for every explanation of the fact that Kennedy and Lincoln both had vice presidents named Johnson. That is what makes this fact coincidental.

To understand what makes it no coincidence that at each of the two moments, a pair of antipodal equatorial points having the same temperature exists, we must recognize that for a fact to qualify as coincidental, it is not enough for its components to have no interesting common *cause*. Rather, it is enough to make a fact coincidental that its components have no interesting common *because*—that is, no interesting common *explainer*. In fact, the similarity between the two moments (in that at both, there is a pair of antipodal equatorial points having the same temperature) is not coincidental. The two components of this fact have interesting common explainers (e.g., that temperature is a continuous function of position)—indeed, enough common explainers to give these two components a common explanation: the distinctively mathematical explanation.

Admittedly, then, there may be a causal explanation of the fact that *at a given moment*, two antipodal equatorial points at the same temperature exist. Nevertheless, there is no causal explanation of the fact that *at every moment* (or: at two arbitrary moments) in Earth's history, two such points exist.[19] This fact is explained only by a non-causal, distinctively mathematical explanation.

If the similarity between the two moments had indeed been coincidental (like the Kennedy-Lincoln fact), then their similarity would be explained by the combination of the two separate causal explanations (one for each moment), if there are such causal explanations. But since the similarity between the two moments is in fact no coincidence, any genuine explanation must so characterize it.

■ 1.3 MATHEMATICAL EXPLANATIONS DO NOT EXPLOIT CAUSAL POWERS

I have just suggested that distinctively mathematical explanations are non-causal, even if some of them appeal to causes of what they explain, because they do not appeal to them *as causes*; they do not exploit their causal powers. In section 1.4, I will suggest that if some association between a cause (in the explanans) and its effect (the explanandum) is invoked by a distinctively mathematical explanation, then that association holds not by virtue of an ordinary law of nature, but by virtue of something modally stronger—typically, by mathematical necessity.[20] In this way, mathematics enters distinctively mathematical explanations.

It may be objected that although a distinctively mathematical explanation appeals to mathematical facts, it also exploits the causal powers of some of the explanandum's causes. In the Königsberg bridge case, for example, the arrangement of bridges and islands initially (i.e., at the start of some attempt to cross them) helps to cause their arrangements later (while the attempt is under way), and this fact is crucial to the distinctively mathematical explanation. (After all, matters would be very different if it were a law of nature that whenever someone starts to traverse a bridge, its beginning and ending points come to be touched by an even number of bridges, perhaps by another bridge's coming into existence.)

Likewise, in the example involving Mother's attempt to distribute strawberries to her children, the numbers of children and strawberries initially (when Mother begins her attempt) are causes of their numbers later. That bridges are not brought into existence or caused to disappear by people traveling over other bridges, that strawberries are not caused to replicate by being distributed, and that (in the trefoil knot example) knotted ropes do not spontaneously break, their ends then tending to reunite, all reflect the causal powers of various things and are matters of contingent natural law, not mathematical necessity. These facts underlie the distinctively mathematical explanations I have given.

I reply that these distinctively mathematical explanations do not exploit these causal powers. Rather, the fixity of the arrangement of bridges and islands, for example, is presupposed by the why question that the explanation answers: Why did this attempt (or every attempt) to cross this particular arrangement of bridges (the bridges of Königsberg in 1735) end in failure? The bridges' arrangement does not function in the distinctively mathematical explanation as an initial condition that in fact persists during all attempts to cross the bridges (partly by virtue of various ordinary laws of nature describing various kinds of causal interactions). Rather, the why question itself takes the arrangement as remaining unchanged over the course of any eligible attempt. If, during the course of an attempt to cross all of the bridges exactly once, one of the bridges happened to collapse before it had been crossed, then the attempt in progress would simply be disqualified from counting as having managed to cross the intended arrangement of bridges. The laws determining the conditions under which the bridges' arrangement would remain fixed thus do not figure in the explanans.

If every distinctively mathematical explanation used no contingent laws of nature, then this feature would nicely distinguish these explanations from many ordinary scientific explanations that use mathematics, such as the explanation (described in section 1.1) of any infinite uniform line charge's electric field strength. However, it would not distinguish distinctively mathematical explanations from all non-causal scientific explanations. For example, the non-causal explanation of the fact that Mark Twain and Samuel Clemens have the same height (namely, because Twain and Clemens are identical) appeals to no contingent natural laws (see the preface). Furthermore, even if every distinctively mathematical explanation used no contingent laws of nature, this feature would not distinguish distinctively mathematical explanations from all causal explanations; some causal explanations likewise appeal to no contingent natural laws. For example, a given biological trait's increasing frequency in some population may be explained by the absence of mutations and migrations together with the trait's fitness exceeding the fitness of any alternative to it that was available to the population. Such an explanation appeals to no contingent natural laws, but rather to the principle of natural selection (PNS)—roughly speaking: that fitter traits are more likely to increase in frequency in the absence of mutation or migration. The PNS is a broadly logical truth. (That is, its modality belongs with narrowly logical necessity, metaphysical necessity, mathematical necessity, conceptual necessity, moral necessity, and so forth.) The PNS is not a contingent natural law, but it is also not a mathematical fact.

That one trait is fitter than another entails that there is selection *of* the fitter trait but not that there is selection *for* that trait (Sober 1984). That is, the fitter trait might not *make* various creatures more likely to have a greater number of viable offspring (so it might not be selected *for*), but might merely tend to be associated with traits that do (so that there is selection *of* creatures with that trait). A selectionist explanation of a trait's increasing frequency (or its current high frequency) might go beyond citing the trait's greater fitness to identify the particular selection pressures at work; it might specify whether or not the given trait has been selected *for* and, if so, why. It might, for instance, explain how the trait represents an optimal solution to some challenge that such a creature faces. Baker (2005, 229–235; 2009b) characterizes one such selectionist explanation as a distinctively mathematical explanation. Although Batterman (2010, 3) finds Baker's example "interesting and persuasive" and Leng (2005, 174) agrees that it qualifies as a distinctively mathematical explanation, I think we must first draw some distinctions before we can find here a distinctively mathematical explanation.

The explanandum in Baker's example is "that cicada life-cycle periods are prime" rather than composite numbers of years (Baker 2009b, 624). One possible explanation that biologists have offered is that a species with a periodic life-cycle maximizes its chance of avoiding predator species that also have periodic life-cycles exactly when the species' period in years is coprime to the most numbers close to it (where natural numbers m and n are "coprime" exactly when they have no common factors except 1). That is because if two species' periods m and n are coprime, then their coincidence is minimized (since mn is their lowest common multiple). Since a *prime* number m is coprime to the most numbers close to it (namely, to every number less than $2m$), it is evolutionarily advantageous for cicada life-cycle periods to be prime and so (if this is the only relevant consideration) they are likely to be prime.

However, if this is the reason why cicada life-cycle periods are prime numbers of years, then (it seems to me) this explanation works by describing the world's network of causal relations—in particular, the natural history of cicadas. Consider the explanation that cicadas have prime periods because prime periods have been selected for (and this is the only relevant selection pressure and the PNS holds). This is a causal explanation, since "selection *for* is the causal concept par excellence" (Sober 1984, 100). Suppose we add that prime cicada periods have been selected for over composite periods because some of the cicada's predators also have periodic life-cycles, the avoidance of predation by these predators is selectively advantageous to cicadas, and prime cicada periods tend to minimize this predation while bringing to cicadas no selective disadvantages that outweigh this advantage. This explanation is also just an ordinary causal explanation. It uses a bit of mathematics in describing the explanandum's causal history, but it derives its explanatory power in the same way as any other selectionist explanation. Taken as a whole, then, it is not a distinctively mathematical explanation, though it appeals to some mathematics and to no contingent natural laws.

But suppose we narrow the explanandum to the fact that in connection with predators having periodic life-cycles, cicadas with prime periods tend to suffer less from predation than cicadas with composite periods do. This fact has a distinctively mathematical explanation (namely, the explanation given above involving coprime numbers).[21] Analogous remarks apply to the selectionist explanation that Lyon and Colyvan (2008, 228–229) characterize as distinctively mathematical: "What needs explaining here is why the honeycomb is always divided up into hexagons and not some other polygons (such as triangles or squares), or any combination of different (concave or convex) polygons" (Lyon and Colyvan 2008, 228). The proposed explanation is that it is selectively advantageous for honeybees to minimize the wax they use to build their combs—together with the mathematical fact that a hexagonal grid uses the least total perimeter in dividing a planar region into regions of equal area (the "Honeycomb Conjecture" proved recently by Thomas Hales; see Hales 2001). Again, this explanation works (purportedly) by describing the relevant features of the selection pressures that honeybees have experienced, so it is an ordinary causal explanation, not distinctively mathematical.[22] But suppose we narrow the explanandum to the fact that in any scheme to divide their combs into regions of equal area, honeybees would use at least the amount of wax they would use in dividing their combs into hexagons of equal area (assuming combs to be effectively planar and the dividing walls' thickness to be negligible). This fact has a distinctively mathematical explanation: it is just an instance of the Honeycomb Conjecture. By the same token, "word problems" in mathematics textbooks are full of allusions to facts that have distinctively mathematical explanations—for example, the fact that if Farmer Brown, with 50 feet of negligibly thin and infinitely bendable fencing, uses his fencing to enclose the maximum area in a flat field, then he makes his fencing into a circle.

■ 1.4 HOW THESE DISTINCTIVELY MATHEMATICAL EXPLANATIONS WORK

In the previous section, I suggested that if every distinctively mathematical explanation used no contingent laws of nature, then this feature would nicely distinguish distinctively mathematical explanations from many (though not all) causal explanations that use mathematics, such as the explanation of an

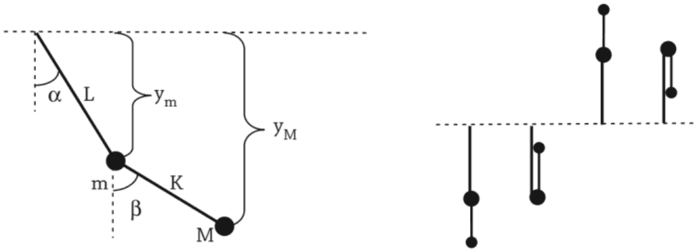

Figure 1.3 The simple double pendulum (left) and its four equilibrium configurations (right); only the first equilibrium configuration is stable.

infinite uniform line charge's electric field strength. However, some distinctively mathematical explanations appeal to contingent natural laws. Here is an example.

Suppose we make a "simple double pendulum" by suspending a simple pendulum from the bob of another simple pendulum and allowing both bobs to move under the influence of gravity (which varies negligibly with height) while confined to a single vertical plane (see fig. 1.3). (By definition, a "simple pendulum" has an inextensible cord with negligible mass and encounters negligible friction and air resistance.) Any simple double pendulum has exactly four equilibrium configurations (see fig. 1.3), where a "configuration" is fixed by the angles *α* and β. (An "equilibrium configuration" is a configuration where the two bobs, once placed there at rest, will remain there as long as the system is undisturbed.)

One way to explain why a simple double pendulum has these four equilibrium configurations is to identify the particular forces on the two bobs (with masses *m* and *M*, as shown in fig. 1.3) and then to determine the configurations in which both bobs feel zero net force. By Newton's second law of motion, they will then undergo no acceleration and so will remain at rest once placed in that configuration.[23] Equivalently, since the force on a system is the negation of its potential energy's gradient,[24] we can express the system's potential energy $U(\alpha,\beta)$ and then solve for the configurations where the energy's gradient is zero, that is, where $\frac{\partial U}{\partial \alpha} = \frac{\partial U}{\partial \beta} = 0$—that is to say, where U is "stationary" (i.e., at a maximum, minimum, or saddle point):

$$U(\alpha,\beta) \;=\; -mg\,y_m - Mg\,y_M$$

$$y_m = L\cos\alpha$$

$$y_M = L\cos\alpha + K\cos\beta$$

$$\text{So } U(\alpha,\beta) \;=\; -mg\,L\cos\alpha - Mg\left(L\cos\alpha + K\cos\beta\right)$$

$$\frac{\partial U}{\partial \alpha} = mgL \sin \alpha + MgL \sin \alpha$$

$$\frac{\partial U}{\partial \beta} = MgK \sin \beta.$$

Hence, U's gradient vanishes exactly when $\sin \alpha = \sin \beta = 0$, which is exactly where $(\alpha,\beta) = (0,0), (0,\pi), (\pi,0)$, or (π,π)—the four equilibrium configurations shown in figure 1.3.

This is a causal explanation.

But there is a non-causal, distinctively mathematical explanation of the fact that a simple double pendulum has at least four equilibrium configurations. Since (α,β) and $(\alpha + 2\pi n,\beta + 2\pi m)$ designate the same configuration (for any integers n, m), the configuration space of any double pendulum can be represented as the points on a toroidal surface (see fig. 1.4). Since $U(\alpha,\beta)$ is everywhere finite and continuous, it can be represented by distorting the torus so that each point (α,β)'s height equals $U(\alpha,\beta)$. Any such distortion remains a surface of genus $g = 1$ (i.e., topologically equivalent to a torus, which is a sphere with $g = 1$ holes in it). For any surface (as long as it is smooth, compact, orientable, etc.), the numbers of minima, maxima, and saddle points obey the equation $N_{min} - N_{sad} + N_{max} = 2 - 2g$, which equals zero for $g = 1$.[25] By compactness, there must be at least one maximum and one minimum, so by this equation, there must be at least two saddle points—and so at least four stationary points in total.

This is a non-causal explanation because it does not work by describing some aspect of the world's network of causal relations. No aspect of the particular forces operating on or within the system (which would make a difference to $U(\alpha,\beta)$) matters to this explanation. Rather, the explanation exploits merely the fact that by virtue of the system's being a double pendulum, its configuration space is the surface of a torus—that is, that U is a function of α and β. This topological explanation is similar to the distinctively mathematical explanation of the fact that there are always antipodal equatorial points of the same

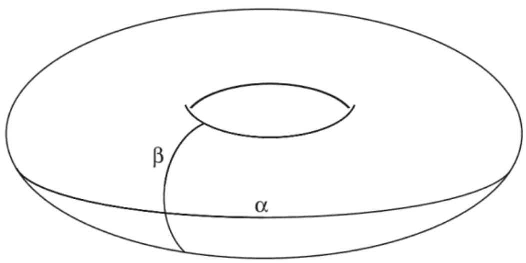

Figure 1.4 The topology of the configuration space of a simple double pendulum.

temperature—except that the relevant surface is in configuration space rather than physical space.

Since the configuration space of *any* double pendulum is a torus, the same explanation applies to any double pendulum, not just to a simple one. For example, the same explanation applies to a compound square double pendulum (see fig. 1.5). It also applies to a double pendulum where the two suspended extended masses are not uniformly dense and to a complex double pendulum under the influence of various springs forcing its oscillation. Each of these has at least four equilibrium configurations, though the particular configurations (and their precise number) differ for different types of double pendulums. Although the *causal* explanation of the system's *particular* equilibrium configurations differs for each of these kinds of pendulum (since their potential energy functions differ), the distinctively mathematical explanation of its having at least four equilibrium configurations is the same in each case.

Perhaps there is a causal explanation of a given double pendulum's having at least four equilibrium configurations (namely, an explanation that first explains why it has certain particular equilibrium configurations rather than different ones, and then points out that those equilibrium configurations number at least four). But even if there is such a causal explanation for a given double pendulum, the combination of two such explanations (e.g., for a simple double pendulum and a complex double pendulum) does not explain why the two pendulums are alike in having at least four equilibrium configurations. That is because this combination appeals to importantly different factors for the two pendulums and so inaccurately depicts as coincidental their similarity in having at least four equilibrium configurations.

The reason I have mentioned the distinctively mathematical explanation of a double pendulum's having at least four equilibrium configurations is that although this explanation does not derive its explanatory power from its

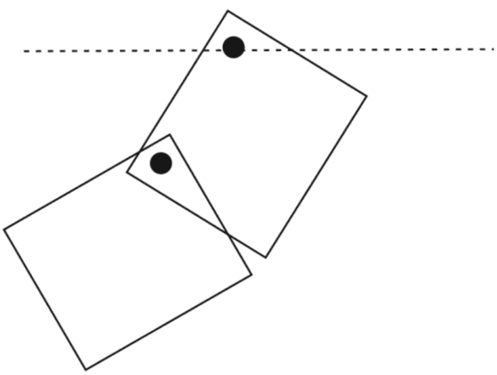

Figure 1.5 A compound square double pendulum

describing the causes operating on the system, it does appeal to a contingent natural law: that a system is at equilibrium exactly when the net force on each of its parts is zero (i.e., when its potential energy is stationary)—a particular case of Newton's second law. Why doesn't this law make the explanation causal? Because, I suggest, Newton's second law describes merely the framework within which any force must act; it does not describe (even abstractly) the particular forces acting on a given situation. Any *possible* force accords with Newton's second law. For example, had gravity been an inverse-cube force, then it would still have operated according to Newton's second law. Had there been some other (physically impossible) kind of forces in addition to the actual kinds, Newton's second law would still have held. Such counterlegals are sometimes invoked in science, as when Ehrenfest (1917) famously showed that had gravity been an inverse-cube force or fallen off with distance at any greater rate, then planets would eventually have collided with the Sun or escaped from the Sun's gravity. Ehrenfest's argument requires that Newton's second law would still have held, had gravity been an inverse-cube force or fallen off with distance at any greater rate. As I will explain further in the next three chapters, Newton's second law—like the conservation laws (to be discussed in chapter 2), the spacetime coordinate transformation laws (in chapter 3), and the parallelogram law for the composition of forces (in chapter 4)—according to many scientists "transcends" (Wigner 1972, 13) the peculiarities of the various kinds of forces that happen to exist (e.g., electromagnetic, gravitational) in that it would still have held even if those forces had been different.

Indeed, even just to say "the peculiarities of the various kinds of forces that *happen* to exist" is to recognize that although these individual force laws are matters of natural necessity, Newton's second law is *more* necessary than they. Compared with it, they *happen* to hold. In other words, although Newton's second law is not a mathematical, conceptual, metaphysical, or logical truth, it stands closer to them modally than an ordinary law does. Thus, an explanation that shows the explanandum to follow entirely from such laws thereby shows the explanandum to be necessary in a way that no explanation that depends on a force law could show it to be—or, more broadly, in a way that no causal explanation could show it to be.

Of course, Newton's second law characterizes the causal relation between force and motion: that the force experienced by a point particle (together with its mass) causes it to undergo a certain acceleration. When Newton's second law (together with the force on and mass of a point body) explains the body's acceleration, it is figuring in a causal explanation. However, that is not what it is doing in connection with the distinctively mathematical explanation of a double pendulum's having at least four equilibrium configurations. That Newton's second law describes a *causal* relation does not matter to that explanation. All that matters is that there is a certain relation between the force (or energy) function and equilibrium, regardless of whether and how one is causally related to the other—and that this relation is modally stronger than the causal details. Recall that the number of Mother's children helps to *cause* her to fail in her attempt to distribute the strawberries evenly. But that *causal* role is not the role that the number of Mother's children plays in the distinctively mathematical explanation. That role is to make Mother's success *impossible* (in a stronger respect than causal considerations underwrite). Likewise, the causal relation between force, mass, and acceleration helps to causally explain many features of a double pendulum's behavior. But that is not its role in the distinctively mathematical explanation of the existence of at least four equilibrium configurations. Its role there is to make the existence of at least four equilibria inevitable (in a stronger respect than causal considerations could). What matters to that explanation is the law's role in the framework that any possible double pendulum must inhabit. What the law contributes to the explanation is its strong variety of necessity, which helps to make it impossible (in that strong respect) for a double pendulum to have fewer than four equilibrium configurations. This strong necessity makes no contribution to a causal explanation of a body's acceleration; as far as such a causal explanation is concerned, Newton's second law of motion might just as well not transcend the various particular force laws.

Any causal explanation in terms of forces must go beyond Newton's second law to exploit the particular forces at work—if not specifying them fully, then at least appealing to their relevant features (such as their proportionality to the inverse-square of the distance). This is not done by the distinctively mathematical double-pendulum explanation (which is why it can apply to double pendulums that differ in the particular forces at work). Therefore, this explanation qualifies as non-causal despite including Newton's second law. By contrast, even an "equilibrium explanation" of why a given ball, released just inside the rim of a concave bowl, ends up at the bowl's bottom must appeal to something about the particular forces acting, such as the existence of friction between the ball and the bowl's surface. So this explanation is causal despite abstracting from the ball's particular trajectory in the bowl.

Any natural laws in a distinctively mathematical explanation, I will suggest, must transcend the laws describing the particular kinds of causes that exist. In chapter 2, I will make this notion of "transcendence" precise. For now, I will say merely that Newton's second law transcends the particular force laws if it is modally stronger than they. By the same token, mathematical truths (such as those figuring in the various distinctively mathematical explanations that I have examined) transcend the particular force laws in that mathematical necessity is a stronger variety of necessity than natural necessity. A distinctively mathematical explanation works (I propose) not by describing the world's actual causal structure, but rather by showing how the explanandum arises from the framework that any *possible* physical system (whether or not it figures in causal relations) must inhabit, where the "possible" systems extend well beyond those that are logically consistent with all of the actual natural laws. Both mathematical truths and contingent natural laws that transcend the force laws are therefore able to figure in distinctively mathematical explanations.

For example, suppose we had two double pendulums: one simple, the other with inhomogeneous, extended masses and oscillations driven by various springs. Why do both of these pendulums have at least four equilibrium configurations? We could specify the energy functions for both pendulums and then derive separately the particular equilibrium configurations for each, thereby showing that each has at least four of them. But this derivation would portray the explanandum as a coincidence since this derivation would fail to identify some important feature common to the two pendulums as responsible for their both having at least four equilibrium configurations. Since there is such a common feature, this derivation fails to explain why both pendulums have at least four equilibrium configurations. The explanandum has only a distinctively mathematical explanation. That these two double pendulums are alike in having at least four equilibrium configurations is no coincidence because any double pendulum, in virtue of being a double pendulum, *must* possess this property. This *must* is stronger than the necessity possessed by the force laws, and so no derivation using those laws could show that these two pendulums, just by virtue of being double pendulums, *must* have this feature. (I made a similar point at the end of section 1.2 regarding the fact that two moments in Earth's history are alike in that at each there exists two antipodal equatorial points having the same temperature; the similarity between these moments also turns out to be no coincidence by virtue of having a distinctively mathematical explanation.)

In like manner, the distinctively mathematical explanation of the repeated failure to cross the Königsberg bridges works not by describing the world's nexus of causal relations, but rather by showing that the task *cannot* be done—where this impossibility is stronger than natural impossibility. The distinctively mathematical explanation thereby reveals it to have been no coincidence that all of the actual attempts failed. The explanans consists not just of various mathematically necessary facts, but also (as I showed in section 1.3) of various contingent facts presupposed by the why question: that the arrangement of bridges and islands is fixed, that any "crossing" consists of a continuous path over them, and so forth. The distinctively mathematical explanation shows it to be necessary (more strongly so than any particular force law) that under these contingent conditions, the bridges are not crossed. By the same token, the distinctively mathematical explanation in the double pendulum example shows it to be necessary (and more than merely naturally necessary) for a given double pendulum to have at least four equilibrium configurations under certain contingent conditions that the why question presupposes (e.g., that the string does not lengthen, the bob does not explode, the pendulum remains confined to a plane). These contingent facts specify the double pendulum arrangement in question just as various contingent facts fixing the bridge arrangement are understood to be presupposed by the task of crossing the Königsberg bridges.

Under the contingent conditions fixing the bridge arrangement in question, that arrangement functions as an abstract, ideal, "mathematical" object: a "graph" (or "network"). Likewise, suppose we ask (in discussing a classic problem of geometric construction) why no one has ever trisected a 60° angle using only a compass and an unmarked straightedge. That such a construction is mathematically impossible explains why no one has ever done it. Once again, the task in question is understood to involve an ideal compass and straightedge (or physical instruments that are not exploiting their departures from the ideal).

■ 1.5 ELABORATING MY ACCOUNT OF DISTINCTIVELY MATHEMATICAL EXPLANATIONS

Let's take stock of the foregoing proposal concerning the way that distinctively mathematical explanations operate. I agree with Mancosu, Lipton, and Kitcher (in the passages cited at the start of section 1.2) that distinctively mathematical explanations are non-causal. But I do not accept Batterman's (2010, 3) diagnosis that what makes them non-causal is that they involve a "systematic throwing away of various causal and physical details." Many causal explanations do that, too—including explanations that appeal to one trait's having greater fitness than another (abstracting away from the detailed histories of individual mating, reproduction, and predation events), explanations that appeal to a peg's squareness and a hole's circularity (abstracting away from the particular intermolecular forces at work), and (as Jackson and Pettit 1992 suggest) explanations of a flask's cracking that appeal to the temperature of the hot water in it (abstracting away from the particular collisions between water and glass molecules that caused the cracking). All of these are causal explanations because they work by offering "information relevant to the causal history of the thing to be explained" (Jackson and Pettit 1992, 11; see 3, 9).

A computer program explains why a computer performs some behavior by ensuring that some electronic event or other occurs to bring about that behavior (without determining every detail of the electronic event). In like manner, a "program explanation" of the flask's cracking specifies a property (the water's temperature) that all but ensures that a molecular collision occurs that causes the cracking, but without determining that collision's every detail (Jackson and Pettit 1990). I agree with Jackson and Pettit that such a "program explanation" is causal. However, I suggest (contrary to Jackson and Pettit) that some scientific explanations (such as distinctively mathematical explanations) are non-causal. Like program explanations, distinctively mathematical explanations supply modal information. But unlike distinctively mathematical explanations, program explanations supply modal information by describing the world's causal structure—for instance, by revealing that even if the molecular collision that actually cracked the flask had not occurred, some other molecular collision would still have cracked the flask. A program explanation works by telling us that something plays a certain causal role, without identifying the specific role-filler. Distinctively mathematical explanations do not work in this way.

Likewise, I agree with Pincock's characterization of the Königsberg bridge explanation as an "abstract explanation" in that it "appeals primarily to the formal relational features of a physical system" (Pincock 2007, 257). But I do not agree with Pincock (273) that "abstract explanations are a species of what are sometimes called 'structural explanations' (McMullin 1978)," since McMullin (1978, 139) regards structural explanations as causal: as working by describing the constituent entities or processes (and their arrangement) that cause the feature being explained.[26] On my view, the order of causal priority is not responsible for the order of explanatory priority in distinctively mathematical scientific explanations. Rather, the facts doing the explaining are eligible to explain by virtue of being modally more necessary than ordinary laws of nature (as both mathematical facts and Newton's second law are) or being understood in the why question's context as constitutive of the physical task or arrangement at issue. The arrangement of bridges, the number of students in Lipton's class, the numbers of Mother's strawberries and children, the numbers of jellybeans and available colors, and so forth are explanatorily prior to the outcomes they help to explain in the various distinctively mathematical explanations I have examined—but *not* by virtue of their being *causes* of those outcomes. Rather, in the contexts of the respective why questions, these facts are explanatorily prior to the explanatory targets by virtue of being understood as constituting the situations at hand. They are the fixed parameters of the cases with which those why questions are concerned.

Plenty of explanations abstract from petty causal influences, emphasizing mathematical structure instead—but are nevertheless causal rather than distinctively mathematical explanations. For instance, let our explanandum be the fact that when something diffuses through a homogeneous, boundless, two-dimensional medium after having been released at the origin (0,0) at time $t = 0$ (so the total finite quantity of the diffusing substance starts out infinitely concentrated),[27] at every subsequent time t the concentration curve $\Phi(x,y,t)$ is a Gaussian (i.e., bell-shaped) curve—in other words, the concentration is proportional to $e^{-(x^2+y^2)/c}$ for some constant c. Here is an explanation of this fact, following an argument discovered by John Herschel (1850, 19–20):

> The concentration function is proportional to the function giving the likelihood of a diffusing parcel's being at a given location. A parcel's likelihood of having managed to make its way to some point or other with a given x-coordinate is equal to its