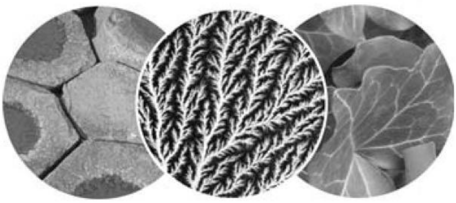
Nature's Patterns: a tapestry in three parts

Branches

PHILIP BALL

Nature's Patterns

A Tapestry in Three Parts



Philip Ball

Nature's Patterns is a trilogy composed of
Shapes, Flow, and Branches

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Preface and acknowledgements

AFTER my 1999 book *The Self-Made Tapestry: Pattern Formation in Nature* went out of print, I'd often be contacted by would-be readers asking where they could get hold of a copy. That was how I discovered that copies were changing hands in the used-book market for considerably more than the original cover price. While that was gratifying in its way, I would far rather see the material accessible to anyone who wanted it. So I approached Latha Menon at Oxford University Press to ask about a reprinting. But Latha had something more substantial in mind, and that is how this new trilogy came into being. Quite rightly, Latha perceived that the original *Tapestry* was neither conceived nor packaged to the best advantage of the material. I hope this format does it more justice.

The suggestion of partitioning the material between three volumes sounded challenging at first, but once I saw how it might be done, I realized that this offered a structure that could bring more thematic organization to the topic. Each volume is self-contained and does not depend on one having read the others, although there is inevitably some cross-referencing. Anyone who has seen *The Self-Made Tapestry* will find some familiar things here, but also plenty that is new. In adding that material, I have benefited from the great generosity of many scientists who have given images, reprints and suggestions. I am particularly grateful to Sean Carroll, Iain Couzin, and Andrea Rinaldo for critical readings of some of the new text. Latha set me more work than I'd perhaps anticipated, but I remain deeply indebted to her for her vision of what these books might become, and her encouragement in making that happen.

Philip Ball

London, October 2007

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A Winter's Tale
The Six-Pointed Snowflake

The followers of Pythagoras believed many strange things, among them that one should not eat beans or break bread, should not pluck a garland and should not allow swallows to land on one's roof. They sound like a bunch of crackpot mystics, but in fact Pythagoreanism has, through its influence on Plato, provided a recurrent theme in Western rationalist thought: the idea that the universe is fundamentally geometric, so that all natural phenomena display a harmony based on number and regularity. Pythagoras is said to have discovered the relationship between proportion and musical harmony, reflected in the way that a plucked string divided by simple length ratios produces pleasing musical intervals. The 'music of the spheres'—celestial harmonies generated by the heavenly bodies according to the sizes of their orbits—is ultimately a Pythagorean concept.

'All things are numbers', said Pythagoras, but it is not easy now to comprehend what he meant by this statement. In some fashion, he believed that integers were building blocks from which the world was constructed. Bertrand Russell is probably imposing too modern a perspective when he interprets the phrase as saying that the world is 'built up of molecules composed of atoms arranged in various shapes', even if, for Plato, those atoms themselves were geometric: cubes, tetrahedra, and other regular shapes that, he said, account for the empirical properties of the corresponding classical elements. All the same, it seems fair to suppose that a Pythagorean would have been less surprised than we are to find spontaneous regularity of pattern and form in the world—five-petalled flowers, faceted crystals—because he would have envisaged this orderliness to be engraved in the very fabric of creation.

The ancient Greeks were not alone in thinking this way. Chinese scholars of long ago were as devoted to the study of nature and mathematics as any of their Western counterparts, and by all appearances they were rather more observant. It was not until the European Middle Ages that the Aristotelian tradition of studying specific natural phenomena for their own sake began to permeate the Western world, prompting the thirteenth-century Bavarian proto-scientist Albertus Magnus to record the 'star-shaped form' of snowflakes, which can be seen with the naked eye. But the Chinese anticipated him by more than a millennium. Around 135 BC, the philosopher Han Ying wrote in his treatise *Moral Discourses Illustrating the Han Text of the 'Book of Songs'* that 'Flowers of plants and trees are generally five-pointed, but those of snow, which are called *ying*, are always six-pointed.' It is a casual reference, as though he is mentioning something that everyone already knew.

Chinese poets and writers in the subsequent centuries took this fact for granted. In the sixth century AD, Hsiao T'ung wrote:

The ruddy clouds float in the four quarters of the caerulean sky
And the white snowflakes show forth their six-petalled flowers.

By the seventeenth century, Chinese scholars had become more systematic and scientific in their approach. 'Every year at the end of winter and the beginning of spring I used to collect snow crystals myself and carefully examined them', wrote Hsien Tsai-hang in his *Five Assorted Offering Trays* (c.1600). He may have used a magnifying glass for this work, which led him to conclude that 'all were six-pointed'.

It was no surprise to the Chinese sages that snow crystals were six-pointed, because many of them held a view of nature that was every bit as numerological as that of the Pythagoreans. Still today, numerical schemes provide a central ordering principle in Chinese thought, from the Eightfold Way of Daoism to the 'Four Greats' of Mao's personality cult. In a system of 'correspondences' analogous to that of the Western mystical tradition, the elements were deemed to have numbers associated with them, and as the great philosopher Chu Hsi wrote in the twelfth century, 'Six generated from Earth is the perfected number of Water.' Thus, according to the scholar T'ang Chin, 'when water congeals into flowers they must be six-pointed', because 'six is the true number of Water'.

The problem with this scheme is that it stifles further enquiry: given such an 'explanation' (which we now see as little more than a tautology), there is nothing more to be said. A profound mystery is reduced to a commonplace fact. And so, in the words of sinologist Joseph Needham, 'the Chinese, having found the hexagonal symmetry [of snowflakes], were content to accept it as a fact of nature'.

Here, then, is a rejoinder to the accusation that a scientific attitude is prone to blunt our wonderment at the world. In the mystic's teleological universe, order and pattern are only to be expected: they are part of the Grand Design. There is nevertheless value in such an outlook, which can help to bring to our notice the regularities that exist in nature—we may not see them at all if we do not expect them. In fact, mysticism in all its guises can lead us to perceive *too much* order, making us prone to seeing significance where there is only the play of chance. The human mind seems to be predisposed to this error, for pattern recognition is an essential survival tool and it seems we must resign ourselves to living with its tiresome side-effects, from numerology to 'faces' on the surface of Mars.

But although the mystical Platonic vision of a geometric, ordered universe helped prepare the ground for early Western science, it needed

Fig. 1.1: The snowflake displays an urge for branching growth played out with exquisite hexagonal symmetry. (Photo: Ken Libbrecht, California Institute of Technology.)

to be replaced by something more empirical, more discerning and sceptical, before we could truly begin to understand how the world works. The snowflake offers a delightful illustration of that process. For it is only when we start to regard these ice crystals as things in themselves, and not as symbols of some deeper principle of nature, that we can truly appreciate how astonishing they are. Their elegance and beauty is, I believe, unrivalled in the natural world, and even Bach would have been silenced by the invention with which they play variations on a simple theme, this interplay of 'sixness' and 'branchingness' in which symmetry seems to be taken about as far as it can tolerate (Fig. 1.1 and Plate 1). They are formed from chaos, from the random swirling of water vapour that condenses molecule by molecule, with no template to guide them. Whence this branchingness? Wherefore this sixness?

Kepler's balls

In the mechanistic worldview that emerged in the West during the wane of the Renaissance, an appeal to numerology could not suffice to account for the remarkable symmetry of the snowflake. The spirit of the age insisted on causative forces that dictated how things happened in their own terms. One could concede that God set the forces at play while insisting that, on a day-to-day basis, they were all He had to work with.

Snowflakes interested the Englishman Thomas Hariot, who noted in his private manuscripts in 1591 that they have six points. Hariot was a masterful mathematician, noted for his contributions to algebra, but his enthusiasms showed the characteristic magpie diversity of the Elizabethan intellectual, among them astronomy, astrology, and linguistics. He tutored Walter Raleigh in mathematics, and when Raleigh set out on a voyage to the New World in 1585 he employed Hariot as navigator. Together they sailed to the land that Raleigh named in honour of his Virgin Queen: Virginia. On the voyage, Raleigh sought Hariot's expert advice about the most efficient way to stack cannonballs on deck.

The question led Hariot to the beginnings of a theory about the close-packing of spheres. Some time between 1606 and 1608 he communicated his thoughts to a fellow astronomer, the German Johannes Kepler, who enjoyed the patronage of the Holy Roman Emperor Rudolph II at his illustrious court in Prague. Most of the correspondence between

Kepler and Hariot concerns the refraction of light and the origin of rainbows, but they also discussed atomism: what are atoms, and can empty space come between them? This was an ancient theme, prompted by the belief that nature abhors a vacuum, but it seemed then to be as irresolvable as ever. The issue of how atoms sat against one another brought Hariot back to Raleigh's cannonballs, and he asked what Kepler thought about the matter. In 1611 Kepler wrote a short treatise in which he speculated that the familiar cannonball stacking, which disports the balls in a hexagonal, honeycomb array, is the densest arrangement there can be. The hexagonal packing 'will be the tightest possible', he wrote, 'so that in no other arrangement could more pellets be stuffed into the same container'.* The booklet in which this assertion was contained was a New Year's gift from Kepler to his patron Johann Matthäus Wacker von Wackenfels: seasonably so, for its title indicates the object towards which Kepler's thoughts on close-packing became directed. It was called *On the Six-Cornered Snowflake*.

'There must be a cause why snow has the shape of a six-cornered starlet', Kepler says. 'It cannot be chance. Why always six? The cause is not to be looked for in the material, for vapour is formless and flows, but in an agent.' But Kepler does not claim that he can solve the mystery; indeed, his booklet is a rather charming study in bafflement, full of false trails and head-scratching. Nonetheless, it contains the seed of an important idea. Prompted by his discussions with Hariot, Kepler began to think about the geometrical shapes that bodies will adopt if their constituent particles are close-packed like cannonballs. He suggested that the hexagonal symmetry he had seen in snowflakes that he collected and observed that very winter might stem from the stacking of 'globules' of water. These globules are not in themselves atoms; rather, he said, 'vapour coagulates into globules of a definite size, as soon as it begins to feel the onset of cold'. They are like little droplets, and, as such, are perfectly spherical.

Yet in the end Kepler rejects this idea, for he notes that balls can be packed into other regular patterns too—notably square arrays—and yet four-pointed snowflakes are never observed. He remarks that flowers commonly display five-pointed heads (a notion I explored in Book I),

*Kepler's conjecture remained just that for nearly four centuries. It was proven to be true by the American mathematician Thomas Hales in 1998.

which he attributes to a 'formative faculty' or plant soul. But 'to imagine an individual soul for each and any starlet of snow is utterly absurd', Kepler wrote, 'and therefore the shapes of snowflakes are by no means to be deduced from the operation of soul in the same way as with plants.' So how does water vapour acquire a formative faculty? It must, in the end, be God's work—which sounds like a capitulation, but in fact reflects the semi-mystical belief common among early seventeenth-century philosophers that nature is imbued with 'hidden' forces that shape its forms. Yet what purpose could be served by this symmetrical expression of a gaseous formative faculty? There is none, Kepler decides: 'No purpose can be observed in the shaping of a snowflake . . . [the] formative reason does not act only for a purpose, but also to adorn . . . [it] is in the habit also of playing with the passing moment.' In this seemingly whimsical conclusion we can discern something valid and profound—for, as I hope this trilogy will show, nature does indeed seem to have an intrinsic pattern-forming tendency that it exercises as though from some irrepressible urge. Kepler even hints inadvertently at the way this impulse can act in living organisms in apparent defiance of the strict utilitarianism that Darwinism later seemed to dictate.

Despite its inconclusiveness, Kepler's treatise on the snowflake established the idea that the geometric shapes of crystals are related to the ordered arrangements of their component units. From this elementary notion came the science of crystallography, beginning in the late eighteenth century, in which the faceted nature of mineral crystals is explained in terms of close-packing of their atoms and molecules (Fig. 1.2). And what is more, his invocation of an almost vitalistic principle behind the growth of snowflakes, redolent of (if not the same as) the 'soul' that guides the growth of plants, captures something of the confusion that snowflakes provoke. The sixness, the hexagonal symmetry, speaks of crystals, of a regularity so perfect that it appears barren. But the branchingness hints at life and growth, at something vegetative and vital.

René Descartes, the arch-mechanist of the early Enlightenment, could not resist the allure of snowflakes. He sketched them in 1637 for his study of meteorology, *Les Météores*, where he recorded rarer varieties alongside the six-pointed stars (Fig. 1.3):

> After this storm cloud, there came another, which produced only little roses or wheels with six rounded semicircular teeth . . . which were quite transparent and

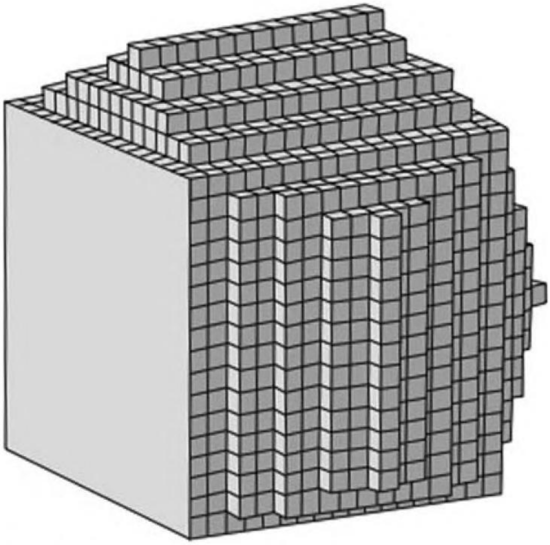

Fig. 1.2: Early crystallographers such as René Just Häuy, from whose book *Traité de Minéralogie* (1801) this illustration comes, explained the faceted shapes of crystals in terms of the packing of their component atoms.

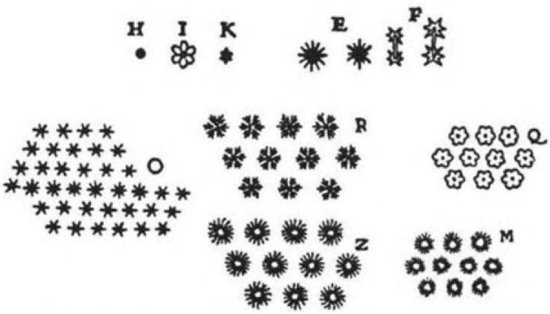

Fig. 1.3: Drawings of snowflakes by René Descartes in 1637.

quite flat . . . and formed as perfectly and as symmetrically as one could possibly imagine. There followed, after this, a further quantity of such wheels joined two by two by an axle, or rather, since at the beginning these axles were quite thick, one could as well have described them as little crystal columns, decorated at each end with a six-petalled rose a little larger than their base. But after that there fell more delicate ones, and often the roses or stars at their ends were unequal. But then there fell shorter and progressively shorter ones until finally these stars

Fig. 1.4: Twelve-pointed snowflakes are formed when two normal six-pointed varieties fuse together at their centres, rotated in relation to one another by about 30°. (Photo: Ken Libbrecht, California Institute of Technology.)

completely joined, and fell as double stars with twelve points or rays, rather long and perfectly symmetrical, in some all equal, in other alternately unequal.

We can recognize in this vivid description some of the unusual forms that have been found in snowflakes, such as prismatic columns with end-caps, like elaborate sundials, and twelve-pointed stars in which two hexagonal flakes have become fused (Fig. 1.4).

The English scientist Robert Hooke had the advantage of a microscope in preparing illustrations of snowflakes for his famous *Micrographia* (1665), where he shows that the 'flowers' are not just six-pointed but branch repeatedly, in a hierarchical manner (Fig. 1.5*a*). The organic associations of these ice crystals are very apparent in the drawings by the Italian astronomer Giovanni Domenico Cassini in 1692, where they look almost leafy (Fig. 1.5*b*). The biologist Thomas Huxley acknowledged this aspect in 1869, when he called snowflakes 'frosty imitations of the most complex forms of vegetable foliage'. Huxley's comments appeared in an essay on 'the physical basis of life', in which he strove

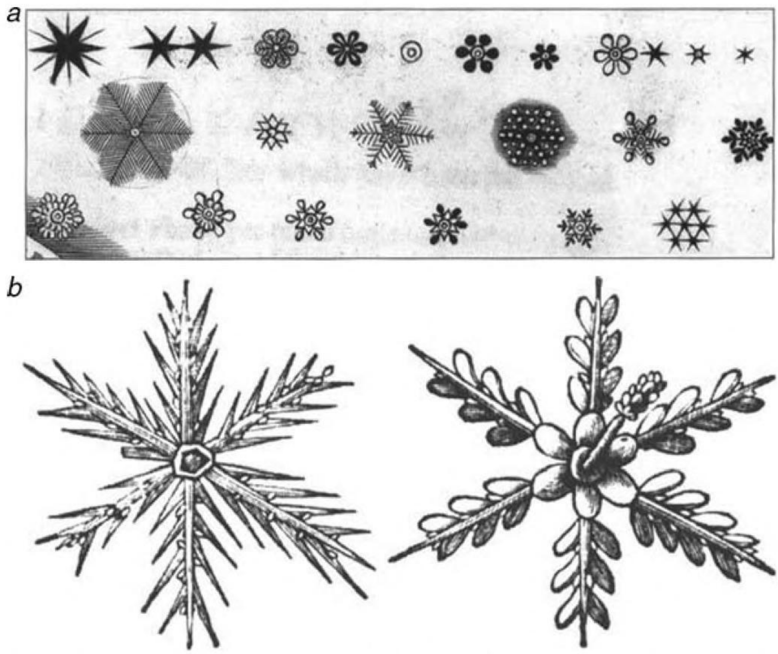

Fig. 1.5: Using an early microscope, Robert Hooke recorded the characteristic 'Christmas-tree' branching patterns of snowflakes (*a*). Giovanni Domenico Cassini's drawings from 1692 seem to make reference to their resemblance to plants (*b*).

like a good positivist to quell any notion of a vital force that animated organic matter and made it fundamentally different from the inorganic world. To Huxley, the 'organic' forms of snowflakes provided evidence that the complex shapes of the biological world need not compel the scientist to invoke some mysterious vitalistic sculpting mechanism, since something of that nature surely did not operate in the simple process of the freezing of water:

> We do not assume that a something called 'aquosity' entered into and took possession of the oxide of hydrogen as soon as it was formed, and then guided the aqueous particles to their places in the facets of the crystal, or amongst the leaflets of the hoar-frost.

It was a reasonable enough assertion, but it surely begs the question: if there is nothing 'organic' about the formation of the snowflake, why then do they look so tantalizingly as though there is?

Flakes frozen on film

As Descartes hinted, the shapes of snowflakes can evolve and mutate as the weather changes. Friedrich Martens, on board a ship travelling from Spitzbergen in Norway to Greenland in 1675, noticed that different meteorological conditions produce different kinds of flake. It takes an Arctic chill to condense the best, most symmetrical snowflakes, as the English explorer William Scoresby noted in his *Account of the Arctic Regions with a History and Description of the Northern Whale-Fishery* in 1820. Scoresby took the observations of snowflakes to a new standard of detail and accuracy, recording a wide range of different shapes (Fig. 1.6). One of the most charming of nineteenth-century records was that produced in 1864 by a minister's wife in Maine named Frances Knowlton Chickering, who used, if not invented, the trick now popular at Christmas of cutting out doily-style snowflakes from folded paper. Chickering's paper flakes were masterpieces of dexterity: from memory of her first-hand observations, she clipped out delicate frond-like branches and pasted the results into her *Cloud Crystals: A Snow-Flake Album*, which implicitly acknowledged the 'artistry' of natural phenomena that the biologist Ernst Haeckel was later to celebrate in his drawings of marine life, as we saw in Book I.

The accuracy of all these visual records of snowflakes was limited not only by the power of the magnifying glass or microscope but also by the artist's inevitable tendency to simplify, idealize, and interpolate these complex geometric forms. That problem was avoided once researchers

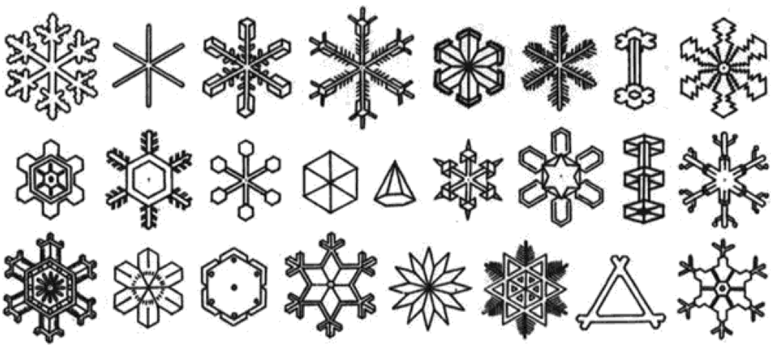

Fig. 1.6: In 1820, explorer William Scoresby made accurate drawings of the snowflakes he observed during a trip to the Arctic.

found a way to marry the new art of photography to the power of the microscope. Microphotography was already a well-established technique by the late nineteenth century, and one of its most inventive practitioners was a Vermont farmer named Wilson Bentley. Between 1885 and 1931, Bentley captured over 5,000 images of snowflakes on photographic plates, constituting one of the most comprehensive surveys of their astonishing variety and beauty (Fig. 1.7). In the late 1920s Bentley compiled 2,000 of his photographs into a book entitled *Snow Crystals* in collaboration with William J. Humphreys, a physicist working for the US Weather Bureau. Bentley died only a few weeks after the book was published in November 1931, allegedly after contracting pneumonia during one of his forays into the New England winter.

Snow Crystals is rightly regarded as a work of wonder, but it is more than that. The scientist, gazing at page after page of seemingly infinite variety on the theme of the six-pointed flower of ice, faces a mystery of an order not previously encountered in the non-living world. Not only were the forms indescribably complex, but there was no end to them.

Bentley's album was pure description, to which Humphreys could add rather little in the way of hard science. But in the 1930s the book inspired a Japanese nuclear physicist named Ukichiro Nakaya, working at the University of Hokkaido, to consider the question of snowflake growth in a rather more analytical spirit. He made the first systematic attempt to discover the factors that influenced snowflake growth, leading to the many different *families* of shapes that had been seen by Scoresby and others in the natural environment. Nakaya realized that snowflakes fall into several distinct categories, and he constructed a laboratory for exploring the conditions that generated these different classes of shape.

It was uncomfortable work: Nakaya's wooden-walled lab could be cooled to $-30\,^{\circ}\mathrm{C}$, and he worked in padded clothing with a mask to protect his face. Snowflakes grow slowly as they fall through the atmosphere, but Nakaya could not recreate this long descent in the lab, so instead he decided to reverse the situation: to hold the snowflake fixed and to let cold, moist air pass over it in a steady stream. The question was, how do you hold onto a snowflake? Nakaya experimented with many different kinds of filament for immobilizing a growing crystal of ice, but most of them simply became coated with frost. He finally found that the experiment worked best with a strand of rabbit hair, on which the natural oils suppressed the simultaneous nucleation of many ice

Fig. 1.7: The collection of snowflake photographs amassed by Wilson Bentley in the four decades after 1885 still stands as the most remarkable record of their endlessly varied forms.

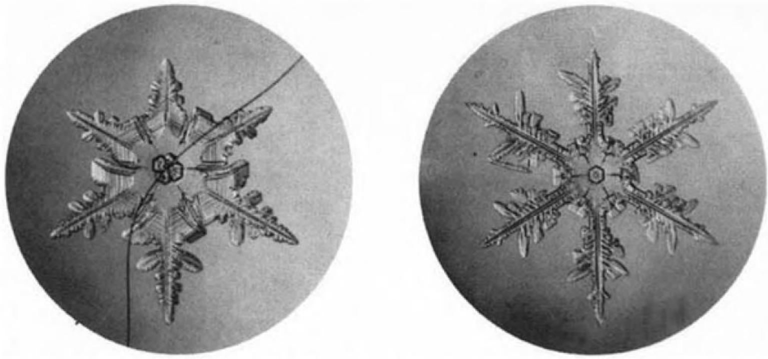

Fig. 1.8: Snowflakes made artificially by Ukichiro Nakaya in the 1930s. In the image on the left, the rabbit's hair on which the crystals are nucleated is still visible.

crystals at once (Fig. 1.8). Using this equipment, Nakaya and his co-workers found that the shapes of the individual crystals changed as two key factors were altered: the temperature and humidity of the air. At low humidity, the crystals did not develop the six frond-like arms of classic snowflakes, but took on more compact forms: hexagonal plates and prisms. These shapes persisted even in moister air if it was very cold (below about −20 °C). At higher temperatures, however, increasing the humidity tended to increase the delicacy and complexity of the snowflakes, giving rise to the highly branched star forms. In a temperature range between about −3 and −5 °C, needle-like crystals appeared instead (Fig. 1.9).

Nakaya collected his findings in an album of images clearly indebted to Bentley and Humphreys, called *Snow Crystals: Natural and Artificial* (1954). His studies brought some order to the ice menagerie, but they did not really bring us any closer to understanding the fundamental mechanism by which a simple process of crystallization, which typically generates a compact prismatic or polyhedral shape, in this case gives us structures that seem to have a life of their own.

As I explained in the previous volumes, the first person to tackle this sort of question about the genesis of complex form within a modern scientific framework was the Scottish zoologist D'Arcy Wentworth Thompson, whose 1917 book *On Growth and Form* set the scene for everything I discuss in this series. Thompson included drawings based on Bentley's photographs in the 1942 revised edition of his book. 'The snow crystal', he wrote, 'is a regular hexagonal plate or thin prism.' But

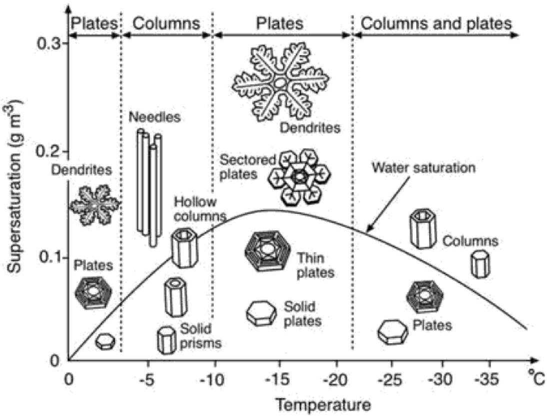

Fig. 1.9: The 'morphology diagram' of snowflakes, showing how their shape changes for different conditions of temperature and humidity (supersaturation).

'ringing her changes on this fundamental form, Nature superadds to the primary hexagon endless combinations of similar plates or prisms, all with identical angles but varying lengths of side; and she repeats, with an exquisite symmetry, about all three axes of the hexagon, whatsoever she may have done for the adornment and elaboration of one.' In other words, all the arms appear to be identical. 'The beauty of a snow-crystal depends on its mathematical regularity and symmetry', Thompson observed,

> but somehow the association of many variants of a single type, all related but no two the same, vastly increases our pleasure and admiration. Such is the peculiar beauty which a Japanese artist sees in a bed of rushes or a clump of bamboos, especially when the wind's ablowing; and such is the phase-beauty of a flowering spray when it shews every gradation from opening bud to fading flower.

Here it is again: flowers and ice. But even Thompson, like Kepler, could say no more. With all his ideas about forces and equilibria and geometry, he, too, was forced to take recourse in metaphors from the organic world.

Endless branches

By the time Nakaya's book appeared, scientists had found a way to attack the problem. Although ice seems to be unique in forming highly symmetrical, isolated flakes, many other substances may crystallize as needle-like protrusions punctuated by regular branches, like a single snowflake arm. These structures, known as dendrites (from the Greek for 'tree') are found when molten metals freeze (Fig. 1.10*a*), when salts precipitate out of a solution, and when metal deposits form on electrically charged electrodes, a process known as electrodeposition and related to electroplating (Fig. 1.10*b*) (see page 30). Dendrites typically have a rounded tip, like the prow of a boat, behind which side-arms sprout and grow in a Christmas-tree pattern. In general, they appear when the solidification process happens rapidly, as for example when a molten metal is quenched ('undercooled') by being plunged into cold surroundings. That's an important clue. We observed in Book I that complex pattern and form is often generated in processes that take place significantly out of thermodynamic equilibrium—which is to say, when the system is highly unstable. A system in equilibrium does not change; a system out of equilibrium 'seeks' to attain such a stable state if left alone to do so, but can be driven away from this goal by a constant influx of energy. We saw in Book II that convection (the flow of a fluid when heated from below) produces such a non-equilibrium state. A liquid that is abruptly cooled far below its freezing point is another non-equilibrium system, being unstable relative to the solid form of the material. That instability makes change happen rapidly, under which conditions pattern is apt to appear. In contrast, crystals that are formed close to equilibrium—very close to their freezing point, say—grow slowly, and tend instead to develop the familiar compact, faceted shapes.

In 1947 the Russian mathematician G. P. Ivantsov showed theoretically that a metal solidifying rapidly from its molten form may develop needle-like fingers. Ivantsov calculated that the needles have a shape mathematicians called parabolic, with gently curving sides that converge on a blunt tip. This is the same shape as the trajectory of a stone thrown through the air and falling under gravity. Ivantsov showed that in fact all possible types of parabolic needles may be formed, but that the thinner they are, the more rapidly they grow; so thin, needle-like tips should shoot rapidly through the molten metal, while fatter bulges make their way forward at a more ponderous pace.

But in the mid-1970s, Martin Glicksman and co-workers at the Rensselaer Polytechnic Institute in New York performed careful experiments which showed that, instead of a family of parabolic tips, only one

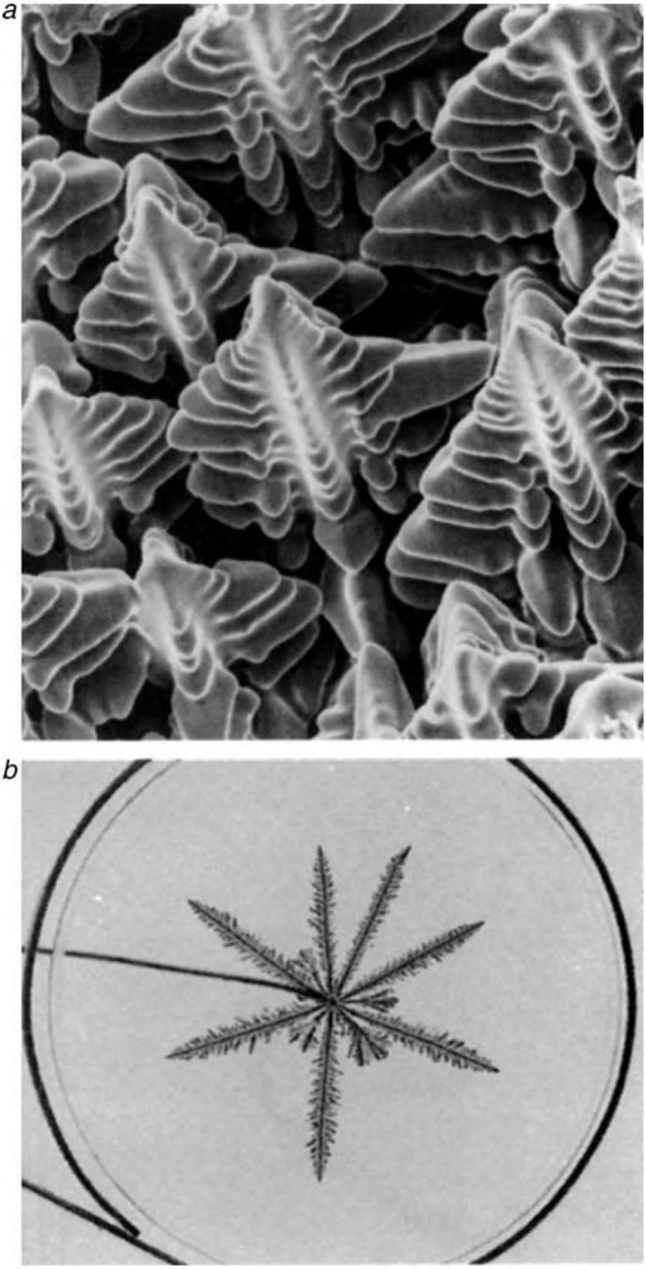

Fig. 1.10: Dendrites formed by rapid solidification of a molten metal (*a*) and in the electrodeposition of a metal (*b*). (Photos: *a*, Lynn Boatner, Oak Ridge National Laboratory, Tennessee. *b*, Eshel Ben-Jacob, Tel Aviv University.)

single tip shape was seen during rapid solidification of metals. For a fixed degree of undercooling, a particular tip is privileged over the others. For some reason, one of Ivantsov's family of parabolas seems to be special.

The puzzle was even more profound, however, because in 1963 two Americans, William Mullins and Robert Sekerka at Carnegie Mellon University in Pittsburgh, argued that *none* of Ivantsov's parabolas should be stable. They calculated that the slightest disturbance to the growth of a parabolic tip will be self-amplifying, so that small bulges that form by chance on the edge of the crystal grow rapidly into thin fingers. This so-called Mullins–Sekerka instability should cause the tip to sprout a jumble of random branches.

The instability is an example of a positive feedback process—again, we have encountered such things already in Books I and II. It works like this. When a liquid freezes, it releases heat. This is called latent heat, and it is the key to the difference between a liquid and its frozen, solid form at the same temperature. Ice and water can both exist at zero degrees centigrade, but the water can become ice only after it has becomes less 'excited'—its molecules cease their vigorous jiggling motions—by giving up latent heat.

So, in order to freeze, an undercooled liquid has to unload its latent heat. The rate of freezing depends on how quickly heat can be conducted away from the advancing edge of the solid. This in turn depends on how steeply the temperature drops from that in the liquid close to the solidification front to that in the liquid further away: the steeper the gradient in temperature, the faster heat flows down it. (It may seem odd that the liquid close to the freezing front is actually warmer than that further away, but this is simply because the front is where the latent heat is released. Remember that in these experiments all of the liquid has been rapidly cooled below its freezing point but has not yet had a chance to freeze.)

If a bulge develops by chance—because of the random motions of the atoms and molecules, say—on an otherwise flat solidification front, the temperature gradient becomes steeper around the bulge than elsewhere, because the temperature drops over a shorter distance (Fig. 1.11). So latent heat is shed around the bulge more rapidly than it is to either side, and the bulge grows, its apex fastest. This in turn sharpens the tip and speeds its advance even more.

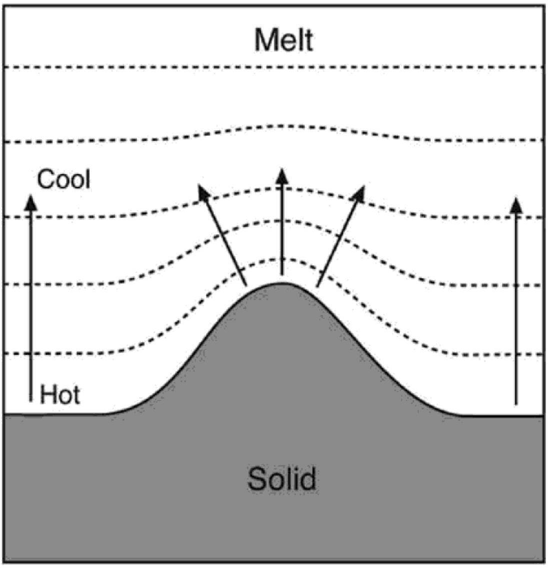

Fig. 1.11: The Mullins–Sekerka instability makes protrusions at the surface of a solidifying material unstable. Because the temperature gradient (shown here as dashed contours of equal temperature) is steeper at the tip of the protrusion, heat is conducted away faster and so solidification proceeds more rapidly here.

In principle, this instability will amplify any irregularity on the solid front into a growing finger, no matter how small it is. But there is another factor that sets a minimum limit to the width of the fingers. The interface between the solid and the liquid has a surface tension, just like that at the surface of water in a glass. As I explained in Book I, the existence of surface tension means that an interface costs energy: the bigger the surface area, the higher the energetic cost. Surface tension thus encourages surfaces to keep their area as small as possible, and here it tends to 'pull' the solidification front flat. Thanks to this smoothing effect, surface tension suppresses bulges smaller than a certain limit. This means that the Mullins–Sekerka instability produces a characteristic branch-tip width, set by the point at which the narrowing of tips caused by positive feedback is counterbalanced by their cost in surface energy. In other words, the front develops fingers with a certain *wavelength*—a regular pattern with a particular size scale to it, determined by a balance of opposing factors.

In 1977 James Langer at the University of California at Santa Barbara and Hans Müller-Krumbhaar in Jülich, Germany, suggested that the Mullins–Sekerka instability might explain Glicksman's observation of a single parabolic tip being selected from all of those allowed by Ivantsov's theory. The instability will make fat fingers break up into a mass of smaller ones, they said, while surface tension sets a limit on how small and narrow they can become. Perhaps, then, there is an optimal tip width at which these two effects balance, favouring a single 'marginally stable' parabolic tip.

But in the early 1980s, Langer and his co-workers showed that surface tension in fact destroys this neat picture. Its influence makes the tip of the dendrite become cooler than the regions to either side. So the tip starts to slow down, and eventually it forks into two new fingers. These also split subsequently, and so on. This repeated tip-splitting results not in a dendritic growth shape at all, but instead a dense mass of repeatedly forking branches, a pattern known as the dense-branching morphology.

This turned out to be a persistent problem with theories of dendrite growth: they seemed prone to instabilities that led to randomly branched fingering patterns, not the orderly, Christmas-tree shapes of snowflake arms. What these theories were neglecting, in the mistaken belief that it was a mere detail, was the most striking aspect of a snowflake's shape: its hexagonal symmetry. It had been suspected ever since Kepler that this was an echo of the underlying symmetry in the arrangement of constituent particles. But no one had guessed that it was to this symmetry that the dendrites owed their very existence.

The joy of six

Why hexagons? Remarkably, the Chinese sages were right: there *is* a sense in which six is the number of water. In 1922 the English physicist William Bragg used his new technique of X-ray crystallography to deduce how water molecules are arranged in an ice crystal. X-rays bouncing off crystals produce a pattern of bright spots that encode the positions of the atoms; Bragg deduced how to calculate backwards from the X-ray pattern to the atomic structure. In this way, he found that the water molecules in ice are linked by weak chemical bonds into hexagonal rings, one molecule at each corner (Fig. 1.12). Thus the

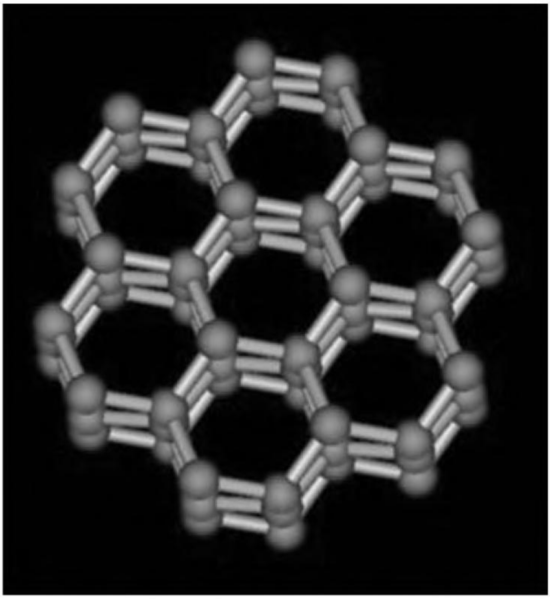

Fig. 1.12: Ice has a structure with hexagonal symmetry at the molecular scale. The water (H_2O) molecules are linked together by weak chemical bonds called hydrogen bonds. In this image the spheres denote the oxygen atoms at the centres of the molecules, and the rods denote the hydrogen bonds that bind molecules together.

crystal structure is dictated not by the shapes of the water molecules themselves but by the way in which they are joined together.

It might seem unlikely that this is the origin of the six-pointed snowflake, since water molecules are *very* much smaller than a snowflake—how could this sixness become so amplified? But as Kepler and the early crystallographers realized, geometric packing of a crystal's constituent units dictates the geometry of the much larger bodies that result. In essence, water's crystal structure imposes an innate hexagonality on the way the ice crystal grows. Or, as D'Arcy Thompson wrote with his customary elegance, 'these snow-crystals seem to give visible proof of the space-lattice on which their structure is framed'.

The presence of the hexagonal 'space-lattice' means that not all directions are the same for the growing crystal. That is why faceted crystals have the characteristic shapes that they do: the flat facets are simply planes of stacked atoms or molecules, but the reason why certain planes and not others define the crystal's form is that some facets grow faster. This non-equivalence of directions is called *anisotropy*; an isotropic substance is one that looks the same, and behaves in the same way, in all directions.

The anisotropy of crystals means that properties like surface tension differ in different directions. In 1984 Langer and his co-workers showed that, for Ivantsov parabolas growing in certain 'favoured' directions picked out by the anisotropy of the material's crystal structure, surface tension no longer induces a tip-splitting instability—the parabolic tip remains stable as it grows. Thus dendritic branches will grow outwards from an initial crystal 'seed' only in these preferred directions: the snowflake grows six arms. This special role of anisotropy in stabilizing the growth of a particular needle crystal was identified independently at the same time by David Kessler, Joel Koplik, and Herbert Levine at the University of California at San Diego.

Anisotropy also explains why a dendrite develops side branches. When, by chance, the parabolic tip develops small bulges on its flanks, these may be amplified by the Mullins–Sekerka instability. But again, only bulges that grow in certain directions will be stable. And there is only one kind of dendrite tip, for a given set of growth conditions, which grows fast enough to avoid being overwhelmed by these side branches. So a particular dendrite, with side branches sprouting in particular directions, is uniquely selected from amongst the possible growth shapes.

How the right arm knows what the left arm is doing

The mind-boggling variety of snowflake forms is therefore the outcome of a tension between chance and necessity. The mechanics of the growth process ensures that the arms will sprout in directions that point to the corners of a hexagon. For any given snowflake, these arms will all grow at the same rate (because, at such a small scale, they all experience the same conditions of temperature and humidity), and so they will have the same length. The side-branches of this six-pointed star are to a degree at the mercy of fate: they may be triggered by the random appearance of tiny irregularities or bulges along the parent arm. Yet they too will always surge outwards in a 'hexagonal' direction. Changes in the prevailing conditions that an individual snowflake experiences as it drifts and falls in the air may trigger simultaneous changes in the growth of all the branches, accounting for how, for example, needle-like arms might develop hexagonal plate-like formations at their tips (Fig. 1.13).