+ - \* /
T U V W

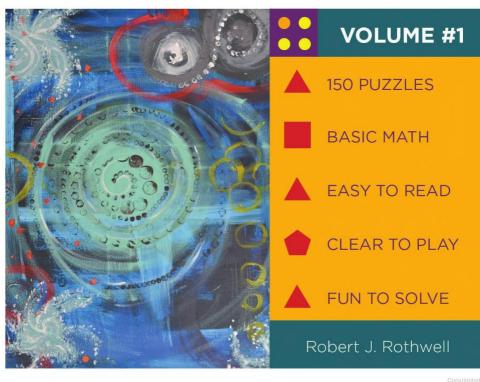
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1 <> 12



# **DODECABUS:**

A New Kind of Math Puzzle



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A New Kind Of Math Puzzle

TUVW

1 <> 12



By Robert J. Rothwell

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# Dodecabus, and Number Theory

Two thousand years ago, the ancient Greeks studied the often magical properties of the counting numbers, using the four basic math operations:

Addition: +, Subtraction: -,

Multiplication: \*, Division: /.

This area of math is now called Elementary Number Theory, and is full of intriguing problems/conjectures, many of which have been studied for hundreds of years by mathematicians, and yet still remain unsolved today.

I've designed this new math puzzle, called Dodecabus, to use some of the ideas from this area of math, and to also be very approachable, and fun to solve:

- >> You will only need basic math skills (+ \* /),
- >> All of the arithmetic will be easy, since it only involves the counting numbers (1,2,3,4,...),
- >> The fonts/sizes of the puzzles have been selected so that they are easy to read, and there is lots of room



to write down any ideas or results as you work on the puzzles.

Even after building a lot of individual Dodecabus puzzles, I am still fascinated at how these simple looking math problems can have many hidden layers and connections. Hopefully, you will discover some of the same fascination, and enjoy developing your own ideas and strategies, while solving them.

# **How to Play Dodecabus:**

- > The object of the puzzle is to find the values of a few unknowns (T,U,V,W,...), knowing that they must be counting numbers, from 1 to 12.
- > Each unknown has an equation (+,-,\*,/).
- >That's it, that's the puzzle.
- > The puzzles each have a difficulty rating:

Easy, Med/Easy, Medium, Med/Hard, Hard.

>>>>> Warm-up Puzzles, with Hints:

Dodecabus # 100, (Easy)

$$[ ] T = 7*U - 32$$

$$[]U = T + 2$$

**Hint:** Try a few values for **U**, and use the fact that both of the unknowns (**T,U**) must have values that are counting numbers, from 1 to 12.

Also, the square bracket areas are for recording the answers for each unknown, as you go along.



Dodecabus # 101, (Easy)

$$[ ] T = (U + 8)/3$$

$$[ ] U = 2*T - 4$$

**Hint:** Try a few values for **U** in the first equation, knowing that the stuff inside the round brackets has to be calculated first, and then divided by 3.

Remember, both **T** and **U** must always have values that are counting numbers, from 1 to 12.

By the way,  ${f T}$  and  ${f U}$  can be the same value.

Dodecabus # 102, (Easy)

$$[ ] T = 5*U - 14$$

$$[ ] U = 3*T - 14$$

**Hint:** Try a few values in either equation, and use the fact that both of the unknowns (**T,U**) must have values that are counting numbers, from 1 to 12.

Also, to solve the puzzle, the values for  ${\bf T}$  and  ${\bf U}$  need to work in both of the equations.

#### Dodecabus # 110, (Easy)

$$[]U = T/6$$

**Hint:** Looking at the division in the second equation, you could start by narrowing down what values **T** could have, knowing that **U** must also be a counting number.

Remember, **T** and **U** must always have values that are counting numbers, from 1 to 12.

#### Dodecabus # 108, (Easy)

$$[ ] T = 11/(U + 1)$$

**Hint:** Looking at the division in the first equation, you could start with figuring out what counting numbers will divide completely into 11, and then figure out what values **U** could have. Remember, **T** and **U** must always have values that are counting numbers, from 1 to 12.

### Dodecabus # 302, (Easy)

[ ] T = V - 4

[]U = V - 6

[ ] V = T\*U

**Hint:** Even though there are now three unknowns, and three equations, the basic ideas are still the same.

Try a few values for  $\mathbf{V}$  in the first two equations, knowing that the resulting values for  $\mathbf{T}$  and  $\mathbf{U}$  must always be counting numbers, from 1 to 12.

Then, see what happens in the third equation.

#### Workspace:

The Counting Numbers/Dodecabus section will also provide some ideas, and helpful hints, for solving the puzzles, and will also introduce you to a little bit of the history behind early Number Theory.

# Counting Numbers, and Dodecabus

Long before the ancient Greek mathematicians, early civilizations were developing written symbols to describe counting, measurement, and to record the results of trading. Some of these civilizations (Babylonians, Egyptians) used puzzle questions to record early mathematical ideas, and as teaching materials. I find it amazing that what we now cover in a few years in school, actually took many hundreds of years to be developed, and communicated from one generation to the next.

The counting numbers (1,2,3,...), and the four basic math operations (+,-,\*,/), are the fundamental building blocks of Dodecabus puzzles, and so it would probably be useful to review these together a bit in order to help solve these puzzles, especially the ones rated Medium, Med/Hard, or Hard.

It is easy to see that adding two counting numbers, or multiplying two counting numbers, will always give you another counting number, and it will be bigger than the two initial numbers.



**Subtracting**, or **dividing**, two counting numbers is more complicated because the relative sizes of the two numbers will now matter a lot.

For example, in the third equation of this example Dodecabus puzzle, there will be some choices for  $\mathbf{T}$  and  $\mathbf{U}$  that will give a value for  $\mathbf{V}$  that will be a counting number, and some choices that will not:

#### Dodecabus Example, (Medium)

 $\Gamma$  1 T = V - W

[ ] U = 3\*T + 2

[]V=U-T

[]W = U/T

If you try U=5 and T=7, then V would be -2, which is not a counting number (1,2,3,...).

Also, if you try U=5 and T=5, then V would be 0, which is also not a counting number. By the way, there is a name for all of the counting numbers, together with zero, and with all the negative versions of the counting numbers:

Integers: ...,-3,-2,-1,0,1,2,3,...

Counting Numbers: 1,2,3,4,5,...

Dividing two counting numbers is even more interesting, because the process of taking a number of items, and separating them into equal smaller piles, sometimes results in some leftover items. If you start out with fifteen items, and try to divide them into four equal piles, then you will end up having three items in each pile, with three items leftover:

$$15 = 4*3 + 3$$
 (leftover).



In the language of Number theory, four does not divide completely into fifteen. Instead, if you try to divide the fifteen items up into three piles, you will get five items in each pile, with no leftovers:

15 = 3\*5.

Also, if you try to divide it up into five piles, you will get three items in each pile, with no leftovers:

15 = 5\*3.

In the language of Number Theory, both three, and five, divide completely into fifteen. Another way of saying the same thing is that fifteen is **divisible** by three, or five.

Going back to the Dodecabus example puzzle on the opposite page, if you look at the fourth equation, the fact that  ${\bf W}$  must be a counting number means that  ${\bf T}$  must divide completely into  ${\bf U}$  (or, equivalently,  ${\bf U}$  must be divisible by  ${\bf T}$ ).

So, if you were trying values for U and T, then U=6, T=3, would give a value for W that is a counting number (W=2), but U=8, T=6 would not work.

By the way, the answer values for this example puzzle are: T=2, U=8, V=6, W=4. (These are all counting numbers, from 1 to 12).

This brings up another important thing to note, namely that the unknown values in Dodecabus puzzles only use the first twelve counting numbers (1,2,...,11,12), but the full range of counting numbers in general goes on forever: 15, 27, 1000 are all counting numbers.

This comes into play while solving the puzzles sometimes, such as in the second equation in the example



puzzle: if you try T=6, then the resulting value for  $\bf U$  will be a counting number (U=20), but it will be too big (bigger than 12).



# **Easier Puzzle Section:**

- >> 18 Puzzles, all rated "Easy"
- >> The Warm-up Puzzles, and the section on Counting Numbers/Dodecabus, should provide a good background for solving these "Easy" puzzles.
- >> Always in Dodecabus, it helps to use the fact that all the unknown values must be counting numbers, from 1 to 12.



#### Dodecabus # 103, (Easy)

$$[ ] U = 4*T - 3$$

#### Dodecabus # 104, (Easy)

$$[ ] T = U + 10$$

$$[ ] U = 2*T - 21$$

#### Dodecabus # 105, (Easy)

$$[ ] T = (T + U)/3$$

$$[ ] U = 4*U - 6$$

#### Dodecabus # 106, (Easy)

$$[ ] T = 20 - T$$

$$[ ] U = 2*T - 14$$

#### Dodecabus # 107, (Easy)

$$[ ] T = U*(U - 3)$$

#### Dodecabus # 109, (Easy)

$$[ ] T = 7*U - 32$$

$$[ ] U = 2*T - 1$$

#### Dodecabus # 113, (Easy)

$$[ ] T = 7*U - 2*T$$

$$[]U = T - 4$$

#### Dodecabus # 142, (Easy)

$$[ ] T = (U + 2)/3$$

$$[ ] U = 2*T + 1$$

#### Dodecabus # 143, (Easy)

$$[ ] T = (U + 1)/3$$

$$[ ] U = 2*T + 1$$

## Dodecabus # 144, (Easy)

$$[ ] U = 5*T - 1$$

### Dodecabus # 145, (Easy)

$$[ ] T = U/9$$

$$[]U = T + 8$$

#### Dodecabus # 146, (Easy)

$$[ ] T = 7*U + 1$$

$$[]U = 9 - T$$

#### Dodecabus # 147, (Easy)

$$[ ] T = 7/U$$

$$[]U = T - 6$$

#### Dodecabus # 300, (Easy)

$$[ ] T = 3*U + 1$$

$$[]U=T-V$$

$$[ ] V = 6*U - 7$$

## Dodecabus # 515, (Easy)

$$[ ] T = V - U - W + 2$$

$$[]U = T + 1$$

$$[ ] V = T + 7$$

$$[ ] W = 2*T - 1$$

#### Dodecabus # 148, (Easy)

$$[ ] T = U*U + 7$$

$$[]U = T - 9$$

#### Dodecabus # 301, (Easy)

$$[ ] T = 2*U + 1$$

$$[ ] U = 4*V - 3$$

$$[ ] V = T - 9$$

#### Dodecabus # 516, (Easy)

$$[ ] T = U*V - W$$

$$[ ] V = 2*T - 5$$

$$[]W=T+5$$