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Effective Theories in Physics

From Planetary Orbits to Elementary Particle Masses

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Acronyms

ALFB	Adjusted Galileo's Law of Falling Bodies
CERN	Centre Européenne pour la Recherche Nucléaire
ESM	Effectified Standard Model of Particle Physics
GeV	Giga electron volts
GLFB	Galileo's Law of Falling Bodies
IBE	Inference to Best Explanation
km	Kilometers
LHC	Large Hadron Collider
m	Meters
s	Seconds
SM	Standard Model of Particle Physics

Chapter 1
The Utility of Effective Theories

1.1 Definition of Effective Theories and Their Purpose

"Effective Theories" are theories because they are able to organize phenomena under an efficient set of principles, and they are effective because it is not impossibly complex to compute outcomes. The only way a theory can be effective is if it is manifestly incomplete. "Everything affects anything" is generally correct, but it saps confidence in our ability to predict outcomes. Effective Theories modify this depressing maxim by pointing out that "most things are irrelevant for all practical purposes." A tree falling in Peru does not appreciably affect a canon ball's flight in Australia. Any good Effective Theory systematizes what is irrelevant for the purposes at hand. In short, an Effective Theory enables a useful prediction with a finite number of input parameters.

With this definition of Effective Theories it appears that all theories are such, and thus giving it a fancy capitalized name is pointless pedantry. However, the proper name is useful to repeat at times as a reminder that the prominent views of science were not always agreeing that theories were necessarily incomplete, and as a reminder to go beyond it when and if the circumstances may arise. Furthermore, the natural tendency of young students entering science is to believe a theory is either right or useless, when they can never be completely right, but rather merely Effective Theories that are "correct enough for our purposes in this domain." Frequent and formalized reminders of this are helpful for newcomers to the field.

The other purpose of emphasizing the name Effective Theories is to force us to confront a theory's flaws, its incompleteness, and its domain of applicability as an integral part of the theory enterprise. The most useful Effective Theories are ones where we know well their domains of applicability, and can parametrically assess the uncertainties induced by ignoring the "irrelevant." They may even have a well-defined procedure for becoming more and more complex as one wishes to compute to higher accuracies. This is the case in many Effective Field Theories of particle physics, such as pion scattering or even graviton scattering. There is

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a science in understanding the circumstances of when questions can be addressed using accurate, convenient Effective Theories, and it is generally acknowledged that scale separation (Hillerbrand 2013) is one important feature of systems that enable an Effective Theory to separate out well the "relevant" from the "irrelevant". Indeed the phrase "irrelevant operator" is a technical term used in particle physics (Cohen 1993) to identify small contributions to phenomena caused by dynamics at a much different energy scale than is being probed. This issue arises in one form or another in all Effective Theories and will be seen in the examples presented.

1.2 Galileo's Law of Falling Bodies as an Effective Theory

Throughout this book we will get progressively more modern in our discussion of how to apply the concepts of Effective Theories to physics. We will move from the harmonic oscillator to Newton to Einstein to Fermi to Higgs and others. Before we do that, let us begin in this introductory chapter with Galileo—one of the first scientists who had what is recognizable as a modern perspective to scientific thought. Galileo was dedicated to knowing what was correct with less care about his or others' preconceived ideas. He was dedicated to experimental verification as an unbiased arbiter of theories. He investigated many things, but we will focus on his theory of falling bodies, and within that context show, as a warm-up to more sophisticated theories later, how the concepts of Effective Theory could have engendered further insight into a more general theory of gravity beyond just describing a falling body.

Let us suppose that we are back in the day of Galileo, well before Newton came along, and we are very mathematically sophisticated for the times. Upon reading Galileo's book the *Two Sciences* we come across the following passage:

> When, therefore, I observe a stone initially at rest falling from an elevated position and continually acquiring new increments of speed, why should I not believe that such increases take place in a manner which is exceedingly simple and rather obvious to everybody? If now we examine the matter carefully we find no addition or increment more simple than that which repeats itself always in the same manner. This we readily understand when we consider the intimate relationship between time and motion; for just as uniformity of motion is defined by and conceived through equal times and equal spaces (thus we call a motion uniform when equal distances are traversed during equal time-intervals), so also we may, in a similar manner, through equal time-intervals, conceive additions of speed as taking place without complication; thus we may picture to our mind a motion as uniformly and continuously accelerated when, during any equal intervals of time whatever, equal increments of speed are given to it.... And thus, it seems, we shall not be far wrong if we put the increment of speed as proportional to the increment of time; hence the definition of motion which we are about to discuss may be stated as follows: A motion is said to be uniformly accelerated, when starting from rest, it acquires, during equal time-intervals, equal increments of speed (Galileo 1638).

In mathematical language Galileo is saying $\delta v = g\delta t$, where v is the speed and g is the constant of proportionality. In differential calculus language $\delta v, \delta t \to dv, dt$.

Bringing dt to the other side of the equation one can rewrite Galileo's Law as $dv/dt = g$. But change in velocity with respect to time is nothing other than the acceleration, and Galileo's law becomes $a = g$, which is "uniform acceleration" as Galileo himself called it. Notice that the mass of the stone falling is not in this equation. More on that later. Another way to write the above equation is

$$\ddot{z} = -g \text{ (Galileo's Law of Falling Bodies)}, \tag{1.1}$$

in the convention that z is the position of the ball with increasing z in the opposite direction of the acceleration vector.

As an aside, every first year physics student has computed the trajectory of a ball in a uniform gravitational field. The equation of motion is usually derived from Newton's Second Law of Motion $F = ma$. In this case the force is $-mg$ where $g = 9.8\,\mathrm{m/s}$ is the acceleration downward due to gravity on the Earth's surface, and $a = \ddot{z}$ is the second time derivative of the ball's motion—the actual acceleration of its trajectory. The equation of motion is then $\ddot{z} = -g$, which is exactly Galileo's Law. Despite everyone knowing this, the reader is here requested to forget the more sophisticated later era of Newton, where this particular equation $\ddot{z} = -g$ is a simple derivation of a deeper law. Instead, I would like to ask the reader to treat $\ddot{z} = -g$ as a law of nature that has no parent—it is something stand-alone discovered by Galileo. That is why I am giving it a fancy name: "Galileo's Law of Falling Bodies", or GLFB for short. Let us press forward with GLFB, and ask what Effective Theories may say about it.

To give us something concrete to talk about with regard to GLFB, let us compute the time it takes for a body at rest to drop from a height h. The position of the body as a function of time is

$$z(t) = h - \frac{1}{2} g t^2. \tag{1.2}$$

Falling a distance h then takes time $T = \sqrt{2h/g}$. Notice, this does not depend on the mass of the body—an interesting conclusion that Galileo understood well. He knew that air friction caused bodies to slow down, and he even understood the concept of terminal velocity,[1] but most impressively he realized that air friction was a complication that was not fundamental to the problem:

> Now seeing how great is the resistance which the air offers to the slight momentum [*momento*] of the bladder and how small that which it offers to the large weight [*peso*] of the lead, I am convinced that, if the medium were entirely removed, the advantage received by the bladder would be so great and that coming to the lead so small that their speeds would be equalized (Galileo 1638).

In other words, in the limit that the density of the body was much higher than the density of the air, the air friction was not important. Galileo repeated this principle

[1] "... there is no sphere so large ... or so dense... that the resistance of the medium, although very slight, would check its acceleration and would, in time reduce its motion to uniformity" (Galileo 1638).

in other places, and understood it well: the fundamental law of falling bodies with resistance-less medium is uniform acceleration.

Another demonstration of Galileo's genius was that he understood better than anyone at that time that scientific claims were not only about deep thoughts that sounded good, but required experiment to test them and that any result was subject to question. At one point he took a swipe at Aristotle for holding what Galileo thought was an unjustified opinion: "... I greatly doubt that Aristotle ever tested by experiment whether it be true ..." (Galileo 1638). Galileo was certainly no respecter of persons, but rather had unswerving loyalty to determining what was correct. Even when he introduced his theory of falling bodies he qualified it by saying, "we shall not be far wrong" if we agree to his theory. Tentativeness, testing and refinement, the hallmarks of science, were important to his approach.

Galileo surely would not have minded any correction to his law that was not in conflict with what appeared to be sacrosanct symmetries of nature, such as invariance under rotations and space and time translations (Arnold 1989). A correction that seems quite reasonable is to disrupt uniform acceleration slightly by adding a correction term that depends on height position z.[2] Thus let us add the correction $\ddot{z} = -g + cz$, where c is some "small" constant.

The constant c is unknown and so this theory is not very predictive. However, we can make some intelligent guesses of roughly what value it could take. For one, we know that somehow we have to make cz have units of acceleration. This requires c to have units of acceleration/length. This is an awkward set of units. However we can simplify it by utilizing the one and only constant of our original theory, which is g and has units of acceleration. Thus, the obvious thing to do is let $c \to g/R$, where R is some unknown fixed constant of length. What could R possibly be? The test bodies are being pulled to earth, and they are all being pulled with (nearly) uniform acceleration independent of the size of the test body,[3] and so it is very reasonably to assume that we need to look to the Earth to provide us with a "natural length scale" to assign R. The radius of the Earth, $R_e = 6400\,\text{km}$, is the obvious candidate.[4]

If we were dogmatic and very arrogant we would say that our choices were "obvious" and that this new law, the Adjusted GLFB (ALFB), is the correct first correction and write $\ddot{z} = -g(1 - z/R_e)$ and then start computing. However, let us be humble scientists and suggest that this correction is perhaps "not far wrong", as Galileo might say, and insert a "constant of tentativeness" η, which is dimensionless

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[2] This is not in conflict with Galilean translation invariance, as z is shorthand for a *difference* in position of the body with respect to the earth's surface $z = r - R_{earth}$.

[3] Furthermore, using the size of a small test body as the parameter R would lead to dramatically too large effects, and for that reason also it can be dismissed as an option.

[4] There are several other length scales that perhaps might be equally justified, including the circumference of the earth ($R = 40{,}000\,\text{km}$), the height of the tallest mountain ($R = 9\,\text{km}$), or the depth of the deepest sea ($R = 11\,\text{km}$). The latter two are perhaps less intuitively relevant and could be dismissed as serious candidates. Nevertheless, if one kept an open mind to them all, the length scales are all within about a factor of 10^3 of each other, which might appear disastrously large to estimate a correction term, but it is decidedly better than not knowing how to estimate within a factor of ∞.

and perhaps not far from 1. Our new ALFB can be written as

$$\ddot{z} = -g\left(1 - \eta\frac{z}{R_e} + \cdots\right) \text{ (Adjusted Galileo's Law of Falling Bodies).} \quad (1.3)$$

Writing theories down with extra terms that have "natural sizes" and are consistent with symmetries is a cornerstone of the Effective Theory approach. This example is intended to demonstrate that a new theory can be generated by having this mindset, and the new theory is more correct, even if a little less predictive.

Ignoring the higher order "$\cdots$" terms, the solution to the problem of position as a function of time now becomes

$$z(t) = h\,\cosh\left(\sqrt{\eta\frac{g}{R_e}}\,t\right) - \frac{1}{2}gt^2 \quad (1.4)$$

and the time it takes to reach $z = 0$ is

$$T = \sqrt{\frac{2h}{g}}\left(1 + \frac{\eta}{2}\frac{h}{R_e} + \mathcal{O}\left(\frac{h^2}{R_e^2}\right)\right). \quad (1.5)$$

A body dropped from 200 m takes about a tenth of a second longer according to ALFB with $\eta = 1$ compared to the 6.5 s predicted by the GLFB.

In an alternative scientific history this effect of longer dropping time could have been measured and the anomaly noted before Newton's theory of gravity was decisively understood. The measurements would have converged on $\eta = 2$ to within experimental uncertainties. A discrepancy with Galileo's pure GLFB would not have been the subject of deep worries about human's ability to understand the laws of the universe since Galileo himself was tentative about his law. In time, Newton's theory would then develop, and the value of η would be computed to be exactly 2, and Newton's law of gravity would then replace GLFB as the overarching theoretical framework by which to understand and compute the trajectories of falling bodies.

We have seen from this simple example that one does not need to know the more fundamental theory of Newtonian gravity to anticipate corrections, compute their effects, and compare with data. The Effective Theory of ALFB is better than Galileo's original law, despite being less predictive, because ultimately it can accommodate the data better and reflects Newton's deeper theory. We will see another example of this in the chain of theories in a later chapter that shows how one could have anticipated phenomenological implications of Einstein's General Relativity by taking a more tentative, Effective Theory approach to Newton's Law of Gravity.

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Chapter 2
Harmonic Oscillator as an Effective Theory

Abstract The concepts of Effective Theories are illustrated allegorically within the context of one of the most ubiquitous models of oscillating physical phenomena—the harmonic oscillator.

2.1 Basics of the Harmonic Oscillator

The concepts and issues related to effective theories can be illustrated quite nicely by the harmonic oscillator problem. The harmonic oscillator is one of the most ubiquitous mathematical models of physics phenomena. It is present in almost every system with a restoring force, which includes the galaxy, solar system, springs, atoms, molecules, and innumerable other configurations.

The main point I would like to illustrate is that the lowest order effective potential for the harmonic oscillator is an excellent approximation to the motion of a system over a wide range of amplitudes. However, at some point it breaks down when the amplitude is large enough, and then control over the system is lost unless a deeper theory is understood. We shall not go into the construction of deeper theories in this chapter, but rather focus on the domain of applicability of the harmonic oscillator effective theory, and show how small corrections can be anticipated and then measured by precise experiments to start building a more complete picture of the potential governing the system.

To keep the illustration simple, we will restrict ourselves to one-dimensional harmonic motion of a particle subject to the restoring potential $V(x) = kx^2/2$. The Lagrangian of the system is then

$$L = \int dt \left(m\frac{\dot{x}^2}{2} - k\frac{x^2}{2} \right). \tag{2.1}$$

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From the principle of least action the equation of motion gives Newton's second law of motion $F = ma$ the form

$$m\ddot{x} = -kx \Longrightarrow m\ddot{x} + kx = 0. \tag{2.2}$$

Defining $\omega^2 = k/m$, we can rewrite this as

$$\ddot{x} + \omega^2 x = 0 \tag{2.3}$$

which has the solution

$$x(t) = A\sin(\omega t) \tag{2.4}$$

where A is the amplitude, and the boundary condition of $x = 0$ at $t = 0$ is enforced.

Let us review a few basic facts about the harmonic oscillator solution. The period is

$$T_{period} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}. \tag{2.5}$$

The amplitude A of motion is related to the initial velocity by equating full potential energy at maximum amplitude to the full kinetic energy at maximum velocity:

$$\frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2 \Longrightarrow A = v_{max}\sqrt{\frac{m}{k}} = \frac{v_{max}}{\omega} = \frac{v_{max}T_{period}}{2\pi}. \tag{2.6}$$

It should also be noted that the period of the harmonic motion is not dependent on the amplitude of the motion. This is clear from Eq. 2.5 where it is shown that the period only depends on the input parameters m and k. The amplitude and maximum velocity conspire with each other such that v_{max}/A is always equal to $\sqrt{k/m}$.

2.2 Ubiquity of the Harmonic Oscillator

The harmonic oscillator problem is ubiquitous in physics, describing small motions of an object attached to a string, molecules vibrating in crystals, electrical circuit response, etc. There is a straightforward reason why there are so many examples that follow simple harmonic behavior. Let us suppose that the equilibrium point (i.e., the minimum of the potential) is about the origin. Then, the potential for motion is a power series of the form

$$V(x) = a_2x^2 + a_3x^3 + a_4x^4 + \cdots. \tag{2.7}$$

We do not write down a constant term or a term linear in x because the first is irrelevant and the second term cannot be present if $x = 0$ is a local minimum. If it