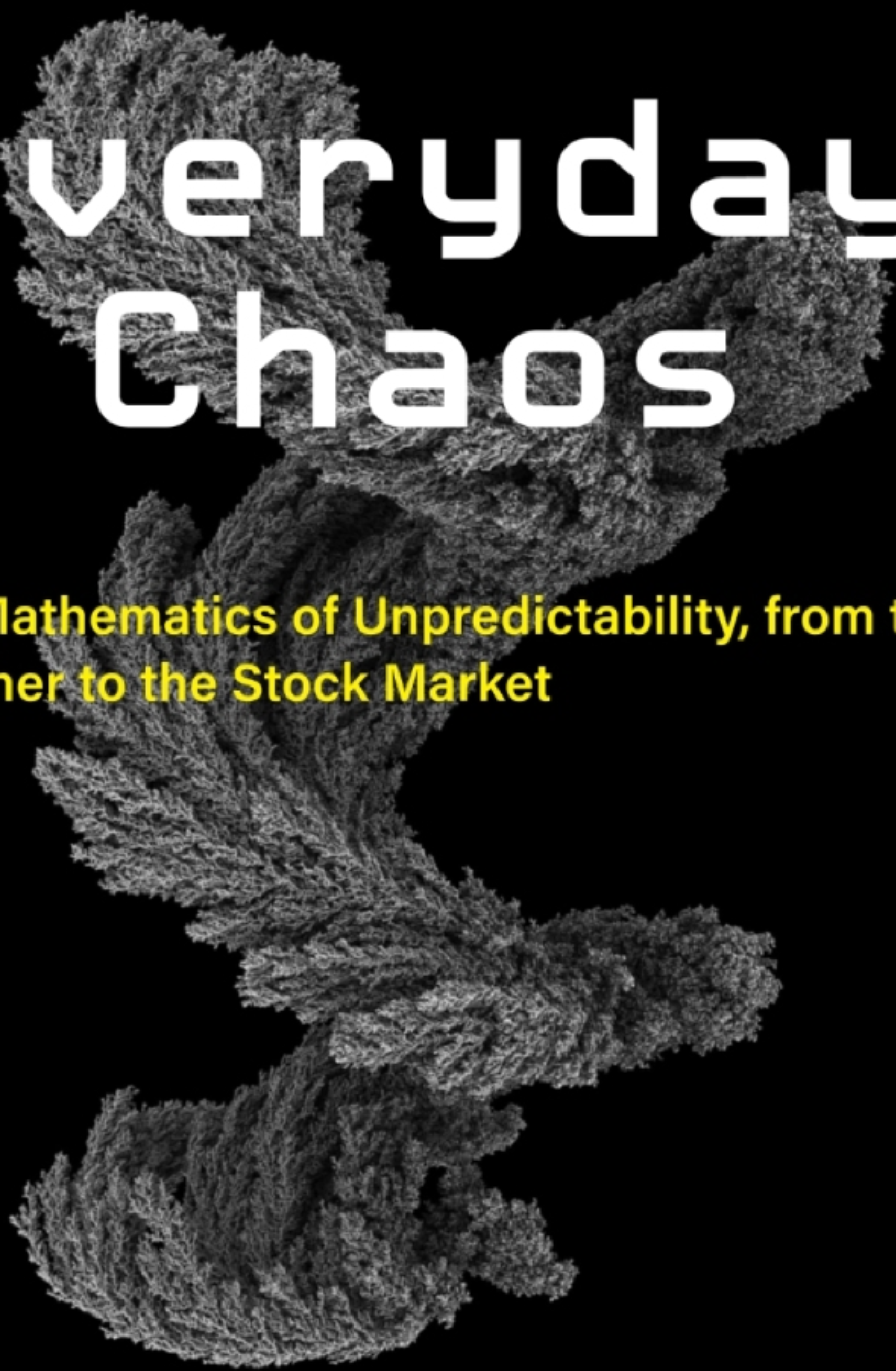
"He still manages to surprise me with something new on every page."—**Alain de Botton**

Everyday Chaos



The Mathematics of Unpredictability, from the Weather to the Stock Market

Brian Clegg

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Everyday Chaos

The Mathematics of Unpredictability, from the Weather to the Stock Market

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Introduction

Scientists and mathematicians have a tendency to take words that are used loosely in the world at large and give them very specific meanings. So, for example, in everyday speech we tend to use "power" and "energy" nearly synonymously. But in physics, power is very specifically the rate of transfer of energy from one location to another. Similarly, the words "chaos" and "complexity," which will be at the heart of this book, are broad descriptive terms in general usage, but in math they have meanings that imply particular characteristics.

In normal usage, when we speak of chaos, we think of a mess. A lack of order. Randomness. That word came to the English language via Latin, taken from the name of what some considered the primal Greek god, who represented the first, formless matter. The original word in Greek could also mean a chasm—but in either case, it denoted a lack of structure. Chaos spread confusion and was a force for destruction. It wasn't on the side of the good guys, which makes it an interesting choice as the term used to describe the mathematics of a surprising number of everyday things around us. First applied to animal population growth and the weather, chaos in the mathematical sense is typified by a system—a collection of things that interact—where very small changes in the way things start out can have huge implications for the way things eventually unfold.

If chaos implies unpredictability—disorder arising from apparently ordered starting points—the mathematical concept of complexity is a kind of *alter ego* (even though chaotic systems can be complex). In a complex system, the interaction of apparently simple components results in outcomes that would not otherwise be possible. Complexity is the ultimate end of "The whole is greater than the sum of the parts."

→
Untangling of Chaos
Sixteenth century engraver Hendrick Goltzius's illustration for Ovid's first-century epic poem *Metamorphoses*.

In ordinary usage, "complexity" simply refers to being made up of large numbers of parts or having an intricate form. But mathematical complexity can emerge from a relatively small

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system, just as much as it can from an intricate mechanism. So, to be mathematically complex, a system doesn't have to be, well, complex.

One mark of a complex system is emergence. This is where the "greater than the sum of the parts" bits comes together. Emergence suggests that new abilities *emerge* spontaneously from the complex system without any guiding force being totally responsible for shaping those abilities. You, for example, are a complex system. If we take a look at the individual atoms that make up your body in its entirety, they are not alive. But you are alive. If we step it up a level, we could describe the cells in your body as alive—but they certainly aren't capable of thinking or feeling or carrying out the actions that your body does. These capabilities are emergent from the complexity that is you.

Perhaps what is most remarkable about chaos and complexity is that they are all around us in nature. They are present in every living thing, in the weather, in the majority of the real-world objects that we interact with. And they are there in many human creations and systems, from the stock exchange to a bookstore. Yet we don't get taught about chaos and complexity at school. They don't feature in a whole lot of the work carried out by scientists either, who often concentrate instead on the small details, producing results that aren't applicable holistically.

Much of science can be described as reductionism—breaking a complicated thing down to its components and seeing how those components work, then building back up from the individual parts to try to understand the whole. For example, a real-world chemical reaction can be chaotic. Anyone who has ever added concentrated sulfuric acid to water will know that the result is highly dependent on how you start out. But when we study chemistry, we break things down to their component atoms and only consider how they interact.

The twin theories of chaos and complexity give us the opportunity to get a better understanding of the real world, rather than the toy universe in which most science takes place. The real world is far more complex, chaotic, and, frankly, interesting than much of the science we were taught in school suggests. We are about to dive under the surface and discover reality.

Welcome to everyday chaos.

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Que será, será

For the past two and a half thousand years we have developed an increasingly scientific viewpoint, often supported by mathematics. In some cases, this approach has proved remarkably effective. However, all too often the real world has confounded the attempts of science to predict what will happen.

It was not until the second half of the twentieth century that we realized what was occurring. The interaction of the components of systems, from an apparently simple jointed pendulum to immensely detailed weather systems, produces unexpected results. At the same time, collections of simple entities are capable of remarkable feats—think, for example, of the abilities of some species of ant that as individuals are totally incapable of any useful action but, by working together, can use their bodies to form bridges, stitch leaves, and carry weighty loads.

To see how chaos and complexity came to be understood, we first have to take a journey back in time to a point where it seemed that the future was entirely within the grasp of our mathematical minds. Thanks to the work of Isaac Newton, his successors were convinced that it would soon be possible to take on the universe and win.

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Newton, Laplace, and the amazing clockwork universe

"An intellect which at any given moment knew all of the forces that animate nature and the mutual positions of the beings that compose it, if this intellect were vast enough to submit the data to analysis, could condense into a single formula the movement of the greatest bodies of the universe and that of the lightest atom; for such an intellect nothing could be uncertain and the future just like the past would be present before its eyes."
Pierre-Simon, Marquis de Laplace, 1749–1827

The fluidity of time

In our time, when technology is such a normal and everyday part of life, it can be easy to forget what a transformative piece of technology the mechanical clock was. Prior to the availability of clocks, time was a thing of approximations, with only the broadest reference points. Available to everyone was the apparent transit of the Sun through the sky, or the motion of the heavens at night (unless it was cloudy). The better-off might have had a sundial, a water clock that measured time through a liquid dripping out of a small orifice, or the progress of a candle's burn. But any sense of exactness with respect to time was not a real thing. This is apparent from certain enduring expressions, such as "the sands of time," referring to an hourglass, or the practice of fixing time by the Sun when using the terms dawn, midday, and dusk.

Our lives are now so tied to technology that precision in time can seem a burden—a thing of deadlines and pressures—but

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when mechanical clocks were first invented, they were instead a wonderful eye-opener. It wasn't just the ability to know what time it was—to be able to meet someone at a particular time, rather than having to wait an hour or two—it was an essential both for the daily observations of religions, which tended to be tied to specific times, and for science. Having a measured approach to the progress of time was crucial to beginning to understand how aspects of the universe worked. It is no coincidence that the great breakthroughs in grasping the physics of motion came about in Europe at the same time as relatively accurate mechanical clocks were becoming more widespread.

The earliest mechanical clocks seem to have been developed in Europe in the fourteenth century. It's difficult to pin down a first, but certainly one of the oldest examples was the tower clock of St. Alban's Abbey in England, constructed by Richard of Wallingford in the 1320s. This example did not survive the Reformation, but another early English clock, in Salisbury Cathedral and dating back to around 1386, is still in action. Like many clocks of the age it had no dial—the point of having the clock was for it to strike a bell on the hour to ensure that religious services, which were scheduled at specific hours of the day, could be performed on time.

The escapement—the mechanism that measures out the units of time—in these early clocks was inaccurate by modern standards. Relatively precise measurements of time were not possible until 1656 when Dutch scientist Christiaan Huygens invented a clock with a regular beat provided by a pendulum. A contemporary of Sir Isaac Newton, Huygens was among those who were driving physics in a more mathematical direction.

A few decades earlier, when Galileo Galilei had needed to time objects in motion, he had had to rely on imprecise measures such as his own pulse. But with Newton, mathematics took on a central role in explaining the universe—requiring the kind of

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accuracy that Huygens's clock and its successors could provide. Clockwork not only provided the beat against which motion would be measured, it gave a mental model on which to base understanding of the universe itself.

Mr. Newton's legacy

Since the ancient Greeks first studied the night sky the universe had been seen as resembling a mechanical structure, but with crystal spheres carrying the planets and stars, rather than the gears of a machine. We know the Greeks constructed a geared model (presumably not the only one) that reflected some of the motions of the heavens in the remarkable Antikythera mechanism, an astronomical calculator from *ca.* 100 BCE, discovered in a shipwreck off the Greek coast in 1901.

More dramatic astronomical clocks, of which one magnificent example is the *Orloj* in Prague, in the Czech Republic, dating from 1410, presented a clockwork analog of the universe, while small-scale devices known as orreries provided heliocentric models of the universe itself (what we would now call the solar system), showing the positions and orbits of the planets and moons, usually driven by a clockwork mechanism.

→
Prague Astrological Clock (Orloj), detail
Dial showing the paths of the Sun and Moon, the phases of the Moon, and more.

→ →
Prague Astrological Clock (Orloj)
Laplace's mathematical view of the universe is likely to have been influenced by the mechanical precision of astronomical clocks, such as this magnificent example from 1410 on the Old Town Hall in Prague.

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Newton's *Principia*
Title page of a first edition of Isaac Newton's masterpiece *Philosophiae Naturalis Principia Mathematica*, published in 1687 (the book should have been published in 1686, but the budget wasn't available).

These were "physical" models in the commonplace usage of that adjective. But with the work of Isaac Newton a new kind of model of the universe became available to natural philosophers (the name by which early scientists were known)—a *mathematical* model. Newton was not the first to describe the physics of motion—Galileo, for example, had made a start by studying the way that balls accelerated down ramps under the influence of gravity. However, Newton turned what had been primarily a descriptive science into one where mathematics could be used to determine the future.

In his masterpiece, *Philosophiae Naturalis Principia Mathematica* (Mathematical Principles of Natural Philosophy), usually known as the *Principia*, Newton used mathematical tools to describe how the attractive force of gravity between two bodies—for example, Earth and the Moon—caused them to move in a particular orbit and caused objects like the famous apple to fall to Earth. He also proposed his three laws of motion, explaining the way that bodies move and how forces cause them to accelerate and interact.

To achieve this feat, Newton developed a new type of mathematics, which he called the "method of fluxions," now better known as calculus, the name used by his competitor, the German polymath Gottfried Leibniz. Equipped with Newton's new mathematical tools, his successors were ready to take on the whole universe—and none more so than his most enthusiastic European supporter, the French natural philosopher Pierre-Simon Laplace.

No need for that hypothesis

Newton, in his work on gravity, had focused on the movements of bodies in the solar system. Laplace had a grander vision. Born into an aristocratic family in 1749, he showed an early talent for mathematics, which would blossom as he took on many of the problems of applying mathematics to the universe and bringing math to practical uses in physics and engineering. From our viewpoint, though, Laplace's greatest contribution was establishing the concept of determinism.

Here, the mathematical description of reality is taken to the extreme. Laplace envisioned the image of a clockwork universe, where everything that happens for all time is determined exactly by what occurred the moment before, proceeding mechanically under Newton's laws. To illustrate the

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implications of such a vision, Laplace dreamed up his "demon." His description in 1814 of this creation, which he described as "an intellect," is quoted at the beginning of this chapter.

According to Laplace's view, if someone knew every detail of the state of the universe at a point in time, then thanks to the mathematical certainties of Newton's and Laplace's mathematics, we would be able to predict perfectly what would happen next from moment to moment. Everything, for eternity, would be preordained.

As human beings, we have always wanted to know the future. Ancient civilizations had their oracles and auguries—mystical and magical means to get an apparent glimpse of what was to come. Astrology, until late medieval times, was considered an acceptable part of the scientific armory, on the assumption that the movements of the planets had an influence over what happened on Earth; it was frequently consulted by kings and commoners alike. By building on Galileo's observations, Newton was able to push aside the mysticism and make mathematical predictions that were of a different order to those of the oracles and astrologers. They worked. Repeatedly, *repeatably*, they foretold what would happen.

Newton's math described not only how things around us moved, but it tied the familiar movements of things on Earth to the apparently grander and totally separate movements of the heavens. He showed how the journey of the Moon around Earth, for example, could be predicted from the simple factors of the masses of the two bodies and their distance apart. Others would take this even farther. Newton's friend and supporter Edmond Halley (it was Halley who ensured that *Principia* was published) used Newton's mathematics to make an accurate prediction of the return of the comet now known as Halley's comet. He would not live to see its triumphant reappearance, but Halley's forecast was sound. The comet came back—on time.

Laplace went farther still. It seemed to him that, given perfect knowledge of how things in the universe were at a particular point in time, for every future time it should be possible to run the mathematical model of the universe forward and see what would happen next, moment by moment. It was a picture of a clockwork universe that ran on unwavering tracks. Yet for many it seemed implausible. How could reality be so far from this mathematical ideal?

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Astrological zodiac
Fourteenth-century Spanish illustration of the signs of the zodiac, describing when the Sun enters each sign.

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Randomness is predictable

"In that case, I would rather be a cobbler, or even an employee in a gaming house, than a physicist."
Albert Einstein, 1879–1955

Randomness in reality
For Laplace's vision to work, everything needed to follow on from what came before, with clear cause and effect, from moment to moment. It was a universe we would now describe as deterministic, meaning that everything that happens now is determined, clearly and unequivocally, by everything that happened the moment before. However, there was a clear problem blocking Laplace's view of the universe—randomness.

The idea that things can happen at random with no prior reason is not one that comes naturally to human beings. We understand the world around us through patterns, finding it difficult to accept that things can happen without a guiding principle—with no reason *why*. This dependence on patterns is excellent for survival when it comes to recognizing a predator or a dangerous situation. But it also means that we see bogeymen when there is nothing there, or assume, for example, that a disaster has to be blamed on the direct action of deity, or fate, or the malevolent intervention of a magical power.

→
Albert Einstein
Despite being a founder of quantum physics, Einstein (pictured here in 1921) was reluctant to accept the randomness at the heart of the theory.

In reality, from the days of antiquity it was realized that some events were, to all intents and purposes, random—the toss of a coin or roll of dice, for example—which is why such events feature in games of chance. But our disinclination to accept

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The power of probability

"Chance favors only the prepared mind."
Louis Pasteur, 1822–1895

What are the chances?
Games that rely on probability for their outcome date back to ancient times. It's not recorded who first spotted that a two-sided coin could be spun in the air and produce a reasonable approximation to a random selection between heads and tails, but coin tosses have certainly been used for random selection, fortune-telling, and games of chance for thousands of years.

Similarly, dice—or their early equivalents—go back a long way. Archaeologists have found astragali, shaped knucklebones that act as crude dice, dating back at least 5,000 years, while backgammon-like "tables" are among the oldest-known board games. For much of the time such methods of chance have existed, good players may have had an instinctive feel for how probability worked, but the rules of probability only began to be quantified when a sixteenth-century Italian physician, Girolamo Cardano, himself an enthusiastic gambler, wrote a book on the subject.

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The Backgammon Players
Painting from 1634 by Flemish artist Theodoor Rombouts showing a lively game of backgammon.

Although Cardano was in his twenties when he wrote *Liber de Ludo Aleae* (Book on Games of Chance) and continued to refine it through his life, it was not published until 1663, nearly a century after it was first penned, because its topic was not deemed suitable for polite society.

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Probability is a concept we seem naturally to struggle with, and Cardano's great contribution to help us understand it was to turn probabilities into more manageable fractions. Let's imagine that you had a fair coin and tossed it 100 times. Heads or tails? You would expect to get heads around 50 times and tails around 50 times. Cardano realized that you could represent the chance of getting a head as $\frac{1}{2}$ and the chance of getting a tail as $\frac{1}{2}$. The bigger the value, the more likely the outcome, with a value of 1 meaning absolute certainty and 0 something that would never happen. As a coin toss (ignoring landing on the edge) has to result in either a head or a tail, the chance of getting either a head or tail is $\frac{1}{2} + \frac{1}{2} = 1$.

Similarly, with a familiar six-sided die, the chance of getting any particular number is $\frac{1}{6}$. Once our gambling scholar took a mathematical approach to probability, he could start to look at how to combine different outcomes to find the chance of a given combination. So, for example, if you want to know the chance of getting either a five or a six, it's simply a matter of adding the probabilities, giving us $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$.

Cardano also dealt with the distinctly trickier problem of combining multiple probabilities. To take a common example, we know the chance of getting a six with a single die is $\frac{1}{6}$. What's the chance of getting a double six with two dice? Cardano showed that this was a simple multiplication problem: $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$. But what about the chance of getting at least one six with a pair of dice? Clearly it's a better chance than getting a six with just one die. But we can't double the $\frac{1}{6}$ chance—otherwise, rolling six dice would guarantee getting a six, which we know isn't the case. Cardano engaged in some lateral thinking. The chance of not getting a six with the first die is $\frac{5}{6}$. The chance of *not* getting a six with the second die is also $\frac{5}{6}$. So the chance of not getting a six with both dice is $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$. As the total of all possible outcomes has to be 1, this means the chance of getting a six is $\frac{11}{36}$.

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The power of distributions

Turning our minds to randomness and probability, the outcome is often less straightforward than it is in games of chance where the probabilities of the different outcomes are known in advance (assuming that the coin isn't double-sided or the dice loaded).

Equipped with the ability to master probability using Cardano's mathematical approach, there's an essential requirement for getting a handle on randomness and probability in the real world: an understanding of distributions, that is, a picture of how possible outcomes are distributed—whether, for example, some outcomes are more likely than others.

If we look at the distribution of possibilities for a coin toss, what we have, effectively, is a bar chart with two possible outcomes. We can approximate to the bar chart by tossing a coin repeatedly and noting down how many heads and tails we toss. Initially there may be significantly more of one than the other, but over time, as tosses are collected, the numbers will get ever closer to the expected distribution.

Similarly, we can produce an equally dull distribution for the possible results of throwing a six-sided die—and again, we could build this distribution without knowing the actual values by repeatedly throwing a die. Things get a little more interesting if we look at the distribution of values we can produce by rolling two dice simultaneously and adding the results together. The outcomes range from 2 to 12—but not all possibilities have an equal chance of turning up. Here the distribution is not only more interesting, it can tell us something—for example, that the most likely outcome of rolling a pair of dice is 7.

Normal distribution
Also known as a Gaussian distribution, the normal distribution is symmetrical on either side of the most likely outcome, with long shallow "tails" to either side showing outcomes that are less likely.

In the world around us, where something is varying randomly, we often find that the distribution comes in the form of a "normal distribution," sometimes called a bell curve because of its shape when plotted. So, for example, if you plot the height of a whole lot of people, you will find that heights of men and women are each distributed in an approximate normal distribution.

Any particular individual will, of course, only have one specific height—but what the distribution enables us to do is to predict the most likely height and how likely it is that a man or woman's height will be within a certain range either side of the most likely height. The normal distribution has a measure known

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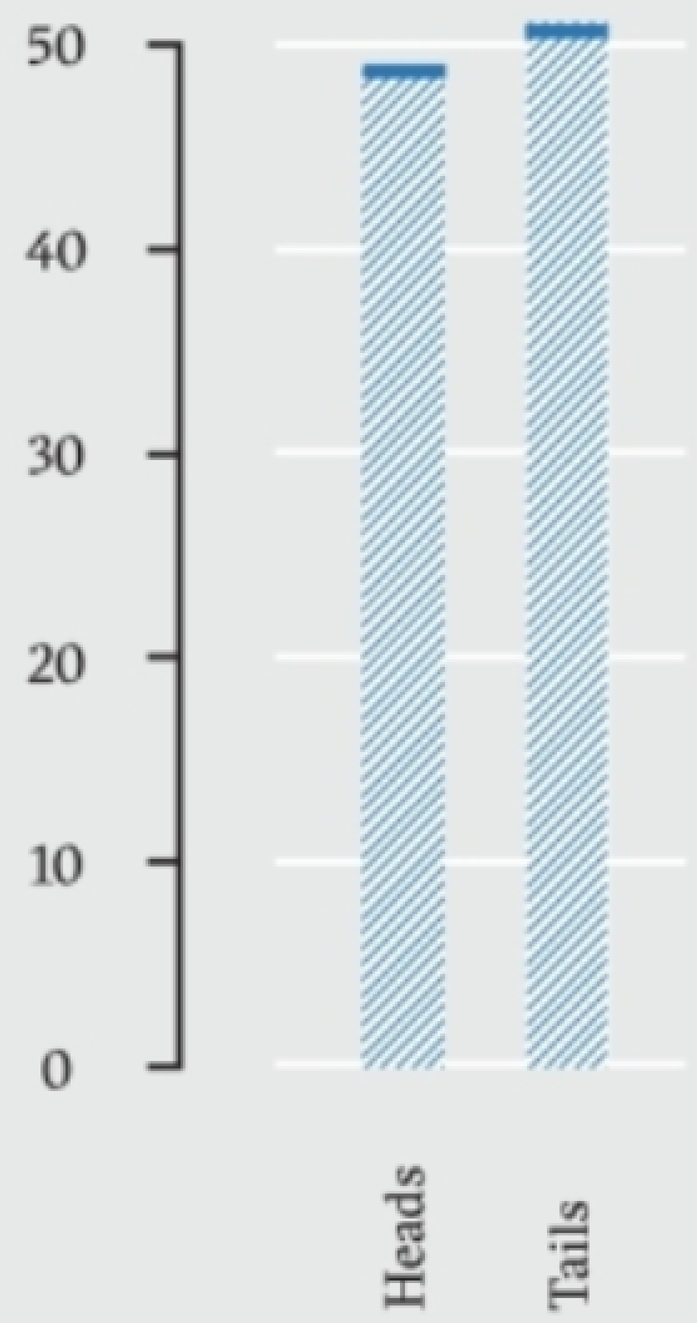
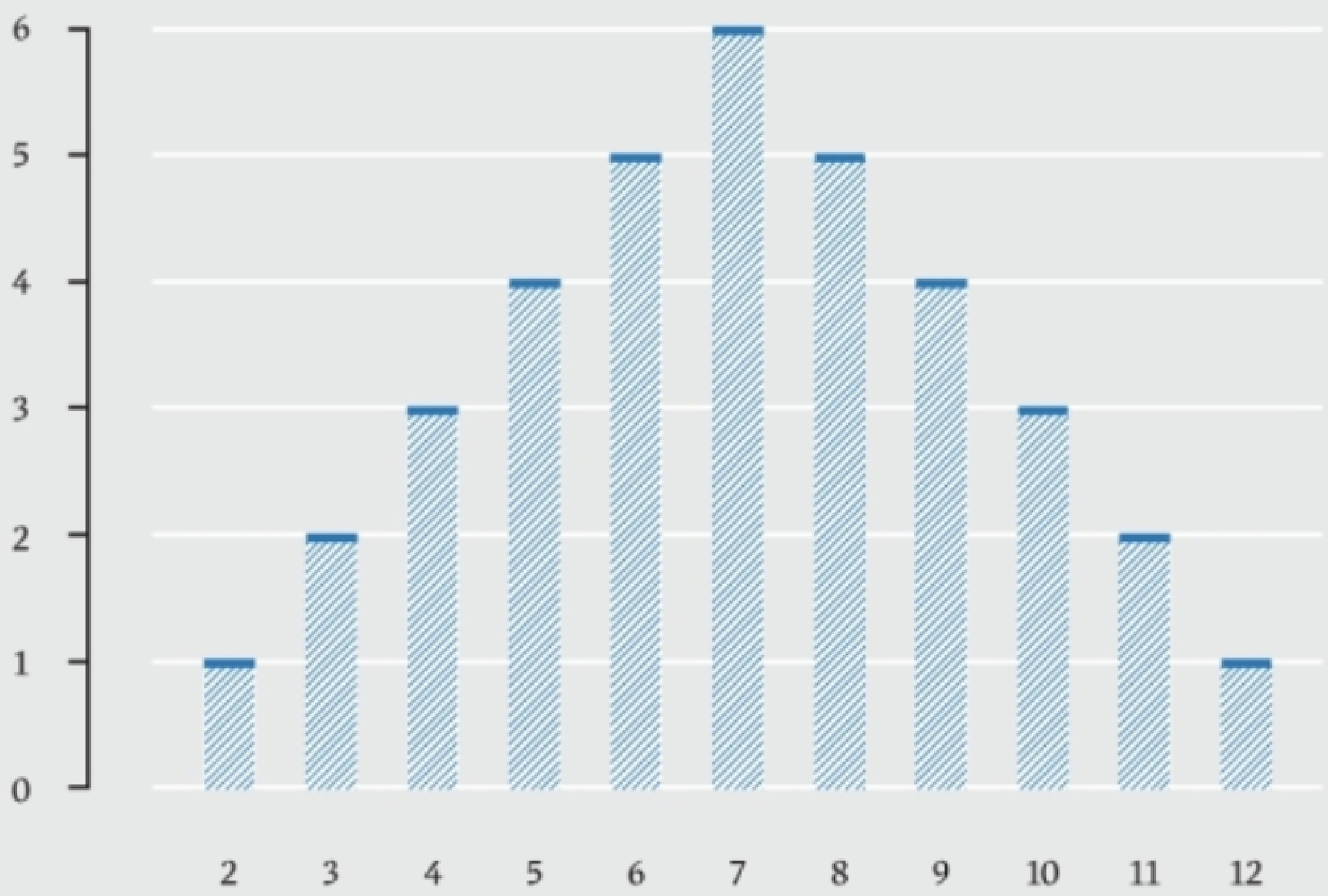
Distribution of coin tosses in an experiment after 100 tosses
As more and more coin tosses are made, the distribution will get closer to 50:50 heads and tails.



Distribution of the number of ways an outcome can be achieved with two dice.
Not all outcomes of throwing two dice are equally likely: the distribution shows relative probabilities.



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Logic
The systematic study of inference, where a conclusion is derived from the combination of a series of facts or statements.

Note that induction is different from deduction—something that is rarely effective in the real world (or in science). Deduction enables us to use logic to form a conclusion. So, for instance, if I know that "All dogs have four legs" and I have an animal in front of me with three legs, I can deduce that this animal is not a dog. The problem with deduction in the real world is that, while I can make a definitive observation of the number of legs the animal has, the deductive process depends on the ability to make the earlier statement "All dogs have four legs."

To be definitive about this, I need to have examined all the dogs that exist beforehand—which simply won't happen in the real world. So, I have to support my deduction with some induction. Perhaps every dog I've ever seen has four legs, so I assume that they all do. But, of course, there are actually three-legged dogs out there. Deduction is only as good as the underlying assumption, and as long as this depends on induction, we can do no more than say that this is our best current theory. That is how science usually works.

There are exceptions, of course. If I do have the opportunity to verify my initial statement, deduction is reliable. If I have a box of buttons, I can check every one of them and be able to say, for example, "All the buttons in this box are blue." If I am then given a red button I can deduce with certainty that this button did not come out of my box. But real life is rarely like my button box.

Patterns can catch us out

Bertrand Russell, the British philosopher who famously commented on the sun rising each morning, also made an observation of the experiences of poultry on a farm, which is sometimes reshaped in terms of a turkey's diary. Imagine that a turkey kept a diary which showed how good a day it was having. If we plotted these ratings for each day, it might well provide us with a nice distribution and, being mathematically minded, we might try to use that distribution to make predictions about

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the future. And then, a few days before Thanksgiving, from the turkey's point of view, its day would go off the scale (in the bad direction) as it was prepared for the table.

Without knowledge of the circumstances, it might seem that the turkey's "bad day" was a totally random event—and certainly without background information it was totally unpredictable. Yet we humans struggle to accept that there can be deviations from our understanding of the world. In the turkey's worldview, based on its previous experience, being trussed and roasted was not a possible outcome. But it happened. Similarly, when the unexpected strikes, we often try to provide an explanation within our current understanding, where it may well be that the cause of a sudden, unpredicted occurrence is outside of our available information and not explainable without changing our view of the world.

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The turkey's diary
Turkeys might logically infer that each day will be pretty much like the previous one. Until Thanksgiving.

In effect, what happens to the turkey—and happens to us all the time—is that the wrong pattern is being used to understand the world. One way this manifests itself is through superstition. If we see an apparent pattern linking one occurrence and another, we assume there to be a causal link. The superstition we associate with mirrors, ladders, or black cats is a near-inevitable outcome of making use of induction, because it can be difficult to separate real causes and events that just happen to coincide in space or time.

Often, like the turkey, we have a limited understanding of what is going on. It is then easy to apply patterns that have been successful in the past for other requirements to situations where they just don't fit. It could be that something is genuinely random, but we expect a different kind of pattern to apply. If I toss a coin and get nine heads in a row, it's hard not to expect it to be more likely that the next toss will be a tail because in my mind there's a pattern that tells me half the tosses should be heads and half tails. But in reality the coin has no memory, it doesn't know what has happened before, so there is still a 50:50 chance of getting a head or a tail.

The same thing is seen with sports records. When a team has a run of luck or a player is described as having a "hot hand," we are applying a pattern that implies some underlying causation to what can be a totally random set of circumstances that certainly will have no effect on what will happen in the future.

Predicting what will happen next depends on having a good enough understanding of what is going on. We need to get a feel for what a system is (more on this comes next) and how the nature of that system enables—or prevents—the ability to predict what is likely to happen. And a good starting point for this is to take a look at the humble pendulum.

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Playing with pendulums

"If one wishes to make the vibration-time
of one pendulum twice that of another,
he must make its suspension four times as long."
Galileo Galilei, 1564–1642
(trans. Henry Crew and Alfonso de Salvio)

Getting systematic
Whenever we are trying to understand the world around us and the impact of chaos on it, the fundamental unit is the system. In everyday life, a system tends to be a way of referring to a social grouping, often in a negative way ("She spends her time fighting the system"), an approach to doing something ("He has a system for winning"), or a piece of technology ("This sound system is amazing"). But for our purposes, a system has a much wider meaning.

A system simply means a collection of interacting components. It can be as basic as a ball and a slope it can roll down, or as complex as the universe. A pen is a system, as is a smartphone, your body, a country's administration, or the weather. A useful classification is to split systems into two broad kinds. An open system can interact with other elements and systems that are outside the system in question, while a closed system cannot. Most systems in everyday life are open, but to simplify matters, we often treat a system that has limited interaction with its surroundings as closed.

One of the simplest systems is the pendulum, which the great Italian natural philosopher Galileo Galilei spent a considerable

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amount of time studying in the sixteenth and seventeenth centuries. A basic pendulum consists of an anchor point—say, a hook in the ceiling—a suspension mechanism—it could be a piece of string—and a bob, which is a weight attached to the end of the suspension mechanism.

A pendulum demonstrates the conversion of energy from one form to another. If we start the pendulum swinging from side to side, at the top of the swing, the bob has some potential energy, the energy caused by being lifted up under the force of gravity, but no kinetic energy—the energy of motion. As the bob starts to swing down, some of the potential energy is lost as the bob gets lower and the kinetic energy increases as the bob moves faster. As it moves from side to side, the pendulum is constantly switching energy between potential and kinetic and back again.

Energy
The aspect of nature that makes things happen. As the American physicist Richard Feynman said, "It is important to realize that in physics today, we have no knowledge of what energy *is*."

This is not a closed system. A small amount of the energy initially given to the system by lifting the bob will go into distortion of the suspension mechanism, generating heat, while a little more will be lost to air resistance, unless the pendulum is enclosed in a vacuum chamber. Crucially, the system can't be considered closed as without the gravitational pull of Earth, there would be no potential energy to be transferred into kinetic.

Soon after Galileo's work, pendulums started to be used for timekeeping—Christiaan Huygens's first pendulum clock constructed in 1656 made use of Galileo's observation that the time it took for the pendulum to complete its swing depended only on the length of the suspension mechanism. It didn't matter how heavy the weight was, and bigger and small swings were accomplished in the same time.

In reality, this last observation was only true for relatively small movements, but the pendulum was a well-behaved system with an easily predicted motion. It was the absolute opposite of chaos. Yet one small change is all it takes to alter that.

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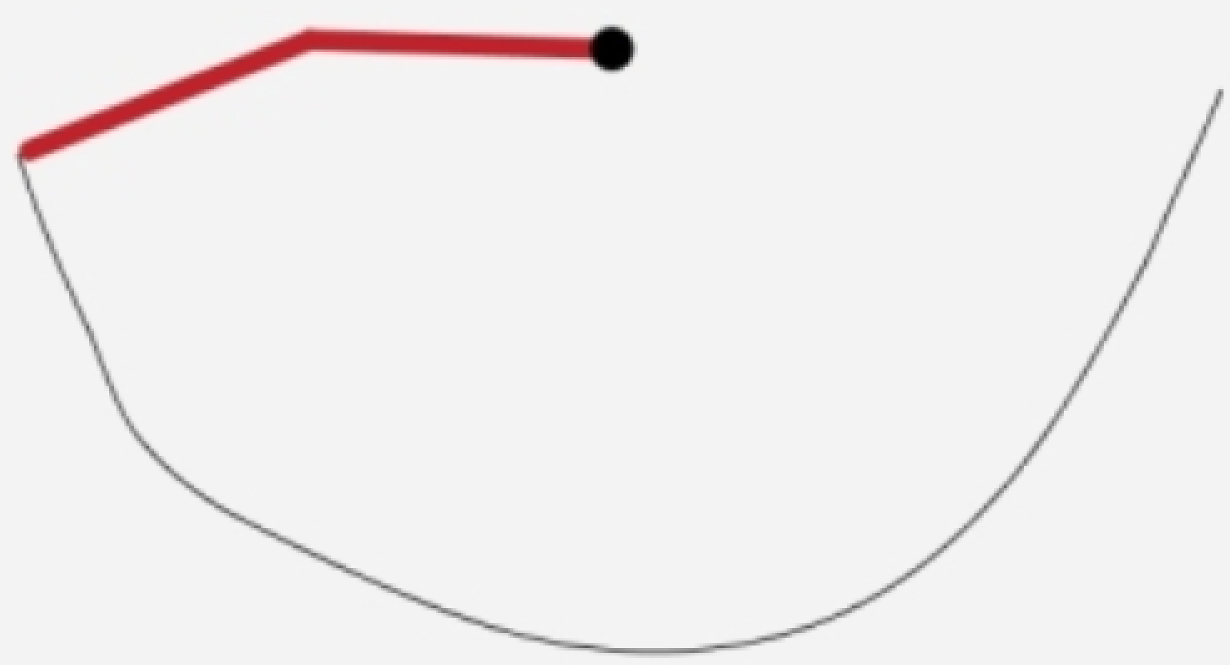
Double pendulum path
One example of the motion over time of a pendulum with a single joint. Starting it again would trace a totally different path.



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