ANTONIO PADILLA

FANTASTIC NUMBERS AND WHERE TO FIND THEM

A COSMIC QUEST FROM ZERO TO INFINITY

allen lane

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Infinity

About the Author

Tony Padilla is a leading theoretical physicist and cosmologist at the University of Nottingham. In 2016, he shared the Buchalter Cosmology Prize for his work on the cosmological constant. He is also a star of the Numberphile YouTube network, where his most popular videos include a discussion of Ramanujan's sum of all positive integers, which has been viewed more than eight million times.

To my girls
(who called me Gilderoy)

A Chapter That's Not a Number

The number lay there, brazen, taunting me from the tatty piece of paper that sat neatly on the ancient oak table: zero. I'd never scored zero in a maths test before but there was no mistaking my mark. The number was scrawled aggressively in red at the top of the coursework I'd handed in a week or so earlier. This was in my first term as a mathematics undergraduate at Cambridge University. I imagined the ghosts of the university's great mathematicians whispering their contempt. I was an imposter. I didn't know it at the time, but that coursework would prove to be a turning point. It would change my relationship with both maths and physics.

The coursework had involved a mathematical proof. These usually begin with some assumptions and, from there, you infer a logical conclusion. For example, if you assume that Donald Trump was both orange and President of the United States, you may infer that there has been an orange President of the United States. My coursework had nothing to do with orange presidents, of course, but it did involve a series of mathematical statements that I'd connected with a clear and consistent argument. The Cambridge don agreed – all the arguments were there – but he had still given me a zero. It turned out his issue was with how I'd laid it all out on the tatty piece of paper.

I was frustrated. I'd done the hard part in figuring out the solution to the coursework problem, and his complaint seemed petty. It was as if I'd scored a spectacular goal, only for the don to check with the Video Assistant Referee and rule it out for a marginal offside. But I now know why he did it. He was trying to teach me about rigour, trying to instil the mathematical pedantry that is an essential part of a mathematician's toolkit. Reluctantly, I became a pedant, but I also realized then that I needed a little more from mathematics. I needed it to have personality. I'd always loved numbers, but I wanted to bring them to life – to give them a purpose – and for that I found that I needed physics. That is what this book is all about – the personality of numbers shining through in the physical world.

Take Graham's number as an example. This is a leviathan, a number so large that it once had pride of place in the *Guinness Book of World Records* as the largest number ever to appear in a mathematical proof. It is named after the American mathematician (and juggler) Ron Graham, who was wonderfully pedantic in making mathematical use of it. But his pedantry is not what brings Graham's number to life. What brings it to life – or perhaps more accurately, death – is physics. You see, if you were to try and picture Graham's number in your head – its decimal representation written out in full – your head would collapse into a black hole. It's a condition known as *black hole head death* and there is no known cure.

In this book, I'm going to tell you why.

In fact, I'm going to tell you more than why. I'm going to take you to a place where you will question things you'd always assumed to be true. This journey through

Fantastic Numbers will begin with the biggest numbers in the universe and a quest to understand what is known as *the holographic truth*. Are three dimensions just an illusion? Are we trapped inside a hologram?

To understand this question, punch the air around you. You should probably make sure you aren't sitting too close to anyone, but punch forwards and backwards, left and right, and up and down. You can punch your way through three dimensions of space, three perpendicular directions. Or can you? The holographic truth asserts that one of these dimensions is a fake. It is as if the world is a 3D movie. The real images are trapped on a two-dimensional screen, but when the audience puts on their glasses a 3D world suddenly emerges. In physics, as I will explain in the first half of this book, the 3D glasses are provided by gravity. It is gravity that creates the illusion of a third dimension.

It was only by taking gravity to its extreme that we became aware of its sorcery. But then this is a book of extremes. Our quest to understand the holographic truth begins, inevitably, with Albert Einstein, his genius, the perverse brilliance of relativity and the underlying structure of space and time. Of course, I have a number for his genius: 1.0000000000000000858. And yes, I'm calling this a big number. I imagine you are sceptical, but hopefully I'll convince you that it is a huge number, at least if you think about the physics it represents: one man's ability to meddle with time. To really understand why, we'll need to run alongside the legendary Jamaican sprinter Usain Bolt. We'll need to plunge to the depths of the Pacific Ocean, to the deepest part of the Mariana Trench. We'll have to go to the edge of physics, dancing dangerously close to a monstrous black hole as it guzzles greedily on the stars and planets at the centre of a distant galaxy.

But relativity and black holes are just the beginning. To find the holographic truth, we will need four more leviathans – genuine numerical gargantua that come to life whenever they collide with the physical world. From a googol to a googolplex, from Graham's number to TREE(3), these are the titanic numbers that will appear to break physics. But the truth is they will guide us in our understanding. They will teach us the meaning of entropy, so often misunderstood, which describes the turbulent physics of secret and disorder. They will introduce us to quantum mechanics, the lord of the microworld, where nothing is certain and everything is a game of chance. The story will be told with tales of doppelgängers in far-off realms and warnings of a cosmic reset, when everything in our universe returns, inevitably, to the way it once was.

In the end, in this land of giants, we will find it: a holographic reality. Our reality.

I am a child of the holographic truth. It was an idea that took off around the time I scored zero in my coursework, although I knew nothing about it back then. By the time I started my doctorate about five years later, it was fast becoming the most important idea to be developed in fundamental physics in almost half a century. Everyone in physics seemed to be talking about it. Everyone is *still* talking about it. They are asking deep and important questions about black holes and quantum gravity and, in the holographic truth, they are finding answers.

There was something else everyone was talking about back then, as we were getting ready to usher in a new millennium: the mystery of our finely tuned and unexpected universe. You see, ours is a universe that simply should not exist. It's a universe that has let us live, that has given us a chance of survival, against all the odds. It's where we will go in the second part of this book, guided not by leviathans but by the mischief-makers – the little numbers.

Little numbers betray the unexpected. To understand this, imagine me winning *The X Factor*. I cannot stress how unexpected this would be, because I'm a terrible singer, so awful that in a high-school musical I was asked by the teachers to stand away from the microphones. With this in mind, I would say that the probability of me winning a national singing competition is somewhere in the region of the following number:

$$\frac{1}{\textit{number of people living in UK}} \approx 0.000000015$$

That's quite a small number. Then again, my success would be quite unexpected.

Our universe is even more unexpected. With little numbers as our guide, we will explore this unexpected world. They don't get any smaller than zero, the ugly number that spread its scorn all over my university coursework. The contempt I felt for zero on that particular day has been repeated throughout history. Of all the numbers, zero has been the most unexpected and the most feared. This is because it was identified with the void, with the absence of God and with evil itself.

But zero is neither evil nor ugly; in fact, it is the most beautiful number there is. To understand its beauty we must understand the elegance of the physical world. To a physicist, the most important aspect of zero is its symmetry under a change of sign: minus zero is exactly the same as plus zero. It is the only number with this property. In nature, symmetry is the key to understanding why things vanish, why they equate to the mythical zero.

Things start to get confusing when we encounter small yet non-zero numbers, since they reflect the absurdity of the way the universe seems to be set up as well as our struggles in trying to make sense of it. We will tell this particular story through two disturbingly small numbers, one that betrays the mysteries of the microworld and the other the mysteries of the cosmos. Through the prism of the alarmingly little 0.0000000000000001, we enter the subatomic world of particle physics: gluons, muons, electrons and taus, dancing around in random abandon. And there we will find the Higgs boson – the so-called God particle – tying them all together. The Higgs boson was discovered in a whirl of particle excitement in the summer of 2012. It was heralded as a triumph for theory and experiment, ending a near-fifty-year wait for confirmation of the particle's existance. But in among the fanfare was a secret: something didn't quite add up. It turns out that the Higgs boson is far too light, 0.0000000000000001 times lighter than it should be. That's a very little number. It tells us that the microworld lurking within you and around you is very unexpected indeed.

When we get to the number 10^{-120}, we will see that the cosmos is even more unexpected. We see it in the light of distant stars exploding out of existence. The light is dimmer than expected, suggesting that the stars are further away than we'd originally thought. It points to an unexpected universe whose expansion is speeding up, the space between galaxies growing at an accelerated rate.

Most physicists suspect that the universe is being pushed by the vacuum of space itself. That might sound strange – how could empty space push galaxies apart? The truth is that empty space is not so empty, not when you factor in quantum mechanics. It is filled with a bubbling broth of quantum particles frantically popping in and out of existence. It is this broth that pushes on the universe. We can even calculate how hard it pushes, and that's when things start to fall apart. As we will see, the universe is pushed only by a tiny amount, a fraction of what we expect based on our current understanding of fundamental physics. The fraction is just 10^{-120}, less than one part in a googol. This tiny number is the most spectacular measure of our unexpected universe.

It turns out that we are incredibly fortunate. If the universe had been pushed as hard as our calculations suggest it should have been, it would have pushed itself into oblivion, and the galaxies, stars and planets would never have formed. You and I would not exist. Our unexpected universe is a blessing but also a cosmic embarrassment, given our inability to properly understand it. It's a puzzle that has dominated my entire career and continues to dominate it.

But there is something beyond all of this, something deeper and even more profound than our quest for a holographic truth or to understand our unexpected universe. To discover it, we will need our final number, a number that isn't always a number and, at the same time, is many different numbers. It is the number that has confounded mathematicians throughout history, driving some to ridicule and others to madness: infinity.

As the German mathematician David Hilbert, a father to both quantum mechanics and relativity, once said: 'The infinite! No other question has ever moved so profoundly the spirit of man.' Infinity will be our gateway to the Theory of Everything – the theory that underpins all of physics and could one day describe the creation of the universe.

It was Georg Cantor, an outcast of German academia in the late nineteenth century, who dared to climb the infinite tower, layer upon layer, to infinities beyond the infinite. As we will see, he developed the careful language of sets, collections of this and that, that enabled him to rigorously reach into the heavens, to categorize one layer of infinity after another. Of course, he was driven quite mad, wrestling with numbers that seem to have more in common with the divine than with the physical realm. But what of the physical realm? Does it contain the infinite? Is the universe infinite?

The quest to understand physics at its most fundamental, at its most microscopically pure, is the quest to conquer its most violent infinities. These are the infinities we encounter at the core of a black hole, at the so-called singularity, where space and time are infinitely torn and twisted and gravitational tides are infinitely strong. These are also the infinities we encounter at the moment of creation, at the instant of the Big Bang. The truth is these infinities are yet to be conquered and fully understood, but there is promise in a cosmic symphony – a Theory of Everything where particles are replaced with the tiniest strings, vibrating in perfect harmony. As we will discover, the song of the strings doesn't just echo through space and time, it *is* space and time.

The big, the small and the frightfully infinite. Together these are the *Fantastic Numbers*, numbers with pride and personality, numbers that have taken us to the edge of physics, revealing a remarkable reality: a holographic truth, an unexpected universe, aTheory of Everything.

I think it's time to find those numbers.

BIG NUMBERS

1.000000000000000858

A BOLT OF RELATIVITY

Among all the usual football-related paraphernalia there was something different under the Christmas tree that year. It was a *dictionary*, one of those classic Collins ones that could serve as a barricade should the need ever arise. I'm not sure why my mum and dad thought fit to buy their ten-year-old son a dictionary when, at that stage, I had shown relatively little interest in words. In those days, I had two passions in life: Liverpool Football Club and maths. If my parents thought this present would broaden my horizons, they were sorely mistaken. I considered my new toy and decided I could at least use it to look up massive numbers. First I searched for a billion, then a trillion, and it wasn't long before I discovered a 'quadrillion'. This game went on until I happened upon the truly magnificent 'centillion'. Six hundred zeroes! That was in old English, of course, before we embraced the short-scale number system. Nowadays a centillion has a less inspiring 303 zeroes, just as a billion has nine rather than twelve.

But this was as far as it went. My dictionary didn't contain a googolplex or Graham's Number or even TREE(3). I would have loved them back then, these leviathans. Fantastic numbers like these can take you to the brink of our understanding, to the edge of physics, and reveal fundamental truths about the nature of our reality. But our journey begins with another big number, one that was also absent from my Collins dictionary: 1.000000000000000858.

I imagine you're disappointed. I've promised you a ride with numerical leviathans, but this number doesn't seem to be very big at all. Even the Pirahã people of the Amazon rainforest can name something bigger, and their number system includes only *hoí* (one), *hói* (two) and *báagiso* (many). To make matters worse, it's not even a very *pretty* or elegant number like pi or root 2. In every conceivable sense, this number appears to be remarkably unremarkable.

This is all true until we start to think about the nature of space and time and the extremes of our human interactions with them. I chose this particular number because it's a *world record* for its size, revealing the limit of our physical ability to meddle with the properties of time. On 16 August 2009 Jamaican sprinter Usain Bolt managed to slow his clock by a factor of 1.000000000000000858. No human has ever slowed time to such an extent, at least not without mechanical assistance. You may remember this event differently, as the moment when the 100-metre world record was shattered at the athletics world championships in Berlin. Watching in the stadium that day were Wellesley and Jennifer Bolt, whose son hit a top speed of 27.8mph (12.42m/s) between the 60- and 80-metre mark of the race. For each second experienced by their son in those moments, Wellesley and Jennifer would experience a little more: 1.000000000000000858 seconds, to be precise.

To understand how Bolt was able to slow time, we need to accelerate him up to the speed of light. We need to ask what would happen if he were able to catch up with it. You can call this a 'thought experiment' if you like, but don't forget that Bolt managed to break three world records at the Beijing Olympics, fuelled by a diet of chicken nuggets. Imagine what he could have achieved if he ate properly.

To have any hope of catching light, we must assume that it travels at a finite speed. That is already far from obvious. When I told my daughter that the light from her book did not reach her eye in an instant she was immediately very sceptical and insisted on conducting an experiment to find out if it was really true. I typically get a nosebleed whenever I stray too close to experimental physics, but my daughter seems to have acquired more of a practical skill set. She set things up as follows: turn the bedroom light off, then turn it on again and count how long it takes for the light to reach you. This is exactly the same sort of experiment carried out by Galileo and his assistant using covered lanterns four hundred years ago. Like my daughter, he concluded that the speed of light 'if not instantaneous … is extraordinarily rapid'. Rapid, but finite.

By the mid-nineteenth century physicists such as the wonderfully named Frenchman Hippolyte Fizeau were beginning to home in on a reasonably accurate – and finite – value for the speed of light. However, to properly understand what it would mean to catch up with light, we need to first focus on the remarkable work of the Scottish physicist James Clerk Maxwell. It will also illustrate the beautiful synergy that exists between maths and physics.

By the time Maxwell was considering the behaviour of electricity and magnetism there were already hints that they could be two different sides of the same coin. For example, Michael Faraday, one of England's most influential scientists, despite his lack of formal education, had previously discovered the law of induction, showing that a changing magnetic field produced an electric current. The French physicist André-Marie Ampère had also established a connection between the two phenomena. Maxwell took these ideas and the corresponding equations and tried to make them mathematically rigorous. But he noticed an inconsistency – Ampère's law, in particular, defied the rules of calculus whenever there was a flux of electric current. Maxwell drew analogies with the equations that governed the flow of water and proposed an improvement on what Ampère and Faraday had to offer. Through mathematical reason, he found the missing pieces of the electromagnetic jigsaw and a picture emerged of unprecedented elegance and beauty. It is this strategy, pioneered by Maxwell, that pushes the frontiers of physics in the twenty-first century.

Having established his mathematically consistent theory, unifying electricity and magnetism, Maxwell noticed something magical. His new equations admitted a wave solution, an *electromagnetic wave*, where the electric field rises and falls in one direction and the magnetic field rises and falls in the other. To understand what Maxwell found, imagine two sea snakes coming straight for you on a scuba dive. They are travelling along a single line in the water, the 'electric' snake slithering up and down, the 'magnetic' snake slithering left and right, and to make matters worse, they are charging towards you at 310,740,000m/s. The last bit of the analogy might be the most terrifying, but it is also the most remarkable part of Maxwell's discovery. You see, 310,740,000m/s really was the speed that Maxwell calculated for his electromagnetic wave – it just popped out of his equations like a mathematical jack-in-the-box. Curiously enough, that figure was also very close to the estimates for the *speed of light* that had been measured by Fizeau and others. Remember: as far as anyone was aware at the time, electricity and magnetism had nothing to do with light, and here they were, apparently consisting of waves travelling at the same speed. Modern measurements of the speed of light through a vacuum place its value at 299,792,458m/s, but the parameters of Maxwell's equations are also known to a greater accuracy and the miraculous coincidence survives. Because of this coincidence, Maxwell realized that light and electromagnetism *had* to be one and the same thing: an astonishing connection between two apparently separate properties of the physical world revealed by mathematical reason.

It gets better. Maxwell's waves didn't just include light. Depending on their frequency of oscillation or, in other words, the rate at which the sea snakes slither from side to side, the wave solutions described radio waves, X-rays and gamma rays, and although the frequencies were different, the speed at which they moved was always the same. It was the German physicist Heinrich Hertz who actually measured radio waves, in 1887. When he was quizzed about the implications of his discovery, Hertz humbly replied, 'It is of no use whatsoever. This is just an experiment that proves Maestro Maxwell was right.' Of course, whenever we tune a radio station to the desired frequency, we are reminded of the real impact of Hertz's discovery. But even if he underplayed his own importance, Hertz was right to describe Maxwell as a maestro. He was, after all, conductor of the most elegant mathematical symphony in the history of physics.

Before Albert Einstein revolutionized our understanding of space and time, it had been widely assumed that waves of light require a medium through which to propagate, much in the way that waves on the ocean need to propagate through a body of water. The imagined medium for light was known as the *luminiferous aether.* Let's assume, for a moment, that the aether is real. If Usain Bolt were to catch up with light, he would have to travel through the aether at 299,792,458m/s. If he *did* get up to speed, then once he is running alongside the light ray, *what would he actually see*? The light would no longer be moving away from him so it would just appear as an electromagnetic wave oscillating up and down and left and right but not actually going anywhere. (Imagine the sea snakes slithering to and fro but ultimately staying in the same place in the ocean.) But there is no obvious way to adapt Maxwell's laws to allow for this sort of wave, which suggests that the laws of physics would have to be radically different for the supercharged version of the Jamaican sprinter.

This is unsettling. When Einstein drew the same conclusions, he knew that something had to be wrong with this idea of catching up with light. Maxwell's theory was much too elegant to abandon just because somebody happened to be moving quickly. Einstein also needed to find a way of taking into account the strange results of an experiment carried out in Cleveland, Ohio, in the spring of 1887. Two Americans, Albert Michelson and Edward Morley, had been trying to find the speed of the Earth through the aether using some clever arrangement of mirrors, but the answer kept coming out as zero. If correct, this would have meant that the Earth, unlike almost all of the other planets in the solar system and beyond, just so happened to be running right alongside this space-filling aether, at *exactly* the same speed and in *exactly* the same direction. As we will come to appreciate later in this book, coincidences like that don't tend to happen without good reason. The simple truth is that there is no aether – and that Maestro Maxwell is *always* right.

Einstein proposed that Maxwell's laws, or indeed any other physical laws, would *never* change, no matter how quickly you move. If you were locked away in a windowless cabin on a ship, there would be no experiment you could do to detect your absolute velocity because *there is no such thing as absolute velocity*. Acceleration is a different story, and we'll come to that, but as long as the captain of the ship set sail at constant velocity relative to the sea, be it at 10 knots, 20 knots or close to the speed of light, you and your fellow experimenters in the cabin would be blissfully unaware. As for Usain Bolt, we now know that his chase would be futile. He would never catch the light ray because Maxwell's laws can never change. No matter how fast he ran, he would always see the light as if it were moving away from him at 299,792,458m/s.

This is all very counterintuitive. If a cheetah runs across the plain at 70mph and Bolt chases after it at 30mph, then everyday logic would suggest that the cheetah will extend its lead on Bolt by 40 miles every hour, simply because its relative speed is calculated as 70mph – 30mph = 40mph. But when we are talking about a ray of light travelling at 299,792,458m/s across the plain, it doesn't matter how fast Bolt runs, the ray of light will still move relative to Bolt at 299,792,458m/s. Light will always travel at 299,792,458 m/s,[1] relative to the African plain, relative to Usain Bolt, relative to a herd of panicking impala. It really doesn't matter. We can sum it up in a single tweet:

The speed of light is the speed of light.

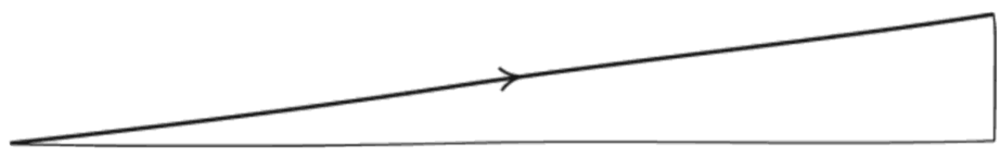
Einstein would have liked this. He always said that his ideas should have been described as 'the Theory of Invariance', focusing on their most important features: the invariance of the speed of light and the invariance of the laws of physics. It was another German physicist, Alfred Bucherer, who coined the phrase 'the Theory of Relativity', ironically while criticizing Einstein's work. We call it the *special* theory of relativity in order to emphasize the fact that all of the above applies only to motion that is uniform, in other words, with no acceleration. For accelerated motion, like a Formula One driver hitting the gas or a rocket being fired into space, we need something more general and more profound – Einstein's *general* theory of relativity. We'll get to that in detail in the next section, when we plunge to the bottom of the Mariana Trench.

For now, let's stick with Einstein's special theory. In our example, Bolt, the cheetah, the impala and the ray of light are all assumed to be moving with constant velocity relative to one another. Those velocities may differ, but they don't change with time, and the most important thing is that, despite those differences, everyone sees the light ray speeding away at 299,792,458m/s. As we have already seen, this universal perception of the speed of light certainly contradicts our everyday understanding of relative velocities, in which one velocity is subtracted from another. But this is only because you aren't exactly used to travelling around at speeds close to the speed of light. If you were, you would look at relative velocities very differently.

The problem is time.

You see, all along you have been assuming that there is a big clock in the sky that tells us all what time it is. You might not *think* you are assuming this, but you are, especially when you start subtracting relative velocities using what you believe to be common sense. I'm sorry to disappoint you, but this absolute clock is a fantasy. It doesn't exist. All that ever matters is the clock on your wristwatch, or on my wristwatch, or the clock ticking along on a Boeing 747 as it flies across the Atlantic. Each and every one of us has our own clock, our own time, and these clocks don't necessarily agree, especially if someone is hurtling around close to the speed of light.

Let's suppose I jump aboard a Boeing 747. Taking off from Manchester, by the time it reaches the British coast at Liverpool, the aircraft is cruising along at several hundred miles per hour. I decide to bounce a ball a couple of metres across the floor of the cabin, to the slight irritation of the other passengers. My sister, Susie (who happens to live in Liverpool), is on the beach as the plane flies over and, from her perspective, the ball moves considerably further, some two hundred metres or more. At first glance, this doesn't seem to require any major revision of our everyday concept of time. After all, the ball just gets a piggyback from the fast-moving aircraft – of course she sees it move further. But now let's play a similar game with light. I switch on a light on the floor of the cabin, shining a ray vertically upwards, perpendicular to the direction of travel of the aeroplane. In a very short time, I see the light climb up to the cabin ceiling. If Susie were able to see inside, she would see the light travel along a diagonal, rising from floor to ceiling but also moving horizontally with the aircraft.



Trajectory of light ray as seen by Susie on the beach.

Her diagonal distance is longer than the vertical distance I measured. That means that she saw the light travel further than I did and yet she saw it travelling at the *same speed*. That can mean only one thing: for Susie, the light took longer to complete its journey; from her perspective, the world inside the aircraft must be ticking along in slow motion. This effect is known as *time dilation*.

The amount by which time is slowed depends on the relative speed, of me with respect to my sister, of Usain Bolt with respect to his parents in Berlin. The closer you are to the speed of light, the more you slow down time. When Bolt was running in Berlin, he hit a top speed of 12.42m/s, and time was slowed by a factor of 1.000000000000000858.[2] That's the record for human relativity.

There is another consequence of slowing down time – you age more slowly. For Usain Bolt, it turns out he aged about 10 femtoseconds less than everyone else in the stadium during the race in Berlin. A femtosecond doesn't seem like much – it's only a millionth of a billionth of a second – but still, *he aged less*, so when he came to rest he had leapt into the future, albeit very slightly. If you aren't much of a runner, you can take advantage of some mechanical assistance to slow down time and, chances are, you will do even better. Russian cosmonaut Gennady Padalka spent 878 days, 11 hours and 31 minutes in space aboard both the Mir Space Station and the International Space Station, orbiting the Earth at speeds of around 17,500mph. Over the course of these missions, he managed to leap forward a record 22 milliseconds in time compared to his family at home on Earth.[fn1]

But you don't have to be a cosmonaut to time-travel in this way. A cabbie driving through the city for forty hours a week for forty years will be a few tenths of a microsecond younger than he would have been had he just stayed put. If you aren't impressed by microseconds and milliseconds, consider what could happen to any bacteria hitching a ride aboard the *Starshot* mission to Alpha Centauri. *Starshot* is the brainchild of billionaire venture capitalist Yuri Milner, who plans to develop a light sail capable of travelling to our nearest star system at *one fifth* of the speed of light. Alpha Centauri is around 4.37 light years away, so we would have to wait more than twenty years on Earth for it to complete its journey. For the light sail and its bacterial stowaway, however, time would slow down to such an extent that the journey would take less than nine years.

At this point, you may have spotted something suspicious. Travelling at one fifth of the speed of light for nine years, the intrepid bacterium will cover less than two light years – which is less than half the distance to Alpha Centauri. It's the same with Usain Bolt. I told you that he ran for 10 femtoseconds less than you might have thought, which suggests he didn't actually run as far. And it's true – he didn't. From Bolt's perspective, the track was moving relative to him at 12.42m/s and so it must have shrunk by around 86 femtometres, which is the width of around fifty protons. You could even argue that he didn't quite finish the race. For the bacterium, the space between Earth and Alpha Centauri was moving very quickly and as a result it shrank to less than half its original length. This shrinking of space, or of the racetrack in Berlin, is known as *length contraction.* So you see, running will not only make you age less, it can also help you look thinner. If you ran close to the speed of light, anyone watching would see you flatten out like a pancake, thanks to the shrinking of the space you occupy.

There is something else you should be worried about. I just said that the track was moving relative to Usain Bolt at 12.42m/s. That means that his parents were *also* moving, relative to their son, at exactly the same speed. But given everything we have established so far, this means that Bolt would have seen his parents' clocks slow down, which is very weird, because I already told you that *they* also saw *his* clock slow down. In fact, this is exactly what happens: Wellesley and Jennifer see their son in slow motion (!), and Bolt sees *them* in slow motion. But here's the *really* troubling part: I also said that Bolt managed to finish the race 10 femtoseconds younger than he would have been had he stood still. Couldn't we flip things around and look at it from Bolt's perspective? Time is ticking more slowly for his parents, so couldn't it be they who age less? It seems we have a paradox. This is known as the *twin paradox*, because of the narrative usually used to explain it, but unfortunately Usain Bolt doesn't have a twin. No matter. The truth is that it is Bolt who ages less, who stays that little bit younger. But why him and not his parents?

In order to answer this question, we have to consider the role of acceleration. Remember, everything we have discussed so far applies to uniform motion when there is *no* acceleration. In those moments where Bolt is running at a constant 12.42m/s, he and his parents are what we would call *inertial.* This is just some fancy jargon that says they aren't accelerating – they don't feel any additional force speeding them up or slowing them down. Whenever this is the case, the laws of special relativity apply and so Bolt will see his parents in slow motion, and vice versa. However, Bolt doesn't run at a constant speed for the entire race: he accelerates from zero up to his top speed before slowing down again at the end. In those periods when he is accelerating or decelerating he is *not inertial*, in contrast to his parents. Accelerated motion is a very different beast. For example, locked away in a cabin of a ship, you would certainly be able to tell if the ship was accelerating because you would *feel* the force acting on your body. Too large an acceleration could even kill you. Bolt was never at risk of death, but his acceleration and deceleration were enough to break the equivalence between him and his parents. This asymmetry takes care of the paradox – a more detailed analysis, carefully factoring in Bolt's accelerated motion, reveals that of all the protagonists it was indeed Bolt who aged that little bit less.

It is important to realize that this isn't just some fun with equations. These are real effects that have been *measured*. Fast-moving atomic clocks have been seen to tick more slowly than their stationary counterparts, 'ageing less', just as Usain Bolt did in Berlin. Further evidence comes from a microscopic particle called the muon and its apparent stay of execution. The muon is very much like the electrons you find orbiting the nucleus of an atom, but it's about two hundred times heavier and it doesn't live anywhere near as long. After about two millionths of a second it decays into an electron and some little neutral particles called neutrinos. There is an experiment at Brookhaven National Laboratory in New York in which muons are accelerated around a 44-metre ring at 99.94 per cent of the speed of light. Given their short life span, you would expect the muons to complete only 15 laps; somehow, though, they make it around 438 times. It's not that they live any longer – if you were travelling alongside one at the same speed, you would still see it decay after two millionths of a second – but then you would also see the circumference of the ring shrink to $^1/_{29}$ of its original size. The muon gets around 438 times because it has less distance to travel, thanks to length contraction.

Length contraction and time dilation help us understand why nothing – not even Usain Bolt – can travel faster than light. As he gets closer and closer to light speed, Bolt's time appears to slow to a standstill and the distances he encounters shrink to nothing. How can time slow down any more? How can distances shrink to any less? There is simply nowhere to go. The speed of light now presents itself as a barrier and the only reasonable conclusion is that no one can go any faster.

As he accelerates towards the speed of light, Bolt takes on more and more calories to try and accelerate faster and faster. The speed of light looms large as a barrier not to be crossed and so eventually his speed begins to plateau and his acceleration slows down. The closer he gets to the speed of light, the harder it becomes. His resistance to acceleration or, in other words, his *inertia*, just gets larger and larger. That is the problem with trying to accelerate up to the speed of light: inertia blows up to infinity.

But *where* is this inertia coming from? Well, the only thing that Bolt is bringing into the system is energy, and so that energy must be the source of Bolt's extra inertia. Energy never goes away, it just changes how it looks, moving from one form to another. So, inertia must be a form of energy, and this must still be true *even when Bolt is resting.* The cool thing is that for a resting Bolt, we know exactly what his inertia is: it's just his mass, because the heavier he is, the harder he is to move. Mass and energy become one and the same or, as Einstein put it[3] : $E = mc^2$. The terrifying thing about this formula is quite how much energy (E) you can get from mass (m), thanks to the enormous value for the speed of light (c). A resting Usain Bolt weighs around 95 kilograms, and if you were to convert all of that mass into energy it would be the equivalent of 2 billion tons of TNT. That is more than a hundred thousand times the energy released by the Hiroshima bomb.

Now let's talk about spacetime.

Wait. What? Where did that come from? The truth is we've been talking about spacetime all along. Length contraction. Time dilation. In the vignettes above, time and space are stretched and squashed in perfect tandem. Little wonder, then, that they should be connected, that they should be part of something greater. It was the

These were the words of Canadian film director James Cameron. They betray a palpable sense of fear, of no longer being in control, of being at the mercy of something greater. They would not be out of place in the script of his most famous movie, *Titanic*, but instead they expressed his emotions upon his return from the Challenger Deep, at the bottom of the Mariana Trench, the deepest known point on the Earth's seabed, almost 11 kilometres below sea level. On 26 March 2012 Cameron journeyed there aboard the deep-sea submersible known as the *Deepsea Challenger* and spent three hours exploring this alien world, all alone in the most hostile environment on the planet.

Cameron was the first person to plunge to such remarkable depths since a US naval team fifty years earlier, and was the first to do so alone. Perhaps the most remarkable fact of all, however, is that he returned from his trip having leapt forward in time by 13 nanoseconds.

Cameron's leap into the future was not due to his high speed, as with Usain Bolt or Gennady Padalka, but due to his *depth*. You see, time also slows as you plunge deeper into a gravitational well; in this case, as you plunge closer to the centre of the Earth. This is an effect of the general theory of relativity – relativity combined with gravity, and the zenith of Einstein's genius. Because James Cameron spent so long exploring the deep, he accumulated an impressive amount of gravitational time dilation. That said, it was the crew of the Arktika 2007 expedition who went closer than any other to the centre of the Earth. On 2 August 2007, pilot Anatoly Sagalevich, polar explorer Artur Chilingarov and businessman Vladimir Gruzdev were the first to descend to the Arctic seabed aboard *MIR-1*, some 4,261 metres below the surface at the North Pole. This might not seem like much compared to the depth of the Mariana Trench, but the Earth is not a perfect sphere. It is an oblate spheroid, bulging out slightly at the equator. As a result, the crew came much closer to its centre than *Deepsea Challenger*. After an hour and a half on the seabed the three men on board *MIR-1* had skipped forward in time by a few nanoseconds. As well as taking soil and animal samples, they planted a Russian flag made of rust-proof titanium metal. The incident sparked fierce objections from other Arctic nations, who saw it as a move to claim the region as Russian territory. The Russians denied this, stating that their goal was simply to prove that the Russian shelf extended as far as the North Pole and comparing it to the moment the *Apollo 11* astronauts planted the American flag on the surface of the Moon.

Although this is not a book about international politics, in this part of the story such things are never too far away. To understand how and why these deep-sea explorers were able to slow down time, we need to position ourselves in the early part of the twentieth century, at a time when the world was at war, the trenches filled with the blood of ordinary men fighting in extraordinary circumstances. At this time there was also a battle raging in the world of science. British physics had been reluctant to embrace Einstein's new ideas about time and space. More than any other community, the British were still invested in the notion of the aether, led, no doubt, by the indomitable Scots-Irish baron Lord Kelvin. They were also invested in Isaac Newton, the legend of British science, whose laws of universal gravitation were still the established model some three hundred years after they were first proposed. Newtonian gravity could explain so much: from the motion of the planets to the trajectory of bullets raining down at the battle of the Somme. But there was also something troubling about Newton's theory, something that Einstein's work brought into sharper focus: instantaneous action at a distance.

To understand why, imagine what would happen if the Sun were to spontaneously disappear in an instant. Of course, we would all die, but how long would it take for us to become aware of our fate? In a world ruled by Newtonian theory, the force of gravity acts instantaneously over large distances, so we would know about the Sun's demise the moment it happened. The trouble is that it takes eight minutes for sunlight to reach us here on Earth. From Einstein's perspective, this means that it should take at least eight minutes for us to receive *any* signal from the Sun, including one that alluded to its demise. Clearly Newton and Einstein are in direct conflict. Although Einstein was far from patriotic, a German challenge to the Newtonian throne was never going to be well received in England against the backdrop of the Great War.

Newton himself had serious misgivings about this action at a distance. In a letter to the scholar Richard Bentley in February 1692 he wrote, 'that … one body may act upon another at a distance through a vacuum wthout [*sic*] the mediation of any thing [*sic*] else … is to me such an absurdity that I beleive [*sic*] no man who has in philosophical matters any competent faculty of thinking can ever fall into it'.

Einstein would eventually address these concerns, but to do so he would deny Newton and refute his greatest discovery. He would deny the existence of gravity altogether.

Gravity is fake.

I like to start my Advanced Gravity class with this little one-liner, even though it upsets some of the students. But the statement *is* true: gravity really is a fake. Even on Earth, you can become weightless; you can eliminate gravity altogether. To see how, take a trip to the opulent desert city of Dubai and climb to the top of the Burj Khalifa, the world's tallest building, stretching almost a kilometre up into the sky. Once there, get inside a large box, something like an old British telephone box with the windows blacked out, and have someone drop you over the edge. As you fall with the box towards the ground, what will happen? You are accelerating towards the Earth at 1g, but *so is the floor of the box.* OK, so there is a small amount of air resistance that will drag on the box, but if the air is thin enough, you will more or less become weightless and gravity will disappear. Now, I appreciate that this is a drastic way to test gravity. But actually, you don't really need to jump off the Burj Khalifa to feel the effects of weightlessness. It is enough to drive down a steep hill in your car. You probably already know that feeling as your stomach starts to perform somersaults. That is gravity starting to disappear as you accelerate down the hill. Whenever it happens, I always remind myself (and anyone who is in the car with me), that they are feeling the effects of Einstein's genius right there in their belly.

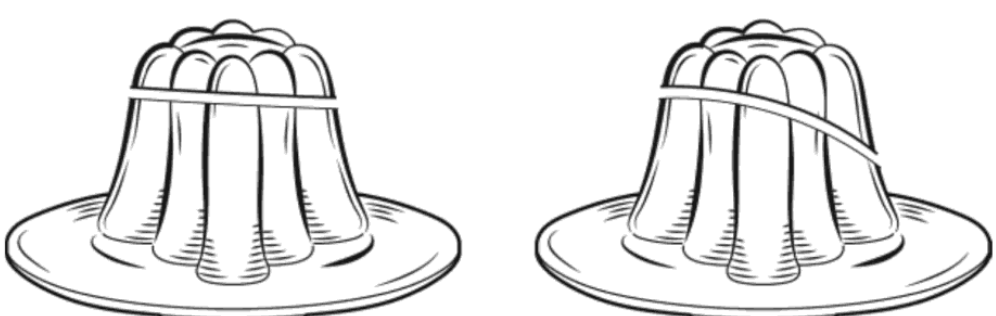
When Einstein saw that he could always eliminate the effects of gravity, he declared it to be the happiest thought of his life. The death of gravity can be traced all the way to Galileo, the genius of the Renaissance and the founder of modern science. According to his student Vincenzo Viviani, Galileo would drop spherical objects of different mass from the top of the Leaning Tower of Pisa, demonstrating to the professors and students how they fell at the same rate. This contradicted Aristotle's ancient claim that heavier objects would fall faster. Whether or not Galileo ever really put on such performances is a matter of some debate,[fn2] but the effect is certainly real. A version of his experiment was even carried out on the Moon, by *Apollo 15* astronaut David Scott. He held a hammer in one hand and a feather in the other then simultaneously dropped them towards the lunar surface.

Without air resistance, the two objects fell at exactly the same rate, just as Galileo had predicted. It is precisely this universal behaviour that guarantees that both you and the telephone box fall from the Burj Khalifa in perfect tandem.

If we can eliminate gravity altogether, in what sense is it real? Can we fake it in outer space? Faking gravity in space is easy – all you need to do is accelerate. If the International Space Station were to switch on its boosters and begin accelerating towards higher altitude at 1g, the astronauts would immediately cease to feel weightless. The ship would push upwards, but to the astronauts it would feel as if they were falling down, just as they would under the influence of gravity. Black out the windows and they could well be fooled into thinking that the ISS had come crashing down to Earth.

The point here is that gravity and acceleration are indistinguishable – in a blacked-out spaceship you have no way of knowing if you are feeling the effects of gravity or if the ship is accelerating through space. This is known as Einstein's *equivalence principle* – the physical equivalence between gravity on the one hand and acceleration on the other. You cannot tell the two of them apart. If you are still not convinced, think about what happens when you are driving your car and you take the corner a little too quickly. Turn left and it's as if you are pulled towards the car door on the right. This is just like a fake force of gravity acting sideways. The truth is that it is the car that is accelerating as it turns the corner while your body wants to carry on in the same direction, the result being that you swing towards the opposite car door.

Let's return to our deep-sea explorers for a moment. To fully appreciate how time is slowed down for them we need to think about light again. How does gravity affect light? Since gravity and acceleration are indistinguishable, we may as well just ask how acceleration affects light. Imagine that you are in a spaceship cruising through empty interstellar space at constant speed and resting in your arms is a plate of jelly.[5] In contrast, your friend is carrying a laser gun. If this were a duel, you would lose, but it's not, it's an experiment. You tell your friend to fire the laser at the jelly. She does as you ask and the laser slices through the jelly in a perfectly straight line. You decide to try again, only this time you fire the engines and start to accelerate the rocket. You and your friend immediately feel the effect of the fake gravity and are able to stand as normal on the floor of the spaceship as it pushes you through space. You tell her to fire the laser, which she does, and again the jelly is sliced through. You take a closer look at the paths that the laser made. While the first path went straight through the jelly, the second is slightly arced, as shown below.



What happens when you fire a laser at a plate of jelly in space, if the spacecraft is travelling at constant speed (left) *and if it is accelerating* (right).

What has happened to the second light ray? Nothing special. It still fired through space in a straight line, as it should, but it did so while the jelly was accelerating 'upwards' with the rocket. From the point of view of you and the jelly, it is as if the light ray is bent. While this is clearly just a consequence of the jelly's acceleration, the equivalence principle suggests that light should also be bent by gravity.

And it is.

The proof arrived not long after the Great War ended. Although few people had embraced Einstein's new ideas in Britain during this difficult time, he did have one advocate. Arthur Eddington was a thoughtful and ambitious astronomer, a pacifist who encouraged British scientists to maintain their pre-war interest in the work of German colleagues. Though it was hard to access German scientific journals, he knew about Einstein's work through the Dutch physicist Willem de Sitter and was determined to test the prediction that starlight would be bent by the Sun's gravity. The trouble with observing starlight passing close to the Sun is that the Sun's glare makes it impossible to see. Eddington realized that he needed a solar eclipse to perform the experiment and his calculations suggested that one was due to take place on 29 May 1919 on the beautiful Portuguese island of São Tomé and Príncipe, off the west coast of Africa, before moving across the Atlantic to northern Brazil. Eddington travelled to the African island with Astronomer Royal Frank Watson Dyson, while a second team was dispatched to observe the eclipse from Sobral in the Brazilian state of Ceará. Despite cloud and rain threatening the success of the experiment, the team were able to photograph several stars in the Hyades cluster during the eclipse. When these were compared to night-time images of the same cluster, the images did not align. The implication was that the starlight passing closest to the Sun had been bent more in the eclipse photograph, creating a mismatch with the night-time images. Einstein's prediction was confirmed and made headline news across the globe. It was the moment he became a superstar.

The bending of light has important implications for time. Far away from a gravitational field, when light is travelling in a straight line, it takes just a few nanoseconds to travel from a lamp on one wall of the ISS to a picture on the other. But if we placed the ISS in orbit around a black hole, this light would be bent by the strong gravitational field. Curved paths are longer than straight ones, so the light would take a little longer to complete its journey from one wall to the other. This means that the same event takes longer to happen when there is more gravity, and so gravity must be slowing down time.

The stronger the gravitational field, the more the light will be bent, and the more time will slow down. This is why James Cameron was able to leap into the future by diving to the bottom of the Mariana Trench. The gravitational field of the Earth is stronger there, albeit by a tiny amount, so clocks tick more slowly. The reverse is also true. Climb high and the gravitational field will weaken slightly, causing clocks to tick more quickly. A second spent at the summit of Mount Everest is about a trillionth of a second longer than the amount of time spent at sea level. After their twelve-and-a-half-day mission, including three days on the Moon, the *Apollo 17* astronauts experienced a record negative time dilation, going back in time by around a millisecond.[fn3]

The effects of gravity on time were measured *directly* in a famous experiment that took place at the Jefferson Tower at Harvard University in 1959. Robert Pound and his student Glen Rebka Jr fired gamma rays – high-energy electromagnetic waves –

from the top of the 22.6-metre-tall tower to a receiver at the bottom. Their clever idea was to use the frequency of the gamma rays as a measure of time, the clock 'ticking' with each new oscillation of the wave. As it turned out, the *same* waves were measured to have higher frequencies at the bottom of the tower than they did at the top. That meant that a single second at the bottom corresponded to more oscillations of the wave than a second at the top. There was only one conclusion – the meaning of 'a second' had to be different at the two ends of the tower. A second at the bottom represented more oscillations of the wave, so it must have been a *longer* second. Time was ticking more slowly at the bottom of the tower than it was at the top, just as Einstein had predicted.

Gravity's ability to bend light and slow time means that the Earth's core is about two and a half years younger than its surface.[6] But how does gravity do this if it's really a fake? How does it cause the bending of light? The truth is that it doesn't. Light always travels in a straight line through space – it is the space itself that is bent. To picture what is going on, go to the fruit bowl and grab an orange. Mark two points on the surface of the orange, reasonably far apart, and then draw the shortest path between the points. If you aren't quite sure which is the shortest path, line the points up so that they are both level at the same height on the orange's 'equator', then draw the line along the equator. Now peel the orange carefully so that the skin remains in one piece. When you have done this, flatten out the peel on the table. What shape is the line that you drew? It is curved, right? This is very weird, because the shortest distance between two points is supposed to be a straight line, but it turns out that this is only true on a flat surface. On a curved surface, the shortest paths are curved, just like the one you drew on the orange. That is what light is doing. It follows the shortest path through space, but because space is curved the path is curved. If you've ever flown long distance from London to New York and sat there watching the flight map, you will have noticed how the aeroplane always looks to be taking a strange, curved trajectory up through the Canadian Arctic. This is because the airline has calculated the shortest path, and it is curved, just like the surface of the Earth.

Of course, it is really the spacetime geometry that is curved. Minkowski told us how to measure distances in a flat spacetime geometry, but when it becomes curved the distance measures get squashed and squeezed, stretched and pulled. What causes this squashing and squeezing? Matter. You. The Sun. The Earth. Anything with mass, energy or momentum causes spacetime to bend and warp. Imagine a rubber sheet stretched out flat. Throw a heavy rock on to the sheet and it causes it to curve. That's a good analogy for what matter does to spacetime.

Light will follow the shortest paths on this curved spacetime. It follows a very special kind of shortest path, so short in fact that its spacetime length vanishes. But that's what makes light special, remember, and it remains true when the spacetime becomes curved. These light-like paths are called *null geodesics*. What about heavier stuff, like planets or suns? What do they do in spacetime? Well, they also follow the shortest paths available to them, the analogue of straight lines. They don't follow the same paths as light rays because they don't travel quite that quickly, but they do take the most economic route through spacetime that is available to them. These paths are known as *timelike geodesics*. In curved spacetime they are curved. In fact, they can appear very curved indeed. The Earth's path is so curved that it loops right back on to itself, mapping out an ellipse on its annual journey around the Sun. In telescope to read a newspaper in New York. That is what it took to capture this astonishing image in such magnificent detail.

But what is it, this horror, this dark source? Pōwehi is a black hole of gargantuan proportions, billions of times more massive than the Sun. It is gravity taken to its terrifying limit. We have already seen how light is bent by gravity. What happens as you ramp up the gravitational field, as you curve the spacetime more and more? You create a prison. Light is bent to such an extent that it becomes trapped, it cannot escape, and if light cannot escape, nothing can. Pōwehi is a cosmic oubliette, an unforgiving hell, a gaol for the forgotten.

It was an English clergyman who first conceived of such horrors. In November 1783 the Revd John Michell proposed the existence of *dark stars*, huge astrophysical objects five hundred times larger than the Sun whose gravitational pull was so strong that light itself could not escape.[fn5] It was an exciting idea at the time, invisible giants hiding in plain sight, although it would soon be forgotten. The reason for this was that it was based on the corpuscular theory, where light is made up of particles, a theory that ultimately gave way to a wave-like model following the experiments of Thomas Young at the turn of the nineteenth century. Although Michell's work on black holes would be ignored for almost two centuries, he would be heralded in science as the father of seismology. His work on the devastating earthquake and tsunami that struck Lisbon in 1755 included the idea that it originated from faults in the Earth's crust rather than from atmospheric disturbances.

Today most scientists are confident that black holes really do exist. Typically, they form when a sufficiently large star – at least twenty times heavier than the Sun – runs out of fuel. Stars power themselves with nuclear fusion, squashing and squeezing atomic nuclei together in their core, a furnace of thermonuclear bombs exploding continuously. This power prevents the star from collapsing under its own weight, exerting outward thermal pressure to counter the effects of gravity. But it doesn't last for ever. Once the star has produced too much iron in its core, the fusion processes become inefficient and it can no longer support its own weight. Star death. Gravity quickly begins to overwhelm the star, crushing it inwards, a garrotte that gets tighter and tighter. And then *bang*! The star fights back, a dramatic counterpunch to gravity's relentless attack. It is the neutrons that carry the fight, subatomic particles in the stellar core, violently repelling one another through a strong nuclear force whenever they are pushed too close together. Outer layers of material fall inwards, strike the immovable core of neutrons and rebound. In an instant, a pressure wave powers its way to the surface of the star and it explodes. A supernova, a cataclysmic event, briefly outshining an entire galaxy.

What is left behind? More than likely a neutron star, an object of tremendous density, so much so that a mere teaspoonful of its matter would weigh as much as a mountain here on Earth. If its total mass can stay below that of about three Suns, the neutron star has a chance of survival. Any heavier, and the gravitational garrotte will begin to tighten once more. There will be nothing the neutrons can do. There will be nothing anything can do. The collapse becomes unstoppable. Eventually, the star becomes so dense that light can no longer escape. Everything that was once the star is hidden behind an *event horizon*, the trapdoor to the cosmic oubliette, a spheroidal surface beyond which there is no return.

About one in every thousand stars is heavy enough to end its life consumed by gravity. These *stellar mass* black holes are everywhere, scattered across the galaxy,

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