IFSR International Series in Systems Science and Systems Engineering

Jeffrey Yi-Lin Forrest

# General Systems Theory

## Foundation, Intuition and Applications in Business Decision Making

Springer

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Volume 32

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ISSN 1574-0463
IFSR International Series in Systems Science and Systems Engineering
ISBN 978-3-030-04557-9 ISBN 978-3-030-04558-6 (eBook)
https://doi.org/10.1007/978-3-030-04558-6

Library of Congress Control Number: 2018961727

Mathematics Subject Classification (2010): 93



This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

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# About the Author

**Dr. Jeffrey Yi-Lin Forrest** also known as Yi Lin, holds all his educational degrees in pure mathematics and had one-year postdoctoral experience in statistics at Carnegie Mellon University. He had been a guest professor of economics, finance, and systems science at several major universities in China, including Nanjing University of Aeronautics and Astronautics. And currently, he is a professor of mathematics at Slippery Rock University, Pennsylvania, and the president of the International Institute for General Systems Studies, Inc., Pennsylvania. He serves either currently or in the past on the editorial boards of eleven professional journals, including *Kybernetes: The International Journal of Systems, Cybernetics and Management Science*, *Journal of Systems Science and Complexity*, *International Journal of General Systems*, etc. Some of his research was funded by United Nations, State of Pennsylvania, National Science Foundation of China, and German National Research Center for Information Architecture and Software Technology. As of the end of 2016, he has published well over 300 research papers and nearly 50 monographs and special topic volumes. Some of these monographs and volumes were published by such prestigious publishers as Springer, World Scientific, Kluwer Academic, Academic Press, etc. Over the years, his scientific achievements have been recognized by various professional organizations and academic publishers. In 2001, he was inducted into the Honorary Fellowship of the World Organization of

Systems and Cybernetics. His research interests are wide-ranging, covering areas like economics, finance, management, marketing, data analysis, predictions, mathematics, systems research and applications, philosophy of science, etc.

# Synopsis

This book is a follow-up edition of my previous book, entitled "General Systems Theory: A Mathematical Approach," initially published in 1999. More than two thirds of the contents in this current volume are based on recent developments in the relevant area of research.

In particular, this new edition consists of three parts, entitled *The Foundation*, *The Systemic Intuition*, and *Applications in Business Decision Making*, respectively. Other than some slight changes and additions, the first part is mainly based on the original edition. The second part introduces the systemic yoyo model to satisfy the desperate need for a practically useful intuition for reasoning in systems science specific contents. The role to be played by this model is expected to be similar to that played by Cartesian coordinate system in the classical science, on top of which all statistical and analytical methods, widely and heavily employed in modern science, are developed. The third part looks at how systemic thinking and the yoyo model can be beautifully applied to address many important problems facing decision makers in business by organically combining methods of the classical science, the first dimension of the two-dimensional science, with those of systems science, the second dimension, as argued by George Klir in the 1990s. As of this writing, important decisions in business are mostly drawn based on data mining or anecdotes. Scientifically speaking, such processes of decision making have been time and again shown to be flawed. Because of this reason, this part of the book is expected to open up a brand new territory of research valuable for working managers and economics professionals.

By presenting a rigorously developed foundation, a tool for intuitive reasoning that is supported by both theory and empirical evidence, and extremely fruitful applications this book demonstrates the theoretical value and practical significance of systems science and its thinking logic. By making use of this science and by employing the systemic intuition, one can produce interesting, convincing, and scientifically sound results. As shown in this book, many of the conclusions drawn on the basis of systems science can be practically applied to produce tangible economic benefits.

By studying this book and by referencing back to it regularly, the reader, who would be either a graduate student, a researcher, or a practitioner in the areas of mathematics, either theoretical or applied, systems science or engineering, economics, and decision science, will master a brand new tool to resolve his/her problems and an intuition from which useful decisions can be made relatively quickly without wasting unnecessarily the valuable time and a lot of the limited financial resources.

# Chapter 1
# Introduction

In our modern times, science develops quickly, technology renews itself in front of our very eyes, and management theories evolve in different directions. The most important and most obvious characteristic of the quickly developing science is that the forest of specialized and interdisciplinary disciplines can be easily seen, yet the boundaries of disciplines become blurred. In terms of the speedy renewing of technology, the most important and most obvious characteristic is that generations of newer gadgets with better functionalities appear with shortening time intervals, while unexpected products are designed and introduced into the marketplace in abundance. Regarding modern management techniques, an increasing number of new theories are developed with forever different insights provided.

That is, the overall trend of modern science is to synthesize all areas of knowledge into a few major blocks, as evidenced by the survey of an American national committee on scientific research in 1985 (Mathematical Sciences 1985), and that of modern technology is to make everything imaginable "smart" through using computer-related technologies and the knowledge of artificial intelligence. On the contrary, the theories of management seem to be more diverse than ever before without any sign of immediate convergence.

Because of the rapid progress of science, technology, and management theories, researchers and practitioners of scientific research and decision-making managers have been faced with unprecedented problems: How can they equip themselves with the newest knowledge? How can they handle the geometrically growing amount of new knowledge? And how can they cope with the faster-changing market environment and much less patient customers? If traditional methods of exploration and learning are employed without modification, then today's unpublished scientific results could become outdated tomorrow, and today's hot products could be out of fashion tomorrow. Facing these challenges, researchers, practitioners, and decision-making managers evidently need to acquire new skills and knowledge regarding how the concepts of wholes and parts and their relationships can be employed to investigate similar problems in different scenarios so that the points of view of interconnection and interdependence can be employed to observe the world in which we live.



J. Y.-L. Forrest, *General Systems Theory*, IFSR International Series in Systems Science and Systems Engineering 32, https://doi.org/10.1007/978-3-030-04558-6_1

In the second decade of the twentieth century, von Bertalanffy (1924) wrote:

> Since the fundamental character of living things is its organization, the customary investigation of the single parts and processes cannot provide a complete explanation of the vital phenomena. This investigation gives us no information about the coordination of parts and processes. Thus the chief task of biology must be to discover the laws of biological systems (at all levels of organization). We believe that the attempts to find a foundation for the theoretical point are a fundamental change in the world picture. This view, considered as a method of investigation, we call "organismic biology" and, as an attempt at an explanation, "the system theory of the organism."

(Note: Similarly, in the study of economics, Rostow (1960) wrote:

> The classical theory of production is formulated under essentially static assumptions which freeze – or permit only once-over change – in the variables most relevant to the process of economic growth. As modern economists have sought to merge classical production theory with Keynesian income analysis they have introduced the dynamic variables: population, technology, entrepreneurship, etc. But they have tended to do so in forms so rigid and general that their models cannot grip the essential phenomena of growth, as they appear to an economic historian. We require a dynamic theory of production which isolates not only the distribution of income between consumption, savings, and investment (and the balance of production between consumers and capital goods) but which focuses directly and in some detail on the composition of investment and on developments within particular sectors of the economy.)

From von Bertalanffy's statement, it can be seen that parallel to the challenge of modern science, technology, and management, formally proposed was a new concept of systems, which can be potentially used by researchers, practitioners, and decision-making managers to face their respective challenges. Tested in the past ninety some years, this concept of systems has been widely accepted by the entire spectrum of science and technology and by working economists and business professionals; for details, see Lin (1999) and Klir (1985).

This chapter consists of five sections. Section 1.1 analyzes the historical background for how systems science has appeared and why it has been widely accepted. Section 1.2 looks at how the evolution of systems should be seen holistically. Section 1.3 explains what systems philosophy and methodology entail. Section 1.4 addresses the reason why we closely look at applications in business areas in this book, and Section 1.5 presents how this book is organized.

## 1.1 Historical Background

Since the time when the concept of systems was formally introduced in the second decade of the twentieth century in biology (von Bertalanffy 1924), the concept has been widely applied either implicitly or explicitly in the entire spectrum of science, be it natural or social, and technology. Although it seems to be a quite recent phenomenon, as all new concepts studied in the world of learning, the ideas and thinking logic of systems have a long history. For example, Chinese traditional medicine, treating each human body as a whole, can be traced to the time of Yellow Emperor about

5,000 years ago, and Aristotle's statement that "the whole is greater than the sum of its parts" has been a fundamental problem in systems theory. In other words, throughout the history, mankind has been studying and exploring nature by using the thinking logic of systems. It is only in modern times that have some new contents been added to the ancient systems thinking. The methodology of studying systems as wholes adequately agrees with the development trend of modern exploration of nature and man itself, namely divide the object, event, and process of consideration into parts as small as possible while studying each and every of them carefully, seek interactions and connections between phenomena, and to observe and comprehend more and bigger pictures of nature than ever before.

In the recorded history, although the word "system" was never emphasized, we can still find many explanatory terms concerning the concept of systems. For example, Nicholas of Cusa, that profound thinker of the fifteenth century, linking medieval mysticism with the first beginning of modern science, introduced the notion of *coincidentia oppositorum*, the opposition or indeed fights among the parts within a whole which nevertheless forms a unity of higher order. Leibniz's hierarchy of monads looks quite like that of modern systems; his *mathesis universalis* presages an expanded mathematics which is not limited to quantitative or numerical expressions and is able to formulate much conceptual thought. Hegel and Marx emphasized the dialectic structure of thought and of the universe it produces: the deep insight that no proposition can exhaust reality but only approaches its coincidence of opposites by the dialectic process of thesis, antithesis, and synthesis. Gustav Fechner, known as the author of the psychophysical law, elaborated, in the way of the natural philosophers of the nineteenth century, supraindividual organizations of higher order than the usual objects of observation, for example, life communities and the entire earth, thus romantically anticipating the ecosystems of modern parlance. Here, only a few names are listed. For a more comprehensive study, see von Bertalanffy (1972).

Even though Aristotelian teleology was eliminated in the development of modern science, problems considered in it, such as the one that "the whole is greater than the sum of its parts," the order and goal directedness of living things, etc., are still among the problems of today's systems research. For example, what is a "whole"? What does "the sum of its parts" mean? Could the sum of parts sometimes be greater than the whole? Although decision-making managers and working professionals know the importance of these and other related problems, none of these problems has been actually studied and addressed in any of the classical branches of science. These classical branches of science have been established on Descartes' second principle and Galileo's method, where Descartes' second principle says to divide each problem of concern into as small parts as possible so that each part could possibly be investigated, and Galileo's method implies to simplify a complicated process into as basic portions and processes as possible so that each portion and process could potentially be comprehended; for details, see Kuhn (1962).

From this superficial discussion, it can be seen that the concept of systems we are studying today is not simply a product of yesterday. As a matter of fact, it is a reappearance of some ancient thought and a modern form of an ancient quest. This quest has been recognized in human struggle for survival with nature and has been studied at various points in time by using the languages of different historical moments.

Ackoff (1959) comments that during the past two decades, we witnessed the appearance of the key concept "systems" in scientific research; however, with the appearance of the concept, what changes have occurred in modern science? Under the name "systems research," many branches of modern science have shown the trend of synthetic development; research methods and results in various disciplines have been intertwined to influence the overall research progress, so one feels the tendency of synthetic development in scientific activities. This synthetic development requires the introduction of new concepts and new thoughts in the entire spectrum of science. In a certain sense, all of this can be considered as the center of the concept of "systems." Hahn (1967, p. 185), an ex-Soviet expert, describes the progress of modern science as follows: Refining specific methods of systems research is a widespread tendency in the exploration of modern scientific knowledge, just as science in the nineteenth century with forming natural theoretical systems and processes of science as its characteristics.

von Bertalanffy (1972) described the scientific revolution in the sixteenth–seventeenth centuries as follows: "(The Scientific Revolution) replaced the descriptive-metaphysical conception of the universe epitomized in Aristotle's doctrine by the mathematical—positivistic or Galilean conception. That is, the vision of the world as a teleological cosmos was replaced by the description of events in causal, mathematical laws." Based on this description, we can describe the change in today's science and technology as follows: At the same time of continuously using Descartes' second principle and Galileo's method, systems methodology is introduced to deal with problems of order and organization.

Should we continue to use Descartes' second principle and Galileo's method? The answer is yes for two reasons. First, they have been extremely effective in scientific research and administration, where all problems and phenomena could be decomposed into causal chains, which could be treated individually. That has been the foundation for all basic theoretical research and modern laboratory activities. In addition, they won victories for physics and led to several technological revolutions. Second, modern science and technology are not Utopian projects as described by Popper (1945), reknitting every corner for a new world, but based on the known knowledge, they are progressing in all directions with more depth, more applicability, and a higher level of difficulty.

On the other hand, instead of treating the world as a pile of infinitely many isolated objects, on which the concept of numbers is introduced with internal structures ignored and modern science is established, not every problem or phenomenon, especially when humans and social events are concerned with, can be simply described by causal relations. The fundamental characteristics of this world are its organizational structure and connections of interior and exterior relations of different matters. The study of either isolated parts or causalities of events and organizations can hardly explain completely or relatively globally our surrounding world. At this junction, the research progress of the three-body problem in mechanics, where the problem is about how three or more bodies would interact with each other, as compared to the two-body scenario as described in Newton's second law of motion, is an adequate example that shows why systems thinking is badly needed. So, as human race advances, studying problems with multi-causality or multi-relation will become more and more significant than ever before.

In the history of scientific development, the exploration of nature has always moved back and forth between specific matters or phenomena and generalities. Scientific theories need foundations and stimulations rooted deeply inside real-life practice, while being used to explain natural phenomena so that human understanding can be greatly enhanced and to provide guidelines for decision makers to take educated actions. In the following, we discuss the technological background for the development of systems science—that is, the need for such a science that emphasizes on order and organization which arose in the development of technology and from the requirement for increasingly higher levels of production and better efficiency of management.

There have had been many advances in technology in the modern history: energies produced by various devices, such as steam engines, motors, computers, and automatic controllers; self-controlled equipment from domestic temperature controllers to self-directed missiles; and the information highway and internet of knowledge and things, which have resulted in increased communication of new scientific results and practical successes. On the other hand, increasing speed of communication furthers scientific development to a different level and helps to intensify the market competition globally. Additionally, societal changes have brought forward more pressing demands for new construction materials and different management strategies. From these examples, it can be seen that the development of technology forces mankind to consider not only a single machine or matter or phenomenon, but also "systems" of machines and "systems" of matters and phenomena. The design of steam engines, automobiles, cordless equipment, etc., can all be handled by specially trained engineers. However, when dealing with the designs of missiles, aircraft, or new construction materials, for example, a collective effort, combining many different aspects of knowledge and know-how, has to be in place. Such joint, collective efforts include the combination of various techniques, machines, electronic technology, chemical reactions, people, etc. Here the relation between people and machines becomes more obvious, and uncountable financial, economic, social, and political problems are intertwined to form a giant, complicated system, consisting of men, machines, and many other components. It was the great political, technical, and personnel arrangement success of the American Apollo project—landing humans on the moon—that hinted about the fact that history has reached such a particular point that all aspects of science and technology have been developed maturely enough so that each rational combination of information or knowledge could result in unexpected consumable products and that different combinations of men and personalities could lead to various unforeseen outcomes.

A great many problems in production require locating the optimal point of the maximum economic effect and minimum cost in an extremely complicated network. This kind problem not only appears in industry, agriculture, military affairs, and business, but politicians are also using similar (systems) methods to seek answers to problems like air and water pollution, transportation blockages, decline of inner cities, and crimes committed by teenage gangs.

In manufacturing production, there exists a tendency that bigger or more accurate products with more profits are designed and produced. In fact, under different interpretations, all areas of learning have been faced with complexity, totality, and "systemality." This tendency betokens a sharp change in scientific thinking. By comparing Descartes' second principle and Galileo's method with systems methodology and considering the development tendency, as described previously, appearing in the world of learning and production, it is not hard to see that because of systems concepts, another new scientific and technological revolution will soon appear. To this end, for example, each application of systems theory points out the fact that the relevant classical theory needs to be further enriched somehow; see Klir (1970), Berlinski (1976); Lilienfeld (1978). However, not all scientific workers have this kind of optimistic opinion. Some scholars believe that it is an omen that systems science itself is facing a crisis; see Wood-Harper and Fitzgerald (1982) for details. Of course, who is right and who is wrong only time and history will have the ability to tell.

## 1.2 Whole Evolutions—Blown-Ups and Revolving Currents

When we enjoy the splendid fruits of modern science and technology, we still need to think about the limitations of and issues existing in the science and technology we inherited from the generations before us. From the primeval to modern civilizations, mankind has gone through a history of development for over several millions of years. However, a relatively well-recorded history goes back only as far as about 3,000 years. During this time frame, the development of science mirrors that of human civilizations. And, each progress of pursuing after the ultimate truth is that of getting rid of the stale and taking in the fresh and that of making new discoveries and new creativities. Each time when the authority was repudiated, science is reborn again. Each time when the "truth" was questioned, a scientific prosperity appears. That is, each time when people praise authorities, they are in fact praising ignorance.

At the turn of the twenty-first century, Shoucheng OuYang (Wu and Lin 2002) proposed the blown-up theory for purpose of addressing and resolving nonlinear evolution problems based on a reversed thinking logic, factual evidence, and over 30 years of reasoning and practice. It is found that in terms of the formalism, nonlinear evolution models represent the singularity problems of mathematical blown-ups of uneven formal evolutions; and in terms of physical objectivity, nonlinear evolution models describe mutual reactions of uneven internal structures of materials, organizations, and events. They no longer stand for a problem of formal quantities. Since uneven structures and organizations symbolize eddy sources, they lead to eddy motions, the mystery of nonlinearity, instead of waves. That mystery has been bothering mankind for over 300 years, is resolved at once both physically and mathematically by OuYang (Wu and Lin 2002). What is shown is that the essence of nonlinear evolutions is the destruction of the initial-value automorphic structures and the appearance of discontinuity. It provides a tool of theoretical analysis for studying objective transitional and reversal changes of materials, organizations, and events.

In the ancient scientific history of the Western civilizations, there existed two opposite schools on the structure of materials. One school believed that materials were made up of uncountable and invisible particles, named “atoms,” the other school that all materials were continuous. A representative of the former school is the ancient Greek philosopher Democritus (about 460–370 BC) and the latter the ancient philosopher Aristotle (384–322 BC). Since abundant existence of solids made it easy for people to accept the Aristotelian continuity, the theory of “atoms” was not treated with any validity until the early part of the nineteenth century when J. Dalton (1766–1844) established relevant evidence. In principle, Leibniz and Newton’s calculus were originated in the Aristotelian thoughts. Along with calculus, Newton constructed his laws of motion on the computational basis of calculus and accomplished the first successes in applications in celestial movements under unequal-quantitative effects. With over two hundred years of development, the classical theory of mechanics has gradually evolved into such a set of methods for analysis based on the concept of continuity that even nearly a century after quantum mechanics and relativity theory were established; the thinking logic and methods developed on continuity are still widely in use today.

Due to differences in environmental conditions and living circumstances, where the west was originated from castle-like environments and the east from big river cultures with agriculture and water conservation as the foundation of their national prosperities, the ancient eastern civilizations were different or even opposite of those of the west. So, naturally, Chinese people have been more observing about reversal and transitional changes of weathers and rivers. Since fluid motions are irregular and difficult to compute exactly, that was why the *Book of Changes* (Wilhalm and Baynes 1967) and *Lao Tzu* (time unknown) appeared in China. The most important characteristic of the *Book of Changes* is its way of knowing the world through images of materials and organizations with uneven internal structures and analyzing changes in events through figurative structures with an emphasis placed on irregularities, discontinuities, transitional, and reversal changes.

As pointed out by Wu and Lin (2002), in terms of symbolism, due to escapes in uneven forms from continuity, the evolution of any nonlinear evolution model is no longer a problem of simply extrapolating the given initial values or the past into the future. What is significant here is that through nonlinear evolutions, the concept of blown-ups can represent Lao Tzu’s teaching that “all things are impregnated by two alternating tendencies, the tendency toward completion and the tendency toward initiation, which, acting together, complete each other,” and agrees with non-initial-value automorphic evolutions as what the *Book of Changes* describes: “At extremes, changes are inevitable. As soon as a change occurs, things will stabilize and stability implies long lasting.”

Since nonlinearity describes eddy motions (Lin 2007), there must exist different eddy vectorities and consequent irregularities. That is, the phenomenon of “orderlessness” is inevitable. When looking at fluid motions from the angle of eddies, one can see that the corresponding quantitative irregularities, orderlessness, multiplicities, complexities, etc., are all about the multiplicity of rotating matters. Therefore, there exist underlying reasons for the appearance of quantitative irregularities, multiplicities, and complexities. Those underlying reasons are the unevenness of "time and space" within the structures of the evolutionary materials, organizations, and events.

From the blown-up theory (Wu and Lin 2002), one can see that all eddy motions, as described symbolically with nonlinearities, are irregular when compared to the standardized tools of analysis. And, all standardized methods become powerless in front of the challenge of solving discontinuously quantified deterministic problems of nonlinear evolution models (Lin and OuYang 2010).

One important concept studied in the blown-up theory is that of equal-quantitative effects, which describes the conclusion of quantitative analysis under quasi-equal acting forces. Although this concept was initially introduced in the study of fluid motions, it represents the fundamental and universal characteristic of materials' movements, organizational developments, and events' evolutions. What is important about this concept is that it reveals the fact that nonlinearities are originated from the structures of materials instead of nonstructural quantities.

On the basis of the blown-up theory and the theory's successes of real-life disaster predictions (Lin and OuYang 2010; OuYang 1994; OuYang et al. 2009), the concepts of inputs, outputs, and converging and diverging eddy motions are coined together in the model as shown in Figure 1.1. This model was initially established in (Wu and Lin 2002) for each object and every system imaginable. In particular, each system or object considered in a study is a multi-dimensional entity that spins about its invisible axis. If we fathom such a spinning entity in our three-dimensional space, we will have a structure as shown in Figure 1.1. The side of inputs sucks in all things, such as materials, information, energy, investment, human resources, etc. After funneling through the short narrow neck, all things are spit out in the form of an output. Some of the materials, spit out from the end of outputs, never return to the other side and some will. For the sake of convenience of communication, such a structure, as shown in Figure 1.1, is called a (Chinese) yoyo due to its general shape. More specifically, what this model says is that each physical entity in the universe, be it a tangible or intangible object, a living being, an organization, a culture, a civilization, etc., can all be seen as a kind of realization of a certain multi-dimensional spinning yoyo with an invisible spin field around it. It stays in a constant spinning motion as depicted in Figure 1.1. If it does stop its spinning, it will no longer exist as an identifiable system.

The theoretical justification for such a model is the blown-up theory (see Chapter 14 for more details). It can also be seen as a practical background for the law of conservation of informational infrastructures (see Chapter 15 for more details). More specifically, based on empirical data, the following law of conservation is proposed (Ren et al. 1998): For each given system, there must be a positive number $a$ such that

$$AT \times BS \times CM \times DE = a \tag{1.1}$$

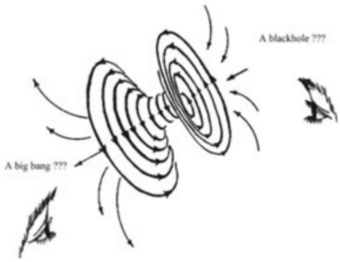


**Figure 1.1** Yoyo model of general systems

where $A$, $B$, $C$, and $D$ are some constants determined by the structure and attributes of the system of concern, and $T$ stands for the time as measured in the system, $S$ the space occupied by the system, $M$ and $E$ the total mass and energy contained in the system.

Because $M$ (mass) and $E$ (energy) can exchange to each other and the total of them is conserved, if the system is a closed one, Equation (1.1) implies that when the time $T$ evolves to a certain (large) value, the space $S$ has to be very small. That is, in a limited space, the density of mass and energy becomes extremely high. So, an explosion (outputs) is expected. Following the explosion, the space $S$ starts to expand. That is, the time $T$ starts to travel backward or to shrink. This end gives rise of the well-known model for the universe derived from Einstein's relativity theory. In terms of systems, we have the following model of systems' evolution: Each system goes through such cycles as: … → expanding → shrinking → expanding → shrinking → … Now, the geometry of this model from Einstein's relativity theory is given in Figure 1.1.

Practically, the yoyo model in Figure 1.1 is manifested in different areas of life. For example, each human being, organization, or civilization, as we now see them, is a three-dimensional realization of such a spinning yoyo structure of a higher dimension. Human feelings, impressions, and silent communications are some of the manifestations of the systemic yoyo structure beyond the entities of human. For details, see Chapter 15 and Lin and Forrest (2011).

The presentation of this section is mainly based on Einstein (1983), Haken (1978), Jarmov (1981), Kline (1972), Liang (1996), Lin and OuYang (1996), Lin (1988, 1989, 1990), Lin et al. (1990), OuYang (1994), Prigogine (1967), Thom (1975), Zhu (1985). For more details, please consult with these references.

## 1.3 Systems Philosophy and Methodology

Philosophically, systems science is concerned with the concepts of "wholes" and "parts" and their relationships by focusing on general systems as its objects in order to uncover isomorphic structures of systems. That is, systems science attempts to unearth the unified laws that govern forms of existence and forms of motion of systems of various characteristics, and the laws all systems comply with. According to the difference between thinghood and systemhood, systems science focuses on the properties of systemhood of systems instead of properties of thinghood as what traditional science studies.

Systems science attempts to understand the world by using its unique logic of thinking, view of nature, view of time and space, and meaning of life. It explores the territories the traditional science has never touched before. It challenges to reveal a much greater and much more truthful world than the one the traditional science has shown.

The essence of all sciences is, to a great extent, to reveal orders within the uncertainties of the world. To this end, the cognitive approach of systems science provides a brand new way to comprehend the world in its original characteristics: structures, organizations, interconnectedness, and interdependence. And the development of systems science is mainly motivated by societal developments and backgrounds prepared by prior scientific, technological, and managerial advances. As a matter of fact, the dialectical principles of universal connections, movements, and developments are the premises of systems science that need to be clearly understood.

As for what the methodology of systems science is, scholars in the field understand it in different ways. However, their fundamental points are the same, loosely speaking. In the following, let us first look at typical opinions of two important figures in modem science: H. Quastler and L. Zadeh.

Quastler (1965) wrote: Generally speaking, systems methodology is essentially the establishment of a structural foundation for four kinds of theories of organization: cybernetics, game theory, decision theory, and information theory. It employs the concepts of a black box and a white box (a white box is a known object, which is formed in certain way, reflecting the efficiency of the system's given input and output), showing that research problems, appearing in the afore-mentioned theories on organizations, can be represented as white boxes, and their environments as black boxes. The objects of systems are classified into several categories: motivators, objects needed by the system to produce, sensors, and effectors. Sensors are the elements of the system that receive information, and effectors are the elements of the system that produce real reactions. Through a set of rules, policies, and regulations, sensors and effectors do what they are supposed to do. By using these objects, Quastler proved that the common structure of the four theories of organization can be described by the following laws:

- Interactions are between systems and between systems and their environments.
- A system's efficiency is stimulated by its internal movements and the reception of information about its environment.

Zadeh (1962) listed some important problems in systems theory as follows: systems characteristics, systems classification, systems identification, signal representation, signal classification, systems analysis, systems synthesis, systems control and programming, systems optimization, learning and adaptation, systems liability, stability, and controllability. The characteristics of his opinion are that the main task of systems theory is the study of general properties of systems without considering their physical specifics. Systems methodology in Zadeh's viewpoint is that systems science is an independent scientific endeavor whose job is to develop an abstract foundation with concepts and frames in order to study various behaviors of different kinds of systems. Therefore, systems science should contain a theory of mathematical structures of systems with the purpose of studying the foundation of organizations and systems structures.

Since the time when these authors made their important marks in the development history of systems science, nearly half a century has gone by quickly. During these past decades, systems science has witnessed major development. So, by summarizing the newest achievements, we will be able to better describe the methodology of systems science as follows.

First, the landscape of knowledge is now seen as two-dimensional with the traditional science as the first dimension, which focuses on the study of thinghood, and systems science as the second dimension, which emphasizes on the investigation of systemhood, as argued by Klir (2001). That is why systems science and research could be and have been naturally transcending all the disciplines of the classical science and become a force making the existing disciplinary boundaries totally irrelevant and superficial. The cross-disciplinary feature of the systems science and research implies that

(1) Researches, done in systems science, can be applied to virtually all disciplines of the classical science.
(2) Issues involving systemhood, studied in individual specialization of the classical science, can be studied comprehensively and thoroughly at the height of systems science.
(3) A unifying influence on the traditional science where a growing number of narrower and narrower disciplines appears is created.

With the advantage of the newly discovered second dimension, the world of learning will be expectedly able to resolve some of the unsettled age-old problems that have been difficult or impossible for the traditional science to conquer.

Second, as the totality of knowledge expands and human understanding of nature deepens, it is discovered that many systems interact nonlinearly with each other and do not satisfy the commonly assumed property of additivity. Systems' emergent irreversibility, sensitivity, and complexity cannot be analyzed and understood by using the methodology of the traditional reductionism. Facing this challenge, systems science takes a hard look at the concept of "emergence": The whole that consists of a large number of individuals that interact with each other according to some elementary rules possesses some complicated properties. That is, the whole is greater than the sum of its parts. By discovering elementary laws that govern complex systems of different kinds, one will be able to explain many complicated and numerous matters and events of the kaleidoscopic world and introduce different control mechanisms.

Third, the rigor of the foundation of systems science is now ideally based on set theory, because such an abstract theory is very flexible in terms of its power to describe and can be readily specified to address particular issues of concern. At the same time, the qualitative theory of differential equations, game theory, gray systems theory are employed to derive various properties of general systems, to study how systems interact with each other, and to discover patterns from available data, be it a very small sample or a flood of information, in order to produce practically useful guidelines. Because of the use of these repeatable and analytical methods, the mostly philosophical works of the earlier stage of systems science have been made rigorous, visible, and tangible, leading to a good amount of convincingly successful applications in the traditional disciplines.

Fourth, based on an understanding of the history of the systems movement and a creative comparison between systems science and calculus, the yoyo model is now available for us to intuitively see how systems would evolve and interact with each other. In the recent development of systems science, this simple model has played several crucial roles; for more details about this model, please consult with (Lin 2009):

(1) It is the intuition and playground for systemic thinking to take place in a fashion similar to that of Cartesian coordinate systems in modern science. To use the model effectively, of course each of the characteristics of the issue of concern needs to be identified with some aspect of the yoyo model.
(2) It provides an intuitive ground for logic thinking so that one could see how to establish models in the traditional fields in an unconventional sense in order to resolve the problem in hand.
(3) It helps understand the issue of concern from an angle that is not achievable from the traditional science alone.

As a matter of fact, the recent development of systems science has clearly shown that this yoyo model has successfully played the role of intuition and playground, as expected, for scholars to investigate the world and explore new knowledge holistically, just as what the Cartesian coordinate system did for the traditional science (Lin 2009; Lin and Forrest 2011; Forrest 2013, 2014; Forrest and Tao 2014; Ying and Forrest 2015). In particular, this yoyo model of general systems has been successfully applied in the investigation of Newtonian physics of motion, the concept of energy, economics, finance, history, foundations of mathematics, small-probability disastrous weather forecasting, civilization, business organizations, the mind, among others.

## 1.4 The Reason for Looking at Applications in Economics and Business Areas

From the presentation in the previous section, it can be readily seen that systems science has made sufficient progress in the past century regarding its foundation, intuitive reasoning, and methodology for rigorous analysis. It now possesses all the necessary characteristics and capabilities to enjoy a glorious and long-lasting life, similar to the lives enjoyed by Euclidean geometry and calculus.

Although all aspects of systems science still need to be developed further, as what is happening in the traditional science, some of the major scientific labors devoted to this area of learning have to be now directed toward producing tangible results and insights that will excite the public and resolve challenges facing the mankind. In other words, it is important for systems scientists and practitioners to locate some weaknesses and incapabilities in the traditional science that affect the effectiveness of practical decision making and challenges the traditional science has not been able to meet successfully. To this end, there are many wonderful choices of materials we can use for this book, as what was done in the first edition of this volume. In particular, in the first edition, we looked at successful applications of systems research and systems thinking in such diverse areas that range from sociology to atomic physics, from Einstein's relativity theory to Dirac's quantum mechanics, from optimization theory to unreasonable effectiveness of mathematics to foundations of mathematical modeling, from general systems theory to Schwartz's distributions. In each of these specific applications, at least one important niche was found and then addressed by using results or methods of analysis of systems science.

Contrary to what is done in the first edition of this volume with a diverse scope of applications, this book will focus on several applications only in areas of economics and business decision making. The reason for me to take this choice is that the currently available theories of the traditional science cannot meaningfully predict near-zero probability disastrous events in advance other than making live reports (Lin and OuYang 2010). For instance, in the area of weather forecasts, the meteorologist relies on satellites, radar systems, and other technology to inform the residents and organizations located on the downstream of an observed moving weather system what to expect and when to expect based on either visual observations or some mathematical extrapolations of the data of rolling availability or both. Even with the employment of all the most advanced technologies and all well-developed scientific theories (of the first dimension), each of his weather forecasts is still attached with a probability in order to theoretically guarantee the accuracy of his forecast. It is because the attached probability basically means no matter what turns out to be the case in reality, his earlier forecast is always correct (probabilistically speaking) (OuYang 1994). In a similar fashion, in economics and business decision making, people need to make educated projections about what to expect and what possible courses of actions to take, unfortunately, without much technology available for live reports to rely on. What is more challenging than the situation of weather forecasting is that in business the decision maker has to deal with people and the possible consequences of their collective endeavors. In particular, when forecasting the possible appearance of a natural event, the accuracy of the forecast does not affect the actual appearance of the event; however, when working with systems involving people, predictions on the consequences of collective actions tend to alter how people behave and what different courses of actions they would actually take. That end practically makes the earlier predictions often incorrect (Soros 2003). Because of this reason, it is no doubt to me that helping economists and business decision makers to improve their abilities to make better predictions and then better decisions on their courses of actions, which has been a challenge impossible for the traditional science to meet, is both practically important and theoretically significant.

Speaking more specifically, in order to produce meaningful results and insights that successfully address challenges facing areas of the traditional science, in this book I will showcase fruitful resolutions of some problems related to business decision making while demonstrating how the systemic yoyo model and related systems reasoning can be practically applied. In particular, we will closely look at why competitive advantages that were sustainable in the past are becoming transient, while markets change faster and customers are less patient than ever before; how nonlinearity appears in demand/supply interactions so that new methods of analysis are needed; what market conditions actually invite new competitions; how mysterious family members behave even under the altruism of the head of the household; why no value system can stay unchanging for long if it is expected to provide the necessary guidance for desirable behaviors; how a leader can potentially achieve management efficiency; why any organization is expected to always experience inefficiencies; how the management can effectively deal with customers' indecisiveness through applying pricing strategies; how the manager can heighten the competitive spirits of her sales' associates, etc.

The literature in economics and business decision making tends to show the frequent use of such words as "believe," "should," and "would." For example, Kotler et al. (2010), Krauss (2011), Stengel (2011) *believe* that today's customers want to be treated as whole human beings and be acknowledged that their needs go beyond pure consumerism. The application of the word "believe" here really indicates, scientifically speaking, that these authors are not quite sure what they are talking about. Considering the fact that Philip Kotler is one of the most influential marketing thinkers (Kaul 2012), what is stated above simply means that most decisions in the area of marketing are made based on anecdotes or data mining or both. However, neither anecdotes nor data mining are recognized to be reliable ways to produce dependable theories beyond potential facts finding in natural science and mathematics, where a pattern or theory derived from anecdotes and data mining cannot be trusted even though the observed pattern can be shown to be true for a very large number of empirical cases, no matter how large the number is, until it is proven for the general case. In terms of data mining, other than possibly discovering facts, the related methods also frequently lead to "realities" that only exist with the particular sets of data used in the analyses. That is why I often describe the approach of using data analysis to develop theories as looking at the clouds in the sky and try to find patterns. For example, let us look at the clouds in Figure 1.2. Do we see an image of two people deeply in love? If you do not see the image, then I or someone else can easily help highlight the pattern out of the background. Now, no matter what story we can make out of this observation about two heavenly figures, no one will believe it—period!

Speaking generally, when a discipline or an area of study frequently uses such words as "believe," "should," and "would," it means that

1. When a rigorous tool of reasoning is used, the author does not know for sure whether or not he/she has considered all related factors.
2. A lot of conclusions are drawn based on one or a few anecdotes without any follow-up rigorous analysis.

**Figure 1.2** A pair of lovebirds in the sky

3. A lot of so-called theories are really established on data mining.

One of the most important reasons for Situation 1 to occur is that there is no readily available playground or intuition for large-scale or global economic reasoning. For example, all calculus-related methods and all statistical approaches are developed and established on the Cartesian coordinate system. However, the Cartesian coordinate system does not really exist in real life. That renders the applied methods and approaches questionable or at least they are only useful locally. That explains why when an analytical or statistical tool is employed, it only models a regional, local phenomenon so that conclusions of any larger scale can only be conjectured. When Situation 2 is the case, the conclusions are really derived inductively instead of deductively. As we mentioned earlier, such conjectured conclusions in general cannot be trusted as reliable. For example, from the anecdotes that $1 < 100$, $2 < 100$, $3 < 100$, can we conclude that any natural number is less than 100? As another example, from the anecdotes that $0^2 = 0$, $1^2 = 1$, can one conclude that for any chosen natural number, its square is equal to itself? Of course, no one can draw such conclusions scientifically from the few anecdotes. Then how can such practice be commonly accepted in business decision making? It is simply because an adequate tool for analysis is desperately needed in these areas, where calculus-related methods and statistical approaches are shown to be inadequate, for more detailed discussion about this end, see Lin and OuYang (2010).

Because we now see where systems science can potentially make a breakthrough, let us next see how we can make at least some meaningful progress little by little. To this end, let us identify the current state of research affairs in economics and business with that in the history of natural sciences right around the time when Isaac Newton developed his laws of physics. By doing so, we can see the similarity between all the facts, partial facts, and nonfacts discovered in the literature of economics and business which are similar to those uncovered by various scholars in natural sciences a few hundred years ago; and the present opportunity created by the development of systems thinking and the systemic intuition in the recent past is quite parallel to that used by Newton and his contemporaries to develop their laws of natural science because of the matrimony of Euclidean geometry and algebra, as a natural consequence of the introduction of the Cartesian coordinate system.

Considering the massive amounts of facts, partial facts, and nonfacts unearthed currently through analyzing anecdotes and data mining, we can confidently expect based on educated analyses of evidence and history that a major breakthrough in areas of economics and business is coming soon. And, because systems science and the yoyo model can provide effective means for large-scale analysis and intuitive thinking, I expect quite surely that when such a breakthrough appears, it will be systems science and systems thinking based, as a consequence of combining the Western thinking with that of the Eastern logic of reasoning.

Consequent to what is discussed in the previous paragraphs in the presentation of the contents in this book special care is given to write the theory and each application in a language appropriate to that field. In particular, the part of the foundation of systems science is written in the language of pure mathematics; the part on the systemic yoyo model is presented in such a way that it is familiar to scholars who are used to developing and employing mathematical models for the purpose of resolving real-life problems; and then each application in economics and business decision making is offered with abstractions and explanations at different levels so that readers in various fields will find it possible to understand and to appreciate the beauty and power of systems science.

Additionally, because of the wide range of topics covered in this book, I try to make each chapter self-contained so that in general, there is no need for the reader to reference a different chapter for his/her reading of any particular chapter.

## 1.5 Organization of Contents in This Book

As an update and expansion of my previous book, entitled "General Systems Theory: A Mathematical Approach," initially published in 1999. This volume contains more than two thirds of the contents on recent developments in the relevant areas of research. In particular, this new edition consists of three parts, entitled *The Foundation*, *The Systemic Intuition*, and *Applications in Business Decision Making*, respectively.

The first part on the foundation of systems science contains only some minor changes and additions compared to the previous edition. It consists of Chapters 2–12, where Chapters 2–7 focus on the rigorous development of the concept of general systems, discovery of elementary properties of systems, construction of large-scale systems, interactions between systems, controllabilities of systems, etc. And Chapters 8–12 look at single-relation systems, where such concepts as chaos, attractors, feedback transformations, decoupling, decomposition, etc., are investigated in depth.

The second part introduces the systemic yoyo model, the desperately needed intuition that is specific to systems science. This part consists of three Chapters 13–15. Here, Chapter 13 introduces the yoyo model, explains what the yoyo field is composed of, and shows how systems interact with each other through their individual yoyo fields. Then the chapter considers how Newton's laws of motions can be rewritten in the language of systems science. Chapter 14 provides several different theoretical supports for why each and every system possesses its specific yoyo structure, and Chapter 15 lists a few empirical evidences for the existence of the yoyo structure behind each and every system.

The third part of the book looks at how systemic thinking and the yoyo model can be beautifully applied to address many important problems facing economists and business decision makers by organically combining methods of the traditional science, the first dimension of the two-dimensional spectrum of science, with those of systems science, the second dimension, as argued by George Klir in the 1990s. In particular, this part of the book consists of Chapters 16–21, where Chapter 16 looks how nonlinearity appears in the interaction of market demand and supply, how economic cycles can be seen through the lens of the systemic yoyo model, and how market competition emerges. Chapter 17 focuses on several issues on competition, explains why coordinated monopoly tends to mean stagnation in expected profits, markets change faster and customers become less patient, and once sustainable competitive advantages become transient. Chapter 18 presents when the famous Becker's rotten kid theorem holds true along with two other mysteries of the household economics, as well as the important never-perfect value system. Chapter 19 shows the reasons why management efficiency is possible in real life, while organizational inefficiency always exists in any business firm. Chapter 20 investigates how indecisive customers can be potentially handled with pricing strategies. And Chapter 21 looks at the relationship between private information and the performance of sales associates. Different of the traditional research in the relevant areas of economics and business, all the conclusions established in this part of the book are first seen intuitively through the yoyo model and then proved rigorously using different methods of the traditional science.

To conclude this chapter, let me remind you, the reader, that the foundation of systems science, presented in Part 1 of this book, is entirely based on a set-theoretical definition of the concept of general systems in order for us to derive rigorously all the elementary properties of systems. As discussed earlier in this chapter, the concept of systems has been investigated in different ways using various languages throughout the recorded history. However, like the concept of Tao (English and Feng 1972; Lao Tzu, time unknown), whenever the concept of systems is defined in a specific fashion, what is described is not exactly the concept, see Section 15.2 for more details. This fact means that each specific technical definition of systems only captures a particular aspect of the desired concept of systems. So, to reach the desired level of understanding, one needs to combine several different versions of the concept in order to achieve the required level of mastery. For our purpose in this volume, the listed versions of definition of systems in Section 15.2 will be adequate.

Along with the convention, established in Chapter 13, that all systems, be they physical objects, or organizations of biological beings, or imaginary entities, can be investigated as rotational pools of "fluids" together with a deep understanding of the concept of systems, the applications in economics and business decision making presented in Chapters 16–22 become quite intuitive and straightforward, developed on the sound scientific means of analytic analysis, laboratory experiments, and formal logic reasoning. To this end, some readers might prefer to lay the applications on a more thorough analysis of human behaviors and economic phenomena themselves. If you are one of such readers, I can fully understand for several reasons:

- All applications are presented in a relatively (natural) scientific fashion. In other words, after a conclusion or model is validated, it is employed throughout each of the applications without repeatedly explaining why the conclusion once again employed is correct.
- My stance is that if reliable thorough analysis of human behaviors and economic phenomena already existed, why do we still have trouble explaining many organizational behaviors and economic events, such as the recent financial crisis? To me, the answer is quite simple: As of this writing, there is no reliable thorough analysis of human behaviors and economic phenomena, especially no analyses have been developed for large-scale phenomena, processes, and events.
- If you, the reader, feel that without enough reliable information, it is impossible to know whether or not specific human phenomena and economic events conform to the yoyo model, the truth of the matter is that in the (natural) scientific history if a model is shown to hold true under certain conditions, then as long as a phenomenon, be it physical or human or economic, satisfies all the listed conditions, it will definitely fit the model or the model applies to the phenomenon. In particular to the applications in Part 3 of this book, as long as we are convinced that the human phenomena and economic events considered herein are systems, then they can be seen as and modeled by the yoyo model without any slightest doubt. As for the reliability of information, it has been shown in practice (Soros 1998) and theory (Liu and Lin 2006) that the more reliable a piece of information is the more chaotic situation it creates. That is, reliability is only relative instead of being absolute.
- Could this study be seen as secondary developed based on the existing scientific literature? The answer to this question is both yes and no. It is yes, because many detailed areas of the applications dig deeply into the existing literature and try to elevate what have been known onto much higher levels. In this sense, all the applications listed in this book in fact should not be treated as secondary to what has already been known in the literature. At the same time, the answer is no, because from my reasonably intensive search of the available literature, it is quite clear that most of the relevant publications are generated on the bases of either anecdotes without much scientific rigor or based on data mining that is really an art. In this sense, what is presented in this volume is more fundamental with scientific rigor than most of the relevant literature and lays the theoretical and intuitive foundation for achieving sound understanding of and arriving at reliable decisions regarding various large-scale organizations, economic processes, and events.

To conclude this chapter, let us look at how the parts and chapters of this book are connected and coordinated with one another.

The three parts present a sequential order of learning and dynamic development of a new theory. In particular, Part 1 presents how a cohesive and systematic theory of general systems can be developed by using a unified language—set theory. From the role successfully played by set theory in mathematics and in science, it is clear that the language of sets is very versatile and universal in terms of its elevated level of abstraction and its superlative power to describe very different phenomena observed in all walks of life. Because of the use of set theory, this first part of this book presents only results and conclusions that can be rigorously derived from a few basic concepts. In other words, results and conclusions presented in this part are universally true no matter which specific area of learning or application is concerned with.

Continuing what is presented in the previous part, while filling a gap existing in the reasoning process of systemic logic, Part 2 establishes a badly needed systemic intuition—the yoyo model—based on a holistic analysis of different understandings (or formulations) of the concept of (general) systems. With this systemic intuition in place, the magnificent successes of Euclidean geometry and calculus throughout the scientific history clearly indicate that the cohesive body of results and conclusions given in Part 1 now becomes a potentially useful knowledge in terms of its capability to resolve problems in real life and to meet challenges in various areas of human endeavor. To demonstrate the fact that what is developed in the previous two parts has become a practically useful theory, as expected, Part 3 focuses on various successful applications in business decision making. And at the end of this book, to show how the large-scale systems, studied in Chapters 3, 5, 7, 9–12 of Part 1, can be practically used jointly with the systemic yoyo model of Part 2 in the area of business decision making, Chapter 22 looks at how to manipulate the flow of speculative hot money and how to introduce monetary policies in order to reduce the severity of the currency attacks. In other words, this final chapter of the book not only shows how the theory developed in this volume can be employed for practical purposes, as what has been done in other chapters of Part 3, but also demonstrates why what is presented in this book is indeed a holistic, organic body of knowledge.

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# Part I
# The Foundation

# Chapter 2
# The Concept of General Systems

This chapter focuses on the basic concept of general systems and then the fundamental properties of general systems. In particular, Section 2.1 introduces the concept of general systems that will be used throughout the rest of this book along with a brief review of other relevant concepts studied by various scholars in different contexts. Section 2.2 investigates the internal properties of general systems. Section 2.3 considers the set-theoretical structures of general systems and L-fuzzy systems along with the existence of centralized systems.

## 2.1 Various Attempts

von Bertalanffy (1934) introduced the concept of systems in biology in the late 1920s, and it has been developed a great deal since then. Starting in the early 1960s, scholars began to establish a theory of general systems in order to lay down a theoretical foundation for all approaches of systems analysis developed in different disciplines. Led by von Bertalanffy and supported by several of the most powerful minds of our time, a global systems movement has been going on for several decades.

Along the development path of the systems movement, many scholars from different disciplines have tried to first define the concept of general systems and then apply the concept and the theory, developed on the concept of general systems, to solving problems in various research areas. Technical difficulties of introducing an ideal definition for general systems, and notably unsuccessful applications of general systems theory [for details, see Berlinski (1976; Lilienfeld 1978)], have taught many painful lessons. Among them are:

(1) There might not exist an ideal definition for general systems, upon which a general systems theory could be developed so that this theory would serve as the theoretical foundation for all approaches of systems analysis, developed in various disciplines.



J. Y.-L. Forrest, *General Systems Theory*, IFSR International Series in Systems Science and Systems Engineering 32, https://doi.org/10.1007/978-3-030-04558-6_2

infinitely divisible"! If the answer to the question is "no," does that mean the world is made up of fundamental elements?

The idea that the world is infinitely divisible can be found in Anaxagoras' "seeds" philosophy (510–428 B.C.). He was unwilling to submerge the tremendous varieties of things into any common denominator and preferred to accept the immediate diversity of things as is. With his philosophy, every object is infinitely divisible. No matter how far an object is divided, what is left would have characteristics of the original substance. The opposite idea that the world is made up of fundamental particles appeared during 500–55 B.C. The Leucippus–Democritus atom combined features of the Ionians' single element, Anaxagoras' seeds, and some thoughts of other schools and yet was an improvement over all of them. "Atom" means "not divisible" in Greek. This term was intentionally chosen by Democritus to emphasize a particle so small that it could no longer be divided. To Leucippus and Democritus, the universe originally and basically consisted entirely of atoms and a "void" in which atoms move.

Upon consideration of the interrelationship between the systems under concern and some environments of the systems, Bunge (1979) gave a model of systems as follows: For nonempty set $T$, the ordered triple $W = (C, E, S)$ is a system over $T$ if and only if $C$ and $E$ are mutually disjoint subsets of $T$ and $S$ is a nonempty set of relations on $C \cup E$; $C$ and $E$ are called the composition and an environment of the system $W$, respectively. Klir (1985) introduced a philosophical concept of general systems, which reads: "a system is what is distinguished as a system." Theoretically, Klir's definition contains the most general meaning of the concept of systems originally posed by von Bertalanffy. Starting in 1976, Xuemou Wu and his followers have studied many different theories, under the name "pansystems." Pansystems analysis is a new research of multi-levels across all disciplines. The theory deals with general systems, relations, symmetry, transformation, generalized calculus, and *shengke* (means survival and vanquishing), called the emphases of pansystems. Based on the research of these emphases, the theory blends philosophical reasoning, mathematical logic, and mechanical structures into one solid body of knowledge (Wu 1990).

The discussion here gives rise naturally to the following question: Why was the concept of systems not studied more before now? One answer is that the development of modern technology, e.g., computer technology, designs of satellites, climate control of giant buildings, etc., reveals one fact: History is in a special moment, where the accumulation of knowledge has reached such a level that each discovery of a relation between different areas of knowledge will materially produce a useful or consumable product.

Closely related to the concept of systems are the following questions:

(1) What is the meaning of a system with two contradictory relations, e.g., $\{x > y, x \leq y\}$ as the relation set?
(2) How can we know whether there exist contradictory relations in a given realistic system?

From Klir's definition of systems, which is one of the definitions with the widest meaning among all definitions of systems, it follows that generally these two questions cannot be answered. Concerning this, a theorem of Gödel shows that it is impossible to show whether the systems representation of mathematics, developed on the ZFC axioms, is consistent. For a detailed discussion of the systems representation of mathematics, see Lin (1989a). This fact implies that there are systems, for example, the system of set theory on ZFC, so that we do not know if propositions with contradictory meanings exist or not. This means that maybe not every system is consistent or that not every system has no contradictory relations.

By rewriting Russell's famous paradox in the language of systems theory, the following is true: There does not exist a system whose object set consists of all systems, where $S$ is a system, if and only if $S$ is an ordered pair $(M, R)$ of sets such that $R$ is a set of some relations defined on the set $M$ (Lin 1987). The elements in $M$ are called the objects of $S$. The sets $M$ and $R$ are called the object set and the relation set of $S$, respectively. Now, for any given system $S = (M, R)$, each relation $r \in R$ can be understood as an $S$-truth. That is, $r$ is true among the objects in $S$. Hence, Russell's paradox implies that there does not exist any statement (or relation) which describes a fact among all systems. If any structure in the world can be studied as a system, as Klir's definition of systems says, does Russell's paradox say that there is no universal truth or absolute truth? It is well known in the mathematics community that no theorem is true in all axiom systems.

Descartes and Galileo developed individually the following methods about scientific research and administration: Divide the problem under consideration into as many small parts as possible, study each isolated part (Kline 1972), and simplify the complicated phenomenon into basic parts and processes (Kuhn 1962). In the history of science and technology, Descartes' and Galileo's methods have been very successfully applied. They guaranteed that physics would win great victories one by one (von Bertalanffy 1972), and not only those, because their methods are still widely used for research of fundamental theories and in modern laboratories. At the same time, due to the tendency of modern science toward synthesis and the transverse development of modem technology, more and more scientists and administrators are forced to study problems with many cause–effect chains (i.e., systems). Some examples are the many-body problem in mechanics, the body structure of living things (but not the particles constituting the living things), etc. In the study of such problems, Descartes' and Galileo's methods need to be modified somehow, because their methods emphasize separating the problem and phenomenon under consideration into isolated parts and processes. But according to von Bertalanffy, it should be recognized that the world we live in is not a pile of innumerable isolated "parts", and any practical problem and phenomenon cannot be described perfectly by only one cause–effect chain. The basic character of the world is its organization and the connection between the interior and the exterior of different things. Thus, the chief task of modem science should be a systemic study of the world.

### 2.1.2 *The Set-Theoretical Definition of General Systems*

In order to produce useful results for theoretical and practical purposes, let us from here on focus on the definition of general systems introduced by Lin (1987).

Let $A$ be a set and $n$ be an ordinal number. An $n$-ary relation $r$ on $A$ is a subset of the Cartesian product $A^n$. In the sequel, $A^n$ means either the Cartesian product of $n$ copies of $A$ or the set of all mappings from $n$ into $A$. From set theory (Lin 1999), it is easy to see that the two definitions of $A^n$ are equivalent.

If $r$ is a relation on $A$, there then exists an ordinal $n = n(r)$, a function of $r$, such that

$$r \subseteq A^n.$$

The ordinal number $n = n(r)$ is called the length of the relation $r$. When $r$ is the empty relation $\varnothing$, assume that $n(\varnothing) = 0$; i.e., the length of the relation $\varnothing$ is 0.

$S$ is a (general) system if $S$ is an ordered pair $(M, R)$ of sets, where $R$ is a set of some relations on the set $M$. Each element in $M$ is called an object of $S$, and $M$ and $R$ are called the object set and the relation set of $S$, respectively. The system $S$ is discrete if $R = \varnothing$ or $R = \varnothing$ and $M \neq \varnothing$. It is trivial if $M = \varnothing$.

The idea of systems, roughly speaking, appeared as long ago as the time of Yellow Emperor in China and the time of Aristotle. For example, Chinese traditional medicine can be dated back to the time of Yellow Emperor and could be seen as the earliest theory of systems, and Aristotle's statement that "the whole is greater than the sum of its parts" could be treated as the first definition of a basic systems problem. Later, many great thinkers used the languages of their times to study certain systems problems. Nicholas of Cusa, for example, a profound thinker of the fifteenth century, linking medieval mysticism with the first beginnings of modern science, introduced the notion of the *coincidentia oppositorum.* Leibniz's hierarchy of monads looks quite like that of modem systems. Gustav Fechner, known as the author of the psychophysical law, elaborated in the way of the native philosophers of the nineteenth century supraindividual organizations of higher order than the usual objects of observation, thus romantically anticipating the ecosystems of modem parlance; for details, see von Bertalanffy (1972).

However, the concept of systems was not introduced formally until the 1920s when Von Bertalanffy began to study the concept of systems formally in biology. Since then, more and more scholars have studied the concept of systems and related topics. For example, Tarski (1954–1955) defined the concept of relational systems. Hall and Fagan (1956) described a system as a set of objects and some relations between the objects and their attributes. They did not define mathematical meanings for objects nor for attributes of the objects. In 1964, Mesarovic began to study the model of general systems in the language of set theory. His final model (Mesarovic and Takahara 1975) reads as follows: A system $S$ is a relation on nonempty sets:

$$S \subseteq \prod \{V_i : i \in I\}. \tag{2.1}$$

After considering the interrelationship between the systems under concern and some environments of systems, Bunge (1979) gave a model of systems as follows: Let $T$ be a nonempty set. Then the ordered triple $W = (C, E, S)$ is a system over $T$ if and only if $C$ and $E$ are mutually disjoint subsets of $T$ and $S$ is a nonempty set of relations on $C \cup E$; the sets $C$ and $E$ are called the composition and an environment of the system $W$, respectively. Klir (1985) introduced a philosophical concept of general systems. The concept of general systems discussed intensively in this chapter and the rest of this book was first introduced by Lin (1987) and Lin and Ma (1987a, b).

In the following, we will conclude this section with three mathematical structures related to the concept of general systems: structure, $L$-fuzzy system, and $G$-system.

### 2.1.3 Structures

Let $A$ be a set and $n$ a nonnegative integer. An $n$-ary operation on $A$ is a mapping $f$ from $A^n$ into $A$. An $n$-ary relation $r$ on the set $A$ is a subset of $A^n$. A type $\tau$ of structures is an ordered pair

$$\left((n_0, \ldots, n_\nu, \ldots)_{\nu<0_{0(\tau)}}, (m_0, \ldots, m_\nu, \ldots)_{\nu<0_{1(\tau)}}\right), \tag{2.2}$$

where $0_{0(\tau)}$ and $0_{1(\tau)}$ are fixed ordinals and $n_\nu$ and $m_\nu$ are nonnegative integers. For every $\nu < 0_{0(\tau)}$, there exists a symbol $f_\nu$ of an $n_\nu$-ary operation, and for every $\nu < 0_{1(\tau)}$, there exists a symbol $r_\nu$ of an $m_\nu$-ary relation.

A structure $U$ (Gratzer 1978) is a triplet $(A, F, R)$, where $A$ is a nonempty set. For every $\nu < 0_{0(\tau)}$, we realize $f_\nu$ as an $n_\nu$-ary operation $(f_\nu)_U$ on $A$; for every $\nu < 0_{1(\tau)}$, we realize $r_\nu$ as an $m_\nu$-ary relation $(r_\nu)_U$ on $A$, and

$$F = \left\{(f_0)_U, \ldots, (f_\nu)_U, \ldots\right\}, \nu < 0_{0(\tau)}, \tag{2.3}$$

$$R = \{(r_0)_U, \ldots, (r_\nu)_U, \ldots\}, \nu < 0_{1(\tau)}. \tag{2.4}$$

If $0_{1(\tau)} = 0$, $U$ is called an algebra and if $0_{0(\tau)} = 0$, $U$ is called a relational system.

It can be seen that the concept of algebras is a generalization of the concepts of rings, groups, and the like. Any $n$-ary operation is an $(n+1)$-ary relation. Therefore, the concept of relational systems generalizes the idea of algebras, and the concept of structures combines those of algebras and relational systems. The fact that any topological space is a relational system shows that the research of relational systems will lead to the discovery of properties that topologies and algebras have in common.

**Example 2.1.** We show that any topological space is a relational system, thus a general system. Let $(X, T)$ be a topological space; for details, see Engelking (1975). Then for each open set $U \in T$, there exists an ordinal number $n = n(U) = 1$ such that $U \in X^n$. Therefore, $(X, T)$ is also a system with object set $X$, and relation set $T$, and the length of each relation in $T$ is 1. Now, applying the Axiom of Choice implies that the topology $T$ can be well-ordered as

$$T = \{U_0, U_1, \ldots, U_\nu, \ldots\}, \nu < |T|, \tag{2.5}$$

where each open set is a 1-ary relation on $X$. So $(X, \varnothing, T)$ becomes a structure, which is a relational system.

A systematic introduction to the study of structures can be found in (Gratzer 1978). Many great mathematicians worked in the field, including Whitehead, Birkhoff, Chang, Henkin, Jonsson, Keisler, Tarski, etc.

### 2.1.4 L-Fuzzy Systems

The concept of fuzzy systems (Lin 1990) is based on the concept of fuzzy sets introduced in 1965 by Zadeh. An ordered set $(L, \leq)$ is called a lattice if for any elements $a, b \in L$, there exists exactly one greatest lower bound of $a$ and $b$, denoted by $a \wedge b$, and there exists exactly one least upper bound of $a$ and $b$, denoted by $a \vee b$, such that

$$\begin{aligned}
&a \vee a = a, a \wedge a = a, a \vee b = b \vee a, a \wedge b = b \wedge a,\\
&a \vee (b \vee c) = (a \vee b) \vee c,\ a \wedge (b \wedge c) = (a \wedge b) \wedge c,\\
&a \wedge (a \vee b) = a, a \vee (a \wedge b) = a
\end{aligned}$$

We call a lattice distributive if the lattice, in addition, satisfies

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c) \quad \text{and} \quad a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c).$$

We call a lattice complete if for any nonempty subset $A \subseteq L$, there exists exactly one least upper bound of the elements in $A$, denoted by $\vee A$ or $\sup A$, and there exists exactly one greatest lower bound of the elements in $A$, denoted by $\wedge A$ or $\inf A$.

We call a lattice completely distributive if for any $a \in L$ and any subset $\{b_i : i \in I\} \subseteq L$, where $I$ is an index set, $\vee\{b_i : i \in I\}$ and $\wedge\{b_i : i \in I\}$ exist and satisfy

$$a \vee (\wedge\{b_i : i \in I\}) = \wedge\{a \vee b_i : i \in I\} \tag{2.6}$$

and

$$a \wedge (\vee\{b_i : i \in I\}) = \vee\{a \wedge b_i : i \in I\}. \tag{2.7}$$

We say that $L$ is a lattice with order-reversing involution if there exists an operator $'$ defined on $L$ such that for any $a, b \in L$, if $a \leq b$, then $a' \geq b'$.

In the following, a completely distributive lattice $L$ with order-reversing involution is fixed; assume that 0 and 1 are the minimum and maximum elements in $L$.

Let $X$ be a set. $A$ is an $L$-fuzzy set on $X$ if and only if there exists a mapping $\mu_A \in L^X$ such that $A = \{(x, \mu_A(x)) : x \in X\}$. Without confusion, we consider that $A$

A chain of object systems of a system $S$ is a sequence $\{S_i : i < \alpha\}$, for some ordinal number $\alpha$, of different-level object systems of $S$, such that for each pair $i$, $j<\alpha$ with $i<j$, there exists an integer $n = n(i, j)$, a function of $i$ and $j$, such that $S_j$ is an $n$th-level object system of $S_i$. A similar proof to that of Theorem 2.2 gives the following.

**Theorem 2.3 [ZFC].** *Each chain of object systems of S must be finite.*

**Theorem 2.4 [ZFC].** *For any system S, there exists exactly one set $M(S)$ consisting of all fundamental objects in S, where a fundamental object in S is a level object of the system S which is no longer a system.*

**Proof:** Suppose that $S = (M_0, R_0)$. Define

$$^*M_0 = \{x \in M_0 : x \text{ is not a system}\}$$

and

$$\widetilde{M}_0 = \bigcup \left\{M_x : x = (M_x, R_x) \in M_0 - {}^*M_0\right\}$$

Then $^*M_0$ and $\widetilde{M}_0$ are sets.

Suppose that for a natural number $n$, two sequences $\left\{^*M_i : i = 0, 1, 2, \ldots, n\right\}$ and $\{\widetilde{M}_i : i = 0, 1, 2, \ldots, n\}$ have been defined such that

$$^*M_i = \left\{x \in \widetilde{M}_{i-1} : x \text{ is not a system}\right\}$$

and

$$\widetilde{M}_i = \bigcup \left\{M_x : x = (M_x, R_x) \in \widetilde{M}_{i-1} - {}^*M_i\right\}.$$

We then define

$$^*M_{i+1} = \left\{x \in \widetilde{M}_i : x \text{ is not a system}\right\} \text{ and}$$
$$\widetilde{M}_{i+1} = \cup\left\{M_x : x = (M_x, R_x) \in \widetilde{M}_i - {}^*M_{i+1}\right\}.$$

By mathematical induction, a sequence $\left\{^*M_i : i \in \omega\right\}$ can be defined. Let $M(S) = \cup\left\{^*M_i : i \in \omega\right\}$. Then Theorem 2.3 guarantees that $M(S)$ consists of all fundamental objects in $S$, and from the uniqueness of each $^*M_i$, it follows that $M(S)$ is uniquely determined by $S$. QED

## 2.3 The Edifice of Systems

The following result describes the overall structure of general systems when each relation is binary.

**Theorem 2.5.** *Let $S = (M, R)$ be a system such that, for any $r \in R$, the length $n(r) = 2$. Then*

$$S \subseteq p^2(M) \cup p\big(p(M) \cup p^4(M)\big), \tag{2.10}$$

*where $p(X)$ is the power set of $X$ and $p^{i+1}(X) = p(p^i(X))$ for each $i = 1, 2, \ldots$.*

**Proof:** For each element, $(x, y) \in r$, $(x, y) = \{\{x\}, \{x, y\}\} \in p^2(M)$ and $r \in p^3(M)$. Therefore, $R \in p^4(M)$. Since $\{M\} \in p(M)$ and $(M, R) \in p\big(p(M) \cup p^4(M)\big)$, it follows that the inclusion in equation (2.10) holds true. QED

**Question 2.1.** Give a structural representation for general systems similar to that in Theorem 2.5.

A system $S = (M, R)$ is called centralized if each object in $S$ is a system, and there exists a nontrivial system $C = (M_C, R_C)$ such that for any distinct elements $x$, $y \in M$, say $x = (M_x, R_x)$ and $y = \big(M_y, R_y\big)$, then

$$M_C = M_x \cap M_y \text{ and } R_C \subseteq R_x|M_C \cap R_y\big|M_C,$$

where $R_x|M_C = \{r|M_C : r \in R_x\}$ and $R_y\big|M_C = \big\{r|M_C : r \in R_y\big\}$. The system $C$ is called a center of the centralized system $S$.

**Theorem 2.6 [ZFC].** *Let $\kappa$ be any infinite cardinality and $\theta > \kappa$ a regular cardinality such that, for any $\alpha < \theta$, $|\alpha^{<\kappa}| < \theta$. Assume that $S = (M, R)$ is a system satisfying $|M| \geq \theta$ and each object $m \in M$ is a system with $m = (M_m, R_m)$ and $|M_m| < \kappa$. If there exists an object that is contained in at least $\theta$ objects in $M$, then there exists a partial system $S' = \big(M', R'\big)$ of $S$ such that $S'$ forms a centralized system and $\big|M'\big| \geq \theta$.*

**Proof:** Without loss of generality, we assume that $|M| = \theta$ and that there is a common element in all the object systems in $M$. Then we have

$$|\cup\{M_x : x = (M_x, R_x) \in M\}| \leq \theta.$$

Since the specific objects in each $M_x$, for each object $x = (M_x, R_x) \in M$, are irrelevant, we may assume that

$$\cup\{M_x : x = (M_x, R_x) \in M\} \subseteq \theta.$$

Then, for each $x = (M_x, R_x) \in M$, the object set $M_x$ has some order type $< \kappa$ as a subset of $\theta$. Since $\theta$ is regular and $\theta > \kappa$, there exists a $\rho < \kappa$ such that $M_1 = \{x \in M : M_x \text{ has order type } \rho\}$ has cardinality $\theta$. We now fix such a $\rho$ and deal only with the partial system $S_1 = (M_1, R_1)$ of $S$, where $R_1$ is the restriction of the relation set $R$ on $M_1$.

For each $\alpha < \theta$, $|\alpha^{<\kappa}| < \theta$ implies that less than $\theta$ objects of the partial system $S_1$ have object sets as subsets of $\alpha$. Thus, $\cup\{M_x : x = (M_x, R_x) \in M_1\}$ is cofinal in $\theta$.

If $x \in M_1$ and $\xi < \rho$, let $M_x(\xi)$ be the $\xi$th element of $M_x$ . Since $\theta$ is regular, there is some $\xi$ such that $\{M_x(\xi) : x \in M_1\}$ is cofinal in $\theta$. Now, fix $\xi_0$ to be the least such $\xi$. Then the condition that there exists a common element in each system in $M_1$ implies that we can guarantee that $\xi_0 > 0$. Let

$$\alpha_0 = \cup\{M_x(\eta) + 1 : x \in M_1 \text{ and } \eta < \xi_0\}.$$

Then $\alpha_0 < \theta$ and $M_x(\eta) < \alpha_0$ for all $x \in M_1$ and all $\eta < \xi_0$.

By transfinite induction on $\mu < \theta$, pick $x_\mu \in M_1$ so that $M_{x_\mu}(\xi_0) > \alpha_0$ and $M_{x_\mu}(\xi_0)$ is above all elements of earlier $x_\nu$; i.e.,

$$M_{x_\mu}(\xi_0) > \max\left\{\alpha_0, \bigcup\left\{M_{x_\nu}(\eta) : \eta < \rho \text{ and } \nu < \mu\right\}\right\}.$$

Let $M_2 = \left\{x_\mu : \mu < \theta\right\}$. Then $|M_2| = \theta$ and $M_x \cap M_y \subseteq \alpha_0$ whenever $x = (M_x, R_x)$ and $y = \left(M_y, R_y\right)$ are distinct objects in $M_2$. Since for each $\alpha < \theta$, we have

$$\left|\alpha^{<\kappa}\right| < \theta,$$

there exists an $r \subset \alpha_0$ and a $B \subset M_2$ with $|B| = \theta$ and for each $x \in B$, $M_x \cap \alpha_0 = r$, $S_2 = (B, R_B)$ forms a centralized system, where $R_B$ is the restriction of the relation set $R$ on $B$. QED

**Corollary 2.1.** *If S is a system with uncountable (or nondenumerable) object set, if each object in S is a system with a finite object set, and if there exists an object that is contained in at least $\aleph_1$ objects in S, then there exists a partial system $S^*$ of S with an uncountable object set and $S^*$ forms a centralized system.*

**Proof:** It suffices from Theorem 2.6 to show that $\forall\alpha < \aleph_1\left(\left|\alpha^{<\aleph_0}\right| < \aleph_1\right)$. This is clear because for each $\alpha < \aleph_1$, $|\alpha| \le \aleph_0$. So, we have

$$\left|\alpha^{<\aleph_0}\right| = \aleph_0 < \aleph_1. \text{ QED} \tag{2.11}$$

A system $S = (M, R)$ is strongly centralized if each object in $S$ is a system and there is a nondiscrete system $C = (M_C, R_C)$ such that for any distinct elements $x$ and $y \in M$, say $x = (M_x, R_x)$ and $y = \left(M_y, R_y\right)$, $M_C = M_x \cap M_y$ and $R_C = R_x|M_C \cap R_y|M_C$. The system $C$ is called an $S$-center of $S$.

**Question 2.2.** Give conditions under which a given system has a partial system which is strongly centralized and has an object set of the same cardinality as that of the given system.

In terms of $L$-fuzzy systems, we have the following result in terms of their structures when each relation is binary.

**Theorem 2.7.** *Suppose that an L-fuzzy system $S = (M, R)$ is such that, for any $r \in R$, $n(r) = 2$. Then we have the following:*

$$S \subseteq p\big(M \cup p^3\big(p^2(M) \cup L\big)\big). \tag{2.12}$$

**Proof:** $S = (M, R) = \{\{M\}, \{M, R\}\} \subseteq p(M \cup R)$, and for any $(x, y, i) \in r \subseteq M^2 \times L$, we have

$$(x, y, i) = ((x, y), i) = \{\{(x, y)\}, \{(x, y), i\}\},$$
$$\{(x, y), i\} = \{\{\{\{x\}, \{x, y\}\}\}, \{i, \{\{x\}, \{x, y\}\}\}\} \in p\big(p^3(M) \cup p^2(M \cup L)\big) =$$
$$= p\big(p\big(p^2(M \cup L)\big)\big) = p^2\big(p^2(M) \cup L\big).$$

So $r \in p^3\big(p^2(M) \cup L\big)$ and $R \subseteq p^3\big(p^2(M) \cup L\big)$. Thus, the inclusion in equation (2.12) holds true. QED

Suppose that $S_i = (M_i, R_i)$, $i = 1, 2$, are $L$-fuzzy systems. The $L$-fuzzy system $S_1$ is a subsystem of the $L$-fuzzy system $S_2$ if $M_1 \subseteq M_2$, and for any $L$-fuzzy relation $r_1 \in R_1$, there exists an $L$-fuzzy relation $r_2 \in R_2$ such that $r_1 \subseteq r_2|M_1$.

**Example 2.2.** An example is constructed to show that the equation $S_1 = S_2$ will not follow from the condition that the $L$-fuzzy systems $S_1$ and $S_2$ are subsystems of each other.

Suppose that the completely distributive lattice with order-reversing involution lattice $L$ is the closed unit interval with the usual order relation. Let $\mathbb{N} = \{1, 2, \ldots\}$ be the set of all natural numbers, and $\{a_i : i \in \mathbb{N}\}$ a sequence defined by $a_i = 1 - 1/i$. We now define two $L$-fuzzy systems $S_1 = (\mathbb{N}, R_1)$ and $S_2 = (\mathbb{N}, R_2)$ as follows:

$$R_1 = \{r_{11}, r_{12}, \ldots, r_{1i}, \ldots\} \text{ and } R_2 = \{r_{20}, r_{21}, \ldots, r_{2i}, \ldots\},$$

where

$$r_{1i}(x) = a_i, \text{ for each } x \in \mathbb{N} \tag{2.13}$$

and for any $i, x \in \mathbb{N}$

$$r_{2i}(x) = \begin{cases} a_{i+1}, & \text{if} x = 1 \\ a_i, & \text{if} x \neq 1 \end{cases} \tag{2.14}$$

and

$$r_{20}(x) = 0, \text{ for all } \in \mathbb{N}. \tag{2.15}$$

Then it is easy to show that $S_1$ and $S_2$ are subsystems of each other, but $S_1 \neq S_2$.

Remarks on references: All the discussions, presented above, are based on the following works by various scholars, Bunge (1979), Engelking (1975), Gratzer (1978), Hall and Fagan (1956), Klir (1985), Lin (1987, 1990, 1999), Lin and Ma (1987a; b, 1989), Mesarovic (1964), Mesarovic and Takahara (1975), Tarski (1954–1955), von Bertalanffy (1972), and Zadeh (1965).

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**Theorem 3.2.** *Let $S = (M, R)$ be a system such that each object in $M$ is a system. Then $S$ is centralizable, if and only if there exists a nontrivial system $C = (M_C, R_C)$ such that $C$ is embeddable in each object of $S$, meaning that $C$ is similar to a partial system of each object of $S$, and each similarity mapping here is called an embedding mapping from $C$ into the object of $S$.*

**Proof:** Necessity. If the system $S = (M, R)$ is centralizable, there exists a centralized system $S^* = (M^*, R^*)$ such that $S$ is one-level homomorphic to $S^*$. Let $C$ be a center of $S^*$. Then $C$ is embeddable in each object of $S$.

Sufficiency. Without loss of generality, we can assume that the collection of all object sets of the objects in $S$ consists of disjoint sets. For each $x = (M_x, R_x) \in M$, the system $C$ is embeddable in $x$; so there exists a one-to-one mapping $h_x: M_C \to M_x$ such that $h_x(C) = (h_x(M_C), h_x(R_C))$ is a partial system of $x$. Define a new system $x^* = ({}^*M_x, {}^*R_x)$ as follows:

$$ {}^*M_x = M_C \cup (M_x - h_x(M_C)) \tag{3.5}$$

and let $g_x: M_x \to^* M_x$ be the bijection defined by

$$g_x(m) = \begin{cases} m, & \text{if } m \in M_x - h_x(M_C) \\ h_x^{-1}(m), & \text{if } m \in h_x(M_C) \end{cases}. \tag{3.6}$$

Then define

$$ {}^*R_x = \{g_x(r): r \in R_x\} \tag{3.7}$$

So $C$ is a partial system of $x^*$. Let $h^*$ be a mapping defined by letting $h^*(x) = x^*$ for each $x \in M$. Then the system $h^*(S)$ is centralized with the system $C$ as a center. Therefore, $S$ is a centralizable system. QED

**Example 3.1.** An example is constructed to show that the hypothesis in Theorem 2.6 that there exists an object belonging to at least $\theta$ objects in $S$ is essential for the result to follow and that not every centralizable system is a centralized system. Consequently, the concepts of centralized systems and centralizable systems are different.

Let $\omega_1$ be the first uncountable ordinal number, and $M$ a collection of $n$-element subsets of $\omega_1$, for some natural number $n > 0$, such that $|M| = \omega_1$ and for any $x$, $y \in M$, $x \bigcap y = \emptyset$ if $x \neq y$.

We now consider a system $S$ with object set $M^*$ such that for any $x^* \in M^*$, $x^*$ is a system with some element in $M$ as its object set, and for any $x \in M$ there exists exactly one $x^* \in M^*$ such that $x$ is the object set of the system $x^*$. It is clear that $S$ has no partial system which forms a centralized system and that $S$ is a centralizable system.

**Theorem 3.3.** *Let $\kappa$ and $\theta$ be cardinalities satisfying the conditions in Theorem 2.6. Assume that $S$ is a system with an object set of cardinality $\geq \theta$ and each object in $S$*

*is a system with an object set of cardinality* $<\kappa$. *There then exists a partial system* $S'$ *of* $S$ *such that the object set of* $S'$ *is of cardinality* $\geq\theta$ *and* $S'$ *forms a centralizable system.*

The proof follows from Theorems 2.6 and 3.2.

A mapping $h$ from a system $S_1 = (M_1, R_1)$ into a system $S_2 = (M_2, R_2)$ is $S$-continuous if for any relation $r \in R_2$,

$$h^{-1}(r) = \left\{h^{-1}(x): x \in r\right\} \in R_1. \tag{3.8}$$

**Proposition 3.2.** *Suppose that* $h$: $S_1 \to S_2$ *is a mapping. Then the following hold true:*

(i) *If the systems* $S_i$, $i = 1, 2$, *are topological spaces,* $h$ *is* $S$*-continuous, if and only if it is continuous from* $S_1$ *into* $S_2$.
(ii) *If the systems* $S_i$, $i = 1, 2$, *are groups, the* $S$*-continuity of* $h$ *implies that* $h$ *is a homomorphism from* $S_1$ *into* $S_2$.
(iii) *If the systems* $S_i$, $i = 1, 2$, *are rings, the* $S$*-continuity of* $h$ *implies that* $h$ *is a homomorphism from* $S_1$ *into* $S_2$.

**Proof:** Conclusion (i) follows from the definition of $S$-continuity of mappings and Example 2.1.

For (ii), let $(X, +)$ be a group. A system $(X, R)$ is defined as follows: $R = \{r_+\}$ and $r_+ = \left\{(x, y, z) \in X^3: x + y = z\right\}$. The system $(X, R)$ will be identified with the group $(X, +)$, and the relation $r_+$ with $+$.

Let $S_i = \left(M_i, \left\{r_{+_i}\right\}\right)$, $i = 1, 2$, be two groups and $h$: $S_1 \to S_2$ an $S$-continuous mapping. Then $h^{-1}\left(r_{+_2}\right) = r_{+_1}$. Thus, for any $x$, $y$, and $z \in M_1$, if $x +_1 y = z$, then $h(x) +_2 h(y) = h(z)$. This implies that $h$ is a homomorphism from $S_1$ into $S_2$.

The proof of (iii) is a modification of that of (ii) and is omitted. QED

**Theorem 3.4.** *A bijection* $h$: $S_1 \to S_2$ *is a similarity mapping if and only if* $h$: $S_1 \to S_2$ *and* $h^{-1}$: $S_2 \to S_1$ *are* $S$*-continuous.*

The proof is straightforward and is left to the reader.

**Theorem 3.5.** *Suppose that* $f: S_1 \to S_2$ *and* $g: S_2 \to S_3$ *are* $S$*-continuous mappings. Then the composition* $g \circ f: S_1 \to S_3$ *is also* $S$*-continuous.*

**Proof:** For each relation $r$ in the system $S_3$, from the hypothesis that the mapping $g$ is $S$-continuous we have

$$g^{-1}(r) \in R_2. \tag{3.9}$$

From the hypothesis that the mapping $f$ is $S$-continuous, it follows that

$$f^{-1}\left(g^{-1}(r)\right) \in R_1. \tag{3.10}$$

Therefore, from equation (3.10), we have

$$(g \circ f)^{-1}(r) = f^{-1} \circ g^{-1}(r) = f^{-1}\big(g^{-1}(r)\big) \in R_1.$$

Thus, $g \circ f$ is $S$-continuous. QED

## 3.2 Free Sums

Let $\{S_i = (M_i, R_i): i \in I\}$ be a set of systems such that $M_i \cap M_j = \emptyset$ for any distinct $i$ and $j \in I$. The free sum of the systems $S_i$, denoted $\oplus\{S_i: i \in I\}$ or $S_1 \oplus S_2 \oplus \ldots \oplus S_n$, if $I = \{1, 2, \ldots, n\}$ is finite, is defined to be the system

$$\oplus\{S_i: i \in I\} = (\cup\{M_i: i \in I\}, \cup\{R_i: i \in I\}). \tag{3.11}$$

The concept of free sums of systems can be defined for any set $\{S_i: i \in I\}$ of systems such that the collection of all object sets of the systems in the set consists of not necessarily pairwise disjoint sets. To show this, we simply take a set $\{{}^*S_i: i \in I\}$ of systems with pairwise disjoint object sets such that ${}^*S_i$ is similar to $S_i$, for each $i \in I$, and define

$$\oplus\{S_i: i \in I\} = \oplus\left\{{}^*S_i: i \in I\right\}. \tag{3.12}$$

Up to a similarity, the free sum of the systems $S_i$ is unique. Therefore, it can be assumed that any set of systems has a free sum (uniquely determined up to a similarity), but in the proofs of some theorems in the future it will be tacitly assumed that the set under consideration consists of systems with pairwise disjoint object sets.

A system $S = (M, R)$ is an input–output system if two sets $X$ and $Y$ are given such that $X \cup Y = M$ and for each relation $r \in R$ there are nonzero ordinals $n = n(r)$ and $m = m(r)$ such that $r \subseteq X^n \times Y^m$. The sets $X$ and $Y$ are called the input space and the output space of the system $S$, respectively.

**Proposition 3.3.** *Let $S_1$ and $S_2$ be two similar systems. If $S_1$ is a free sum of some systems, then so is $S_2$.*

**Proof:** Suppose that $S_1 = \oplus\left\{S^i: i \in I\right\}$, where $\{S^i: i \in I\}$ is a set of systems $S^i = \left(M^i, R^i\right)$, and $h$: $S_1 \to S_2$ is a similarity mapping. Then we have

$$S_2 = \oplus\left\{h\left(S^i\right): i \in I\right\}, \tag{3.13}$$

because $h\left(R^i\right) \subseteq R_2$ for each $i \in I$, and for each $r \in R_2$ there exists a relation $s \in R^i$, for some $i \in I$, such that $h(s) = r$. QED

**Theorem 3.6.** *Suppose that* $\{S_i = (M_i, R_i): i \in I\}$ *is a set of systems, where I is an index set. Then the free sum* $\oplus\{S_i: i \in I\}$ *is an input–output system if and only if each system* $S_i = (M_i, R_i)$ *is an input–output system,* $i \in I$.

**Proof:** Necessity. Suppose that $\oplus\{S_i: i \in I\}$ is an input–output system with input space $X$ and output space $Y$. Then for any fixed $i \in I$ and any relation $r \in R_i$, there are nonzero ordinals $n = n(r)$ and $m = m(r)$ such that $r \subseteq X^n \times Y^m$. Hence, $S_i$ is an input–output system with input space $M_i \cap X$ and output space $M_i \cap Y$.

Sufficiency. Suppose that $S_i = (M_i, R_i)$ is an input–output system with input space $X_i$ and output space $Y_i$. Then for each relation $r$ in $\oplus\{S_i: i \in I\}$, there exists an $i \in I$ such that $r \in R_i$; therefore, we have

$$r \subseteq X_i^{n(r)} \times Y_i^{m(r)} \subseteq (\cup_{i\in I} X_i)^{n(r)} \times (\cup_{i\in I} Y_i)^{m(r)}. \tag{3.14}$$

Hence, the free sum $\oplus\{S_i: i \in I\}$ is an input–output system with input space $\bigcup_{i\in I} X_i$ and the output space $\bigcup_{i\in I} Y_i$. QED

**Theorem 3.7.** *If a system S is embeddable in the free sum* $\oplus\{S_i: i \in I\}$ *of some systems* $S_i$*, then S is also a free sum of some systems.*

The proof is straightforward and is omitted.

**Theorem 3.8.** *If the free sum* $\oplus\{S_i: i \in I\}$ *of nontrivial systems* $S_i$ *is a centralized system, then each* $S_i$ *is centralized, for* $i \in I$.

**Proof:** The theorem follows from the fact that each center of the free sum $\oplus\{S_i: i \in I\}$ is a center of each system $S_i$, $i \in I$. QED

**Theorem 3.9.** *Suppose that* $\{S_i: i \in I\}$ *is a set of centralizable systems such that each system* $S_i$ *has a center* $C_i = \left(M_{C_i}, R_{C_i}\right)$, $i \in I$, *and that* $\left|M_{C_i}\right| = \left|M_{C_j}\right|$, *for all i and j* $\in I$. *Then the free sum* $\oplus\{S_i: i \in I\}$ *is a centralizable system.*

**Proof:** Let $\left|M_{C_i}\right| = \aleph$, for some $i \in I$. Then the system $(\aleph, \emptyset)$ is embeddable in each object in the free sum $\oplus\{S_i: i \in I\}$. Theorem 3.2 implies that the free sum is centralizable. QED

It is easy to construct an example to show that the free sum of two centralized systems may not be a centralizable system.

**Theorem 3.10.** *Let* $h: S \to \oplus\{S_i: i \in I\}$ *be a homomorphism. Then the system S is also a free sum of some systems.*

**Proof:** Suppose that $S = (M, R)$ and $S_i = (M_i, R_i)$ for each $i \in I$. Then

$$M = \cup\left\{h^{-1}(M_i): i \in I\right\}. \tag{3.15}$$