LITTLE *Ways* *to* LIVE *a* BIG LIFE

HOW TO COUNT TO INFINITY

MARCUS DU SAUTOY

The bestselling author of *The Music of the Primes*

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Contents

Ready, Steady, Go

How to count to infinity? It couldn't be simpler. You start at 1 and then keep going. 1, 2, 3, 4 . . . The only trouble is it's going to take you quite a long time . . . especially towards the end (to steal a Woody Allen joke). In fact, you're never going to get there. Time will run out. The Polish artist, Roman Opalka, tried to paint all the numbers from 1 to infinity. He started in 1965. He got as far as 5,607,249, and then died in 2011 before he painted the next number.

Even if you said the numbers out loud rather than painting them, the likelihood is that in a lifetime you might make it as far as one billion and then you'll gasp your last breath and fail to make it to one billion and one. And that's provided no one interrupts you. Lose your place and it's back to square (or number) one. But even if you only make it to one billion, you'll know there's always another number waiting out there for anyone who can make it a bit further. A trillion. A zillion. A gazillion. A Brazilian. A googol (that's a 1 with 100 zeros). A googolplex (that's a 1 with a googol number of zeros). A googolplex plus 1!

Perhaps humankind can form a relay race. As one person gives up, the next person could take over where the last left off. But it turns out that even this strategy is doomed. The universe itself is going to run out of time. (Time, we believe, began ticking at the Big Bang. Once it had started ticking it was believed that it would go on for ever. But recent discoveries about the way the universe is expanding imply that at some point it will be stretched out so much that there will be nothing left to keep the count to

infinity going. In fact, there will be nothing left even to keep track of time itself. Time will run out. Time has an end. It too is finite. But that's another story.)

Nevertheless, mathematicians have found some cunning new ways to navigate infinity without having to count all the way to the end. Using ingenious stratagems cooked up at the end of the 19th century, they've discovered not only how to count to infinity but that there are different sorts of infinity. Some bigger than others. It is one of the most extraordinary feats of human endeavour. Getting to the top of Everest only requires a finite number of steps. But, mathematicians have shown how, using the finite equipment in your head, you can hit dizzying heights that make the finite height of Everest pale into insignificance.

So here I am, your mathematical Sherpa, to guide you in our quest to count to infinity and beyond.

You might ask why we want to get there even if we can. You're only ever going to need a finite number of numbers in your day-to-day life. So why worry about getting to infinity? During your lifetime, there will be a biggest number that you will think of and after that you will never think of a bigger number because your finite life will stop you getting any further.

But that is precisely why contemplating the infinite is worth the effort. The infinite provides an escape from the miserable finiteness of our mortal existence. To conceive of the infinite gives those who achieve this feat a sense of transcendence. As the famous German mathematician, David Hilbert, said of the 19th-century mathematician Georg Cantor, who first gave us a view of the infinite, 'No one will drive us from the paradise which Cantor created for us.' It is into that paradise that I wish to take you.

Like a Buddhist monk achieving a state of otherworldliness through meditation, the journey to infinity is going to require tapping into a

mathematical Zen-like state of acceptance. There will be moments when that might feel unsettling, but remember that we are trying to access something which may not have a physical reality. The portal to infinity is to be found deep inside the neurons of your mind. But the finite amount of grey matter inside your head is all you will need to reach this infinite mathematical nirvana.