Graham Priest

LOGIC

A Very Short Introduction

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Chapter 1

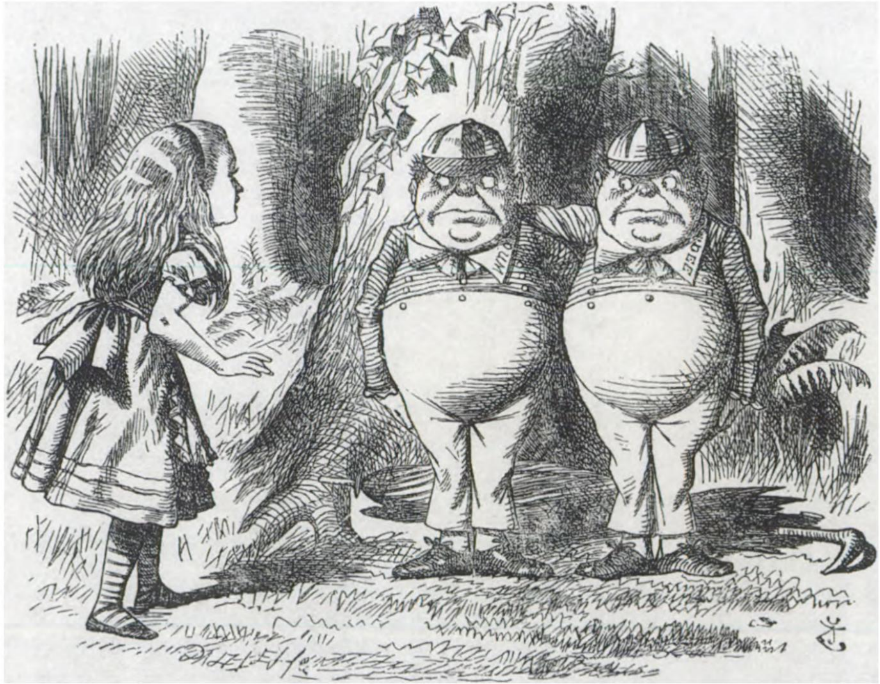
Validity: What Follows from What?

Most people like to think of themselves as logical. Telling someone 'You are not being logical' is normally a form of criticism. To be illogical is to be confused, muddled, irrational. But what is logic? In Lewis Carroll's *Through the Looking Glass*, Alice meets the logic-chopping pair Tweedledum and Tweedledee. When Alice is lost for words, they go onto the attack:

> 'I know what you are thinking about', said Tweedledum: 'but it isn't so, nohow.'
>
> 'Contrariwise,' continued Tweedledee, 'if it was so, it might be; and if it were so, it would be: but as it isn't, it ain't. That's logic.'

What Tweedledee is doing – at least, in Carroll's parody – is reasoning. And that, as he says, is what logic is about.

We all reason. We try to figure out what is so, reasoning on the basis of what we already know. We try to persuade others that something is so by giving them reasons. Logic is the study of what counts as a good reason for what, and why. You have to understand this claim in a certain way, though. Here are two bits of reasoning – logicians call them *inferences*:



1. Tweedledum and Tweedledee debate the finer points of logic with Alice

1. Rome is the capital of Italy, and this plane lands in Rome; so the plane lands in Italy.
2. Moscow is the capital of the USA; so you can't go to Moscow without going to the USA.

In each case, the claims before the 'so' – logicians call them *premisses* – are giving reasons; the claims after the 'so' – logicians call them *conclusions* – are what the reasons are supposed to be reasons for. The first piece of reasoning is fine; but the second is pretty hopeless, and wouldn't persuade anyone with an elementary knowledge of geography: the premiss, that Moscow is the capital of the USA, is simply false. Notice, though, that if the premiss had been true – if, say, the USA had bought the whole of Russia (not just Alaska) and had moved the White House to Moscow to be nearer the centres of power in Europe – the conclusion would indeed have been true. It would have followed from the premisses; and that is what logic is concerned with. It is not concerned with whether the premisses of an inference are true or false. That's somebody else's business (in this case, the geographer's). It is interested simply in whether the conclusion follows from the premisses. Logicians call an inference where the conclusion really does follow from the premisses *valid*. So the central aim of logic is to understand validity.

You might think this a rather dull task – an intellectual exercise with somewhat less appeal than solving crossword puzzles. But it turns out that this is not only a very hard matter; it is one that cannot be divorced from a number of important (and sometimes profound) philosophical questions. We will see some of these as we go along. For the moment, let us get a few more of the basic facts about validity straight.

To start with, it is common to distinguish between two different kinds of validity. To understand this, consider the following three inferences:

1. If the burglar had broken in through the kitchen window, there

would be footprints outside; but there are no footprints; so the burglar didn't break in through the kitchen window.
2. Jones has nicotine-stained fingers; so Jones is a smoker.
3. Jones buys two packets of cigarettes a day; so someone left footprints outside the kitchen window.

The first inference is a very straightforward one. If the premisses are true, so must the conclusion be. Or, to put it another way, the premisses couldn't be true without the conclusion also being true. Logicians call an inference of this kind *deductively valid*. Inference number two is a bit different. The premiss clearly gives a good reason for the conclusion, but it is not completely conclusive. After all, Jones could simply have stained his hands to make people think that he was a smoker. So the inference is not deductively valid. Inferences like this are usually said to be *inductively valid*. Inference number three, by contrast, appears pretty hopeless by any standard. The premiss seems to provide no kind of reason for the conclusion at all. It is invalid – both deductively and inductively. In fact, since people are not complete idiots, if someone actually offered a reason like this, one would assume that there is some extra premiss that they had not bothered to tell us (maybe that someone passes Jones his cigarettes through the kitchen window).

Inductive validity is a very important notion. We reason inductively all the time; for example, in trying to solve problems such as why the car has broken down, why a person is ill, or who committed a crime. The fictional logician Sherlock Holmes was a master of it. Despite this, historically, much more effort has gone into understanding deductive validity – maybe because logicians have tended to be philosophers or mathematicians (in whose studies deductively valid inferences are centrally important), and not doctors or detectives. We will come back to the notion of induction later in the book. For the present, let's think some more about deductive validity. (It is natural to suppose that deductive validity is the simpler notion, since valid inferences are more cut-and-dried. So it's not a bad idea to try to understand this first. That, as we shall see, is hard enough.) Until further notice 'valid' will simply mean 'deductively valid'.

So what is a valid inference? One, we saw, where the premisses can't be true without the conclusion also being true. But what does that mean? In particular, what does the *can't* mean? In general, 'can't' can mean many different things. Consider, for example: 'Mary can play the piano, but John can't'; here we are talking about human abilities. Compare: 'You can't go in here: you need a permit'; here we are talking about what some code of rules permits.

It is natural to understand the 'can't' relevant to the present case in this way: to say that the premisses can't be true without the conclusion being true is to say that in all situations in which all the premisses are true, so is the conclusion. So far so good; but what, exactly, is a situation? What sorts of things go into their makeup, and how do these things relate to each other? And what is it to be *true*? Now, there's a philosophical problem for you, as Tweedledee might have said.

These issues will concern us by and by; but let us leave them for the time being, and finish with one more thing. One shouldn't run away with the idea that the explanation of deductive validity that I have just given is itself unproblematic. (In philosophy, all interesting claims are contentious.) Here is one problem. Assuming that the account is correct, to know that an inference is deductively valid is to know that there are no situations in which the premisses are true and the conclusion is not. Now, on any reasonable understanding of what it is to be a situation, there are an awful lot of them: situations about things on the planets of distant stars; situations about events before there were any living beings in the cosmos; situations described in works of fiction; situations imagined by visionaries. How can one know what holds in *all* situations? Worse, there would appear to be an infinite number of situations (situations one year hence, situations two years hence, situations three years hence, . . .). It is therefore impossible, even in principle, to survey all situations. So if this account of validity is correct, and given that we can recognize inferences as valid or invalid (at least in many cases) we must have some insight into this, from some special source. What source?

Do we need to invoke some sort of mystic intuition? Not necessarily. Consider an analogous problem. We can all distinguish between grammatical and ungrammatical strings of words of our native language without too much problem. For example, any native speaker of English would recognize that 'This is a chair' is a grammatical sentence, but 'A chair is is a' is not. But there would appear to be an infinite number of both grammatical and ungrammatical sentences. (For example, 'One is a number', 'Two is a number', 'Three is a number', . . . are all grammatical sentences. And it is easy enough to produce word salads *ad libitum*). So how do we do it? Perhaps the most influential of modern linguists, Noam Chomsky, suggested that we can do this because the infinite collections are encapsulated in a finite set of rules that are hard-wired into us; that evolution has programmed us with an innate grammar. Could logic be the same? Are the rules of logic hard-wired into us in the same way?

Main Ideas of the Chapter

- A valid inference is one where the conclusion follows from the premiss(es).
- A deductively valid inference is one for which there is no situation in which all the premisses are true, but the conclusion is not.

Chapter 2
Truth Functions – or Not?

Whether or not the rules of validity are hard-wired into us, we all have pretty strong intuitions about the validity or otherwise of various inferences. There wouldn't be much disagreement, for example, that the following inference is valid: 'She's a woman and a banker; so she's a banker'. Or that the following inference is invalid: 'He's a carpenter; so he's a carpenter and plays baseball'.

But our intuitions can get us into trouble sometimes. What do you think of the following inference? The two premisses occur above the line; the conclusion below it.

The Queen is rich.   The Queen isn't rich.

Pigs can fly.

It certainly doesn't seem valid. The wealth of the Queen – great or not – would seem to have no bearing on the aviatory abilities of pigs.

But what do you think about the following two inferences?

The Queen is rich.

Either the Queen is rich or pigs can fly.

Either the Queen is rich or pigs can fly.   The Queen isn't rich.
____
Pigs can fly.

The first of these seems valid. Consider its conclusion. Logicians call sentences like this a *disjunction*; and the clauses on either side of the 'or' are called *disjuncts*. Now, what does it take for a disjunction to be true? Just that one or other of the disjuncts is true. So in any situation where the premiss is true, so is the conclusion. The second inference also seems valid. If one or other of two claims is true and one of these isn't, the other must be.

Now, the trouble is that by putting these two apparently valid inferences together, we get the apparently invalid inference, like this:

The Queen is rich.
____
Either the Queen is rich or pigs can fly.   The Queen isn't rich.
____
Pigs can fly.

This can't be right. Chaining valid inferences together in this way can't give you an invalid inference. If all the premises are true in any situation, then so are their conclusions, the conclusions that follow from *these*; and so on, till we reach the final conclusion. What has gone wrong?

To give an orthodox answer to this question, let us focus a bit more on the details. For a start, let's write the sentence 'Pigs can fly' as p, and the sentence 'The Queen is rich' as q. This makes things a bit more compact; but not only that: if you think about it for a moment, you can see that the two particular sentences actually used in the examples above don't have much to do with things; I could have set everything up using pretty much any two sentences; so we can ignore their content. This is what we do in writing the sentences as single letters.

The sentence 'Either the Queen is rich or pigs can fly' now becomes 'Either q or p'. Logicians often write this as $q \vee p$. What of 'The Queen isn't rich'? Let us rewrite this as 'It is not the case that the Queen is rich', pulling the negative particle to the front of the sentence. Hence, the sentence becomes 'It is not the case that *q*'. Logicians often write this as $\neg q$, and call it the *negation* of *q*. While we are at it, what about the sentence 'The Queen is rich *and* pigs can fly', that is, '*q* and *p*'? Logicians often write this as $q \,\&\, p$ and call it the *conjunction* of *q* and *p*, *q* and *p* being the *conjuncts*. With this machinery under our belt, we can write the chain-inference that we met thus:

$$\frac{\dfrac{q}{q \vee p} \quad \neg q}{p}$$

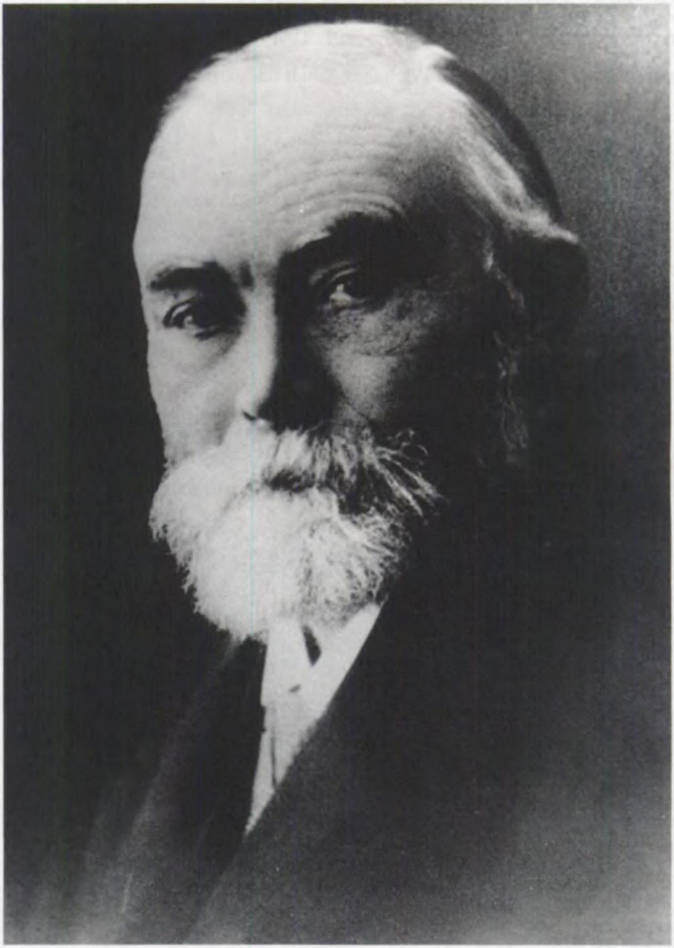
What are we to say about this inference?

Sentences can be true, and sentences can be false. Let us use *T* for truth, and *F* for falsity. After one of the founders of modern logic, the German philosopher/mathematician Gottlob Frege, these are often called *truth values*. Given any old sentence, *a*, what is the connection between the truth value of *a* and that of its negation, $\neg a$? A natural answer is that if one is true, the other is false, and vice versa. Thus, if 'The Queen is rich' is true, 'The Queen isn't rich' is false, and vice versa. We can record this as follows:

> $\neg a$ has the value *T* just if *a* has the value *F*.
> $\neg a$ has the value *F* just if *a* has the value *T*.

Logicians call these the *truth conditions* for negation. If we assume that every sentence is either true or false, but not both, we can depict the conditions in the following table, which logicians call a *truth table*:

a	$\neg a$
T	*F*
F	*T*



2. Gottlob Frege (1848–1925), one of the founders of modern logic

If a has the truth value given in the column under it, $\neg a$ has the corresponding value to its right.

What of disjunction, ∨? As I have already noted, a natural assumption is that a disjunction, $a \vee b$, is true if one or other (or maybe both) of a and b are true, and false otherwise. We can record this in the truth conditions for disjunction:

> $a \vee b$ has the value T just if at least one of a and b has the value T.
> $a \vee b$ has the value F just if both of a and b have the value F.

These conditions can be depicted in the following truth table:

a	b	$a \vee b$
T	T	T
T	F	T
F	T	T
F	F	F

Each row – except the first, which is the header – now records a possible combination of the values for a (first column) and b (second column). There are four such possible combinations, and so four rows. For each combination, the corresponding value of $a \vee b$ is given to its right (third column).

Again, while we are about it, what is the connection between the truth values of a and b, and that of a & b? A natural assumption is that a & b is true if both a and b are true, and false otherwise. Thus, for example, 'John is 35 and has brown hair' is true just if 'John is 35' and 'John has brown hair' are both true. We can record this in the truth conditions for conjunction:

> a & b has the value T just if both of a and b have the value T.
> a & b has the value F just if at least one of a and b has the value F.

These conditions can be depicted in the following truth table:

a	b	$a \& b$
T	T	T
T	F	F
F	T	F
F	F	F

Now, how does all this bear on the problem we started with? Let us come back to the question I raised at the end of the last chapter: what is a situation? A natural thought is that whatever a situation is, it determines a truth value for every sentence. So, for example, in one particular situation, it might be true that the Queen is rich and false that pigs can fly. In another it might be false that the Queen is rich, and true that pigs can fly. (Note that these situations may be purely hypothetical!) In other words, a situation determines each relevant sentence to be either T or F. The relevant sentences here do not contain any occurrences of 'and', 'or' or 'not'. Given the basic information about a situation, we can use truth tables to work out the truth values of the sentences that do.

For example, suppose we have the following situation:

$p : T$
$q : F$
$r : T$

(r might be the sentence 'Rhubarb is nutritious', and '$p : T$' means that p is assigned the truth value T, etc.) What is the truth value of, say, $p \& (\neg r \vee q)$? We work out the truth value of this in exactly the same way that we would work out the numerical value of $3 \times (-6 + 2)$ using tables for multiplication and addition. The truth value of r is T. So the truth table for $\neg$ tells us that the truth value of $\neg r$ is *F*. But since the value of *q* is *F*, the truth table for $\vee$ tells us that the value of $\neg r \vee q$ is *F*. And since the truth value of *p* is *T*, the truth table for & tells us that the value of $p \,\&\, (\neg r \vee q)$ is *F*. In this step-by-step way, we can work out the truth value of any formula containing occurrences of &, $\vee$, and $\neg$.

Now, recall from the last chapter that an inference is valid provided that there is no situation which makes all the premisses true, and the conclusion untrue (false). That is, it is valid if there is no way of assigning *T*s and *F*s to the relevant sentences, which results in all the premisses having the value *T* and the conclusion having the value *F*. Consider, for example, the inference that we have already met, $q / q \vee p$. (I write this on a single line to save Oxford University Press money.) The relevant sentences are *q* and *p*. There are four combinations of truth values, and for each of these we can work out the truth values for the premiss and conclusion. We can represent the result as follows:

q	*p*	*q*	$q \vee p$
T	*T*	*T*	*T*
T	*F*	*T*	*T*
F	*T*	*F*	*T*
F	*F*	*F*	*F*

The first two columns give us all the possible combinations of truth values for *q* and *p*. The last two columns give us the corresponding truth values for the premiss and the conclusion. The third column is the same as the first. This is an accident of this example, due to the fact that, in this particular case, the premiss happens to be one of the relevant sentences. The fourth column can be read off from the truth table for disjunction. Given this information, we can see that the inference is valid. For there is no row where the premiss, *q*, is true and the conclusion, $q \vee p$, is not.

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