Man vs Mathematics

Timothy Revell has a masters in mathematics and a PhD in computer science, but despite this he still struggles with long division. He has worked with BBC *Horizon*, *The Naked Scientists* radio show – one of the world's most popular science shows, and writes for *New Scientist*. Timothy learnt his trade running an award-winning science communication group in Glasgow called theGIST, producing articles, videos and podcasts. He loves mathematics and thinks that everybody else should too.

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Mathematics can quickly get out of hand. Often you start with something as simple as thinking about shuffling a pack of cards and then all of a sudden you've worked out a conclusion that is just a little mind-blowing.

With fifty-two cards in a standard deck there are many different possible shuffles. In fact, there are so many different ways that any time you give a pack a decent mix it's likely that it will end up in an arrangement that has never been seen before. Never.

After a shuffle there are fifty-two possible cards that could end up being at the top of the deck, all of which are equally likely. Once one has been chosen there are fifty-one possibilities for the second card in the deck, fifty for third position and so on. This means that the total number of different ways to shuffle a pack of cards is found by nothing more than a quick multiplication.

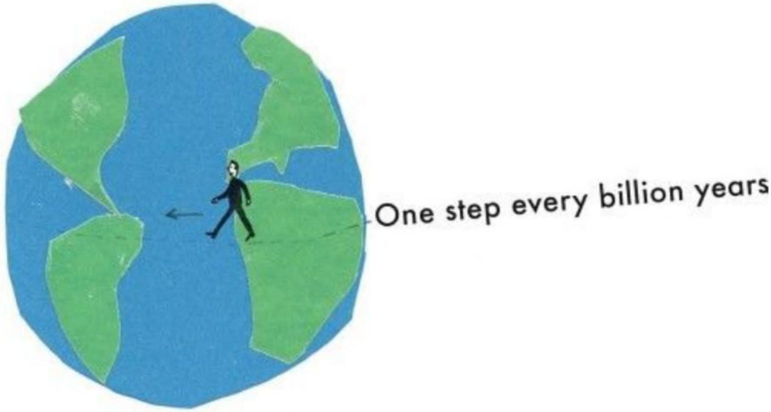
$$\text{Number of ways to shuffle a pack of cards} = 52 \times 51 \times \ldots \times 3 \times 2 \times 1$$

Which written out is, 80658175170943878571660636856403766975289505440883277824000000000000, but normally to save paper we write this as '52!' and pronounce it 'fifty-two factorial'. Using just a little bit of mathematics we've managed to work out the number of different possible shuffles, but comprehending just how many there are is another task altogether.

52! is so big that if every person who has ever lived had been shuffling a pack of cards once every second since the start of the universe, we still wouldn't expect to have seen any repeats yet. We wouldn't even be close.

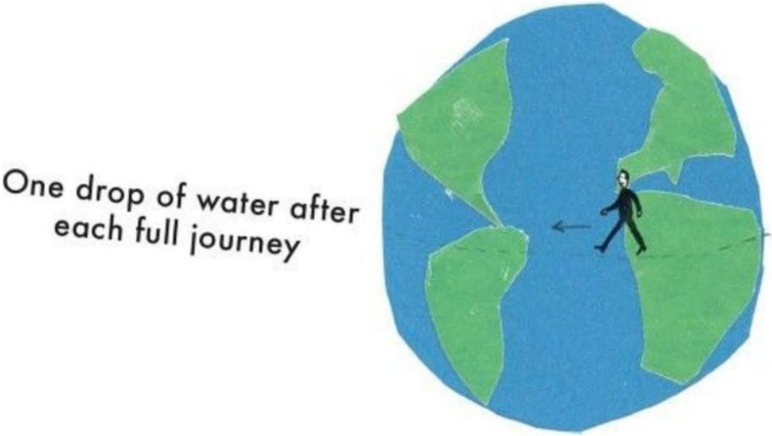
In an attempt to really understand the monstrously big 52! data analyst Scott Czepiel's decided to play a little game. He begins by starting a timer that counts down from 52! seconds all the way down to zero, with the aim of the game being to keep himself amused while he waits for the alarm to go off.

Czepiel starts his wait at the equator, staying in the same spot for a billion years before eventually taking a step forward. He then waits another billion years before taking a second step. Billion years. Step. Billion years. Step. Billion years. Step ...



Eventually Czepiel makes it the whole way round the equator, ending up back where he started, and wonders, 'How much time do I have left?' So he takes another look at the clock. But the timer has hardly moved at all. It seems that time doesn't fly when you're walking the equator at one step per billion years.

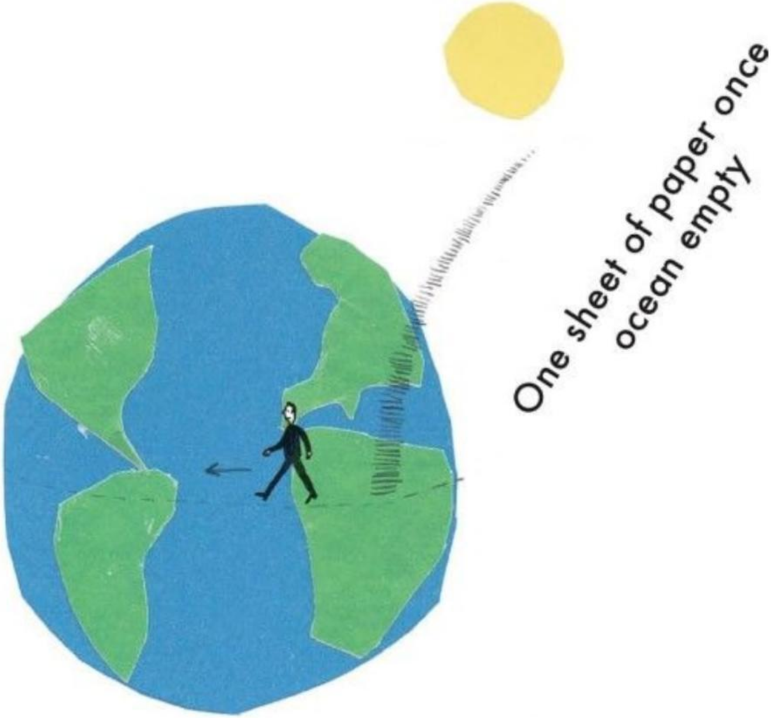
As Czepiel still has plenty of time left, he decides to remove one single drop of water from the Pacific Ocean, before walking round the equator again at the same pace and removing a drop of water from the Pacific whenever he completes a round trip.



Round trip. Remove drop. Round trip. Remove drop. Czepiel continues relentlessly until eventually all the water from the Pacific Ocean has been removed. Czepiel is feeling pretty proud of all of his hard work and so decides to check the clock again, but it's *still* hardly changed at all. He hasn't even dented the first three digits (806 ...). Taking a deep breath, Czepiel decides to place a single piece of paper on the ground and refill the ocean.

He then takes another step-every-billion-years walk, emptying the Pacific Ocean drop by drop after every round trip, and placing a piece of paper on the pile whenever he empties the ocean, before refilling it and starting again.

Czepiel eventually has so many pieces of paper in his pile that it reaches all the way to the sun, and he wonders, 'How much time do I have left now? Surely the timer has run out?' But, nope. The first three digits still haven't even moved. 52! seconds really is a very long time.



Czepiel recovers his energy and repeats this whole process 1,000 more times, walking round the earth, emptying the ocean and adding sheets of paper, until he eventually has 1,000 piles each large enough to reach the sun. Then he allows himself to have one more look at the clock.

Czepiel exhales in relief, as he is now a third of the way there. Two more of everything and Czepiel will have finally used up all 52! seconds.

What is mathematics?

Working out the number of possible card shuffles was pretty easy using a little bit of mathematics, but getting an intuitive understanding of the scale was almost impossible, making the conclusions a little surprising. This often happens when working with mathematics; rigorous logical arguments lead to unexpected conclusions.

If when solving a problem we follow only our intuition and gut feelings then we might arrive at the right answer, but just as often we might not. The only real way to find certainty is through mathematics, and this is what makes it so useful.

There are a lot of different definitions about what mathematics really is, but my favourite comes from the incredible YouTuber and self-described 'recreational mathemusician' Vi Hart who says, 'mathematics is about making up rules and seeing what happens'. There is no one true mathematics, there are just the rules that we make up and the games that we play.

The sheer number of different possible theorems and proofs in mathematics is far greater than 52! In fact there is an infinite number. For mathematicians this is great news as it means that there is no shortage of new mathematics and there never will be. If everyone on the planet decided to become a mathematician there would still be plenty of theorems to go round. With such abundance, the job of the mathematician is not just to prove theorems but to work out which ones are actually interesting or useful and then find out if they are true.

At the moment, there are more mathematicians, producing more new mathematics than ever before. It's true that we still use much of the mathematics that was discovered thousands of years ago, but that doesn't mean we are done. We are not even close.

From cities to computer viruses, from matchmaking to movies and sport, mathematics can explain and illuminate. Why adding a road can make traffic worse; why your friends have more friends than you; and why the universe prefers one number over the rest – it all comes down to mathematics. And all of this will be explored in this book.

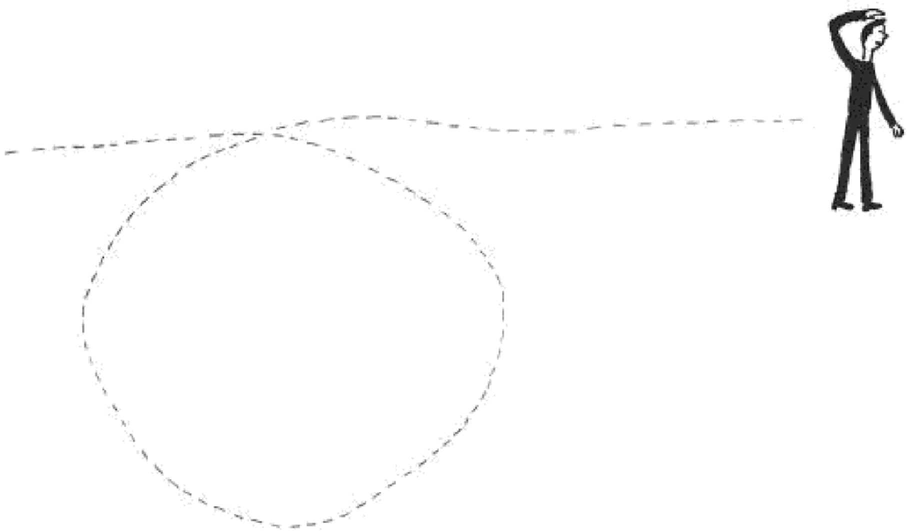
You vs Mathematics

For so many subjects, ranging from astronomy to zoology, mathematics is the most versatile tool, playing a role in nearly every major discovery. But for some reason mathematics has a bit of a bad reputation.

Yes, mathematics can be hard, but that doesn't mean understanding the results and what makes them great has to be complicated. Is grammar hard? Sure. But you don't need to know the difference between present perfect and future perfect to understand why a sentence is useful, interesting or beautiful. And the same goes for mathematics.

It doesn't take a PhD to understand that if someone finds a nineteenth-century ship filled with gold by using an equation, then that equation must be pretty cool. And this is how this book will read. We'll focus on the ideas rather than the details and follow the stories rather than the subtleties.

If at any point you get lost in a mathsplanation, don't panic. All of the mathsy parts of this book are just to give a flavour of what's going on and are not crucial to the overall flow. None of the mathsplanations need much technical expertise. Just give them a go. I'm sure you'll find something useful ...



It's 3 September 1857. The SS *Central America* has just set sail from the port of Colón in Panama after a brief stay and is on its way to New York City. The ship is filled with millions of dollars' worth of gold straight out of the California gold rush. It's a beautiful day and a great time of year for sailing.

After nearly a week at sea, SS *Central America* is making good progress and is now heading up the east coast of the US, when, all of a sudden, the weather starts to turn. A storm is brewing and there is nothing that SS *Central America* can do about it other than just try to ride it out.

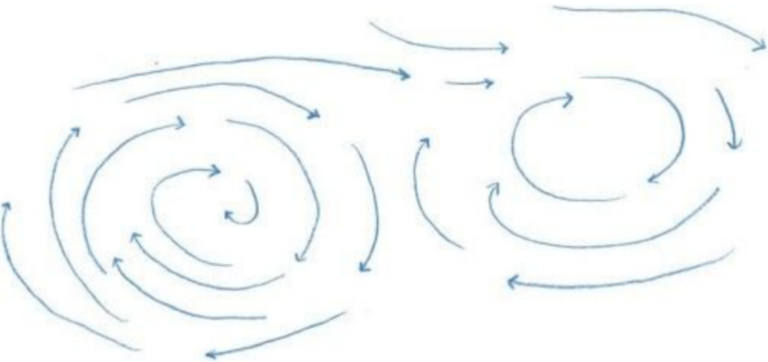
As the hours pass, the storm turns into a hurricane. The wind picks up so much speed that it splits the ship's sails wide open. Waves smash into its wooden hull and water surges onto the deck, breaching the interior. The engine becomes so waterlogged that it no longer works, leaving SS *Central America* completely powerless.

After three days of being pulled along in the eye of the storm and pummelled from either side by the wind and the waves, the ship can't take any more. Captain William Lewis Herndon knows it is only a matter of time before his ship will suffer a fatal blow that will sink it and so orders the flag to be flown upside down to signal distress and emergency flares to be fired into the sky. Miraculously, two ships eventually see the signal and come to help, but too much damage has already been done.

On the evening of 12 September 1857 one wave too many clatters into SS *Central America*, bursting open the ship's timbers. The wood growls before shattering into hundreds of pieces, water surges into the ship and begins dragging it below the surface.

Around a quarter of the ship's passengers were rescued, but the rest, including the captain, tragically went down with the ship. SS *Central America*, battered, bruised and ultimately beaten, sank to the bottom of the ocean, thousands of metres below the surface, off the coast of North Carolina.

The shock of the loss of life filled the newspapers of the day and was one of the worst American sea tragedies in history. But as time went by people started to wonder about the huge quantity of gold hidden within the remains of the ship on the ocean floor. In the years that followed, many people would dream of finding the SS *Central America* and claiming its golden treasures. Its notoriety grew and grew, as the quest of finding the gold became like an old pirate's tale, but nobody ever managed to track it down. It seemed that the SS *Central America* would be lost forever, until, that is, modern mathematics got involved.



The mathematics of searching for stuff, brought to you by Larry Stone

'I'm surprised that they didn't throw me overboard,' says Stone. 'I was this twenty-five-year-old kid and I was telling these navy captains where to search, and they weren't too happy about it.' The search for USS *Scorpion* was pretty difficult and it took a long time to eventually find the vessel but Stone loved every minute of it.

After USS *Scorpion*, Stone became an expert in searching for stuff with mathematics by pioneering the field known as Bayesian search theory. The field focuses around a mathematical result called Bayes' theorem, which allows mathematicians to weigh up different pieces of evidence based on their strength and then rigorously work out the probability that something is true. This could be anything from a computer calculating the probability that a particular email is spam (using pieces of evidence such as 'dollar signs in subject heading' and 'sent by foreign prince') to whether an online shopper would like to purchase this very book (using evidence like 'user bought *Principia Mathematica*' and 'user bought *Twilight*'). And in Bayesian search theory, Bayes' theorem is used to help find missing objects, including lost treasure.

In the 1980s, an American engineer named Tommy Thompson was keen to start a fresh search for SS *Central America*. Thompson was as charismatic as he was clever and by the summer of 1987 he had charmed 161 different investors into funding his hunt for the long-lost ship.

Taking your likes, dislikes and personal preferences into account, surely love could be the only outcome.

Operation Match had some success but faded out after a few years. The project was ultimately a little too far ahead of its time. But fast-forward fifty years to today and the industry really is serious. Billions of pounds a year serious.

With the addition of the internet and the skyrocketing of cheap computing power, mathematical matchmaking has become massive, and it is estimated that there are currently ninety-one million people signed up to online dating services. This has meant overcoming a major issue: computers are logical, straightforward, black-and-white thinkers, but finding love is often surprising, irrational and fifty shades of pink. The solution, as always, is mathematics.

To even consider using a computer, determining the compatibility of two people needs to be distilled into a single mathematical recipe – an algorithm. Given an algorithm a computer can then follow the steps to work out which users might have a good date and which ones might not.

Many of the dating site algorithms are hidden behind closed doors as trade secrets, but Christian Rudder, a mathematician and founder of the online dating site OKCupid, gave a great TED-Ed talk where he outlined some of the basic elements. These basic elements give a simple approach to the problem that allows us to understand the mathematics involved, which can then be tweaked over and over again to make it ever more complicated, and ever more accurate. But, as with many good ideas, the basic elements really are simple – first ask love seekers some questions and then compare their answers.

Just as with Operation Match, users begin by answering a questionnaire. For some questions two people are more compatible if they agree, like 'Do you like sci-fi films?', or 'Would you like to get married?', but for other questions it's not so easy. A question like 'Do you like to be in charge?' is a complicated one. If two people in a relationship both like to be in charge that could cause serious problems. And so to ensure the right level of wanting-to-be-in-chargeness when suggesting matches, the algorithm needs to know both the users' answer and which answer they would like a potential partner to give.

But this still isn't quite enough information to determine a good romantic match. Suppose you have answered that you like sci-fi films and would like to get married, and that you would like a partner to agree with both. But you are quite happy to watch sci-fi films with your friends or alone so you don't mind if your partner doesn't like the genre, but marriage is a must. So for every question the algorithm needs to know your answer and the answer you would most like a potential partner to give, along with how important it is to you.

To actually compute whether you and another person are a match, numbers need to be assigned to each of your answers. If a question is *unimportant* to you it scores 0 points, if it is *somewhat important* then it scores 10, if it's *very important* it scores 50, and if it is *mandatory* then it scores a whopping 250 points. That way it's incredibly hard to be a match with someone if you disagree on a crucial issue.

> Three parts to a question:
>
> *1. What's your answer to the question? Yes or No.*
>
> *2. How would you like a potential partner to answer the question? Yes or No.*
>
> *3. How important is this question to you? Unimportant/Somewhat important/Very important/Mandatory. (Please choose one.)*

Let's see how this works. Meet Al. He's a man who hates sci-fi and it's *somewhat important* to him that a partner would too. He would also like to get married and he would like that to be to his partner, so they would have to want to get married as well, and that's *very important* to him.

The compatibility of you with Al is worth a total of 60 points, 10 for sci-fi since it's *somewhat important* to him and 50 for marriage since it's *very important*. Unfortunately, you score 0 out of 10 for sci-fi since you're a fan, but you do agree on marriage so you get the full 50 points, meaning that your compatibility with Al is 50 points out of a possible 60, or 83%.

So far so good, but this is only half of the story. We still need to know how compatible Al is with you. Let's pretend that you like sci-fi, although it's *unimportant* that a partner would, too, but getting married is *mandatory*. This means that Al's compatibility with you is worth a total of 250 points, 0 for sci-fi and 250 for marriage.

Al hates sci-fi, but that doesn't matter as there are 0 points available for the first question, but as he's keen to get married he gets the full 250 points for marriage, making his compatibility to you 250, or 100%. Things are looking pretty good.

We know that you have a compatibility score with Al of 83% and Al has a compatibility score with you of 100%, and so by combining these two scores we can calculate a mutual compatibility score. One way of doing this would be simply to take the average of the two numbers (add them together and then divide by two), which would give you a mutual compatibility score of 92%. This would make the two of you a pretty good match, and we could stop there. But using the average is just soooo old school, dude. And also not quite the best thing to do.

If you have two people who are both a 50% match with each other, and if you have two other people who have compatibility scores of 0% and 100%, by using the average both pairs would have the *same* mutual compatibility score of 50%.* But love *is mutual*. Two 50%-ers should be a better match for each other than a 0%-er and a 100%-er. The 50%-ers are in the 'we seem to like some of the same stuff, shall we give it a try?' category. But the 0%-er and the 100%-er are in the 'I love them so much and want them to marry me, but they don't even know who I am' category. It's an important category, but our algorithm really shouldn't put these two people together. This is the restraining orders category. So instead we look to the two-number combination method *du jour*, the geometric mean.

To compute the geometric mean we multiply our two scores together, then take the square root (that's the symbol √ on your calculator, folks). This means that the 50%-ers receive a mutual compatibility score of 50%, but pairings in the restraining order category receive a mutual compatibility score of 0%, as they should. For Al and you, the geometric mean gives a new mutual compatibility score of 91%, now accounting for the fact that you are slightly more compatible with Al than he is with you. Cupid's algorithmic love arrow strikes again.

$$\sqrt{(50 \times 50)} = 50\%$$

Maybe	Maybe	Maybe Together

$$\sqrt{(100 \times 0)} = 0\%$$

Hell yes!	Hell no!	No chance!

By repeating this process for more and more questions a more accurate compatibility score can be generated, allowing algorithms like this to have a huge impact on dating worldwide.

Why is being ‘reasonably attractive’ no good for dating?

With the huge amounts of data that we now have on singles the dating problem should be completely solved. Every person looking for a date should be paired with the perfect partner immediately using various algorithms. Within weeks new couples should have declared their everlasting devotion to each other, started finishing each other’s sentences, got matching tattoos, and, perhaps the clearest sign of their newfound love, created a couples playlist. But, sadly, it just doesn’t seem to work this way. The problem is not with the mathematics, though, it’s with the matchmaking questions. And the answers.

Matchmaking mathematics can give you a chance to specify some of the qualities that you are looking for in a partner, but it can’t possibly ask you every possible question. That would be impossible. Instead, love seekers can only portray certain aspects of their personalities and those parts will be what determine a good match or not. Algorithms simply cannot know everything about you.

To get around this problem many dating sites allow love seekers to create profiles. The profiles can display any information you like. This means that if you are desperate for a similarly enthusiastic model railway fanatic, you can include it in your profile, even though there wasn’t a chance to mention it in the questionnaire. And often not only can you write about your lust for locomotives, you can include pictures of you and your best train set. Everyone is a winner. Unless, that is, nobody is interested.

In an ideal world your dating profile would appeal to a huge variety of people, all of whom would be vying for your attention. You could then pick one of these people based on their

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