MATHEMATICA® BEYOND MATHEMATICS

THE WOLFRAM LANGUAGE IN THE REAL WORLD

José Guillermo Sánchez León

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Preface

About *Mathematica* and the *Wolfram Language*

If you have used *Mathematica* occasionally or heard of it, you may have the false impression that it is a program for performing complicated calculations, usually for academic purposes. However, this idea is far from the truth. Actually, *Mathematica* is much more than that. By putting together the computational power and ease of use of the Wolfram Language, *Mathematica*'s high-level general-purpose programming language, the program can be used in any scientific or technical field: Aerospace engineering, environmental sciences, financial risk management, medical imaging and many others. (You can get an idea by visiting: http://www.wolfram.com/solutions).

Mathematica can be considered a tool that empowers non-professional programmers to develop applications although if you are a professional programmer, you will see that the software provides a development environment similar to the ones available for C++ or FORTRAN. You can even use the program to control external devices.

About the Book

Although many books have been written about *Mathematica*, very few of them cover the new functionality added to the most recent versions of the program. This text introduces the new features using real-world examples, based on the experience of the author as a consultant. In the process, you will also learn more about the Wolfram Language and how you can use it to solve a wide variety of problems. Both are the most important objectives of the book. To accomplish that, the author raises questions from a wide range of topics and answers them by taking full advantage of *Mathematica's* latest features. For example: What is the hidden image in "The Ambassadors" painting by Holbein? What sources of energy does the world really use? How can we calculate tolerance limits in manufacturing processes? Are our cities getting warmer? Is the novel "El Quijote" written in Pi? How do we know how old our planet is? How can we find planets outside our solar system? How big is our galaxy? And the universe? How do we know it? How can we model the distribution of radioactive isotopes in the human body? And a tsunami? What are and how can we create Mandelbrot fractals? How can we measure the genetic distance between species? How can we perform financial calculations in real time? How do we value financial derivatives? How can we make entertaining simulations for teaching mathematics, physics, statistics, ... ? Why are there no free quarks?

The answers to the previous questions will not only help you master *Mathematica*, but also to become more familiar with the corresponding topics themselves.

About the Book Objectives

This book will not only be useful to newcomers but also to those familiar with the program and interested in learning more about the new functionality included in the latest versions.

Those readers with minimal or no knowledge of *Mathematica* are strongly advised to read chapter 1 along with Stephen Wolfram's book *An Elementary Introduction to the Wolfram Language* available from within the program documentation: **Help** ▸ **Wolfram Documentation** ▸ **Introductory Book ≫**

This text will also make it easy to start programming using the Wolfram Language and to learn how to take full advantage of its capabilities.

The final objective of *Mathematica Beyond Mathematics* is to help you avoid feeling overwhelmed by the software's vast capabilities. The author has explored a significant part of them choosing the most relevant parts and illustrating them with examples from many different sources including the program documentation. Links to additional resources will also be provided. The main aim of all this is to reduce significantly the amount of time required to master the tool. The commands used will be explained using short sentences and simple examples.

About the Use of the Book

Although this book is not a manual, inexperienced readers should at least read the first four chapters in consecutive order to gain a solid understanding of how *Mathematica* works. Regarding the rest of the chapters, you should keep in mind the following: a) Chapter 5 covers the most innovative features of the program such as how to access data from many different fields and how to use natural language to interact with the software; b) Chapter 6 deals with relatively advanced probability and statistics topics; c) Chapters 7 to 11 explore topics related to a single area of knowledge (mathematics, astronomy, nuclear physics, biokinetic modeling and economics and finance). You can read them according to your preferences; and d) Chapter 12 is for those readers facing problem requiring big computational resources (parallel computing, grid-enabled calculations, etc.) and/or the ones interested in other programs related to *Mathematica* such as web*Mathematica* for adding computational capabilities to websites or Workbench, a complete software development environment using the Wolfram Language.

The principal theme of each chapter is used as the motivation to illustrate certain features of *Mathematica* that you may find useful for solving a great variety of problems. For example: Chapter 6 teaches you how to build a *Mathematica* package; in Chapter 8, related to astronomy, you learn how to create dynamic images; Chapter 10, covering the modeling of biological systems, discusses the resolution of differential equations in *Mathematica* using concrete examples.

About the First English Edition and the Supplementary Materials

Everything shown in the text has been done using *Mathematica*, including the access to information sources.

The book has been written using *Mathematica* 10 and 11 and edited using *Mathematica* 11, although it should still be useful with future versions of the program as well.

Mathematica Beyond Mathematics is based on *Mathematica más allá de las matemáticas, 2ª Edición* but its contents have been improved and updated. In this endeavor, Ruben Garcia Berasategui, a lecturer in Jakarta

International College, has played a fundamental role, not only by translating the text but also by making insightful comments and suggestions.

The text doesn't display "In[*n*]:=" and "Out[*n*]:=", the default symbols that *Mathematica* adds to Input (where the commands are entered) and Output (where the results are shown) cells.

In a few cases, the book uses specific files available for downloading from the author's website: http://diarium.usal.es/guillermo.

At the end of each chapter there is a section containing additional resources carefully selected that can be accessed most of the time from either *Mathematica* or through the Internet.

The author would appreciate any comments or suggestions. Please send them to: guillermo2046@gmail.com, with the subject: '*Mathematica* Beyond Mathematics'.

Salamanca (Spain), January 2017

About the Author

J. Guillermo Sánchez León (http://diarium.usal.es/guillermo). Engineer, physicist and mathematics PhD holder, is an associate professor in the University of Salamanca and works in the energy industry. He has conducted research in a variety of fields: Modeling, optimization, medical physics, astronomy, finance and others. In 1999 he was awarded a research grant at Wolfram Research Inc. headquarters in Champaign (Illinois, USA) after his statistical applications with *Mathematica* project won a competition sponsored by the company. Since then he has been an active *Mathematica* and web*Mathematica* alpha and beta tester. He's also a Wolfram certified instructor and has extensive experience in teaching and developing programs with *Mathematica* and web*Mathematica*. Some of them are cited as references in *Mathematica's* official web page and are available online in both English and Spanish. Among his more than 100 articles, there are several where *Mathematica* and web*Mathematica* have been used extensively.

About the Translator

Rubén García Berasategui is a math, finance and statistics lecturer in Jakarta International College. He holds a bachelor's degree in business administration and an MBA. He's been using *Mathematica* since 1997 and in 2012 became the first Wolfram certified instructor in Southeast Asia. He has been training hundreds of newcomers to *Mathematica* and sharing his passion for the program with them ever since.

Acknowledgments

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The author would also like to say thank you to Addlink Software Científico (http://www.addlink.es), sponsor of the first Spanish edition; Antonio Molina and Juan Antonio Rubio for their editorial guidance; and J. M. Cascón (USAL), J. López-Fidalgo (UCLM), S. Miranda (Solventis), R. Pappalardo (US), J. M. Rodríguez (USAL) and C. Tejero (USAL) for their comments and corrections.

1

Getting Started

In this chapter we will take a brief tour of Mathematica, a program for modern technical computing based on the Wolfram Language. It is important to read this chapter carefully since its contents are essential to be able to follow the rest of the book. Even if you are already a Mathematica user, you may find the following pages useful to get familiar with the new functionality included in the most recent program releases.

1.1 *Mathematica*, an Integrated Technical Computing System

> *Mathematica* is a software application that goes beyond numeric and symbolic calculations. It is a modern technical computing system based on a high-level general-purpose programming language, the *Wolfram Language*. You will be able to start using it after a few minutes but, if you want to take full advantage of its capabilities, you will have to spend time to master its language although probably less than with other programming languages.

Mathematica took a revolutionary step forward in Version 8 with the introduction of the **free-form linguistic input**, that consists of writing in plain English a request that the program will try to answer. Using this format ensures that newcomers can get started very quickly. For experienced users, it means help in finding the most appropriate functions to perform a desired calculation. To get an idea of *Mathematica*'s capabilities before beginning to use the program, let's take a look at the following examples (for now, you just need to read what follows, later we will show you how to write inputs and run commands):

- To solve an equation such as $3x^2 + 2x - 4 = 0$, you can literally type what you want *Mathematica* to do. The output shows the correct *Mathematica* input syntax and the answer, which in this case includes both, the exact solution and the decimal approximation.

```
Solve 3 x^2 + 2 x – 4 = 0
↳ Results (1 of 2)
  {Reduce[-4 + 2*x + 3*x^2 == 0, x],
   N[Reduce[-4 + 2*x + 3*x^2 == 0, x]]}
```

{x=13(−1−13)∨x=13(13−1),x=−1.53518∨x=0.868517}

- To add quantities in different currencies, just enter them and the result is shown in the currency that was used first; if you execute the command, the result will most likely be different due to changes in the exchange rates from the time the input was first executed.

```
$500 + £307 + 127 € + 1520 ¥
$500. + £307. + €127. + ¥1520
```

$1061.78

- We can even ask very specific questions: To learn more about the temperature in a location, just write down the name of the place followed by "temperature". The most important advantage is that the information from the answer can be used inside the *Mathematica* environment.

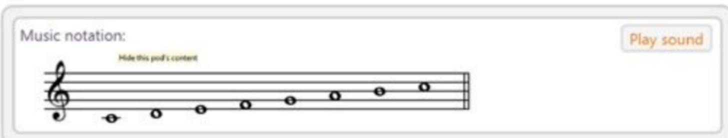
```
Salamanca temperature
AirTemperatureData[ Salamanca (city) ]
```

17.3°C

The *Mathematica* free-form input is integrated with Wolfram|Alpha, http://www.wolframalpha.com, a type of search engine that defines itself as a computational knowledge engine and that instead of returning links like Google or Bing, provides an answer to a given question in English. If you want, you can access Wolfram|Alpha directly from within *Mathematica*, and we will show you how to do that later on.

- To represent the Do major music scale, you just need to write:

```
Do major
```



Another example of the new functionality recently incorporated in *Mathematica* is the possibility of accessing scientific and technical databases through the Internet from different fields already formatted by Wolfram Research for their direct manipulation inside the application.

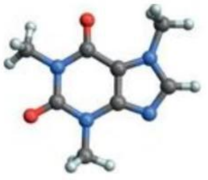
- In this example, we obtain the value of the Euro in US dollars after executing the command.

```
FinancialData ["EUR /USD"]
```

```
1.0661
```

- We can choose a chemical compound and get many of its properties. In this case, we get the plot of the caffeine molecule.

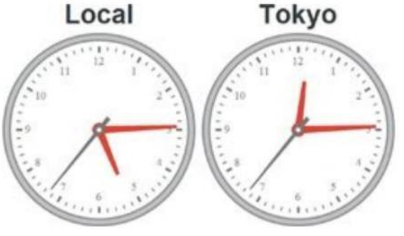
```
ChemicalData ["Caffeine", "MoleculePlot"]
```



With very brief syntax we can build functions that in other programming languages would require several lines of code.

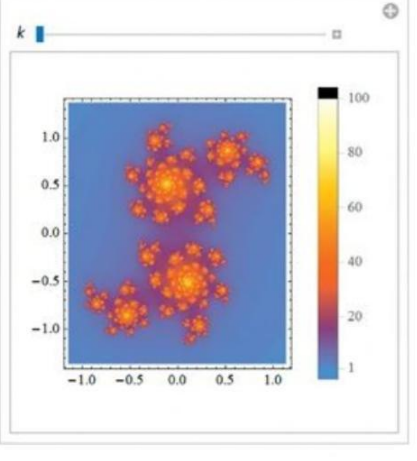
- With just two lines of commands we can generate a dynamic clock that is synchronized with the time in our computer and in a different time zone.

```
Dynamic [Refresh [Row[
   {ClockGauge[AbsoluteTime[], PlotLabel -> Style["Local", Large,
Bold]],
    ClockGauge[AbsoluteTime[TimeZone] → +9],
     PlotLabel -> Style["Tokyo", Large, Bold]]}], (UpdateInterval ->
1)]]
```



- In less than half a line we can write the necessary instructions to make an interactive model of the Julia fractal set.

```
Manipulate[JuliaSetPlot [0.365 - k ⅈ,
  PlotLegends → Automatic, ImageSize → Small], {k, 0.4, 0.5}]
```

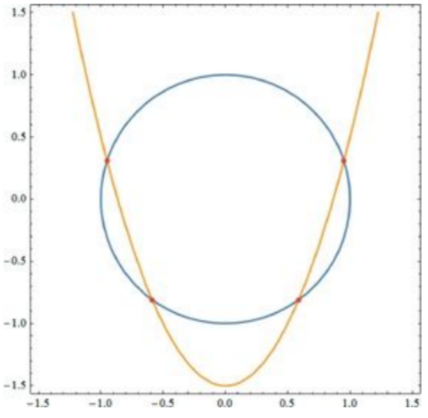


- We find the intersection points of a circumference and a parabola and graphically represent them. To write special symbols such as "∧", the best thing to do is to use Palettes, which we will discuss later on.

pts = Slove[x2+y2=1∧y−2x2+32=0,{x,y}]{{x→−1212(5−5),y→14(−1−5)},
{x→1212(5−5),y→14(−1−5)}, {x→−1212(5+5),y→14(5−1)},

{x→1212(5+5),y→14(5−1)}}

```
Show[
 {ContourPlot[{x2+y2=1,y-2x2+32=0}, {x, -1.5, 1.5}, {y, -1.5, 1.5}],
  Graphics[{Red, PointSize[Medium], Point[{x, y} /. pts]}]}]
```



Apart from the examples given above showcasing some of *Mathematica*'s new functionality, it is also worth mentioning: (i) The *Wolfram Predictive Interface*, that makes suggestions about how to proceed after executing a command, (ii) The context-sensitive input assistant to help us choose the correct function, (iii) The possibility of running the program from the browser (WolframCloud.com), including specific cloud-computing related commands, (iv) The machine learning capabilities that, for example, enable us to analyze a handwritten text and keep on improving its translation through repetition, (v) Very powerful functions for graphics and image processing. We will refer to these and other new capabilities later on.

1.2 First Steps

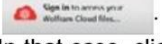
The program can be run locally or in the cloud accessing the **Wolfram Cloud** (http://www.wolframcloud.com/) with a browser. From here on, except mentioned otherwise, it will be assumed that you have installed *Mathematica* locally and activated its license. When installing it, an assistant will guide you through the activation process. If this is the first time, it will take you to the user portal (https://user.wolfram.com), where you will have to sign up. The website will ask you to create a Wolfram ID, use your email address, and your own password. It will also automatically generate an activation key. Remember your Wolfram ID and password because they will be useful later on if, for example, you would like to install the program in a different computer or have an expired license. In both cases, you will need a new activation key. The Wolfram ID and password will also be necessary to access the Wolfram Cloud, and we will give a specific example at the end of the chapter.

To start the program in Windows, under the programs menu in your system, click on the *Mathematica* icon. Normally there are two icons; choose *Mathematica* and not *Mathematica* Kernel.

Under OS X, the program is located in the applications folder and you can launch it either locating it with Finder, using the Dock or from the launchpad (OS X Lion or more recent versions).

By default, a welcome screen (Figure 1.1) similar to this one will appear:

Figure 1.1 The Welcome Screen in *Mathematica* 11.

This screen contains several external links (hyperlinks). For example, by clicking on **Resources**, you will be able to access a broad collection of reference materials about how to use *Mathematica*, including videos: http://www.wolfram.com/broadcast/. You can always go back to this screen from the menu bar: **Help ▸ Welcome Screen...** . In the upper left-hand corner (in *Mathematica* 10/11) you will see this icon:  . If you are a registered user, chances are you already have a Wolfram ID. In that case, click on the icon and sign in. Once the process has been completed, the icon will be replaced with the name that you used when you signed up, as shown in the previous image. The advantage of this is to gain access from within *Mathematica* to your files in the cloud (WolframCloud) but for now, you can skip this step if desired. If you click on `NewDocument`, below the icon shown at the beginning of the paragraph, a new blank *Notebook* will be created. Notebooks are the fundamental way to interact with *Mathematica*. You can also create them by clicking on **File ▸ New ▸** Notebook.nb.

A *notebook* is initially a blank page. We can write on it the text and instructions that we want. It is always a good idea to save the file before we start working on it and, as in most programs, this can be done in the menu bar: **File ▸ Save As ▸** "Name of the File" (*Mathematica* by default will assign it the extension .nb).

It is always convenient to have access to the toolbar from the beginning of the session; to do that, select in the menu bar **Window** and check **Toolbar ▸ Formatting** (or **Show Toolbar** in Version 9 and earlier). Please note the drop-down box to the left of the toolbar (Figure 1.2), that we'll refer to as: *style box*. It will be useful later on.

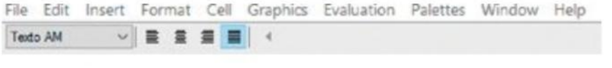


Figure 1.2 *Mathematica*'s Formatting Toolbar.

Once you begin to type, a new cell will be created. It will be identified by a blue right-bracket square (]) ("cell marker") that appears on the right side of the *notebook*. Each cell constitutes a unit with its own properties (format, evaluable cell, etc.). To see the cell type, place the cursor inside or on the cell marker. The type will appear in the *style box* (Figure, Input, Text, Title, Program...). We can also change the format directly inside the box. Type in a text cell and try to change its style.

```
This is a cell formatted using the Program style.
```

When a blank *notebook* is created, it has a style (cell types, fonts, etc.) that assigns to each type of cell certain properties (evaluable, editable, etc). The first time a *notebook* is created, *Mathematica* assigns a style named *Default* (by default). Later on we will see how to choose different styles. In the *Default* style, when a new cell is created, it will be of the *Input* type, which is the one used normally for calculations. When we evaluate an *Input* type cell, "In[*n*]:= *our calculation*" will be shown and a new cell of type *Output* will be generated where you will see the calculation result in the form of "Out[*n*]:= *result*" (*n* showing the evaluation order). However, in this book we sometimes use an option to omit the symbols "In[*n*]:=" and "Out[*n*]:=". These and other options to customize the program can be accessed through **Edit ▸ Preferences ...**.

> To execute an *Input* cell select the cell (placing the cursor on the cell marker) and press SHIFT + ENTER or ENTER on the numeric keypad. You can also access the contextual menu by right-clicking with the mouse and selecting "Evaluate Cell".

```
2 + 2
```

4

Figure 1.3 The Suggestions Bar.

Since *Mathematica* 9, when a cell is executed, a toolbar appears below its output as shown above (Figure 1.3). This bar, named the **Suggestions Bar**, provides immediate access to possible next steps optimized for your results. For example, If you click on `range` you will see that a new input is being generated and a new suggestions bar will appear right below the result:

```
Range [4]
```

{1, 2, 3, 4}

The **Suggestions Bar** is part of the predictive interface. It tries to guide you with suggestions to simplify entries and ideas for further calculations. It is one more step to reduce the time it takes to learn how to use the program.

All written instructions inside a cell will be executed sequentially. If at the end of an instruction we type semicolon ";", the instruction will be executed but its output will not be shown. This is useful to hide intermediate results. In an evaluation cell (*Input*) we can include one or more comments, writing them in the following format: (* comment *). Nevertheless, it is recommended to include the comments in separate cells in text format. Remember that you can do that by choosing *Text* in the *style box*.

To create a new cell we just need to place the cursor below the cell where we are and then start writing. We can also create a new cell where the cursor is located by pressing ALT + ENTER.

To facilitate writing, *Mathematica* provides **Palettes**. These can be loaded by clicking on **Palettes** in the menu bar. There are several palettes to make it easier to type mathematical symbols such as the **Basic Math Assistant** palette or **Other ▸ Basic Math Input**. It would be useful from now on to keep one of them open.

Alternatively, you can use keyboard shortcuts to write special symbols, indexes, subindexes, etc. The most useful ones are: subindex CTRL + _, superindex CTRL + ^ or CTRL + 6, fraction CTRL + / and the square root symbol CTRL + 2. Depending on your keyboard configuration, to use some of the previous symbols you may have to press first + SHIFT (key CAPS) for example: superindex CTRL + SHIFT + ^.

- Using a palette or the keyboard shortcuts write and execute the following expression. Please remember: an empty space is the equivalent of a multiplication symbol: that is, instead of 4×5 you can write 4 5.

  4×55+4−32−3

The previous *Input* cell was written in the standard format (*StandardForm*). It is the one used by default in *Input* cells. To check it, position the cursor on the cell marker and in the menu bar choose: **Cell ▸ Convert To ▸**. *StandardForm* will appear with a check mark in front of it.

Expressions can also be written without the need for special symbols by using the form: *InputForm*. It is very similar to the one used by other programming languages such as C, FORTRAN or BASIC. For instance: multiplication = " * ", division = " / ", addition = " + ", subtraction " = ", exponentiation =" ^ ", and square root of a = "`Sqrt [a]`". We can replace the multiplication symbol by an empty space. It is very important to keep that in mind since *Mathematica* interprets differently "`a2`" and "`a  2`": "`a2`" is a unique symbol and "`a  2`" is `a*2`.

- In *InputForm* the previous expression can be written as:

```
(4*5)/5 + Sqrt[4] - 3^2
```

  −3

Initially, it was its symbolic calculation capabilities that made *Mathematica* popular in the academic world.

- As an example, write and execute the following in the standard form in an input cell (use a palette to help you enter the content).

  ∂x,y Sin [x y]cos(xy)2y−12x sin(xy)

A customized style, to which we will refer later, has been defined in this notebook so that the outputs are shown in a format that corresponds to traditional mathematical notation. In *Mathematica* it is called *TraditionalForm*. We could have defined a style so that the inputs also use the traditional notation as well, but in practice it is not recommended. However, you can convert a cell to the *TraditionalForm* style anytime you want.

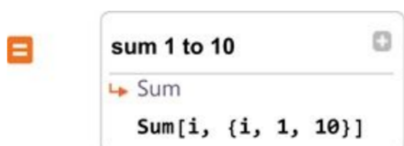
- Copy the *input* cell from above to a new cell and in the menu bar select **Cell ▸ Convert To ▸ TraditionalForm**. Notice that the cell bracket appears with a small vertical dashed line. You will be able to use the same menu to convert to other formats.

  ∂2 sin(xy)∂ x ∂ ycos(xy)2y−12x sin(xy)

For Windows, since Windows 7, you can use the **Math Handwriting Input**, available under accessories, to write the entries manually using a tablet or an electronic board.

The **linguistic** or **free-form** format (available since *Mathematica* 8) allows us to write in plain English the operation we want to execute or the problem we want to solve. To start using it press **=** at the beginning of the cell (input type) and you will notice that ⊟ will appear. After that write your desired operation. Your instruction will automatically be converted into standard *Mathematica* notation, and after that the answer will be displayed. Let's take a look at some examples.

- To add from 1 to 10 you can write "sum 1 to 10" or other similar English expressions such as "sum from 1 to 10".



If you follow the previous steps, the entry proposed may be different from the one shown here. The reason behind this is that when using the free-form format, the program connects to Wolfram|Alpha, Wolfram's computation knowledge engine, that is constantly evolving.

Move the cursor over `Sum {i, 1, 10}]` and the message "Replace cell with this input" will appear. If you click on the symbol + located in the upper right-hand corner, additional information about the executed operation will be shown.

If you have received any message related to connection problems, it is because the free-form format and the use of data collections that we will refer to later on, requires that your computer is connected to the Internet. If it is and you still have connectivity problems it may be related to the *firewall* configuration in your network. In that case you may have to configure the *proxy* (see http://support.wolfram.com/kb/12420) and ask for assistance from your network administrator.

> When you don't know the correct function syntax, you can use the free-form format to write what you want to do. *Mathematica* probably will show you the correct syntax immediately, click on it and only the input in the correct notation will remain with the free-form format entry disappearing.

The free-form input format generally works well when it's dealing with brief instructions in English. In practice it is very useful when one doesn't know the specific syntax to perform certain operations in *Mathematica*.

Many more examples can be found in:

http://www.wolfram.com/mathematica/new-in-8/free-form-linguistic-input/basic-examples.html.

It's also very useful to use **Inline Free-form Input**.

- In this example, we assume we don't know the syntax to define an integral. In an input cell we click on **Insert ▸ Inline Free-form Input** and will appear. Inside this box we type `integrate x^2`.

  ```
  Integrate x^2
  ```

  x33

- The entry is automatically converted to the standard syntax in *Mathematica*.

  ```
  Integrate[x^2, x]
  ```

  x33

- By clicking on the above *input* over the symbol, the cell will go back to its free-form format .

  ```
  Integrate x^2
  ```

  x33

- In the following example we have used **Inline Free-form Input** writing **integrate cos x from 0 to x**.

  ```
  Integrate[Cos[x], {x, 0, x}]
  ```

sin(*x*)

- By clicking in the ✓ input is automatically converted to the standard syntax in *Mathematica*. It can be used as a template.

```
Integrate [Cos[x], {x, 0, x}]
```

sin(*x*)

- The command below returns the average acceleration of the gravity *g* on earth at sea level.

```
earth gravity
```

9.80 m/s²

By default the units used are the ones from the international system (SI):

- Here we combine a standard function inside the **Inline Free-form Input** with `QuantityMagnitude` that shows the numeric value, without including the units:

```
QuantityMagnitude[earth gravity]
```

9.80

As we have indicated in the introduction, you can directly access Wolfram|Alpha from *Mathematica*. To do that just press the equal sign on your keyboard twice ("==") at the beginning of an input cell. The symbol will appear, then type what you want.

Pi with 200 decimals

Input interpretation:

π	200 decimal places

Result:

3.141592653589793238462643383279502884197169399375105820974944592307816406·.
286208998628034825342117067982148086513282306647093844609550582231725359·.
408128481117450284102701938521105559644622948954930381 96

WolframAlpha

- Let's go back to one of our introductory examples:

Do major

From the output, not shown, we selected the part that we were interested in ("Music notation") by clicking on the symbol ⊕ and choosing the desired option ("Formatted pod"). A new entry was generated showing only the musical scale.

```
WolframAlpha ["Do major",
 IncludePods → "MusicNotation", AppearanceElements → {"Pods"},
 TimeConstraint → {30, Automatic, Automatic, Automatic}]
```

If you click on **Play sound** you will hear the sound.

The most important difference between using the free-form input notation and the Wolfram|Alpha one, is that in the first case the entry that we write is sent by the program to a Wolfram server that will return the syntax in *Mathematica* and it will be executed locally, while by using the Wolfram|Alpha notation, all the processing is done in the server and our computer only displays the output. If we have *Mathematica* it is better to use the free-form notation to get the *Mathematica* syntax and then we will be able to use the output easily in subsequent calculations.

Wolfram|Alpha is very useful from a browser since we don't need to have the program installed: There are applications -*Apps*- for smart phones and tablets that facilitate its use and *Widgets* developed with it.

Besides all the previously described ways to define the cell type, there is another one that consists of placing the cursor immediately next to the cell to which you want to add another one. You will see "+" below the cell to its left. Click on it and choose the cell type that you want to define: *Input*, , , Text, etc.

1.3 The Help System

If you have doubts about what function to employ, the best approach in many cases is to use the free-form input format and see the proposed syntax. However, if you would like to deepen your knowledge about the program you will have to use its help system.

The help system consists of different modules or guides (**Help ▸ Wolfram Documentation**) from where you can access an specific topic with its corresponding instructions (**guide/...**). Additionally, you will find tutorials and external links. Clicking on any topic will show its contents.

Figure 1.4 The Wolfram Language Documentation Center.

In the screen-shot shown above (Figure 1.4), after clicking on **Data Manipulation & Analysis**

▸ **Importing & Exporting** *Mathematica* displays the window below (Figure 1.5). Notice that besides giving us the functions related to the chosen topic, we can also access tutorials and even external resources such as video presentations.

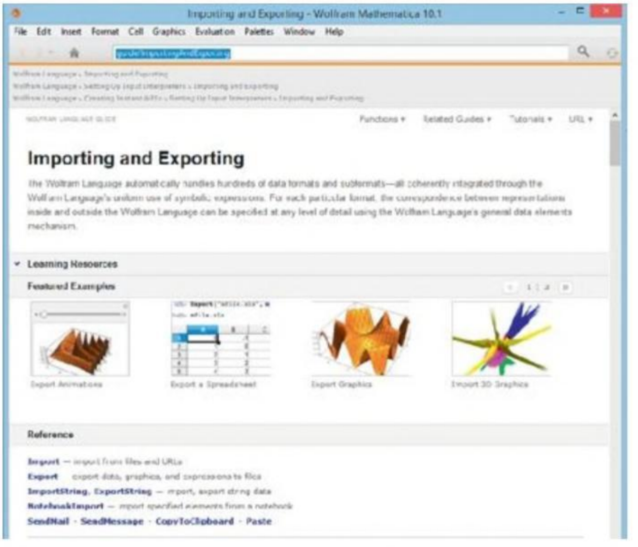


Figure 1.5 How to import and export using *Mathematica*.

If we don't have a clear idea of the function we want to use or we remember it vaguely, we can use the search toolbar (Figure 1.6) with two or three words related to what we are looking for.

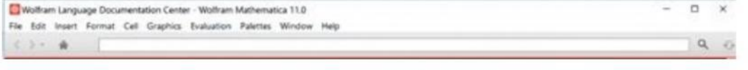


Figure 1.6 The Search Toolbar.

Another type of help is the one given when a mistake is made. In this case we should pay attention to the text or sound messages that we may receive. For example, if we try to evaluate the following text cell, we will receive a beep.

"This is a text entry"

With **Help ▸ Why the Beep?...** we will be able to know the reason behind the beep.

Another additional source of help is the Input Assistant with its context-sensitive autocompletion and function templates features: when you start writing a function, *Mathematica* helps you automatically complete the code (similar to the system used by some mobile phones for writing messages). For example: If you start typing `Pl` *Mathematica* will display all the functions that start with those two letters right below them (Figure 1.7). Once you find an appropriate entry, click on it and a template will be added.

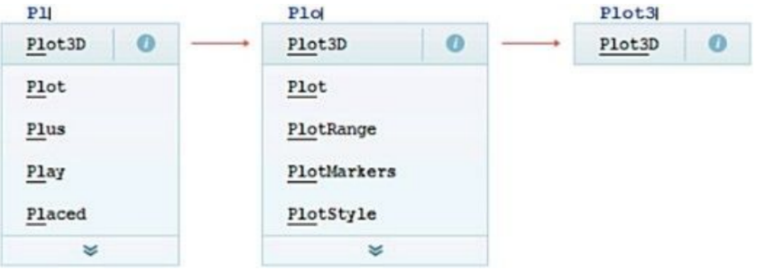


Figure 1.7 The Input Assistant in action.

Sometimes a down arrow will be shown at the end of the function offering us several templates (Figure 1.8). Choose the desired one.

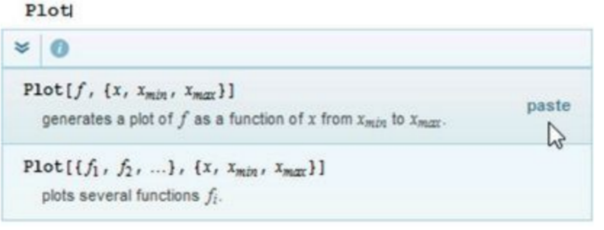


Figure 1.8 Function templates for the **Plot** function.

Additionally, a color code enables syntax error detection. If you have written a function and it is shown in blue, probably it has not been typed correctly. All *Mathematica* and user-defined functions will be shown in black. For example: Write in an input cell **Nmaximize**, it will be shown in blue, write **`NMaximize`** and you will see that it is shown in black and a template is displayed.

1.4 Basic Ideas

Users with prior knowledge of other programming languages such as C or FORTRAN, very often try to apply the same style (usually called procedural) in *Mathematica*. The program itself makes it easy to do so since it has many similar commands such as: For, Do, or Print. Although this programming style works, we should try to avoid it if we want to take full advantage of *Mathematica*'s capabilities. For that purpose we should use a *functional* style, that basically consists of building operations in the same way as classical mathematics.

This section is for both the uninitiated into *Mathematica* and those *Mathematica* users that still use procedural language routines. Because of that, if you have ever programmed in other languages please try not to use the same programming method.

In this brief summary we will refer to some of the most frequently used commands and ideas. We will usually not explain their syntax or we'll do it in a very concise way. For additional details, just type the command, select it with the cursor and press <F1>. If you don't understand everything that you read, don't worry, you will be given more details in later chapters.

> When you don't know the syntax of a function, type it, select it with the cursor and press the <F1> key.

1.4.1 Some Initial Concepts

- Let's begin by writing the expression below.

```
expr1 = 5! 6^2/2^(1/3) π Cos[3 π] e^-7 Sin[1]
```

$$-\frac{2160 \times 2^{2/3} \pi \sin(1)}{e^7}$$

We have assigned it the name **expr1**, to be able to use the result of the calculations later on. To write the transcendental numbers π and e we have used special symbols (e is not the same as e, e is a letter without any other meaning) located in one of the palettes. If we prefer to use the keyboard we can write Pi for π and E^ or Exp[] for e. Notice that we use a single equal sign "=" to indicate sameness or equivalence in equations. For comparisons, we will use the double equal sign "==".

- The same operations can be written as in the cell below, using the typical *InputForm* notation.

```
(5! 6^2/2^ (1/3) Pi Cos[3 Pi] Sin[1]) / E^7 (*This is a commment*)
```

$$-\frac{2160 \times 2^{2/3} \pi \sin(1)}{e^7}$$

  We have included a comment inside the actual cell "(* comment *)". This comment will be neither considered in the computation nor shown in the output.

Naturally, in both cases we obtain the same result. There is something that should draw our attention in this output. Apparently, there are terms that have not been evaluated and that appear again as such: π, Sin[1], y e^7. This is because *Mathematica* does not simplify or make approximations if that means losing precision. It literally works with infinite precision. Nevertheless, we can force it to compute the decimal approximation, the usual approach in other programs. To do that we will use the function `N` as shown in the next function. A similar result can be obtained by including decimals in some of the numbers.

- In this example we use the % symbol that recalls the last output. `//N` or `N`[*expr*] will use in the calculations machine precision even though the output may show fewer decimals.

```
% //N
```

−8.26548

- Another option is to substitute the number with a decimal approximation. You can try in the previous example using `Sin[1.0]` instead of `Sin[1]`
- By default 5 decimals are shown, but with `N[expr, n]` we can use a precision `n` as large as desired.

```
N[expr1, 30]
```

−8.26547962656679448059436738241

Mathematica contains thousands of prebuilt commands or functions (Functions) that are always capitalized. The program differentiates between lower and upper cases. For example: `NMinimize` is not the same as Nminimize. Arguments are written inside square brackets [arg]. However, users can define a function using lower case.

Lists are a fundamental *Mathematica* concept, frequently used in many contexts. In a list elements are inside curly brackets {}. Later on we will refer to them.

> Remember: All functions in *Mathematica* are capitalized: `Log`; arguments are written inside square brackets: `Sin[2 x + 1]`; list elements are inside curly brackets: {a_1, ...,

a_n}; ranges are written as lists: `Plot[x, {x, 0, 2 Pi}]` ; parentheses are used to group operations: `(Log[a x] + Sin[b x])/(a - b)`.

Use the help to investigate the various functions and their arguments. Remember that you can select a function with the cursor and press <F1> to get information about it. Besides its syntax, examples, tutorials and other related functions are displayed.

- Write `Pl` in an *input* cell. The context-sensitive input assistant will show you all the functions that start with Pl. Select Plot. Next to Plot a new down arrow will appear. Click on it and a template like the one shown below will be created.

```
Plot[f, {x, xmin, xmax}]
```

Figure 1.9 Template for the **Plot** function.

The cell above is an *input* type cell. If you'd like to avoid its evaluation: **Cell ▸ Cell Properties** and uncheck Evaluable. That is what we have done in this case in the original document since the idea is to see the cell content but not to execute it.

- In the previous template, all the basic function arguments are displayed. Now you can fill them in using the options as the next example shows. Execute the command.

```
Plot[Cos[x], {x, 0, 30}]
```

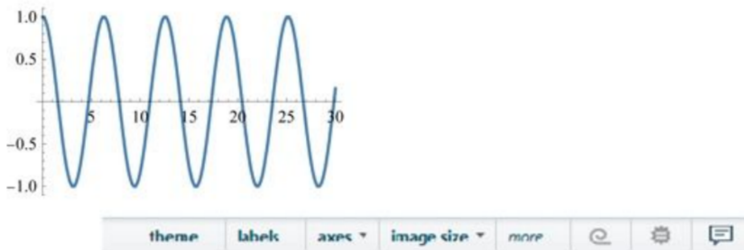


Figure 1.10 The Suggestions Bar for plots.

- The resulting output is a plot with the suggestion bar (Figure 1.10) appearing right below it.

  Since Version 9, thanks to the WolframPredictiveInterface, the Image Assistant and Drawing Tools provide point-and-click image processing and graphics editing. Click on **theme...** and a menu with several choices to customize the plot will unfold (Figure 1.11).

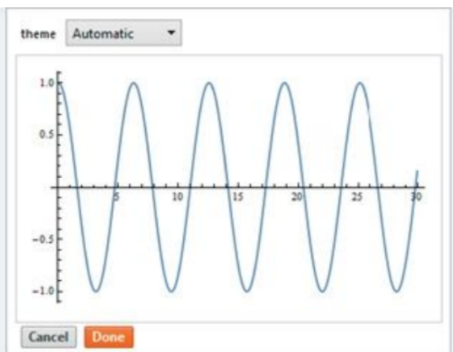


Figure 1.11 Customizing a plot using the "**theme...**" option in the Suggestions Bar.

- Alternatively, or if you are using a version prior to 9, you can customize the plot using the Plot options directly (type Plot, select it with the mouse and press <F1> to see them).

Besides arguments, commands also have options. One way of seeing those options directly is `Options`[*func*]:

- `Solve` options:

```
Options [Solve]
```

{Cubics → True, GeneratedParameters →*C*, InverseFunctions → Automatic,
MaxExtraConditions → 0, Method → Automatic, Modulus → 0,
Quartics → True, VerifySolutions → Automatic, WorkingPrecision → ∞}

Next, we'll show some examples. Replicate them using the **Basic Math Assistant** that includes templates for hundreds of commands.

- Definite integral example:

$$\int \mathrm{Sin}[x]^4\,\mathrm{Cos}[x]^3\,dx$$

$$\frac{3\sin(x)}{64} - \frac{1}{64}\sin(3x) - \frac{1}{320}\sin(5x) + \frac{1}{448}\sin(7x)$$

- Use `Simplify` anytime you need to simplify a complex expression.

```
Simplify [x^8 - 4x^6 y^2 + 6x^4 y^4 - 4x^2 y^6 + y^8]
```

$(x^2 - y^2)^4$

There are also other commands to manipulate expressions. You can find them in the palette **Other ▸ Algebraic Manipulation**. Note that you can manipulate an entire expression or just part of it.

- Try to use it with this example simplifying the Sin and Cos arguments separately.

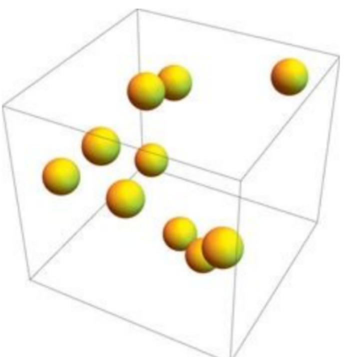
```
Sin[x^8 - 4x^6 y^2 + 6x^4 y^4 - 4x^2 y^6 + y^8] [Cos[x^4 + 2x^2 y^2 + y^4]
```

- To get:

```
Sin[((x^2 - y^2)^4)]/Cos[(x^2 + y^2]^2]
```

- In this example a random distribution of spheres in a three-dimensional space is shown.

```
Graphics3D[
 {Yellow, Sphere [RandomInteger [{-5, 5}, {10, 3}]], ImageSize → 250}]
```



You can rotate the image to see it from different angles by clicking with the mouse cursor inside.

If you haven't used *Mathematica* previously, the last command may seem strange. Let's analyze it step by step:

- Highlight `RandomInteger` and press <F1>. The help page for the command will be displayed. In this case with `RandomInteger [{-5, 5}, {10, 3}]` we are generating 10 sublists with 3 elements, each element being a random number between -5 and 5. We are going to use them to simulate the coordinates {*x*, *y*, *z*}.
- The previous command is inside `Sphere` (do the same, highlight `Sphere` and press <F1>) a command to define a sphere or a collection of spheres with the coordinates `[{{`x_1`, `y_1`, `z_1`}, {`x_2`, `y_2`, `z_2`}, ...}, `*r*`]`, where *r* is the radii of the spheres. If omitted, as in this example, it's assumed that *r* = 1.

```
Sphere[RandomInteger [{-5, 5}, {10, 3}]]
```

Sphere[(23041−14−4−3−524−23−2−4−52−1−3315−5−4−2−15−4−1)]

Note: In this particular notebook, list outputs are sometimes shown as matrices.

We use the command `Graphics3D` to display the graphs. We have added `Yellow` to indicate that the graphs, the spheres in this case, should be yellow and the option `ImageSize` to fix the graph size and make its proportions more adequate for this example. Finally we arrive at the command: `Graphics3D[{Yellow, Sphere[RandomInteger[{-5, 5}, {10, 3}]], ImageSize → 250}]`.

> When you don't understand an instruction consisting of several different commands, decompose it into its individual instructions and analyze each one separately using the help system.

- In the next cell we use `Rotate` to show a rotated output, in this case by 1 radian, compared to the usual display.

```
Rotate[1/Sqrt[1 + x], 1]
```

Type inside the output and you'll see that it retains its properties.

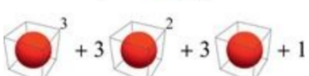
- Practically anything can be used as a symbol. Generate a small red sphere.

```
Graphics3D [{Red, Sphere[]}, ImageSize → 30]
```

- Substitute the symbol thus obtained, , pasting it below replacing b and evaluating the cell.

Expand$[(1+b)^3]$

$b^3 + 3\,b^2 + 3\,b + 1$



1.4.2 Replacements

A replacement consists of a rule to substitute one or several symbols with other symbols or values. To do that you apply the following syntax: "*exp /. rule"* that will replace *exp* with the contents of *rule*.

- In the expression *a x* + *b y, a* is replaced with 3 and *b* with 5, using a replacement rule.

```
exp1 = ax + by / . {a → 3, b → 5};
```

- From now on anytime we call `exp1` we will see that the replacement has taken place:

```
exp1
```

$3\,x + 5\,y$

If you apply several replacement rules to the same expression in succession, the replacement takes places consecutively:

- We first apply the rule (`/.a→b`) to the expression *a x* + *b y* and then the second rule (`/.b→c`) is applied to the previous result.

```
a x + b y / . a → b / . b → c
```

$c\,x + c\,y$

- In the expression below we make an assignment that consists of assigning the symbol `sol` to the solution of $a\ x^2 + b\ x + c == 0$.

```
sol = Solve[a x^2 + b x + c == 0, x];
```

- The output is a list.

```
sol
```

{{x→−b2−4a c−b2 a},{x→b2−4 a c−b2 a}}

> Remember that for assignments a single equal sign "=" is used, while to indicate an equality in an equation or a comparison, we will use a double equal sign "==" that when typed will be automatically converted to "==".

- To verify that the previous result represents the solutions to the equation $ax^2 + bx + c == 0$ we use the replacement "*exp* `/.` *rule"* that will replace in *exp* whatever is specified in *rule*.

```
a x^2 + b x + c = 0 /. sol
```

{(−b2−4 a c−b)24 a+12 ab(−b2−4 a c−b)+c=0, (b2−4 a c−b)24 a+12 ab(b2−4 a c−b)+c=0}

We can see that the replacement has taken place, but it's convenient to simplify it. For that we'll do as follows: we use `Simplify [%]` where % enables us to call the result (*Out*) of the last entry and evaluate the cell.

- We will check that effectively the equality holds and therefore the solution is correct. In this case it was evident, but we can apply the same method to more complex situations.

```
Simplify [%]
```

{True, True}

When an assignment is not going to be used later on, it may be convenient to delete it using the command `Clear`.

- The following command removes the assignment previously associated to `sol`.

```
Clear[sol]
```

- Now, when we type `sol` we see that it has no assignment.

```
sol
```

sol

1.4.3 Functions

Mathematica makes it easy to define functions in a way very similar to regular mathematics. The usual syntax in the case of a function of one variable is: `f[x_]:= expr[x]`. The *blank* ("_") next to the variable is used to specify the independent function variable.

- Let's see it with an example.

```
f[x_] := 0.2 Cos[0.3 x^2]
```

- Now we can assign a value to the independent variable and we'll get the function value.

```
f[3]
```

−0.180814

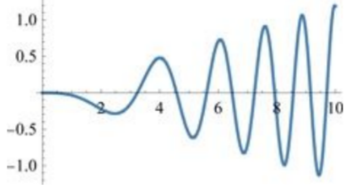
- We could have also typed the previous two operations in the same cell (don't forget in this case to enter ";" at the end of each function). However, it's better to use one cell for each function until you have enough practice. It will help you find mistakes.

```
f[x_] := 0.2 Cos[0.3 x^2];
f[3]
```

−0.180814

- Now we are ready to visualize the `Derivative` (') of `f[x]` in a specific interval.

```
Plot[f' [x], {x, 0, 10}]
```

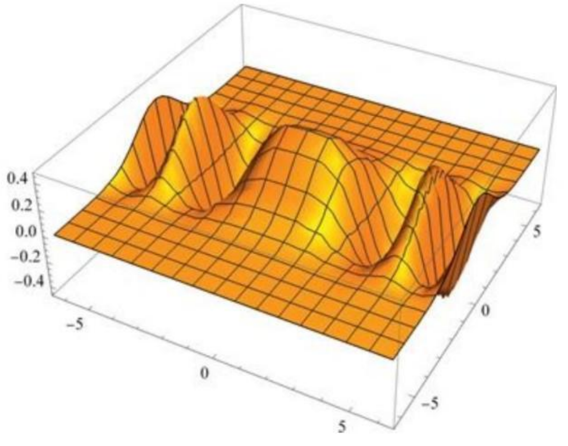


- The same approach can be extended to several variables. In this example we use the previously defined *f*(*x*) function.

```
g[x_, y_] := f[x] 2.3 Exp[- 0.3 y^2]
```

- Now we can present the result in a 3D graph using `Plot3D`. We use the option `PlotRange -> All` to display all the points in the range for which the function is calculated, remove the option to see what happens.

```
Plot3D[g[x, y], {x, -2π, 2π}, {y, - 2π, 2π}, PlotRange -> All]
```



1.4.4 Dynamic Assignment of Variables

We can make dynamic assignments in which the symbol (make sure that in the menu bar **Evaluation ▸ Dynamic Updating Enable** is checked) returns an entry that changes dynamically as we make new assignments to it.

```
Clear [a]
```

- Let's create a variable.

```
Dynamic [a]
```

a

- If we now assign to `a` different values or expressions, you'll see that the output above keeps on changing.

a=255a=Integrate[x^2,x]x33

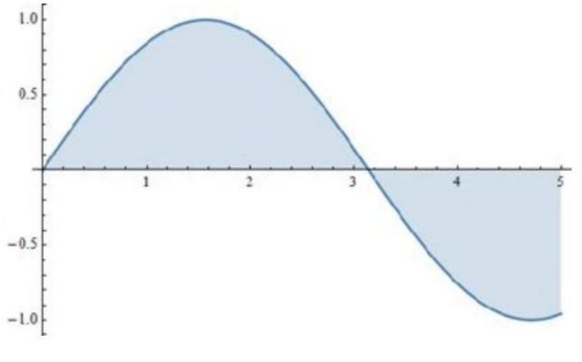
- We can also create a box with `InputField` and write any *input* in it.

```
InputField[Dynamic[a]]
```

a

- Try to use any f[x] in the box above, for example `Sin[x]`, and you will see how the next graph is updated. The process will repeat itself anytime you write a new function in the box.

```
Dynamic[Plot [a, {x, 0, 5}, Filling → Axis]]
```



1.4.5 List and Matrices

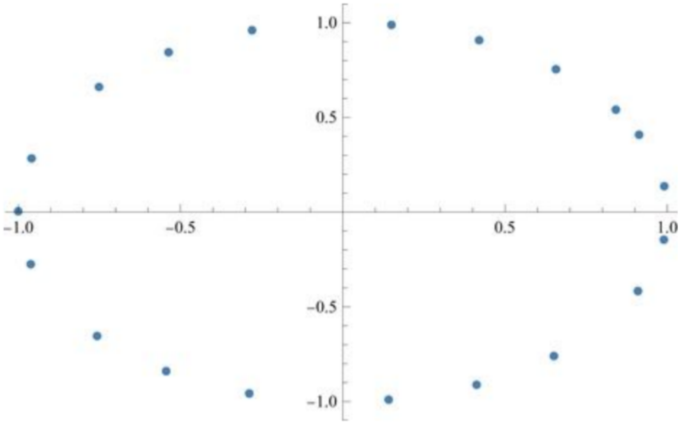
Lists (List) are a fundamental concept in *Mathematica*. Lists or domains are written inside curly brackets with the elements separated by commas: {a, b, c}. For a description of list operations see the documentation: guide/ListManipulation. We'll describe them in more detail in later chapters.

- In the example below we generate a list with `Table`.

```
list1 = Table[{Sin[n], Cos[n]}, {n, 20}];
```

- We see that these are pairs of numbers corresponding to an ellipse that we can use to represent it graphically.

```
ListPlot[list1]
```



Matrices have list format and consist of lists of sublists.

- Let's create a matrix of random numbers.

```
mat = RandomInteger[10, {5, 5}];
```

- We add `InputForm` so you can see the output displayed in the internal format used by *Mathematica*.

```
mat // InputForm

{{4, 6, 3, 9, 8}, {3, 7, 0, 8, 6}, {5,
6, 10, 8, 10}, {3, 1, 10, 3, 8}, {0, 5,
0, 4, 5}}
```

- If desired, matrices can be presented in standard mathematical notation.

```
MatrixForm[mat]
```

(46398370865610810311038050 45)

- Since they are lists, matrices can be handled as such. For example, here we extract the second element of the first sublist:

```
mat [[1, 2]]
```

6

For further details about matrix operations you can consult the documentation: guide/MatrixOperations.

1.4.6 Graphics

One of the most remarkable aspects of *Mathematica* is its graphical capabilities. The program's tool for 2D graphics can be accessed by pressing CTRL+D or by selecting in the menu bar **Graphics ▶ Drawing Tool**.

The most commonly used functions to represent graphics are `Plot`, `Plot3D` and `ListPlot`, of which we have already seen some examples. However, there are many more.

- Since *Mathematica* 9, when typing `Plot` the context-sensitive input assistant will show you all the words that include the word Plot, browse them. Alternatively type the following command and execute it (its output is omitted):

```
?*Plot*
```

The names that are shown can give you a hint about the type of plot they generate. Click on the chosen name to get more detailed information. In some cases you will be referred to the plot options.

- The graphical functions don't end there. The ones that include the word `Chart` are usually related to statistical graphs. Similarly to the previous case you can use the command:

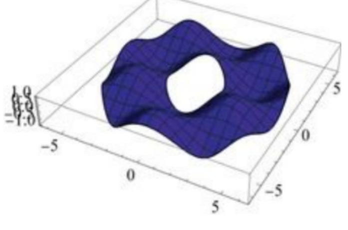
```
?*Chart*
```

- Graphics-related functions include numerous options, as you can see in the case of `Plot`. Use the following command (its output is omitted):

```
Options[Plot3D]
```

- The next cell displays the surface cos(x) sin(y) bounded by the ring $3 \leq x^2 + y^2 \leq 30$. Note that to set the region where the function exists we use the `RegionFunction` option.

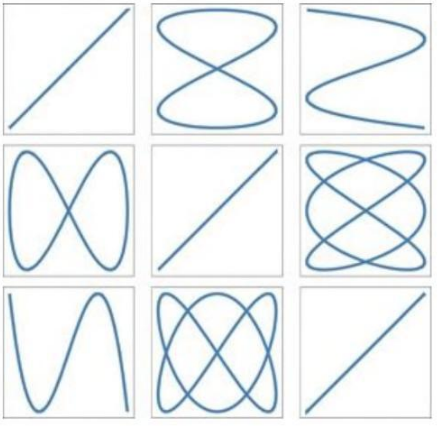
```
Plot3D[Cos[x]Sin[y], {x, -2π, 2π},
 {y, -2π, 2π}, RegionFunction → (3 ≤ #1^2 + #2^2 ≤ 30 &),
 BoxRatios → Automatic, PlotStyle → Blue, ImageSize → Small]
```



You can interact with 3D graphics: Click on the graph and you'll see that you can rotate it, with CTRL + click (o ⌘-click) you can zoom, and with SHIFT+click you can move it around.

- The code below generates the famous Lissajous curves. Using `Grid` we can show a two-dimensional mesh of objects, in this case graphs. We also use the `Tooltip` function to display labels related to objects (not only graphics) that will be shown when the mouse pointer is in the area of such objects. Here, when the mouse pointer hovers over the chosen curve, you'll see the function that generated it.

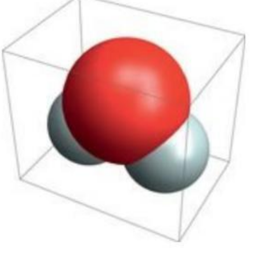
```
Grid[Table[Tooltip[ParametricPlot[{Sin[nt], Sin[mt]}, {t, 0, 2Pi},
 ImageSize → 70, Frame → True, FrameTicks → None, Axes → False],
 {Sin[n t], Sin[m t]}], {m, 3}, {n, 3}]]
```



Other functions that do not include **Plot** or **Chart** but that are also useful for creating graphs are **Graphics**, **Graphics3D** and **ColorData**, as the following example shows.

- Representation of the water molecule.

```
Graphics3D[{Specularity[White, 50], ColorData["Atoms", "H"],
 Sphere[{0, 0, 0}, .7], Sphere[{1.4, 0, 0}, .7], ColorData["Atoms",
"O"],
 Sphere[{.7, 0, .7}]}, Lighting → "Neutral", ImageSize → Small]
```



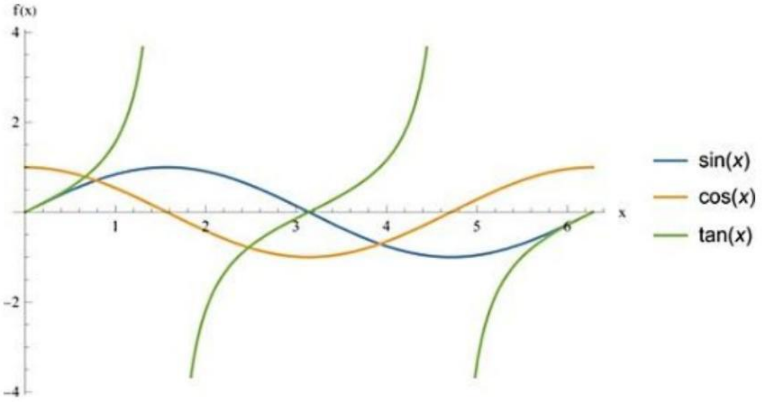
If you want to learn more about graphics, you can select the text with the cursor and press <F1>. For example: Select the text shown below and press <F1>:

tutorial/GraphicsAndSoundOverview

Graphics can be completed with legends using **PlotLegends** (if you are using a version prior to *Mathematica* 9 you will need to first execute **Needs ["PlotLegends`"])**

- In the following example we use **AxesLabel** to add labels to the axes and **PlotLegends → "Expressions"** to show the represented functions.

```
Plot[{Sin[x], Cos[x], Tan[x]}, {x, 0, 2 Pi},
 AxesLabel → {"x", "f(x)"}, PlotLegends → "Expressions"]
```



1.4.7 Images

Image processing has also experienced a significant improvement in the most recent versions of *Mathematica*. Starting with *Mathematica* 9, when clicking on an image you will see a menu with numerous options (Figure 1.12).

- The following command loads an image so we can play with it.

```
ExampleData[{"TestImage", "Lena"}]
```

Figure 1.12 The Image Assistant.

- Since version 9, a row of icons, the Image Assistant, appears below the image. Click on any of them. For example, after clicking on **more...** the following menu (Figure 1.13) will be deployed:

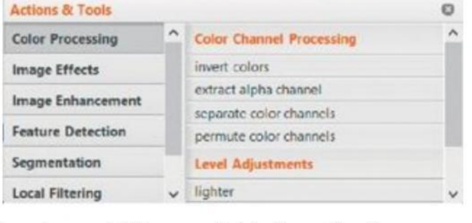


Figure 1.13 Additional capabilities available from the "**more... option**" in the Image Assistant.

- Play with the controls. When you click on one of them, a miniature version of the image will be shown where you will be able to check the effect on the original image. For example: When clicking on **invert colors** you'll see Figure 1.14.

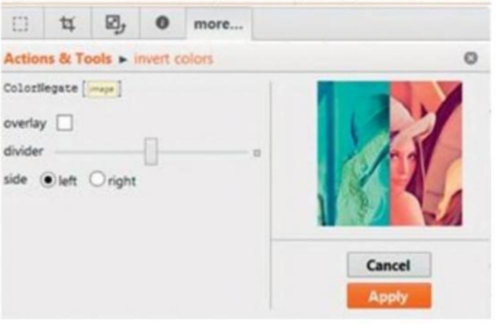


Figure 1.14 Inverting the colors using the Image Assistant.

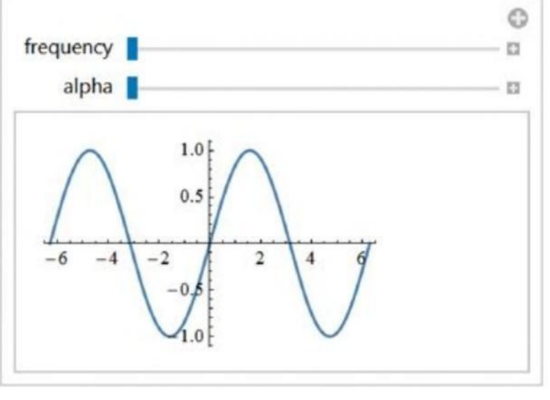
After getting the desired effect press **Apply** and the change will be applied to the image. An alternative way is to copy the command that appears in the menu: `ColorNegate`[*image*] to obtain the same result. Copy it to an *input* cell and execute it. This last procedure is the appropriate one if you want to create a program or explain how the image has been modified. We will be using this method when we refer to image processing in later chapters.

1.4.8 Manipulate

Among the improvements in the most recent versions of *Mathematica* is the addition of instructions to enable dynamic and interactive operations. One of the most significant examples of this is the `Manipulate` command, as can be seen in the following examples.

- Type the command shown below. Note how sliders are created for each of the parameters being defined. Click on the + symbol included in the output to show the buttons created by `Manipulate`.

```
Manipulate[Plot[Sin[frequency x + alpha], {x, -2π, 2π},
 ImageSize -> Small], {frequency, 1, 5}, {alpha, 0, Pi/2}]
```

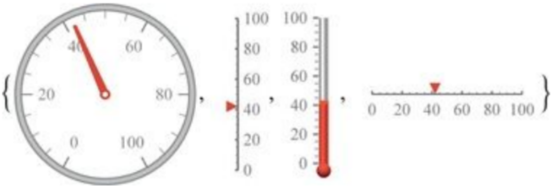


In the previous example we have used the **ImageSize** → **Small** option for printing purposes. We will use the option throughout the book but if you repeat the example in your computer you can ignore it.

1.4.9 Gauges

Since *Mathematica* 9 different types of gauges have been included that can behave dynamically and be customized:

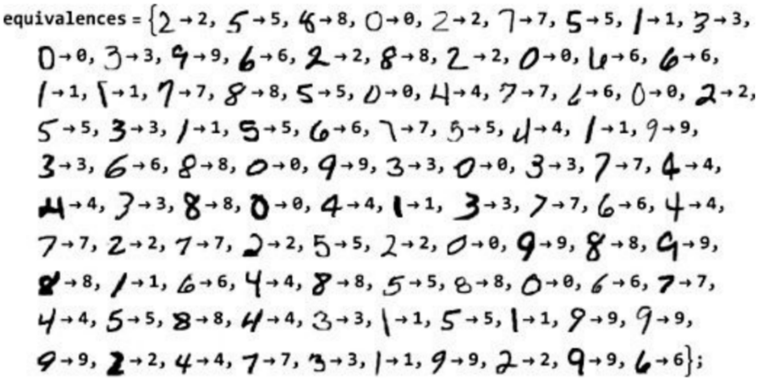
```
Through[{AngularGauge, VerticalGauge, ThermometerGauge, HorizontalGauge}[
  42, {0, 100}, ImageSize → Tiny]]
```



1.4.10 Handwritten Text Recognition

Among the latest new features is the inclusion of machine learning functions. Let's see an example.

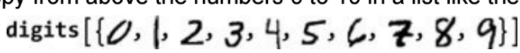
- Handwrite different numbers on a white piece of paper, scan them or type them in a touch screen (in MS Windows you can use the **Crop** application) and establish their equivalence as shown in the next function.



- Use the command `Classify` to establish equivalences based on the examples. Note that for a given number, its handwritten equivalent may not be the same. This function uses statistical criteria to assign a weight to each established equivalence.

```
digits = Classify[equivalences]
```

- Copy from above the numbers 0 to 10 in a list like the following:



{0, 1, 2, 3, 4, 5, 4, 7, 8, 9}

- Note that the identification has been done correctly except in the case of 6 that has been mistaken with 4. The program assigns probabilities based on the stored data. You can check that the probability assigned to the symbol  is higher for 4 than for 6. Even a person may not be sure whether the handwriting represents a 6 or a 4.

```
digits[
```

```
, "TopProbabilities"]
```

{4 → 0.375915, 6 → 0.354657, 0 → 0.235305}

- If 6 is written in a less equivocal way the probability of a correct identification improves substantially.

```
digits[
```

```
, "TopProbabilities"]
```

{6 → 0.992954}

1.4.11 Things to Consider

Don't forget that for everything to work correctly, it's necessary that all the commands in the same session are executed sequentially. Cells are executed based on the order of evaluation and not on the order they are shown on the screen. This means that if a function calls previous functions, those functions should have been executed in advance. It's not enough to see them already typed. If you modify an assignment that is used in a later function, you will have to execute again all the related cells.

- Execute sequentially the following three cells:

```
a = 3 (*first*)
```

3

```
b = x^a (*second*)
```

x^3

```
a b (*third*)
```

$3x^3$

Now type `a = 2` in the first cell. Execute it and then the third one (`a b`) What has happened?

> ***Mathematica* retains information about all the variables defined in a session, even after starting a new *notebook***. Because of that, when starting a new section that will not be using the variables previously defined it would be a good idea to clear them first.

- One way to remove all the values and definitions is as follows:

```
Clear ["Global`*"]
```

Another way to achieve the previous result is to limit the application of the variables to a certain context through the menu bar: **Evaluation ▸ Notebook Default Context**. This offers the possibility to limit the defined variables to the active *notebook* or to certain cells grouped following a specific criterion, for example: all the cells belonging to the same section. As a matter of fact this chapter has been created using this option. The *input* counter will reset itself when a new group of cells is created (here it cannot be seen because we have suppressed the symbol In[*n*]=). Nevertheless, this method is less straightforward than `Clear ["Global` *"]`. We should evaluate the most appropriate way based on the situation.

If you really want to remove all the information and not only the variables, the best way is to quit the session. This requires that you exit *Mathematica* (actually it requires that you exit the program *Kernel*). You can do that in several ways: With **File ▸ Exit** or if you don't want to exit the notebook use either **Evaluation ▸ Quit Kernel** or in an input cell type **Quit[]**. Once you have exited, the program removes from memory all the assignments and definitions. When you execute the next command the kernel will be loaded again.

Although *Mathematica* tries to maintain compatibility with *notebooks* created in previous versions, the compatibility is not always complete so you may have to make some modifications. When you open a notebook created in a previous *Mathematica* version for the first time, the program will offer you the possibility of checking the file automatically; Accept it and read the comments that you may receive carefully. In many cases *Mathematica* will modify expressions directly and let you know about the changes made giving you the option to accept or reject them. In other cases it will offer suggestions.

Remember: If you don’t understand a function use the Documentation Center.

1.5 Computational Capabilities

Early versions of *Mathematica* were mainly oriented toward symbolic computations. However, the latest versions of the program, apart from having significantly increased those capabilities, have complemented them with powerful functions for numeric calculations. An example of this combination is the large collection of probability related functions that provide seamlessly either symbolic or numeric answers depending on the inputs.

1.5.1 Equation Solving

We are going to show several available functions for solving different types of equations. In the examples we will use only one variable but the same functions can be used with a higher number of variables. Don’t forget to use “==” to indicate equality in an equation.

- It is interesting to compare `Solve` and `Reduce`.

```
Solve[{x + ay + 3z == 2, x + y - z == 1,
2x + 3y + az == 3}, {x, y, z}]
```

{{x→1,y→−1−a−3,z→−1−a−3}}

- `Reduce` generates all the solutions depending on the value of the parameter a and uses a more formal notation in its output.

```
Reduce[{x + ay + 3z == 2, x + y - z == 1,
2x + 3y + az == 3}, {x, y, z}]
```

(a=2∧y=15(5−4x)∧z=x5)∨(a−2≠0∧x=1∧a+3≠0∧y=1a+3∧z=y)

- `Reduce` and `Solve` accept constraints regarding the variables domain.

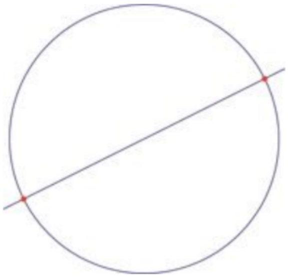
Reduce[272x-1x==92x−1,x,Reals]x=12∨x=3

- `Solve` can solve geometrically formulated problems. In this case the solution to the intersection of a line passing through the points {0,0},{2,1}, with a circumference of radius *r* = 1 centered at {0, 0}(*Mathematica* uses `Circle` to refer to the circumference and `Disk` to refer to the circle, by default the commands assume a circle or circumference of *r* = 1, and centered at {0, 0}):

```
Solve[{x, y} ∈ InfiniteLine[{{0, 0}, {2, 1}}] && {x, y} ∈ Circle[],
{x, y}]
```

{{x→−25,y→−15},{x→25,y→15}}

```
Graphics [{{Blue, InfiniteLine[{{0, 0}, {2, 1}}], Circle[]},
  {Red, Point [{x, y}] / . %}}]
```



An analytical solution may not exist in many cases or we may be interested in a numeric result. In those situations we can use `FindRoot` or `NSolve`.

- With `FindRoot` you must define, for a given equation, the variable whose value you want to find and an initial starting point at which the search for the solution starts.

```
FindRoot [Cos[x] == x + Log[x], {x, 1}]
```

{$x \to 0.840619$}

```
NSolve [x^5 − 6 x^3 + 8x + 1 == 0, x]
```

{{$x \to -2.05411$}, {$x \to -1.2915$}, {$x \to -0.126515$}, {$x \to 1.55053$}, {$x \to 1.9216$}}

1.5.2 Integration

- The symbolic integration capabilities of *Mathematica* are very powerful as shown in the example below.

```
Integrate[Exp[1 - x^2], x]
```

12eπ erf(x)

- However, there are times when symbolic integration is not possible and is necessary to integrate using numerical methods (`Log` refers to the base e logarithm, to indicate the base 10 logarithm type Log[10, *expr*]).

```
NIntegrate[Log[x + Sin[x]], {x, 0, 2}]
```

0.555889

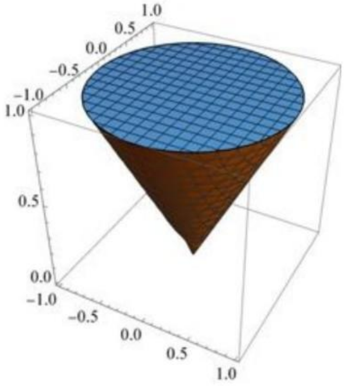
- It is also possible to limit the integration to a certain region by using the `Boole` function. In this example this region is a cone with radius 1.

```
Integrate[(x^2 + y^2) Boole[0 ≤ z ≤ 1 && x^2 + y^2 ≤ z^2],
  {x, -1, 1}, {y, -1, 1}, {z, 0, 1}]
```

π10

- With `RegionPlot` we can represent the integration region, a feature that from an educational point of view may be of interest.

```
RegionPlot3D[0 ≤ z ≤ 1 && x^2 + y^2 ≤ z^2,
 {x, -1, 1}, {y, -1, 1}, {z, 0, 1}, ImageSize → Small]
```



We can even specify assumptions, with `Assumptions`, about the parameters of the function that we wish to integrate.

- In this example we specify that when integrating x^n, n is greater than 1.

```
Integrate[x^n, {x, 0, 1}, Assumptions -> n > 1]
```

1n+1

- We can also integrate over a region. In this example, over a sphere centered at {0,0,0} and with radius *r*>0. Obviously, we are calculating the volume of the sphere.

```
Integrate[1, {x, y, z} ∈ Ball[{0, 0, 0}, r], Assumptions -> r > 0]
```

4 π r33

1.5.3 Sums, Logical Operations, Simplifications and Differential Equations

- We can evaluate sums and products with finite and infinite terms.

Σk=1∞1K6π6945

- We can also perform logical operations.

Log[2]<1.4<2

True

- To check whether two expressions are equivalent "===" can be used. Note that 3 y 3.0 in *Mathematica* are different numbers.

```
1 === 3/3
```

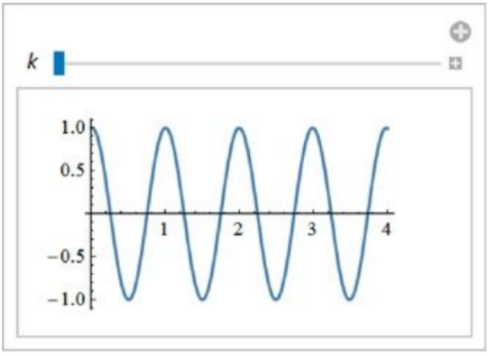
True

```
1 === 3.0/3
```

False

- Next, we solve a differential equation for a pendulum with damping constant k. Using `Manipulate` we can make the value of k vary from 0 to 3 and plot the solution. By using `Evaluate` we are forcing *Mathematica* to solve the differential equation first, and then substitute x and k with the given values. Note that the use of the pattern "**y[x]/.** *something*". " /." means "replace with" (in the example: y[x] is replaced by the equation solution).

```
Manipulate[Plot[Evaluate[y[x] / .
    DSolve[[y"[x] + k y' [x] + 40 y[x] == 0, y[0] == 1, y'[0]==13},
y[x], x]],
  {x, 0, 4}, ImageSize → Small], {k, 0, 3}]
```



1.5.4 Application: The Lorenz System

There are many functions available for numerical methods, such as **NDSolve**, used when solving differential equations numerically.

- An example of the use of **NDSolve** is its application to the well-known Lorenz system, a simplified model to study atmospheric convection movements that exhibit chaotic behavior.

```
eqs = {x'[t] == -3 (x[t] - y[t]),
    y'[t] == -x[t] z[t] + 26.5x[t] - y[t], z'[t] == x[t] y[t] - z[t]};
ics = {x[0] == z[0] == 0, y[0] == 1};
sol = NDSolve [{eqs, ics}, {x, y, z}} {t, 0, 200}, MaxSteps -> ∞]
```

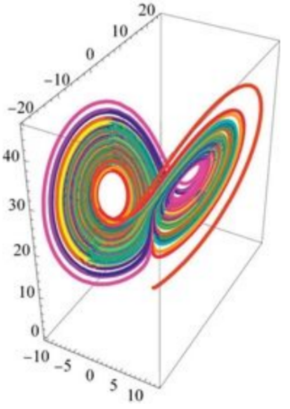
{{x → InterpolatingFunction[Domain: (0. 200.) Output: scalar],

y → InterpolatingFunction[Domain: (0. 200.) Output: scalar],

z → InterpolatingFunction[Domain: (0. 200.) Output: scalar]}}

- The solution to the previous equation in the phase space is displayed using **ParametricPlot3D**.

```
ParametricPlot3D[{x[t], y[t], z[t]} /. sol[[1]], {t, 0, 200},
  PlotPoints → 1000, ColorFunction → (Hue[#4] &), ImageSize → Small]
```



1.5.5 Statistical Calculations

Mathematica probably includes as many functions, if not more, than other specialized statistical software programs. The downside is that users must have a deep understanding of what they are trying to accomplish.

- This example shows the probability density function (pdf) and the cumulative density function (cdf) of a Poisson distribution with mean μ = 10. An option in **PlotLegends** is included to place the legend in the desired position.

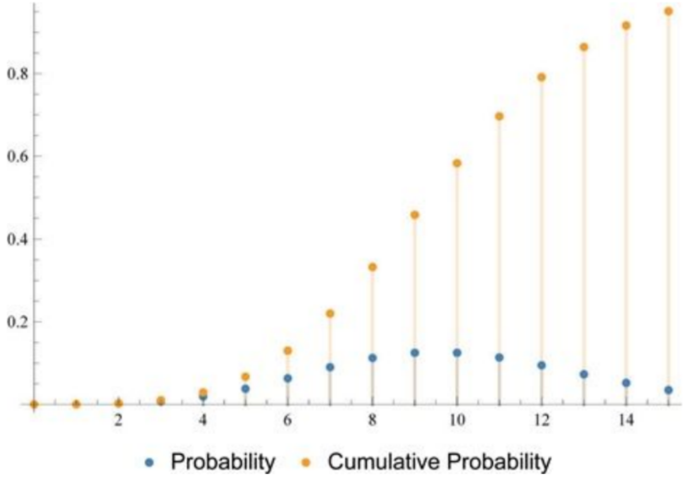
```
pdf = PDF[PoissonDistribution[μ], x]
```

{e−μ μxx!x≥00True

```
cdf = CDF[PoissonDistribution[μ], x]
```

{Q([x]+1,μ)x≥00True

```
DiscretePlot[Evaluate[{pdf, cdf} /. μ → 10], {x, 0, 15}, PlotRange →
All,
  PlotLegends → Placed[{"Probability", "Cumulative Probability"},
Below]]
```



1.6 Utilities

1.6.1 Example Data

The installation of the program includes a set of files with data from different fields that can be used to test some of the functions. To download them use the command `ExampleData`.

- Let's download the original text of Charles Darwin's book *On the Origin of Species*:

```
txt = ExampleData[{"Text", "OriginOfSpecies"}];
```

- To find out how many times the word "evolution" appears compared to the word "selection" we use the function `StringCount`, specifying the chosen word or text.

```
StringCount[txt, "evolution"]
```

4

```
StringCount[txt, "selection"]
```

351

1.6.2 Accessing External Data

One of the most important features of *Mathematica* since Version 8, already mentioned, is its integration with **Wolfram|Alpha** (tutorial/DataFormatsInWolframAlpha). This enables us to access information about practically anything and once downloaded it can be used in the *Mathematica* environment to perform further calculations.

- Type `Japan vs. Germany`. Several frames (pods) will be displayed, each of them containing information about an aspect of the question. In the right corner of each frame the symbol appears. Click on one of them and you will see a menu with several options. In this example, we chose the **Geographic Properties** frame and clicked on the **Subpod content**. The following entry was generated afterward:

```
WolframAlpha["Japon vs. Germany",
 IncludePods → "GeographicProperties:CountryData",
 AppearanceElements → {"Pods"},
 TimeConstraint → {30, Automatic, Automatic, Automatic}]
```

Geographic properties: More | Show non-metric

	Japan	Germany
total area	377 835 km² (square kilometers) (world rank: 62nd)	357 022 km² (square kilometers) (world rank: 63rd)
land area	374 744 km² (square kilometers) (world rank: 62nd)	348 672 km² (square kilometers) (world rank: 63rd)
continent	Asia	Europe

- We repeat the same steps for "China vs USA" but in this case instead of **Subpod content** we choose **ComputableData**. The output is shown below. This has the advantage of being generated as a list allowing easy subsequent manipulation.

```
WolframAlpha["China vs USA",
 {{"GeographicProperties:CountryData", 1}, "ComputableData"}]
```

(ChinaUnited Statestotal area9.597×106 km29.631×106 km2land area9.326×106 km29.162×106 km2continentAsiaNorth America)

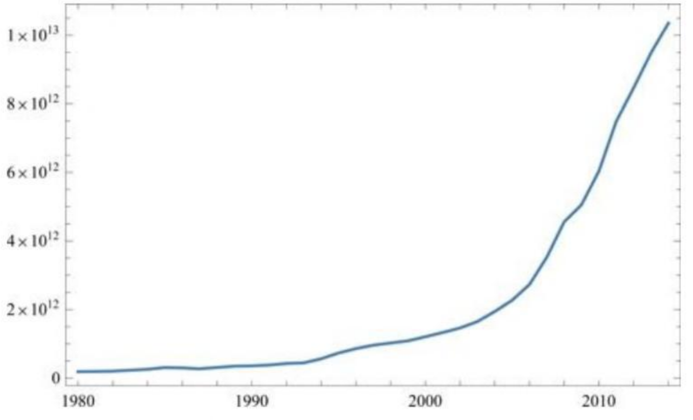
In the **WolframAlpha** style you can select the information you are interested in by choosing from the menu that appears after clicking on ▸ **Subpod content** or

ComputableData in the desired frame. An *input* will be generated to get only the desired information.

You can also access external data through Computable Data (guide/ComputableDataOverview), a group of functions specific to *Mathematica*, to obtain the desired information. We will cover them in more detail in Chapter 5.

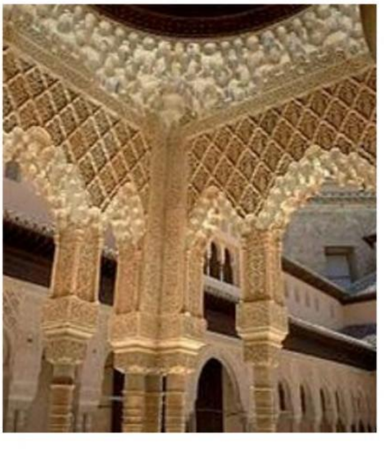
- The graph below shows the evolution of the gross domestic product, GDP, of China from 1980 to 2015.

```
DateListPlot [CountryData["China", {{"GDP"}, {1980, 2015}}]]
```



- We can even obtain information from Wikipedia using the new function (available since *Mathematica* 10.1): `WikipediaData`.

```
WikipediaData["Alhambra", "ImageList"][[5]]
```



1.6.3 Application: Everybody Out

We would like to estimate the amount of energy required to move the entire population off-planet: https://what-if.xkcd.com/7/. Without considering the weight of the rockets, the idea is to see how we can solve this problem accessing external data from within *Mathematica*. The same approach can be useful to solve other types of problems.

The data that we need to know are: the kinetic energy equation (it's been such a long time since we took high-school physics that we have forgotten it) and the values of its components. We will need to know the world population as well.

- The kinetic energy equation can be found using `FormulaData`.

```
FormulaData["KineticEnergy"]
```

- Or the free-form input format: typing **Kinetic Energy Equation**.

```
Kinetic Energy Equation
```

K=12 m v2

- From the equation we can see that we need to know: *m* (mass per person) and *v* (escape velocity from Earth). Both values can be obtained with the free-form input format typing: **average adult weight** and **earth's escape velocity** or similar statements.

```
m = average weight adult
```

82 kg

```
v = earth's escape velocity
```

1.118×10^4 m/s

- With these data we obtain the energy per person. We need to multiply them by *n*, the world population.

```
n = QuantityMagnitude[CountryData["World", "Population"]]
```

7.13001×10^9

- Now, after evaluating the kinetic energy equation, we get the necessary kinetic energy to put the entire world population in orbit. Naturally this is just a toy example that has nothing to do with reality since we would need to consider the mass of the launcher and the fact that people would have to go in spaceships that have masses much bigger than that of their passengers. Until now we have sent about 600 people into space. The trip in a Soyuz spacecraft costs about 20 million dollars per person. Multiply that figure by the number of people living on earth and the result is thousands of times bigger than the world's GDP. Let's hope that we will not need to put the entire human population into orbit!

```
UnitSimplify[12 m v^2*n]
```

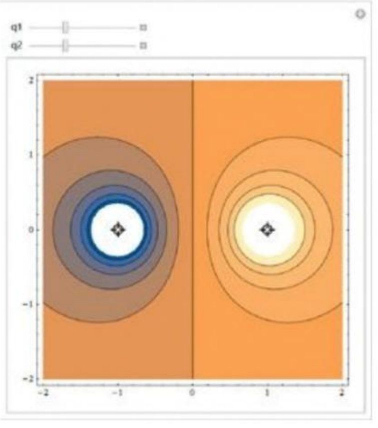
3.65391×10^{19} J

1.6.4 Interactive Visualizations (Demonstrations)

In http://demonstrations.wolfram.com you can find thousands of small interactive applications of great educational value. In this book we'll refer to them using the word "demonstrations". If you don't know how to build them just download and open the files; later on you'll be able to modify them to suit your needs or even make new ones. In Chapter 4 we'll learn how to create them.

- This example explores *Coulomb's law*. This law describes the interaction between two charges (based on '*Potential Field of Two Charges*' created by Herbert W. Franke: http://demonstrations.wolfram.com/PotentialFieldOfTwoCharges/).

```
Manipulate[ContourPlot[q1/Norm[{x, y} - p[[1]]] + q2/Norm[{x, y} -
p[[2]]],
   {x, 2, -2,}, {y, 2, -2}, Contours → 10],
 {{q1, - 1}, -3, 3}, {{q2, 1}, 3, -3},
 {{p, {{-1, 0}, {1, 0}}}, {-1, -1}, {1, 1}, Locator}, Deployed → True]
```



Move the locators around with the mouse to see how the potential fields change.

1.6.5 Packages

Commands can be used directly or can be put together to develop applications for specific purposes (*packages*). The packages included in *Mathematica* can be seen in **Documentation Center ▸ StandardExtraPackages** and **Documentation Center ▸ InstalledAddOns**. StandardExtraPackages are the ones included in the program installation. Many of those functions now in the packages will probably be part of the kernel of the program in the future.

Installed AddOns are packages developed by users for specific purposes. Some of them are available in http://library.wolfram.com/infocenter. This book's author has developed several packages, some of them available in http://diarium.usal.es/guillermo.

Normally these packages will be copied in Addons/Applications, located in the *Mathematica* installation directory.

To load a package you can type Needs["package name`"] or <<package name`. For example, the following package includes various functions related to the properties of the radiation emitted by a black body.

```
Needs["BlackBodyRadiation`"]

?"BlackBodyRadiation`*"
```

▼ BlackBodyRadiation`

BlackBodyProfile	MaxPower	PeakWavelength	TotalPower

- The following package function calculates the peak wavelength of a black body for a given temperature.

```
PeakWavelength[5000 Kelvin]
```

5.79554×10^{-7} Meter

In a later chapter we'll give a brief introduction to package development.

1.7 Editing Notebooks

We've seen that the interaction with *Mathematica* is done through the Front End (what we usually see when using *Mathematica*), and this generates files named "*Mathematica notebooks*". What you are currently reading is one such *notebook*.

Each *Mathematica notebook* (file.nb) is a complete and interactive document containing text, tables, graphics, calculations and other elements. The program also enables high-quality editing so notebooks can be ready for publication (some journals directly admit papers in *nb* format).

1.7.1 Notebook Structure

Each cell has a style. For example, the previous cell has the style **Subsection** associated to it. To do that we just need to select its cell marker (]) and in the *style box* choose Subsection.

Notebooks are organized automatically into a hierarchy of different cell types (title, section, input, output, figures, etc). If you click on the exterior cell marker, the cells will be grouped. For example, if you click on a section, all the cells associated to that section will collapse and you will only see the title of the section. You just need to click again to see them back.

This is an ordinary text with the color formatted Format ▸ Text Color.

Click on a text cell and modify its format.

With Insert ▸ Hyperlink we can create hyperlinks to jump from one location to another inside the same *notebook*, to another *notebook* or to an external link.

Using **Format ▸ Text Color** we can choose the `font`, **face**, size, color, etc. We can even use special characters such as .

This *notebook*, like any other, has a predefined style. The style tells *Mathematica* how to display its contents. There are many such styles. To choose the *notebook's* overall style you can use **Format ▸ Stylesheet**. With **Format ▸ Style** you can define the style of a particular cell. Additionally, inside a style, we can choose the appropriate working environment **Format ▸ Screen Environment**. Try changing the appearance of a *notebook* by modifying the working environment.

1.7.2 Choosing Your Style

To edit an article, book or manual the **Writing Assistant** palette is of great help. Load it. Note that is divided into three sections: *Writing and Formatting*,*Typesetting* and *Help and Settings*. See the help included in the palette.

If you click inside the palette on the button **Stylesheet Chooser**... (alternatively you can go to **Format ▸ Stylesheet ▸ Stylesheet Chooser**...) a separate window with icons representing different styles will appear (Figure 1.15). If you press on the upper part of any of the icons (**New**) a new sample *notebook* formatted according to that style will be generated. You will be able to use it and modify its contents to suit your needs. If you click on the bottom part of the icons (**Apply)**, the style will be applied to the currently active *notebook*. For example, in the next screenshot (Figure 1.15), by clicking on the upper part of the Textbook icon, a *notebook* with a Textbook template style will be opened.

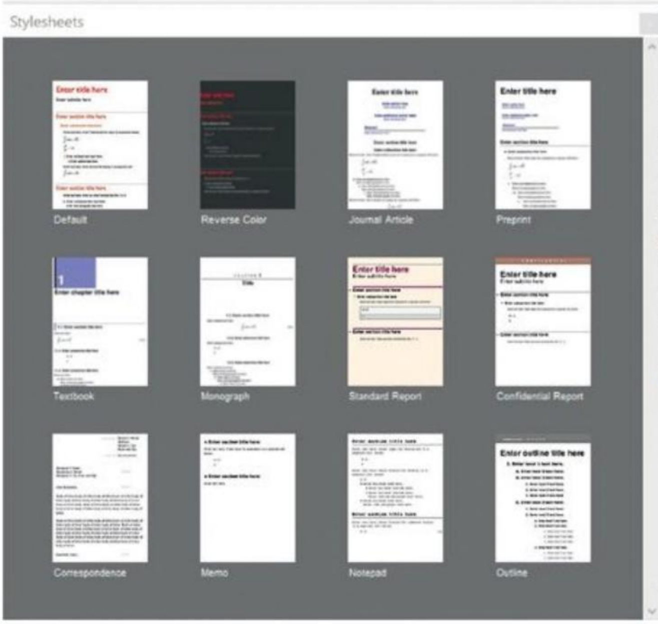


Figure 1.15 The Stylesheet Chooser.

When a new cell is created, depending on the stylesheet, by default it will be of a certain format type, for example with the *Default* stylesheet new cells are of type *Input*.

> When typing in a new *notebook* choose the stylesheet from the beginning with **Format ▸ Stylesheet ▸ Stylesheet Chooser...** . After clicking on the upper side of the icon representing the desired style (New), you will be able to use and modify the template according to your needs.

1.7.3 Typesetting Advice

If you are interested in emphasizing text containing formulas using classical mathematical notation, begin by typing in a text cell. Select the formula and define it as Inline Math Cell by pressing CTRL + 9 or, in the Writing Assistant palette, choose **Math Cells ▸ Inline Math Cell**. Finally, in the same palette in Cell Properties click on Frame and select the appropriate one . That's how the following example was done:

> ¿What is the domain of the function f(x)=x2−1?

If you'd like to type a sequence of formulas and align them at the equal signs, in Writing Assistant, choose **Math Cells ▸ Equal Symbol Aligned Math Cell**.

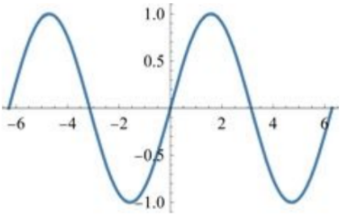
$$\begin{aligned} a+b&=c \\ a&=c-b \end{aligned}$$

Figure 1.16 Aligning equations at the equal sign.

Sometimes it may be useful to type both, the formula and its result, as part of the text. This can be done by writing and executing the function in the same text cell. For example: using a palette write $\int x^3\ dx = \int x^3\ dx$. Now select the second term of the equation and press SHIFT+CTRL+ENTER. Or, in the menu bar, choose **Evaluation ▸ Evaluate in place**, and the previous expression will become ∫x3dx=x44.

When creating a document, you may be interested in seeing only the result. In that case you can hide the Input cell.

- To hide the input just click directly on the output marker cell. The following example was created by executing `Plot[Sin[x], {x, -2 Pi, 2 Pi}]` and then hiding the command to display only the graph.



All *Mathematica* commands are in English but the menus and palettes can also be displayed in any of the additional 12 languages offered since version 11. The alternatives include Chinese, French, German, Italian, Japanese, Korean and Spanish among others. Users can take advantage of this functionality by going to **Edit ▸ Preferences ▸ Interfaces**. Once an option different from English is selected, every function will be automatically annotated with a "code caption" in the chosen language. *Mathematica* 11 also includes a real-time spell checker that works with any of the supported languages. Alternatively, users of previous *Mathematica* versions can review the English contents of a notebook with **Edit ▸ Check Spelling...**.

With **Edit ▸ Preferences** and with **Format ▸ Option Inspector** (see tutorial/OptionInspector) you will be able to customize many features of *Mathematica* that can be applied globally, to a *notebook* or to a particular cell. For example: with **Format ▸ Option Inspector ▸ Global Options ▸ File Locations** you can change *Mathematica* default directories.

1.7.4 The Traditional Style

We've seen that *Mathematica* by default uses the style: `StandardForm` in the input cells (*Input*) by default. However, if we'd like to create better looking documents, it's recommended to use the `TraditionalForm` style that is similar to the one used in traditional mathematical notation.

If the main purpose of a document is to be read by others, it may be convenient to present the inputs and outputs using the traditional notation. It's likely that the reader will not realize that it was created with *Mathematica*. This is specially true if the document is saved using the cdf format to which we will refer in the following section.

> You can write your inputs in `StandardForm`. Once you verify that they work you can convert them to the traditional style.

- Example: Let's type:

  Limit[(x + Δx)2−x2Δx,Δx→0]2x

- We convert the cell to the traditional form by selecting it and clicking on: **Cell ▸ Convert to ▸ TradicionalForm**. Then the cell above will be shown as follows:

  limΔx→0(x+Δx)2−x2Δx2x

If an entry is in the traditional style, to see how it was originally typed convert it to the standard style by selecting the cell and choosing from the menu bar **Cell ▸ Convert to ▸ StandardForm**.

We can make an output to be shown in the traditional form by adding `// TraditionalForm` at the end of each *input*. There are several ways to apply this style to all the outputs in a notebook. In **Edit ▸ Preferences ▸ Evaluation** you can specify for all the outputs to be displayed in the traditional form (`TraditionalForm`). Another easy one is to use the following command:

```
SetOptions[EvaluationNotebook[],
 CommonDefaultFormatTypes -> {"Output" -> TraditionalForm}]
```

It's recommended to include the command above in a cell at the end of the notebook and define it as an initialization cell (click on the cell marker and check **Cell ▸ Cell Properties ▸Initialization Cell**). This way, the cell will be executed before any other cell.

In many of the styles available in **Stylesheet Chooser...** outputs are displayed using the standard form. In this chapter we have used a customized stylesheet that shows outputs in the traditional format. In the rest of the book, we will almost always use the standard form since what we are trying to highlight is how to create functions with *Mathematica* and for that purpose the classical format is not adequate.

1.7.5 Automatic Numbering of Equations and Reference Creation

Textbook and scientific articles frequently use numbered equations. This can be done as follows:

- First open a new notebook (**File ▸ New ▸ Notebook**) and select one of the stylesheets available in the format menu. For instance **Textbook: Format ▸ Stylesheet ▸ Book ▸ Textbook**
- Then create a new cell and choose **EquationNumbered** as its style (**Format ▸ Style ▸ EquationNumbered**). The cell will be automatically numbered, then you can write the equation, for example: $a + b = 1$.

a+b=1(1.1)

If you have opened a new notebook using the Textbook style, you will probably see (0.1) instead of (1.1). It's possible to modify the numbering scheme but it is beyond the scope of this chapter. When you need to number equations, sections, and so on, remember that it is a good idea to select the stylesheet directly from the Writing Assistant palette: **Palettes ▸ Writing Assistant ▸ Stylesheet Chooser...** . After selecting **Textbook** (**New**), a new notebook will be generated with many different numbered options such as equations, sections, etc. You can then use that notebook as a template.

If you wish to insert an automatic reference to the equation proceed as follows:

- Write an equation in a cell with the **EquationNumbered** style. Then go to **Cell ▸ Cell Tags ▸ Add/Remove...** and add a tag to the formula. Let's give it the tag: "par" (we write "par", from parabola, although we could have used any other name). A good idea would be also to go to **Cell ▸ Cell Tags** and check **▸ Show Cell Tags**, keeping it checked while creating the document, and only unchecking it after we are done. This will enable us to see the cell tags at all times. The first equation done this way is written below:

par

x2=1(1.2)

- If you wish to refer to this equation: Type the following: (1.), put the cursor after the dot and go to **Insert ▸ Automatic Numbering**. In the opening dialog (Figure 1.17) scroll down to **EquationNumbered** and choose the tag of the equation, in our case: "par" then press OK (see the screen below). After this has been done the number of the equation appears between the parentheses: (1.2). If you click on the "2" the cursor will go immediately to the actual equation.

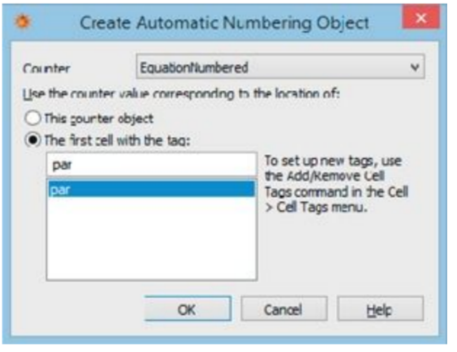


Figure 1.17 Creating an automatically numbered object.

1.7.6 Cell Labels Display

As we have mentioned previously, any time we execute an input cell, a label in the form In[*n*] is shown, but we can choose not to display it. This can be done from **Edit ▸ Preferences** or typing **SetOptions[SelectedNotebook[], ShowCellLabel → False]**. You can also reset the counter (set In[*n*] to 1) in the middle of a session by typing the following in an input cell: $Line=1.

1.8 Sharing Notebooks

1.8.1 Save as...

Documents created with *Mathematica* are saved with extension *.nb, *Mathematica's* own format, by default. If you share any of these documents with someone, that person will require access to *Mathematica*.

However, *Mathematica* can also save documents in other formats. To do that, in the menu bar, choose **File ▸ Save As ▸**. Then, from the list of options, select the desired one.

Very often we may be interested in saving a document in *pdf* format to reach a broad audience. We can also use *TeX* (widely used in professional journals in mathematics) or *HTML/XML* if we want to use it in a website. Probably it would be better to save the web files in *XML* since everything can be included in a single file in contrast to *HTML*, where there is a main file and numerous additional files associated to it. However, if we'd like to keep *Mathematica's* interactive capabilities the most appropriate format would be *cdf* (Computable Document Format).

1.8.2 The Computable Document Format (*.cdf)

As previously discussed, if you'd like other people without access to *Mathematica* to read your documents without losing their interactivity, you should save them in the cdf format: **File ▸ CDF Export ▸ Standalone** or **File ▸ Save As ▸ Computable Document** (*.*cdf*).

The main advantage of this format is that it keeps the interactivity of the objects created with `Manipulate`. In this case your documents can be read with the Wolfram *CDF Player* (available for download for free from http://www.wolfram.com/cdf-player). When sharing a cdf file, to ensure that the intended reader can open it, we recommend that when sending it the link above is included as well.

You can also create a cdf file (normally a section of a notebook) to be embedded in a web page. In the menu bar, select **File ▸ CDF Export ▸ Web Embeddable**.

For more detailed information see in the help system: "Interactivity in .cdf Files" or, even better, visit: http://www.wolfram.com/cdf.

The Wolfram *CDF Player* is not just a reader of *Mathematica* documents for users without access to the program. The player can also avoid displaying the functions used in the inputs showing only the outputs (remember the trick that we have seen previously to hide the inputs and only show the outputs). Furthermore, it can include dynamic objects created with `Manipulate` so that readers can experiment with changing certain parameters and visualizing the corresponding effects in real time. Additionally, readers will not even know that the original document was generated in *Mathematica*. To see all these capabilities in action take a look at the following cdf document:

http://www.wolfram.com/cdf/uses-examples/BriggsCochraneCalculus/BriggsCochraneCalculus.cdf

Many of the Wolfram Research websites, such as Wolfram|Alpha (http://www.wolframalpha.com), allow visitors to generate and download documents in *cdf format*. The same happens with the Wolfram Demonstrations Project (http://demonstrations.wolfram.com) where the files are available in cdf format and you can download and run them (for example, in a digital board connected to a PC with the Wolfram *CDF Player*).

1.9 The Wolfram Cloud

As stated at the beginning of the chapter, the program can be executed in the cloud, that is: in a server accessible through the Internet using a browser, in what is known the Wolfram Cloud (http://www.wolframcloud.com/). The Wolfram Cloud can also be used to store files remotely, as a matter of fact all the files generated locally are interchangeable with the cloud-generated ones. If you don't have access locally and you would like to start familiarizing yourself with *Mathematica* you can do so directly using the Wolfram Development Platform, part of the Wolfram Cloud, for free. If you are planning to use this option heavily, the cost will depend on your requirements and whether you already own a *Mathematica* license or not. In any case, you must register as a user using a Wolfram ID.

If you already registered in the user portal https://user.wolfram.com, mentioned previously, you can use the same Wolfram ID. After signing up you can access the development platform by visiting http://www.wolframcloud.com and then clicking on the **Wolfram Development Platform** icon (http://develop.wolframcloud.com/app).

After entering your Wolfram ID and password, the screenshot shown in Figure 1.18 will appear although you might see an initial screen that is somewhat different since the Wolfram Cloud is constantly evolving.

Figure 1.18 Welcome Screen in the Wolfram Development Platform.

You can now create a new notebook by clicking on: **Create a New Notebook**, and be able to reproduce most of the examples discussed in the chapter. You can also save the notebook in Wolfram Cloud, and when accessing it again, no matter from what location, you will be able to continue working on it. Additionally, you will have the possibility of accessing cloud files from a local *Mathematica* installation: **File ▸ Open from Wolfram Cloud...** or **File ▸ Save to Wolfram Cloud...** . *Mathematica* has specific functions related to the Wolfram Cloud. In later chapters we will refer to them. You can get an idea by clicking on: **Getting Started...** and watching the short video.

1.10 Additional Resources

To access the following resources, enter the web addresses in a browser:

An Elementary Introduction to the Wolfram Language by Stephen Wolfram is in **Help ▸Wolfram Documentation ▸ Introductory Book** ≫, or free on the web: http://www.wolfram.com/language/elementary-introduction/ (there is also a print version).
Complete program documentation: http://reference.wolfram.com/language/
Explanatory video guides: http://www.wolfram.com/broadcast
Introductory program tutorials: http://reference.wolfram.com/language/tutorial/IntroductionOverview.html
Interactive applications: http://demonstrations.wolfram.com
Mathematics reference website: http://mathworld.wolfram.com/
Paid and free courses: http://www.wolfram.com/training/courses/
Information about the *Computable document format* (CDF): http://www.wolfram.com/cdf
News and ideas: http://blog.wolfram.com
Author's website: http://diarium.usal.es/guillermo

2
Data Analysis and Manipulation

In this chapter we are going to delve into list operations, essential for data manipulation and for understanding how Mathematica works; show how to import and export data and files in different formats; have our first contact with the statistical analysis of data; and see how to connect to big databases using ODBC.

2.1 Lists

The basic structure in *Mathematica* is the list. The elements of a list can be numbers, variables, functions, strings, other lists or any combination thereof; they are written inside braces `{}` and are separated by commas. If we feel comfortable with the classical terminology of mathematics we can interpret a simple list as a vector, a two-dimensional list (a list of sublists) as a matrix, and a list of three or more dimensions as a tensor, keeping in mind that the tensor elements can be sublists of any dimension.

- Below we define a list consisting of three sublists.

```
list = {{2, 3, 6}, {3, 1, 5}, {4, 3, 7}};
```

- The previous list can be shown in traditional notation as a matrix but we should not forget that internally *Mathematica* treats it as a list.

```
list // TraditionalForm
```

(236315437)

If the notebook has been configured to display outputs using the traditional format (as in this case), it's not necessary to use //MatrixForm.

- We generate a list of lists of sublists.

```
tensor = Table[i + j + k - l, {i, 2}, {j, 3}, {k, 4}, {l, 2}]

{{{{2, 1}, {3, 2}, {4, 3}, {5, 4}}, {{3, 2}, {4, 3}, {5, 4}, {6, 5}},
  {{4, 3}, {5, 4}, {6, 5}, {7, 6}}}, {{{3, 2}, {4, 3}, {5, 4}, {6,
5}},
  {{4, 3}, {5, 4}, {6, 5}, {7, 6}}, {{5, 4}, {6, 5}, {7, 6}, {8,
7}}}}
```

It's important to distinguish operations between lists and note that they don't behave in the same way as operations between matrices, especially with respect to multiplication.

- This is an example of list multiplication. We write * to emphasize that it's a multiplication, but if we leave an empty space instead, the result would be the same.

```
{{a, b}, {c, d}}*{x, y}

{{ax, bx}, {c y, d y}}
```

- Here we multiply matrices, or if you prefer, a matrix by a vector. To indicate that this is a matrix multiplication we use `Dot` (or "." which is the same). Note that "*" and "." are different operations.

```
{{a, b}, {c, d}} . {x, y}

{ax + b y, c x + dy}
```

- In the mathematical rule for multiplying matrices a row vector is multiplied by a column vector. *Mathematica* is less strict and allows the multiplication of two lists that do not meet the mathematical multiplication rule. It chooses the most appropriate way to perform the operation.

```
{x, y, z} . {a, b, c}

ax + by + cz

{x, y, z} . {{a}, {b}, {c}}

{ax + by + cz}
```

- In general, the sum of lists of equal dimension is another list of the same dimension whose elements are the sum of the elements of the initial lists based on their position.

```
{1, 2, 3} - {a, b, c} + {x, y, z}

{1 - a + x, 2 - b + y, 3 - c + z}
```

- If we multiply a list by a constant, each element of the list will be multiplied by it. The same happens if we add a constant to a list.

```
{{1, 2}, {3, 4}} k

{{k, 2 k}, {3 k, 4 k}}

{{1, 2}, {3, 4}} + c

{{1 + c, 2 + c}, {3 + c, 4 + c}}
```

To extract a list element based on its position *i* in the list we use Part[*list, i*] or, in compact form [[*i*]].

- In this example we show two equivalent ways of extracting the second element of a list:

```
Part[{a, b, c}, 2]

b

{a, b, c}[[2]]

b
```

In the case of a list of sublists, first we identify the sublist and then the element within the sublist.

- In the following example we extract element 3 from sublist 2:

```
{{a1, a2}, {b1, b2, b3}, {c1}}[[2, 3]]
```

```
b3
```

- In the example below we extract elements 2 to 4. Note that the syntax `[[i;;j]]` means to extract from *i* to *j*.

```
{a, b, c, d, e, f}[[2 ;; 4]]
```

```
{b, c, d}
```

- Here we extract elements 2 and 3 from sublists 1 and 2.

```
{{a1, a2, a3}, {b1, b2, b3}, {c1, c2, c3}}[[{1, 2}, {2, 3}]]
```

```
{{a2, a3}, {b2, b3}}
```

- In this case we use **`ReplacePart`** to replace a list element, specifically from sublist 2 we replace element 2 that is b2 with c2.

```
ReplacePart[{{a1, a2, a3}, {b1, b2, b3}}, {2, 2} → c2]
```

```
{{a1, a2, a3}, {b1, c2, b3}}
```

- Next, we use `[[{ }]]` to rearrange the elements in a list.

```
list = {a, b, c, d, e, f};
```

```
list[[{1, 4, 2, 5, 3, 6}]]
```

```
{a, d, b, e, c, f}
```

The solutions (*out*) are often displayed in list format. If we need a specific part of the solution, we can use the previous command to extract it.

- To extract the second solution of this second degree equation:

```
sol = Solve[1.2 x^2 + 0.27 x - 0.3 == 0, x]
```

```
{{x → - 0.625}, {x → 0.4}}
```

- We can proceed as follows:

```
x /. sol[[2]]
```

```
0.4
```

Other functions very useful for list manipulation are **`Flatten`**, **`Take`** and **`Drop`**.

- We apply **`Flatten`** . With **`Take`** `[`*list, i*`]` we extract the first *i* elements of the list, and with -*i* the last *i*. With **`Take`** *[list*, {*i,j*}] we can extract elements *i* to *j*.

```
Take[Flatten[{{a, b}, {c, d, e}, f, g}], 5]
```

```
{a, b, c, d, e}
```

- With **`Drop`** `[`*list, i*`]` we remove the first *i* elements of the list, and with -*i* the last *i*. **`Drop`** [*list, {i, j}*] removes elements *i* to *j*.

```
Drop[{a, b, c, d, e, f, g}, - 3]
```

```
{a, b, c, d}
```

Some functions, when applied to lists, operate on the individual list elements, e.g. f[{*a, b*, ...] → {f[*a*], f[*b*], ...}. In this case the function is said to be `Listable`.

- We can see below how a listable function operates. `Sin` in this case.

```
Sin[{a, b, c}]
```

```
{Sin [a], Sin [b], Sin [c]}
```

- Using Attributes, we can check that `Sin` is listable.

```
Attributes[Sin]
```

```
{Listable, NumericFunction, Protected}
```

- An alternative way to type the previous example is:

```
Sin@{a, b, c}
```

```
{Sin [a], Sin [b], Sin[c]}
```

- There are commands particularly useful when operating on lists. It's easy to identify them since they generally contain `List` as part of their names. You can check using the help system:

```
?*List*
```

We omit the output due to space constraints.

There are many more functions used to manipulate lists. We recommend you consult the documentation: guide/ListManipulation.

The most recent versions of *Mathematica*, starting with *Mathematica* 10, include datasets (`Dataset`). These constructions can be considered an extension to the concept of lists. With this new addition, the program has a very powerful way to deal with structured data.

- The example below shows a simple dataset with 2 rows and 3 columns. It consists of an association of associations. Most datasets are displayed in tabular form.

```
data = Dataset[<| "1" → < | "A" → 3, "B" → 4, "C" → 2|>,
   "2" → <| "A" → 4, "B" → 1, "C" → 3|>|>]
```

	A	B	C
1	3	4	2
2	4	1	3

- You can extract parts from datasets in a similar way to the one used with lists. To extract the element from "row 2" and "column C":

```
data["2", "C"]
```

```
3
```

In a later chapter we will see some examples using Datasets.

2.2 Importing/Exporting

- As mentioned before, we will often use as a precaution the command `Clear` ["Global`*"] at the beginning of a section to avoid problems with variables previously defined:

```
Clear["Global` *"]
```

A fundamental feature of *Mathematica* is its capability for importing and exporting files in different formats. The commands we need for that are **`Import`** and **`Export`**. They enable us to import or export many different types: **`XLS`**, **`CSV`**, **`TSV`**, **`DBF`**, **`MDB`**, ...

In this section we are going to use, among others, the files available as examples in a subdirectory named **ExampleData** created during the program installation. Some of these examples are downloaded from www.wolfram.com (Internet connection required).

A file can be imported or exported by typing the complete source or destination path.

- In Windows, to type the file location for import or export purposes use the following syntax:

  "...*directory**file.ext*", as shown in the following example:

```
Import["C:\\Users\\guill\\Google Drive\\Mathbeyond\\Data\\file.xls"]
```

> An easy way to type the file path is: In an *input* cell write **`Import`**, place your cursor inside **`Import[]`**. On the menu bar click: **`Insert ▶ File Path...`** Locate the file and click on it. The file path will automatically be copied to **`Import[`**"path"].

If we want to use the working directory in which the files are located for importing or exporting purposes, normally the easiest way will be to create a subdirectory inside the directory where the notebook is.

- The following function checks the directory of the active notebook.

```
NotebookDirectory[]
```

```
C:\Users\guill\Google Drive\Mathbeyond\
```

In the directory above, create a subdirectory named **Data**.

- With the following command we set the subdirectory **Data** as our default working directory, the one that the **`Import`** and **`Export`** functions will use during this session.

```
SetDirectory[FileNameJoin [{NotebookDirectory[], "Data"}]]
```

```
C:\Users\guill\Google Drive\Mathbeyond\Data
```

We make the previous cell an **Initialization Cell** to ensure its evaluation before any other cell in the notebook by clicking on the cell marker and choosing from the menu bar: **Cell ▸Properties ▸Initialization Cell**). This means that all the data will be imported/exported from the **Data** subdirectory. We may even consider "hiding" the initialization cell by moving it to the end of the notebook. Remember that the order of operations is not the same as it appears on screen. It is the order in which the cells are evaluated. Therefore, even if an initialization cell is located at the end it will still be the first one to be executed.

- The total number of available formats can be seen using **`$ImportFormats`** and **`$ExportFormats`**.

```
{$ImportFormats // Length, $ExportFormats // Length}
```

```
{178, 148}
```

2.2.1 Importing

To import files use `Import` *["path/file.ext", "Elements"*] where *file* is the name of the file you want to import. If the file is in the **Data** directory, previously defined, you only need to type the file name. If it is located somewhere else you will need to enter the complete path (remember that you can use `Insert ▸ File Path...`). You can also import files directly from the Internet. To do that, specify the desired URL in `Import["http://`*url*`"]`. As an option you can import just part of the file, the one defined by *Elements*.

If your computer uses a non-American configuration remember that *Mathematica* uses the dot "." as a decimal separator and the comma "," to distinguish between list elements. If you have another configuration you may have problems when importing files. The program enables the configuration of the format in **Edit ▸ Preferences... ▸ Appearance** to suit the user needs. Nevertheless, it is better to modify the operating system global settings so that all programs use the same setup. In Windows that can be done by selecting in the **Control Panel** "." as decimal separator and the comma "," as list separator. In OS X, the same can be done by going to **System Preferences... ▸ Language & Region ▸ Advanced...**

- Next, we import an *xls* file containing data about the atomic elements. Note that the program can figure out the type of file from its extension. In this case it's an *xls* file, and the program identifies it as a Microsoft® Excel file.

```
Import ["ExampleData/elements.xls"]

{{{AtomicNumber, Abbreviation, Name, AtomicWeight},
  {1., H, Hydrogen, 1.00793}, {2., He, Helium, 4.00259},
  {3., Li, Lithium, 6.94141}, {4., Be, Beryllium, 9.01218},
  {5., B, Boron, 10.8086}, {6., C, Carbon, 12.0107},
  {7., N, Nitrogen, 14.0067}, {8., O, Oxygen, 15.9961},
  {9., F, Fluorine, 18.9984}}}
```

- Notice that the previous output has the following structure {{{...}}}. The reason is that *Mathematica* assumes the file is an Excel spreadsheet that may contain several worksheets. Using `{"Data", 1}` we tell the program to import the data from worksheet 1 (do not confuse this Data with the working subdirectory Data that we have created earlier):

```
examplexls=Import["ExampleData/elements.xls", {"Data", 1}]

{{AtomicNumber, Abbreviation, Name, AtomicWeight},
 {1., H, Hydrogen, 1.00793},
 {2., He, Helium, 4.00259}, {3., Li, Lithium, 6.94141},
 {4., Be, Beryllium, 9.01218}, {5., B, Boron, 10.8086},
 {6., C, Carbon, 12.0107}, {7., N, Nitrogen, 14.0067},
 {8., O, Oxygen, 15.9961}, {9., F, Fluorine, 18.9984}}
```

- We show below two ways to extract the first 5 rows of the worksheet and display them in a table format.

```
TableForm[Take[examplexls, 5]]

AtomicNumber  Abbreviation  Name        AtomicWeight
1.            H             Hydrogen    1.00793
2.            He            Helium      4.00259
3.            Li            Lithium     6.94141
4.            Be            Beryllium   9.01218
```

```
TableForm[examplexls [[;;5]]]
```

```
AtomicNumber  Abbreviation  Name        AtomicWeight
1.            H             Hydrogen    1.00793
2.            He            Helium      4.00259
3.            Li            Lithium     6.94141
4.            Be            Beryllium   9.01218
```

- Here we see how to import a single column. In this example, we get the fourth column containing the atomic weights.

```
Import["ExampleData/elements.xls", {"Data", 1, All, 4}]
```

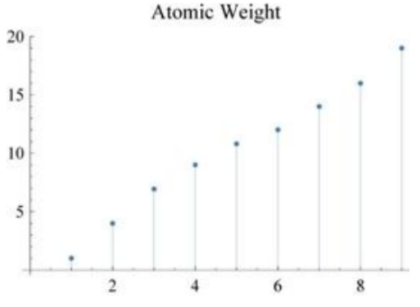
```
{AtomicWeight, 1.00793, 4.00259, 6.94141,
 9.01218, 10.8086, 12.0107, 14.0067, 15.9961, 18.9984}
```

- We use `Rest` to remove the first sublist containing the headings (the same could have been done using `Drop` [data, 1]).

```
data = Rest[Import["ExampleData/elements.xls", {"Data", 1}]];
```

- We show the imported data. `Tooltip` is included so that when the cursor is over a point in the graphic, its corresponding atomic weight can be seen.

```
ListPlot[Tooltip[data[[All, 4]]],
 Filling → Axis, PlotLabel → "Atomic Weight"]
```



- In our next example we find out the elements available in general for this type of format. It refers to all the possible elements. It doesn't mean that this file contains all of them.

```
Import["ExampleData/elements.xls", "Elements"]
```

```
{Data, FormattedData, Formulas, Images, Sheets}
```

- We choose "Sheets" that returns the number of sheets that the file contains including the labels for each one. In this case it only contains one.

```
Import["ExampleData/elements.xls", "Sheets"]
```

```
{Spreadsheet1}
```

- To display data in a typical spreadsheet format, like Excel, we can use `TableView`, a builtin function that as of *Mathematica* 11.0 remains undocumented.

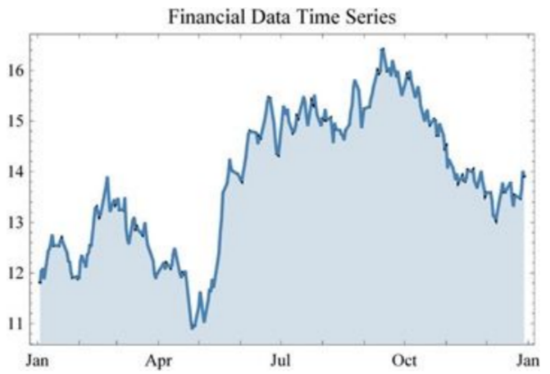
```
TableView[Import["ExampleData/elements.xls", {"Data", 1}]]
```

	1	2	3	4	5
1	AtomicNumber	Abbreviation	Name	AtomicWeight	
2	1.	H	Hydrogen	1.00793	
3	2.	He	Helium	4.00259	
4	3.	Li	Lithium	6.94141	
5	4.	Be	Beryllium	9.01218	
6	5.	B	Boron	10.8086	
7	6.	C	Carbon	12.0107	
8	7.	N	Nitrogen	14.0067	
9	8.	O	Oxygen	15.9961	
10	9.	F	Fluorine	18.9984	
11					
12					
13					
14					
15					
16					
17					
18					
19					

Note: Undocumented functions in *Mathematica* are not guaranteed to be included in future releases of the program. Additionally, some users might experience software stability issues when evaluating them. For these reasons, we recommend caution when using them.

- We can use **`DateListPlot`**, a very useful function when the values in the x-axis are dates.

```
DateListPlot[
 Import["ExampleData/financialtimeseries.csv"], Filling → Axis,
 PlotLabel → "Financial Data Time Series", ImageSize → Medium]
```



- Try to copy and paste links to any of the numerous images available in: earthobservatory.nasa.gov/Features/BlueMarble/.

```
Import[
 "http://earthobservatory.nasa.gov/Features/BlueMarble/Images/BlueMarble_
   2005_SAm_09_1024.jpg"]
```

- This function returns the size of an image.

```
Import[
 "http://earthobservatory.nasa.gov/Features/BlueMarble/Images/BlueMarble_
   2005_SAm_09_1024.jpg", "ImageSize"]
```

```
{1024, 1024}
```

Note: The examples using files imported from the Internet may not work if their web addresses have been changed or the files themselves deleted.

Importing from databases is a frequent data import operation. For example: importing from an Access *mdb* file.

- We import the complete structure of the database:

```
Import[ "ExampleData/buildings.mdb", {"Datasets", "Buildings",
"Labels"}]
```

```
{Rank, Name, City, Country, Year, Stories, Height}
```

- We use `Short` or `Shallow` to display just a few data elements after importing the entire database (this is practical if there's only one table and it's not very big).

```
Import[ "ExampleData/buildings.mdb" ] // Short
```

```
{{{1, Taipei 101, Taipei, Taiwan, 2004, 101, 508}, ≪19≫}}
```

- We import the records associated to several fields and display them in a table format:

```
Text[Style[Grid[Transpose[Import[
     "ExampleData/buildings.mdb", {"Datasets", "Buildings",
"LabeledData",
       {"Rank", "Name", "City", "Height"}} ]], Frame → All], Small]]
```

1	Taipei 101	Taipei	508
2	Petronas Tower 1	Kuala Lumpur	452
3	Petronas Tower 2	Kuala Lumpur	452
4	Sears Tower	Chicago	442
5	Jin Mao Building	Shanghai	421
6	Two International Finance Centre	Hong Kong	415
7	CITIC Plaza	Guangzhou	391
8	Shun Hing Square	Shenzhen	384
9	Empire State Building	New York	381
10	Central Plaza	Hong Kong	374
11	Bank of China	Hong Kong	367
12	Emirates Tower One	Dubai	355
13	Tuntex Sky Tower	Kaohsiung	348
14	Aon Centre	Chicago	346
15	The Center	Hong Kong	346
16	John Hancock Center	Chicago	344
17	Shimao International Plaza	Shanghai	333
18	Minsheng Bank Building	Wuhan	331
19	Ryugyong Hotel	Pyongyang	330
20	Q1	Gold Coast	323

For big databases (ORACLE, SQLServer, etc.), with a multitude of tables and a large number of records, it's better to access them with an ODBC connection as we will show later.

From the URL address of a website we can limit the information selected to only what we are interested in. A good practice is to first open the URL in a browser and see what information suits our needs. After importing the web page you will notice that it will appear in *Mathematica* as a list of lists. Analyze how the information is structured. You need to find out what lists contain the information you want and then use `[[...]]` to extract it.

- The following command opens the specified URL using the default browser on your system.

```
SystemOpen["http://www.uxc.com/review/UxCPrices.aspx"]
```

- From the previous web page (http://www.uxc.com/review/UxCPrices.aspx) we only import the information included in the frame: Ux Month-End Spot Prices as of MM, YYYY (Month-end spot price data), in US dollars and euros. The change from the previous month is included between parentheses. The following command was created after trial and error by downloading the entire web page and trying different combinations until getting the desired results. We tell *Mathematica* to show the results using small fonts with the command Style.

```
Style[
 TableForm[Import["http://www.uxc.com/review/UxCPrices.aspx", "Data"]
[[
   2, 2]]], Small]
```

```
Ux Month-End Prices as of November 28, 2016 [Change from previous
month]
1 US$=                              0.94226 €‡
U 3 O 8 Price (lb)                  $18.25         ( -0.50 )
NA Conv. (kgU)                      $5.85          ( Unch. )
EU Conv. (kgU)                      $6.40          ( Unch. )
NA UF 6 Price (kgU)                 $53.40         ( -1.45 )
NA UF 6 Value § (kgU)               $53.53         ( -1.31 )
```

```
EU UF 6 Value $ (kgU)                      $54.08          ( -1.31 )
SWU (SWU)                                  $47.00          ( -2.00 )
```

The previous command will work as long as the web page keeps the same structure as when we accessed it (December 7, 2016).

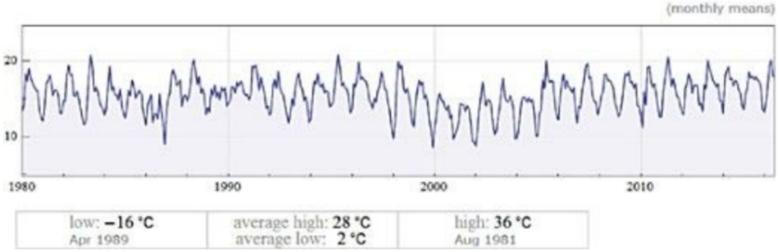
If you know HTML then you might be interested in accessing the contents of a web page by using the function: **`URLFetch`**.

You can also import data through **`WolframAlfa`** using the free-form input.

- In this example we analyze the evolution of the temperatures in Mexico City.

- We can expand the output by clicking on the + that appears in the upper right-hand corner. It will show us several sections available for selection. Here we have chosen the plot with the historical evolution of the average monthly temperatures by selecting from the contextual menu: **Paste input for ▸ Subpod Content**.

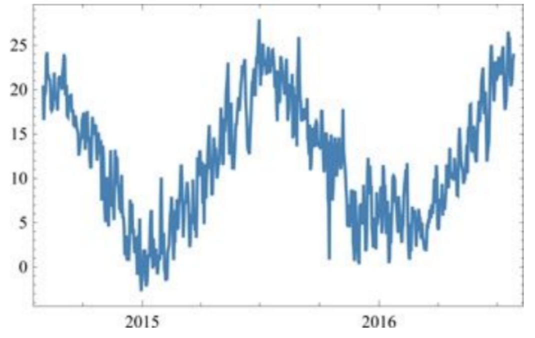
```
WolframAlpha ["Temperatures in Mexico City since 1980 to today",
 {{"TemperatureChart:WeatherData", 1}, "Content"}]
```



To access multiple data from several fields is better to use the computable data functions that we will discuss in Chapter 5. These functions will always return the same output structure, while WolframAlpha outputs to the same question do not always return the data in the same format.

- In this example we download and display the average temperature in a weather station (LESA) located in the province of Salamanca in Spain during a specific time period. We use the computable data function **`WeatherData`** combined with **`DateListPlot`**.

```
DateListPlot[WeatherData["LESA", "MeanTemperature",
  {{2014, 8, 1}, {2016, 7, 31}, "Day"}], Joined → True]
```



2.2.2 Semantic Import

Since *Mathematica* 10 there's a new function named **`SemanticImport`** that detects and interprets the type of objects imported.

- We import sales data for different cities and dates.

```
sales = SemanticImport["ExampleData/RetailSales.tsv"]
```

Date	City	Sales
1 Jan 2014	Boston	198
1 Jan 2014	New York City	220
1 Jan 2014	Paris	215
1 Jan 2014	London	225
1 Jan 2014	Shanghai	241
1 Jan 2014	Tokyo	218
2 Jan 2014	Boston	189
2 Jan 2014	New York City	232
2 Jan 2014	Paris	211
2 Jan 2014	London	228
2 Jan 2014	Shanghai	242
2 Jan 2014	Tokyo	229
3 Jan 2014	Boston	196
3 Jan 2014	New York City	235
3 Jan 2014	Paris	221
3 Jan 2014	London	229
3 Jan 2014	Shanghai	238
3 Jan 2014	Tokyo	236
4 Jan 2014	Boston	194
4 Jan 2014	New York City	237

showing 1–20 of 600

- If you move the cursor over the table above, you will see that the program identifies whether the data points are dates or cities, with the later ones displaying additional information. In *Mathematica*, objects with these properties are called entities (**`Entity`**) and have certain characteristics that we will explore later on. For example: If you hover the mouse over Boston, the following message will appear on the screen: