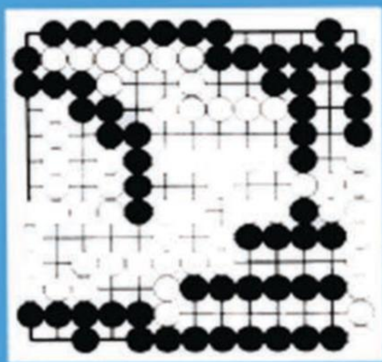
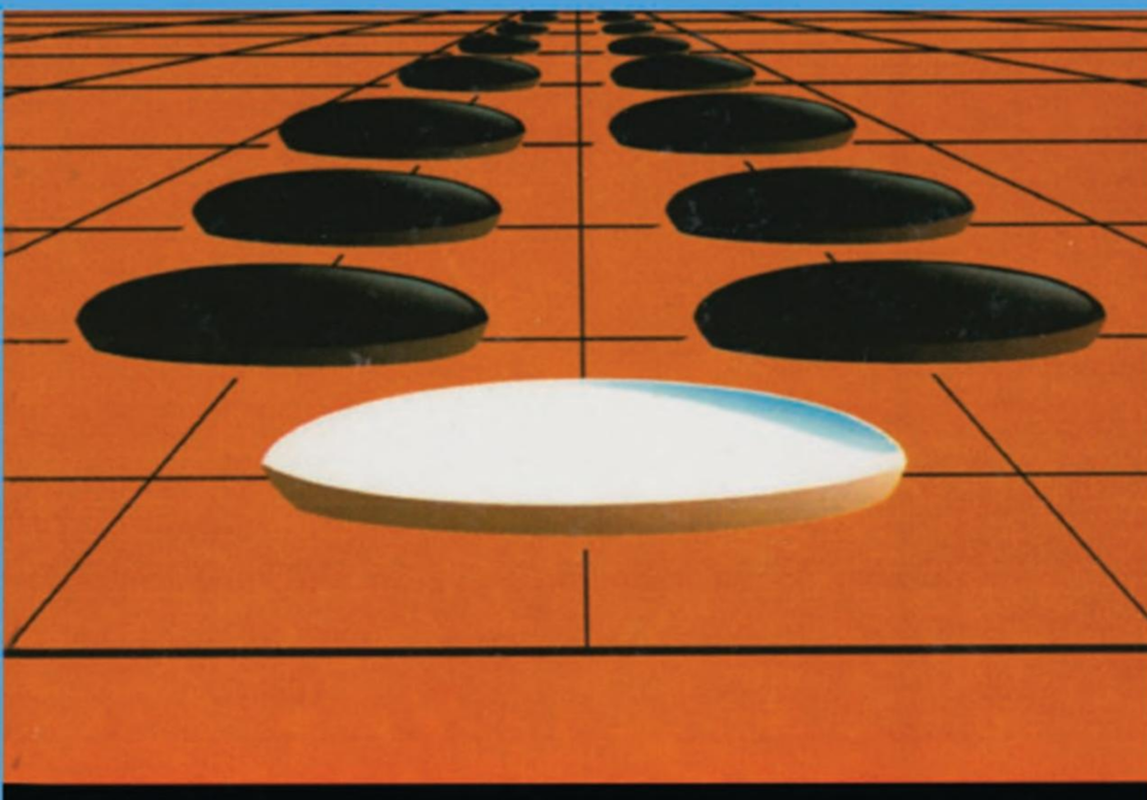
Mathematical Go
Chilling Gets the Last Point





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Foreword

This is an extraordinary book. The senior author (Elwyn Berlekamp) plays Go at only the 10-kyu level, and his colleague David Wolfe is rated an amateur shodan, yet they have developed techniques to solve late-stage endgame problems that stump top-ranking professional players. The problems typically offer a bewildering choice of similar-looking moves, each worth only one or two points, but with subtle priority relationships that cannot be adequately described by sente and gote. The solutions come out of combinatorial game theory, a branch of mathematics that Berlekamp helped develop. A Go player who masters its techniques can extract a one-point win from positions where the uninitiated will almost invariably lose or draw.

The theory presented in this book assigns each active area on the board an abstract value, then shows how to compare these values to select the optimum move, or add them up to determine the optimum outcome. Some of the values are familiar numbers or fractions, but most are more bizarre objects (arrows, stars, trees) quite unlike anything in the existing Go literature. From these abstractions, the reader will learn that positions seeming to have the same numerical value can be crucially different, while positions that appear completely different can be mathematically identical.

It should be emphasized that this book will not help the reader improve his opening or midgame strategy. Nor does it reveal any tactical secrets; locally, the tactics are all of the simple kind that a beginner can read out easily. In terms of practical benefit on the board, the most the reader can hope for is to get stronger by one point. (Of course, a lot of games are decided by one point.)

A Go player with an interest in mathematics, however, or a mathematician interested in Go, will relish this book, because it describes substantial connections between the two subjects which have hitherto been largely unrecognized. The theory developed is precise, rigorous, intellectually appealing, and demonstrably successful. As a bonus, there is a novel discussion of the mathematical rules of Go.

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Preface

History of combinatorial game theory and its role in Go

As a typical game of Go approaches its conclusion, the active regions of play become independent of one another. Play in each region is not affected by play in the others; although one player may make several successive plays in the same region while his opponent chooses to play elsewhere. The overall board position therefore may be regarded as a *sum* of disjoint partial board positions.

Combinatorial game theory has long been concerned with such sums of games. The first nontrivial result was perhaps the solution of the game of Nim by Bouton at Harvard in 1901. In the 1930s, his work was extended to a broader class of games by Sprague and Grundy; later, Guy and Smith [GS56] [Guy89] masterfully exploited this theory to obtain complete, closed-form solutions to a wide variety of games. Most of their work dealt only with *impartial* games, in which all pieces on the board have the same color and can be moved by either player. These games have no scores; rather, the outcome depends only on who got the last move. Nonetheless, the impartial theory later played a major role in the analysis of Dots and Boxes [BCG82, pp. 507–550], even though it is a contest over scores.

In the early 1970s, Conway discovered and axiomatized the theory of combinatorial games, including *partisan* games, which generalized impartial games so that players might control pieces or stones of opposite colors. Conway also discovered several theorems about the special role of games called *numbers*, including the *mean value theorem*, which provided a new formulation of earlier work by Hanner [Han59] and Milnor [Mil53] on sums of games with scores. Many results were published in *On Numbers and Games* [Con76]; more results plus numerous examples appeared in *Winning Ways* [BCG82], a two-volume treatise on which Berlekamp, Conway, and Guy had collaborated since the late 1960s.

These results made it possible to analyze games in which score determined the winner, and so it was tempting to apply the theory to Go. Initial attempts to apply combinatorial game theory to Go endgames in the 1970s failed, partly because certain crucial notions (particularly, *chilling* and *warming* operators) were not yet sufficiently well understood.

By the early 1980s, combinatorial game theory had developed powerful techniques for handling many other games in which board positions tend to break up into disjoint active regions. These included Hackenbush, Toads and Frogs, and Domineering. Here, the theory assigns an abstract *value* to each active region. In a simple inactive region, this value coincides with a numerical count of the score. In an active region, this value embodies the traditional Go player's notion of the *count*, as well as a considerable amount of additional information about the local situation. Each value depends only on a local analysis of the relevant partial board position. The value does *not* depend on who moves next; it takes all competent sequences of local play into account. And fortunately, the values of most simple positions all belong to a relatively small set of common simple values. Each value can occur in many different positions. So, to analyze the entire board position, one can compute the mathematical *sum* of all of the mathematical values representing the various regions of the board.

However, the practical utility of such a theory depends very much on the properties of the values which occur. If the local positions are sufficiently complicated that their values are intractable, then the sum can be very hard to analyze. Until the late 1980s, it had appeared as though the values which occur in even simple Go positions were so strange and intractable as to limit the theory's usefulness.

In 1984, Robin Pemantle [Pem84] calculated values for many $2 \times n$ and $3 \times n$ Domineering positions. Efforts to understand the properties of these values led to refined *warming* and *overheating* operators, and to a new game called Blockbusting, which can be viewed as a simplified variation of $2 \times n$ and $3 \times n$ Domineering. Chilled Blockbusting values are well-understood numbers, and, in Blockbusting, an appropriate warming operator inverts chilling.

This book

In 1989, it became clear that *chilling* reduces many common Go positions to familiar values, such as numbers, infinitesimals, and switches, and that chilling can be inverted by an appropriate warming operator. The authors of this book met and our collaboration began. Much of this book is based on Wolfe's dissertation [Wol91].

After chilling, many values which are common in late-stage Go endgames turn out to be very tractable, even though these values and their mathematical properties are not yet well known outside a small community of specialists. The values which appear are universal; they apply directly to Winning Ways games and to the ancient Hawaiian game of Konane. They facilitate a complete, precise, and thoroughly rigorous analysis of sums of Go endgames.

Acknowledgements

Since 1990, significant contributions to Mathematical Go have been made by a number of students from four incarnations of Berlekamp's combinatorial game theory courses, including Raymond Chen, David Moews, David Moulton, and Yonghoan Kim.

Our work on this book has also benefitted from the advice, criticism, and novel ideas of many others. We received support from the Massachusetts Institute of Technology, from the Center for Pure and Applied Mathematics of the University of California at Berkeley, and from Mitsubishi Electric Research Laboratory, where Berlekamp was a Visiting Fellow in the spring of 1992. During the past 3 years, both of us have given a number of formal and informal seminars and lectures on this subject. We have received some very constructive ideas and comments from many persons, including Eric Baum, David Kent, Kiga Yasuo, Martin Mueller, Jurg Nievergelt, Nick Patterson, Lisa Stewart, Herb Taylor, and Takizawa Takenobu. Stephen Parrott, James Davies, and Richard Bozulich reviewed early versions of this book and suggested so many major improvements that it was substantially rewritten. Richard Bozulich has also served as the facilitator, coordinator, and translator for our visits to Japan, and Fujisawa Kazunari has coordinated our interactions with the professional Go community at the Nihon Ki'in.

We have grown increasingly appreciative of the very different perspectives from which games are viewed by Go players, mathematicians, and computer scientists. People who routinely visualize the subject from more than one angle are rare, but they do exist. One such person was Bob High, who was President of the American Go Association at the time of his untimely death in a white water rafting accident on January 8, 1993. He was a strong amateur Go player who also had a keen interest in mathematics. He attended several of our seminars and learned a great deal about this subject by studying the papers and notes that we sent him. He then wrote two papers, [Hig91] and [Boz92, pp. 218–224], [Boz92, pp. 218–224], aimed at making some version of this material accessible to the Go player who had no

training in advanced mathematics. These manuscripts played a pivotal role in persuading us of the feasibility of this book.

We believe the study of mathematical Go is still in its infancy. Generalizations of the notions of *temperature* and *mean values* to include many types of Kos are already known. Progress is continuing rapidly, and we are hopeful that others will refine and extend the results presented in this book.

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Chapter 1

Introduction

Since you are reading this book, you are likely to fall into one of three categories:

- A mathematician interested in the applications of mathematics to games and Go.[1]

- A Go player interested in how mathematics might improve your game.

- A computer scientist interested in how to design or improve Go playing programs.

Mathematicians will find theorems about Go both subtle and surprising. Combinatorial game theory provides general methods for analyzing games, particularly games in which typical positions have separate pieces with limited interaction. This decomposition into pieces gives *group* structure, and this structure can be exploited to precisely analyze Go endgame positions that don't yield to the methods of the strongest Go players.

Go players can find quicker ways to improve their game than to read this book. Many of the positions analyzed do not tend to come up in contexts which take full advantage of the subtleties of the results. However, there are some wonderful lessons for the Go player. Sente (the worth of keeping the initiative), and gote (giving up initiative) are issues which are only vaguely understood by all but the finest Go players. This is in part because the

[1]A top-down overview of the rules to Go appears in Appendix A. In this book, Japanese or American style scoring rules are assumed (i.e., scoring territory minus prisoners). Extensions of the results to rules which score territory plus stones on the board are discussed in Appendices A and B. Also, a glossary of terms appears in Appendix F.

concepts are subtle, but also because the meaning of initiative is, by nature, vague and amorphous. The game theory does away with these concepts by providing clearer and more concrete methods, and these in turn will give the Go player a better understanding of sente and gote. Lastly, the technique of "playing the difference game" is basic to combinatorial game theory, and does not seem to exist in traditional Go theory; the Go player is encouraged to concentrate much of his/her attention on this technique given in Section 3.3.

For the computer scientist, the mathematical theory provides general and precise methods for simplifying local game trees in the endgame. These methods take into account the possibility that a player may move several times in a row in a local position when the opponent chooses to move elsewhere. Unfortunately, a virtually complete description of local game trees is required; the methods are therefore most applicable at a point when the typical game can be analyzed by brute force. Nonetheless, these methods are a start in a promising new direction. A good program will require a strong understanding of how to integrate what is known about nearly separated positions.

1.1 Why study Go

People want to understand the things that people like to do, and people like to play games. Perhaps that's all the justification needed to study games. But games also provide a concrete, self-contained framework in which to study mathematical and programming methods. In a game not only are the rules (and therefore the model) clear, but one measure of success is clear: How does a human or computer play relative to an experienced player with established records or ratings?

Go is particularly attractive to study because:

- Go's three or four thousand year old history and popularity throughout the world (particularly in Asia) means there are many experts on whom theories can be tested and from whom insights can be gained.

- Simple rules shorten the process of designing models, and make the game susceptible to mathematical analysis.

- Go poses new and more formidable challenges for the sorts of programming methods which have had great success in chess. Go is therefore an excellent testing ground for new artificial intelligence techniques.

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