Mathematics Education as a Research Domain: A Search for Identity

An ICMI Study
Book 1

Edited by

Anna Sierpinska and Jeremy Kilpatrick

Springer-Science+Business Media, B.V.

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TABLE OF CONTENTS

Part VI: Mathematics Education and Mathematics

FOREWORD

No one disputes how important it is, in today's world, to prepare students to understand mathematics as well as to use and communicate mathematics in their future lives. That task is very difficult, however. Refocusing curricula on fundamental concepts, producing new teaching materials, and designing teaching units based on 'mathematicians' common sense' (or on logic) have not resulted in a better understanding of mathematics by more students. The failure of such efforts has raised questions suggesting that what was missing at the outset of these proposals, designs, and productions was a more profound knowledge of the phenomena of learning and teaching mathematics in socially established and culturally, politically, and economically justified institutions – namely, schools.

Such knowledge cannot be built by mere juxtaposition of theories in disciplines such as psychology, sociology, and mathematics. Psychological theories focus on the individual learner. Theories of sociology of education look at the general laws of curriculum development, the specifics of pedagogic discourse as opposed to scientific discourse in general, the different possible pedagogic relations between the teacher and the taught, and other general problems in the interface between education and society. Mathematics, aside from its theoretical contents, can be looked at from historical and epistemological points of view, clarifying the genetic development of its concepts, methods, and theories. This view can shed some light on the meaning of mathematical concepts and on the difficulties students have in teaching approaches that disregard the genetic development of these concepts.

All these theories are interesting and important for a mathematics educator, but the knowledge needed to understand (and act upon) the phenomena of teaching and learning mathematics in the school institution must deal simultaneously with cognition, social institutions, and mathematics in their complex mutual relationships. Theories cannot just be imported ready-made from other domains: Original theories have to be elaborated if we are to understand better the problems of mathematics education.

Efforts in this direction have given rise to a growing body of research that is called research in mathematics education or research in the didactics of mathematics. People who do this research, whether working in university mathematics departments or not, are always interested in maintaining good relations with other communities, including the community of mathematicians. But good relations can only be built on mutual understanding. The work reported herein has been aimed at contributing to the growth of such understanding.

The present book is one of a series of publications resulting from ICMI Studies. The first such study was launched by the International Commission on Mathematical Instruction (ICMI) in 1984. Its theme was The Influence of Computers and Informatics on Mathematics and Its Teaching. Subsequent studies were concerned with such themes as: School Mathematics in the 1990s, Mathematics as a Service Subject, Mathematics and Cognition, Assessment in Mathematics Education, and Gender and Mathematics. Reports of these studies have been published by Cambridge University Press, Kluwer Academic Publishers, and the University of Lund Press. A study on the problems of teaching geometry has been conducted, and studies are planned on the relations between the history and the pedagogy of mathematics and on the specific problems of tertiary mathematics education.

The theme of the ICMI Study reported in this book was formulated as a question: 'What is Research in Mathematics Education and What are Its Results?' No single agreed-upon and definite answer to the question, however, is to be found in these pages. What the reader will find instead is a multitude of answers, various analyses of the actual directions of research in mathematics education in different countries, and a number of visions for the future of that research. Given this diversity, the reader may ask why a study on this theme was held.

In his letter of invitation to the International Program Committee for the Study Mogens Niss (the Secretary of ICMI and an ex-officio member of the committee) wrote:

> Mathematics education has been in existence as a field of academic research for roughly a century. Its growth within the last three decades has been enormous not only in the number of research studies undertaken but also in the number of researchers, places in which scholarly work is being done, and of academic fields represented in that work. It is time to review the state of the field and to begin a dialogue with other scientific communities, in particular the mathematical research community. The International Commission on Mathematical Instruction is undertaking a study on research in mathematics education that will describe the work being done, analyze developments and trends within the field, and assess what research in mathematics education has produced. Major outcomes of the study will be presented at the International Congress of Mathematicians in Zürich in 1994.

Have the goals as defined above been met? As it turned out, the participants at the Study Conference were more interested in looking at the evolution of ideas and at the problems yet to be solved than in taking stock of the field's achievements or results. The questions of assessing what research in mathematics education has produced and of developing criteria for the quality of research were particularly difficult, and the divergence of opinion on these questions was particularly wide. Moreover, concerning the question of a dialogue with other scientific communities, the program committee found that in some countries, the mathematical research community is not the main interlocutor of mathematics education researchers. For example, in the USA, much mathematics education research is conducted within colleges, schools, or departments of education.

Preliminary results from the study were presented at the ICM 1994 in Zürich. Although it was then too early to synthesize the major outcomes of the study, the presentation clarified for many of the mathematicians in the audience some of the differences between research in mathematics education and such domains of activity as the popularization of mathematics, historical studies of mathematics, inventing challenging problems for more gifted students, writing textbooks, or teaching in an innovative way. A subsequent presentation and discussion at the Eighth International Congress on Mathematics Education in Seville in 1996 was valuable in putting the final chapters into context.

The study officially started with a meeting of the program committee (for which we served as co-Chairs) on 19 August 1992, during the Seventh International Congress on Mathematical Education in Québec. Members of the program committee then collaborated to produce a 'Discussion Document' (reprinted in Part I), describing the reasons for the study and its aims, and generally laying a framework for discussion in the form of five questions: 'What is the specific object of study in mathematics education?'; 'What are the aims of research in mathematics education?'; 'What are the specific research questions or *problématiques* of research in mathematics education?'; 'What are the results of research in mathematics education?'; and 'What criteria should be used to evaluate the results of research in mathematics education?'. The Discussion Document appeared in the December 1992 *Bulletin of the International Commission on Mathematical Instruction* (No. 33). In early 1993, it was also published in a number of international mathematics education journals, including *L'Enseignement Mathématique*, *Educational Studies in Mathematics*, *Recherches en Didactique des Mathématiques*, and the *Zentralblatt für Didaktik der Mathematik.* A shortened version appeared in the *Journal for Research in Mathematics Education.*

The Discussion Document contained a call for papers to be submitted by 1 September 1993. The papers and other expressions of interest were used to develop a program and invitation list for a Study Conference organized in Washington, DC, from 8 to 11 May 1994. The conference was sponsored by the ICMI together with the Mathematical Sciences Education Board of the U.S. National Academy of Sciences and the University of Maryland. The participants included more than 80 people from around the world (see the participant list in Part I). Most of the sessions were held at the Adult Education Center of the University of Maryland in College Park, but part of the conference was a half-day symposium for invited U.S. mathematicians and mathematics educators at the National Academy of Sciences building in Washington, DC.

The conference comprised plenary sessions, working groups that addressed the five questions posed in the Discussion Document, and paper sessions in which specific examples of research were discussed in the light of the questions of the study. The plenary sessions were devoted to general themes: definitions of the domain, problems of balancing theory and practice in research, the place of teaching design in research in mathematics education, the training of researchers, and views of mathematics education research held by mathematicians. The conference did not end with a final resolution of the type: *This is what research in mathematics education should be.* Rather, the outcome was more like: *Yes, in spite of all the differences that divide mathematics education researchers (in terms of theoretical approaches, views on relations between theory and practice, philosophies of mathematics, etc.), they still constitute a community, and it is necessary to search for what constitutes its identity.*

The present book is not a set of proceedings from the conference. Most of the authors of the chapters in Parts II to VI, however, participated in the conference, and their chapters reflect the discussions there. It was not possible to organize the book along the five questions posed in the Discussion Document because the authors often addressed all five. In fact, there are two categories of chapters: those that respond, in a way, to the whole Discussion Document, and those that focus on some specific question.

The book is organized as follows: After a section containing materials related to the ICMI Study Conference (Part I), several chapters offer a broad perspective on the definition of mathematics education as a research discipline (Part II). Within this broad view, Part III focuses on the goals, orientations, and envisioned or actual outcomes of mathematics education research. The chapters in Part IV then provide examples or counterexamples for the general theses offered in the previous two parts. They contain descriptions and analyses of the evolution of different research paradigms in mathematics education in different countries or cultures.

Parts V and VI are shorter. Part V is devoted to the question of criteria for judging the quality of research in mathematics education, and Part VI is concerned with the interactions, particularly along social and epistemological dimensions, of mathematics education with mathematics and mathematicians. Both of these treat difficult topics. In fact, one important outcome of the study was a greater recognition of the reasons for the difficulty of the questions that the study posed, leading possibly to another set of questions better suited to the actual concerns and research practices of mathematics education researchers.

ACKNOWLEDGMENTS

We would like to express our appreciation to the other members of the International Program Committee for their help in organizing the Study Conference and preparing the present volume:

Nicolas Balacheff, IMAG & Université Joseph Fourier, Grenoble, France
Willibald Dörfler, Universität Klagenfurt, Austria
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Toshio Sawada, National Institute for Educational Research, Japan
Anna Sfard, University of Haifa, Israel
Heinz Steinbring, Universität Bielefeld, Germany

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Anna Sierpinska and Jeremy Kilpatrick

PART I

THE ICMI STUDY CONFERENCE

THE ICMI STUDY CONFERENCE

In this part of the volume, materials are presented that pertain to the Study Conference 'What is Research in Mathematics Education, and What Are Its Results?' held in Washington, DC, in May 1994. The chapters in subsequent parts were initially prepared in response to a Discussion Document, which is reproduced in the next section. The document also served as a point of departure for the conference, and most of the chapters were revised, at times extensively, in light of discussions there. The second section below lists the conference participants. The remainder of Part I contains the reports of the five working groups, which met for several hours over three days of the conference. Each group was asked to discuss and respond to one of the five principal questions in the Discussion Document. The working group reports are followed by a chapter by Alan Bishop that is based on a paper he and Dudley Blane prepared for the conference and on the summary remarks he made at the final session.

DISCUSSION DOCUMENT

The following people have contributed to the present document: N. Balacheff, A. G. Howson, A. Sfard, H. Steinbring, J. Kilpatrick, and A. Sierpinska.

As mathematics education has become better established as a domain of scientific research (if not as a scientific discipline), exactly what this research is and what its results are have become less clear. The history of the past three International Congresses on Mathematical Education demonstrates the need for greater clarity. At the Budapest congress in 1988, in particular, there was a general feeling that mathematics educators from different parts of the world, countries, or even areas of the same country often talk past one another. There seems to be a lack of consensus on what it means to be a mathematics educator. Mathematics education no longer means the same as *didactique des mathématiques* (if it ever did). French *didacticiens* refuse to translate their *didactique des mathématiques* into 'mathematics education': a special English edition of the journal *Recherches en Didactique des Mathématiques* bears the title 'Research in *Didactique* of Mathematics.' *Die Methodik* (or the Polish *metodyka,* the Slovak *metodika,* and the like) have become obsolete. Does *research* mean the same as *recherche* or *investigación?* How do these words translate into other languages? Standards of scientific quality and the criteria for accepting a paper vary considerably among the more that 250 journals on mathematics education published throughout the world.

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Despite this lack of consensus, publications appear that endeavor to depict the 'state of the art' in mathematics education research. Individuals try to construct didactical theories. But reviewers never have trouble demonstrating the one-sidedness or incompleteness of such publications. Attempts to describe research in mathematics education or *didactique des mathématiques* or whatever other name is used may resemble the accounts of the legendary blind men exploring the legs of a huge elephant.

The ICMI Study *What is research in mathematics education, and what are its results?* does not seek to describe the state of the art. Nor does it intend to tell anyone what research in mathematics education is or is not, or what is or is not a result. Instead, the organizers of the study propose to clarify the different meanings these ideas have for mathematics educators – to pinpoint the different perspectives, goals, research problems, and ways of approaching problems. The study will bring together representatives of the different groups of researchers, allow them to confront one another's views and approaches, and seek a better mutual understanding of what we might be talking about when we speak of research in mathematics education.

SOME QUESTIONS ABOUT RESEARCH

Such a wide-ranging discussion is badly needed in a community increasingly divided into specialized groups and cliques that are not always tolerant of each other. Besides mutual understanding within the community, however, there is also a need to explain the domain to representatives of other scientific communities, among which the community of mathematicians seems to be the most important. Nicolas Balacheff has observed:

> Most of us want to develop this research field within the academic community of mathematicians; this implies both the explanation of our purpose on a social ground (is there any need to develop such research?) and its relevance within the narrow academic world. For this reason, although it is not my sole concern, I have in mind the question of scientific standards, theses, publications, congresses, the employment of young academics in the field, and the connection between our research and research done in other fields.

Thus we need an 'inner' identification of the research domain of mathematics education, as well as an outer vision from the perspective of other domains.

One external domain, for example, is sociology. How is mathematics education organized and institutionalized? Where is research on mathematics education conducted? Where are theses on mathematics education defended? If a mathematics educator employed by a mathematics department has acquired his or her habilitation degree in, say, a department of pedagogy or philosophy (such a degree being unavailable at the employing institution), is he or she accepted as a full member of the community of mathematicians that awards doctoral or master's degrees in mathematics? Are mathematics educators viewed as a part of the mathematics community? Similar questions arise when research in mathematics education is surveyed from other domains, including history, philosophy, anthropology, and psychology.

An approach from both within and outside the field of research in mathematics education raises the following questions, among others, to be discussed:

(1) What is the specific object of study in mathematics education?
The object of study (*der Gegenstand*) in mathematics education might be, for example, the teaching of mathematics; the learning of mathematics; teaching–learning situations; didactical situations; the relations between teaching, learning, and mathematical knowledge; the reality of mathematics classes; societal views of mathematics and its teaching; or the system of education itself.

If a mathematics educator studies mathematics, is it the same object for him or her as it is for a mathematician who studies mathematics? What is mathematics as a subject matter? What is 'elementary mathematics'? Analogous questions could be asked concerning the learner of mathematics as an object of study. Is it the same object for a mathematics educator as it is for a psychologist or a pedagogue? Is the mathematics class or the process of learning in the school viewed in the same way by a mathematics educator and a sociologist, anthropologist, or ethnographer? Are questions of knowledge acquisition viewed the same way by a mathematics educator and an epistemologist?

The variety of activities offered at the ICMEs certainly distinguishes these congresses from, say, the international congresses of mathematicians. ICME 7 was compared by some to a supermarket. Is there a unity in this variety? What gives unity to different kinds of study in mathematics education? Is this the object of research? Or is the object of research perhaps not even something held in common? Might the commonality lie in the pragmatic aims of research in mathematics education?

(2) What are the aims of research in mathematics education?
One might think of two kinds of aims: pragmatic aims and more fundamental scientific aims. Among the more pragmatic aims would be the improvement of teaching practice, as well as of students' understanding and performance. The chief scientific aim might be to develop mathematics education as a recognized academic field of research.

What might the structure of such a field be? Would it make sense to structure it along the lines of mathematical subject matter (e.g., the didactics of algebra or the didactics of geometry), of various theories or approaches to the teaching and learning of mathematics, or of specific topics or *problématiques* (research on classroom interaction and communication, research on students' understanding of a concept, etc.)?

Both kinds of aims seem to assume that it is possible to develop some kind of professional knowledge, whether that of a mathematics teacher, a mathematics educator, or a researcher in mathematics education. The question arises, however, whether such professional knowledge can exist at all. Is it possible to provide a teacher, say, with a body of knowledge that would, so to say inevitably, ensure the success of his or her teaching? In other words, is teaching an art or a profession (*un métier*)? Or is it perhaps a personal conquest? As Luigi Campedelli used to say, '*La didattica è, e rimane, una conquista personale*'.

What does successful teaching depend on? Are there methods of teaching so sure, so objective, that they would work no matter who the teacher and students were? Are there methods of teaching that are teacher-proof and methods of learning that are student-proof? If not, is there anything like objective fundamental knowledge for a researcher in mathematics education – something that any researcher could build upon, something accepted and agreed upon by all? Or will the mathematics educational community inevitably be divided by what is considered as belonging to this fundamental knowledge, by philosophies and ideologies of learning, by what is considered worth studying?

Many mature domains of scientific knowledge have become highly specialized into narrow subdomains. Is this the fate of mathematics education as well? Or rather, in view of the interdisciplinary nature of mathematics education, must every researcher necessarily be a 'humanist,' knowing something of all domains and problems in mathematics education?

Although we aim at clarifying the notion of research in mathematics education as an academic activity, we should be careful not to fall into needlessly 'academic' debates. After all, the ultimate goal of our research may be for a specific teacher in a specific classroom to be better equipped to guide his or her students as they seek to understand the world with the help of mathematics.

(3) What are the specific research questions or problématiques *of research in mathematics education?*

Mathematics education lies at the crossroads of many well-established scientific domains such as mathematics, psychology, pedagogy, sociology, epistemology, cognitive science, semiotics, and economics, and it may be concerned with problems imported from these domains. But mathematics education certainly has its own specific *problématiques* that cannot be viewed as particular cases or applications of those from other domains. One question the ICMI study might address is that of identifying and relating to each other the various *problématiques* specific to mathematics education.

There are certainly two distinct types of questions in mathematics education: those that stem directly or almost directly from the practice of teaching and those generated more by research. For example, the question of how to motivate students to learn a piece of mathematics (inventing interesting problems or didactic situations that generate a meaningful mathematical activity), or how to explain a piece of mathematics belong to the first kind. The question of identifying students' difficulties in learning a specific piece of mathematics is also directly linked to practice. But questions of classifying difficulties, seeing how widespread a difficulty is, locating its sources, or constructing a theoretical framework to analyze it already belong among the research-generated questions. The problem is, however, that a difficulty may remain unnoticed or poorly understood without an effort to answer questions of the latter type; that is, without more fundamental research on students' understanding of a topic. Is it, therefore, possible to separate so-called practical problems from so-called research-generated problems?

Is it possible to admit the existence of two separate types of knowledge: the theoretical knowledge for the scientific community of researchers and the practical knowledge useful in applications for teachers and students? It might be helpful to reflect on the nature of these two types of knowledge, on relations between them, and on whether it would be possible to have a unified body of knowledge encompassing them both.

(4) What are the results of research in mathematics education?
Any result is relative to a *problématique,* to the theoretical framework on which it is directly or indirectly based, and to the methodology through which it was obtained. This relativity of results, though commonplace in science, is often forgotten. One often interprets findings from biology, sociology, or mathematics education as if they were a kind of absolute truth. The reason may be that in these domains we really want to know the truth and not simply whether, if one proposition is true, some other proposition is also true. Questions of biology, sociology, or mathematics education can be of vital importance and fundamental to our survival and well-being.

Two types of 'findings' can be distinguished in mathematics education: those based on long-term observation and experience and those founded on specially mounted studies. Are the former less 'scientific' than the latter? Geoffrey Howson offers an example:

> In the seventeenth century, Spinoza set out three levels of understanding of the rule of three (which, incidentally, can be viewed as an elaboration of the instrumental-relational model of Skemp and Mellin-Olsen expounded over three centuries later). This, like the well-known levels of the van Hieles, was based on observation and experience. On the other hand, for example, CSMS [Concepts in Secondary Mathematics and Science] used specially mounted classroom studies to develop and investigate similar hierarchies of understanding. Do we rule out the work of Spinoza as research in mathematics education? If we do, then we lose much valuable knowledge, especially that resulting from curriculum development. If we do not, then it becomes difficult to find a workable definition [of research in mathematics education].

Balacheff points out that it may be difficult to contrast, in this way, the hierarchies obtained by the van Hieles and the CSMS group. Besides the different ways in which these hierarchies were obtained, the van Hieles and the CSMS group may not have been asking the same kind of question. 'What are these questions?' asks Balacheff. 'What is the validity of the answers they provide? How is it possible to relate them?'

Can a new formulation of an old problem be a research result? Can a problem be a result? Or a questioning of the theory related to a problem, a methodology, or a whole *problématique?* Can a concept be a result? It might be useful to have a definite categorization of the things we do in mathematics education, and of the things we thereby 'produce'.

Most people would probably agree that *making empirical investigations* is research. But is the *doing of practical things* research? Is *thinking* research? Can these activities be separated? Can a result be obtained without thinking and the doing of practical things? Should mathematics education be considered a science? Perhaps it is a vast domain of thought, research, and practice. What qualifies a domain of activity as scientific is the kind of validation and justification methods it uses. Proofs and experiments are considered scientific. But there are thoughts not validated in either of these ways that are valuable because they are filled with meaning.

What examples are there of what we consider results in mathematics education to be? What do we know today that we did not know before? What have we learned about the processes of learning and teaching? What do we know about mathematics that mathematicians were not aware of before?

Can we identify some categories of results? One category might be *economizers of thought.* Any facts, laws, methods, procedures, or theories that are general enough to direct our experience and predict its results will give us increased power over our teaching and learning. Another category might be *demolishers of illusions.* Results that undermine our beliefs and assumptions are always valuable contributions to the field. A third category might be *energizers of practice.* Teachers welcome research that helps them understand what they teach and provides them with ideas for teaching. The development of teaching materials, activities, and challenging problems belongs to this category. Other categories of results might emerge from epistemological, methodological, historical, and philosophical studies.

(5) What criteria should be used to evaluate the results of research in mathematics education?

How do we assess the validity of research findings? How do we assess their worth? Should we use the criterion of relevance? What about objectivity? Or originality? Should we consider the influence research has had on the practice of teaching? What other criteria should we use?

The first problem is to clarify the meaning of terms such as *truth, validity,* and *relevance* in the context of mathematics education. A related issue is the question of what is knowledge as such. This is an even more fundamental question than that of validation. If we knew what kind of knowledge mathematics education aims at, we would be better equipped for answering the question of methods of validation.

It is also useful to understand the ways in which research results are used. How have the results of research in mathematics education been applied? How do teachers use the research? How do policy-makers use it? By clarifying the uses to which research is put, can we develop better criteria for assessing its validity?

PARTICIPANTS

The following people took part in the conference:

Josette Adda (France)
Gilbert Arsac (France)
Michèle Artigue (France)
Ferdinando Arzarello (Italy)
Nicolas Balacheff (France)
Mariolina Bartolini-Bussi (Italy)
Jerry Becker (USA)
Alan Bishop (Australia)
Ole Björkqvist (Finland)
Dudley Blane (Australia)
Morten Blomhøj (Denmark)
Lenore Blum (USA)
Paolo Boero (Italy)
Guy Brousseau (France)
Margaret Brown (UK)
Ronnie Brown (UK)
Jere Confrey (USA)
Beatriz D'Ambrosio (USA)
Willibald Dörfler (Austria)
Tommy Dreyfus (Israel)
Nerida Ellerton (Australia)
Paul Ernest (UK)
James Fey (USA)
Claude Gaulin (Canada)
Gunnar Gjone (Norway)
Juan Godino (Spain)
Pedro Gómez (Colombia)
Koeno Gravemeijer (Netherlands)
Miguel de Guzmán (Spain)
Gila Hanna (Canada)
Kath Hart (UK)
Yoshihiko Hashimoto (Japan)
Gillian Hatch (UK)
Milan Hejny (Czech Republic)
Patricio Herbst (USA)
James Hiebert (USA)
Bernard Hodgson (Canada)
Geoffrey Howson (UK)
Brian Hudson (UK)
Bengt Johansson (Sweden)
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Iman Osta (Lebanon)
Fidel Oteiza (Chile)
Michael Otte (Germany)
Erkki Pehkonen (Finland)
David Pimm (UK)
Susan Pirie (UK)
Norma Presmeg (USA)
Thomas Romberg (USA)
Linda Rosen (USA)
Toshio Sawada (Japan)
Yasuhiro Sekiguchi (Japan)
Anna Sfard (Israel)
Shizumi Shimizu (Japan)
Christine Shiu (UK)
Anna Sierpinska (Canada)
Edward Silver (USA)
Claudie Solar (Canada)
Judith Sowder (USA)
Lynn Steen (USA)
Heinz Steinbring (Germany)
Hans-Georg Steiner (Germany)
Juliana Szendrei (Hungary)
Gérard Vergnaud (France)
Virginia Warfield (USA)
David Wells (UK)
Erich Wittmann (Germany)

In addition, invited officials from the National Academy of Sciences, the National Science Foundation, the University of Maryland, and various organizations in the Washington, DC, area attended plenary sessions.

WHAT IS THE SPECIFIC OBJECT OF STUDY IN MATHEMATICS EDUCATION?

REPORT OF WORKING GROUP 1

Leader: Jere Confrey
Reporters: Dudley Blane and Anna Kristjánsdóttir[1]

The work of the group started with two presentations, one by Anna Sfard and the other by Christine Keitel. Sfard spoke on whether mathematics is the same object for the mathematics educator as it is for the mathematician.[2] For a mathematician, she said, mathematics is a *disciplina mentis,* a way of understanding and knowing. The views of mathematics educators on what mathematics is have been changing over the years: Platonism, constructivism, social constructivism, situated cognition, and others. The question is: What are the educational implications of Platonism or of any other philosophy of mathematics? 'Misconception' makes sense within a Platonistic frame of mind; 'learning in groups' is a consequence of a belief in social constructivism, and so on. The much-debated situated-cognition approach to the learning and teaching of mathematics views learning as 'legitimate peripheral participation in a social practice'. It postulates that mathematics will be learned by more students if taught in integration with other subjects and in real-world contexts and if practical learning apprenticeships are developed. Each of these philosophies poses its own problems and dilemmas. In particular, situated cognition leads to the question: Doesn't contextualization contradict abstraction, and isn't mathematical thinking based mainly on abstraction? Sfard concluded her talk by saying that, because philosophical positions towards mathematics have implications for the decisions we make in research on teaching, learning, and creating mathematics, we have to be aware of these positions. We must make them explicit and known to ourselves and others.

Christine Keitel spoke on common sense and mathematics. One of the questions she posed was: How are mathematics and common sense related? It is known that mathematics shapes common sense (e.g., by the use of numbers in speaking about values, so that 80% on a test means *good;* 20% means *bad*). Does learning mathematics mean overcoming common sense? Is it true that scientific thinking is based on undoing commonsense thinking? On the other hand, maybe it makes sense to speak of a *mathematical common sense* that would be worthwhile developing in students. She proposed a change from seeing the task of mathematics education as modifying, preparing, or transforming ('elementarizing') scholarly mathematics for teachers and students to seeing it as an investigation starting from an analysis of the social system, including the educational system, in which the relationship between mathematics and society is embedded. This analysis is then a basis on which a new understanding, organization, and systematic conceptualization of the 'objects' of research in mathematics education could be obtained. In particular, this new approach would lead us to studying 'the literature' of mathematics instead of its 'grammar'.

Mathematics as an Object of Research in Mathematics Education

The theme of the group was designed broadly, but, perhaps as a result of the questions raised by the two presentations in the group, the discussion focused on whether mathematics can be regarded as an object of research in mathematics education, and if so, in what sense?

The participants generally agreed with Sfard's point that if mathematics is an object of research in mathematics education, it is not the same object for mathematics educators as it is for mathematicians, philosophers, and logicians. The discussion brought up several aspects and consequences of this difference.

Borrowing Charles Morris's distinction between syntactics, semantics, and pragmatics, one could say that while philosophers, logicians, and mathematicians may make abstractions from the learning and uses of mathematics and occupy themselves only with its semantic and syntactic aspects, mathematics educators have to practice the 'pragmatics' of mathematics: they have to study the relations between mathematics and its learners and users. That implies the necessity, in mathematics education, to take into account the social, cultural, and institutional contexts of learning and teaching mathematics. It also requires us to broaden the notion of mathematics to encompass mathematical practices that are relevant for the society at large but are other than those of university mathematicians. There is, in certain cultures, a kind of mathematical common sense (e.g., the notion of zero forms a part of it) shared by the society. Is this common sense worth developing, or is it so incompatible with the mathematics that we want to teach children that it needs to be regarded as an enemy?

'What is the nature of mathematics?' was a much debated question in this group. Some participants voiced their doubts as to whether this is a relevant question for mathematics education, which challenged the opponents to formulate stronger arguments.

It was not obvious to everyone that the notion of mathematics should be broadened to the extent of encompassing, for example, the mathematics of carpenters (who, according to Wendy Millroy's study, reported in a 1992 monograph of the *Journal for Research in Mathematics Education,* do not themselves consider what they are doing as mathematics). But, it was said, if we see mathematics as 'the science of pattern and order', then there is a reason to accept certain professional practices and tacit competences as mathematics. The debate on the nature of mathematics continued as the topic of situated-cognition learning theory was discussed.

Situated Cognition

The situated-cognition theory mentioned by Sfard in her presentation raised a debate in the group. The question was: Is situated cognition a learning theory that can be reasonably adopted by mathematics education? What might our reservations be about such a theory?

Situated-cognition theory has become known in North America through the works of Jean Lave. It has remained relatively unknown for example, in France. The reactions of the French participants to Sfard's presentation of its basic tenets were the following: C. Margolinas said that similar ideas had been proposed 20 years before by G. Brousseau in France (she mentioned particularly the role attributed to the milieu in Brousseau's theory of didactic situations). G. Vergnaud people use mathematical ways of thinking for organizing their personal experience and how they organize and develop those ways of thinking and reflecting.

Sfard used a comparison, and her conclusion was that it is necessary for mathematics education to clarify its position with respect to mathematical knowledge:

Our ultimate objective is the enhancement of the learning of mathematics. However, as researchers, we are producing knowledge (about how people create mathematics for themselves), and as educationists, we are inducing certain knowledge in others. Therefore, we are faced with the crucial question: What is knowledge, and, in particular, what is mathematical knowledge for us? Here, we find ourselves caught between two incompatible paradigms: the paradigm of human sciences (to which we belong as mathematics education researchers) and the paradigm of mathematics. These two are completely different: Whereas mathematics is a bastion of objectivity, of clear distinction between *TRUE* and *FALSE* (for practicing mathematicians at least), there is nothing like that for us. For us, mathematics is social, intersubjectively constructed knowledge.... But we feel somewhat schizophrenic between these two paradigms because our commitment to teach *MATHEMATICS* makes us, to some extent, dependent on [the philosophies of mathematics held by mathematicians]. Therefore, we must make the problem explicit and cure the illness by making clear where we stand with respect to the issue of mathematical knowledge.

R. Mura gave a very personal view of the topic of the group and the discussions:

I came to this group because I had written a paper for this conference on mathematics educators' definitions of *mathematics education*. This experience, in a sense, makes it more difficult for me now to give my own definition. In general, I would say that the object of mathematics education is less problematic than the object of mathematics. Maybe what we want to concentrate on is the border cases. Some of us have had our own work challenged as not being research in mathematics education. The first issue (work not really being *research*) is common to all the social sciences and humanities. The second issue (not research in *mathematics education*) is for us to decide. Some of us have been criticized by people saying that our work is in linguistics, in women's studies, in philosophy, and so forth, rather than in mathematics education. Could we behave in a way similar to our colleagues in mathematics and say that mathematics education is what mathematics educators do?

WHAT ARE THE AIMS OF RESEARCH IN MATHEMATICS EDUCATION?

REPORT OF WORKING GROUP 2

Leader: Ole Bjorkqvist
Reporters: Pedro Gómez and Thomas Romberg

This group was asked to consider at length possible answers to the question of aims in order to clarify the notion of research in mathematics education as an academic activity. In particular, the group was asked to examine 'two kinds of aims: pragmatic aims and more fundamental scientific aims'.

The issue was addressed by first considering two papers; then each was discussed. The two papers served to focus the groups' thoughts on the question. Gilah Leder addressed the diversity of research aims in the field of mathematics education.[3] She argued that the purposes for doing research have changed during the past half century; that scholars who conduct research have diverse, often pragmatic, and occasionally scientific perspectives about the aims of their research; that the perspectives have been shaped throughout history by the Eurocentric male-dominated majority culture; and that three pragmatic considerations (esteem for research within academic circles, social or cultural priorities, and allocation or availability of resources) often shape the kinds of research carried out.

Julianna Szendrei presented a classification of four different kinds of 'results' produced by research in mathematics education. The paper had been jointly prepared with Paolo Boero.[4] Furthermore, she related each type of result to pragmatic or fundamental scientific aims and to three intended outcomes: energizers of practice, economizers of thought, and demolishers of illusions. The first type of a result, which she labeled *innovative patterns,* would include teaching materials, reports about projects, and so forth. Obviously, such results have practical consequences and are designed to 'energize practice'. The second type of result is *quantitative information* about the choices concerning the teaching of a peculiar mathematical content, general or specific learning difficulties, possible relationships with factors influencing learning, and so forth. Such results have both practical and scientific aims and are designed to both 'energize practice' and 'demolish illusions' about current practices or beliefs. The third type of result is *qualitative information* about the consequences of some methodological or content innovation, and so on. These results are related primarily to scientific aims and designed to 'demolish illusions'. The final type of result is *theoretical perspectives* regarding reports that reflect on descriptions and classifications or interpretations of phenomena, models, historical or epistemological analyses of content, and so on. Obviously, such results have scientific aims designed to 'economize thought' and perhaps to 'demolish illusions'.

Following these presentations, the members of the working group entered on three occasions into a spirited discussion of the ideas presented in the conference position paper and these papers. The discussion was also fueled by the plenary talks on the balance between theory and practice, the social and cultural conditions under which each of the members of the working group operate, and the other sessions and discussions each member participated in during the conference. The contents of the discussions ranged over several issues related to all five working groups. Working group members submitted written comments to summarize thoughts; a first-draft synthesis was written, points discussed, and format agreed upon, and after two chances to revise the report, this final version was completed.

Throughout the sessions some issues related to aims emerged again and again. These can be summarized under three headings:

Research as a Human Process

The term *research* refers to a process – something people do, not objects one can touch or see. Furthermore, research cannot be viewed as a set of mechanical procedures to be followed. Rather it is a craft practiced by scholarly groups whose members have agreed in a broad sense on what procedures are to be followed and on the criteria for acceptable work. These facts led us to the following assertions:

- An important aim of all research should be 'to satisfy the curiosity of the researcher about some situation'. [Note, first, that *situation* is used here to refer to all the objects of study being specified by Working Group 1, and second, that the researcher's 'interest' is often influenced by policymakers, school boards, and so forth.]
- That curiosity should lead to an understanding of situations. Many situations involve the teaching and learning of mathematics in classrooms with the expectation that understanding such situations could lead to improved practice. Other situations may be outside schools and may lead to improvements in the workplace. In this regard, we recognize that there are several levels of understanding such as describing or explaining.
- The actual situations a researcher might investigate are embedded in the institutional, social, political, and cultural conditions in which the researcher operates. The personal aims of different researchers will differ because of different beliefs and their membership in particular scholarly groups with differing notions about disciplined inquiry. [Note also that these groups may have differing aims.] And there may be a difference in the aims for a particular study and a set of studies or a research program.

One member of the group proposed that Figure 1 be used to illustrate the variety of things a scholar may be influenced by when deciding on the aim of a particular study.

Diversity of Aims

The teaching and learning of mathematics in schools at any level in any country is complex. When one also considers mathematics outside schools and in adult education, the complexity is compounded. These facts, when added to individual curiosity, make it clear that there has been and will be a diversity of aims. Individual studies and even research programs conducted by different persons or groups will inevitably have different aims. The concern of the group was that such diversity might make impossible any coherent compilation of findings.

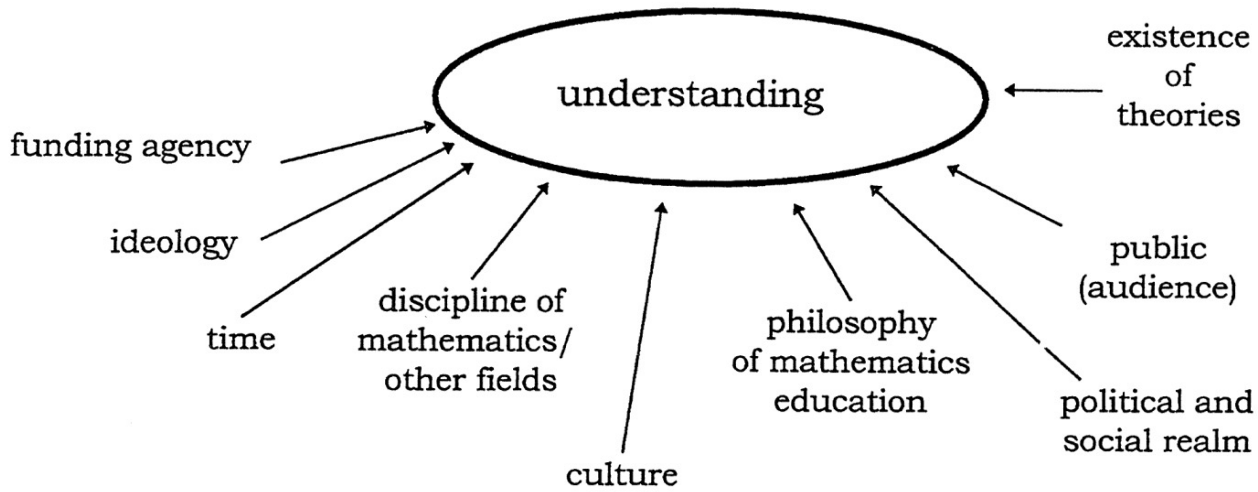

Figure 1 *Things to consider about the aims of a study*

Nevertheless, some factors were considered by the group as helpful in making the specific understandings useful:

- The situations we aim to investigate must include mathematics.
- We need to differentiate specific aims between short-term and long-term aims.
- We need to consider the possible alignment of personal aims with the external aims of professional groups, policy reports, or funding agencies.

It became clear in the discussions that the community of mathematical sciences education needs to become politically active in order to shape external expectations for research.

Practical Aims or Theoretical Aims

In the group's view, given the diversity of aims and the fact that the results of attempting to understand a situation can have a variety of implications, both the differences between theoretical knowledge, professional knowledge, practical knowledge, and their interrelatedness should be appreciated. The group also understood that such knowledge is provisional. Nevertheless, pragmatically it should be obvious that some research studies will have been designed to have practical implications (i.e., energize practice), and others to contribute to theory. In fact, the group agreed that in either case all research should eventually have a positive impact on practice. John Dewey's dictum 'that there is nothing as practical as a good theory' should be remembered. In addition, one should recognize that:

- There are differences between theory-driven research and theory-generating research.
- Some studies should aim to establish the limits of a theory.
- Some studies should identify and contribute to the elimination of obstacles to the growth of research and to the acceptance of research results.

Finally, one member of the group expressed the belief that there are, in the present meeting, different uses of the words *practice* and *theory* corresponding to different points of view that can be summarized as follows:

Point of view	*Practice*	*Theory*
level (role)	teaching	researching
person (status)	teacher	researcher
place	classroom	laboratory
product	technique	knowledge
methodology	collect data	analysis
situation	natural	experiment
time	short-term	long-term
generality	projects	fundamental knowledge
research vs. development	development	research

Complexity of Research Questions

This point was raised by several participants: From the two examples presented by the speakers (QUASAR and proof), it was clear that each research question addresses a complex relationship between different components (e.g., disciplinary, political, social, psychological, and didactical issues). This complexity requires making reference to and drawing upon different research fields (namely, mathematics, epistemology, psychology, sociology, didactics, and so on).

Some research questions deal with only one of the components (e.g., epistemology in the case of proof), but most deal with relationships among several components. It is necessary to avoid 'unjustified reduction of complexity'. What about 'justified reduction of complexity'?

One of the causes of complexity is, for instance, the relationships between mathematics education and mathematics. We cannot reduce research in mathematics education to part of research in mathematics, as researchers in mathematics education are responsible for their own methods, theories, and *problématiques.* But mathematicians and researchers in mathematics education share responsibility for the relevance of the research done with respect to mathematics.

Considering the practice of mathematics education in the fullest sense (planning and reflection, curriculum design and development, teacher preservice and inservice education and not only classroom teaching–learning situations) provides examples of the complexity of relationships between mathematics and mathematics education. (See also the problem of theory and practice below.)

A Dichotomy

A major (false?) dichotomy (THEORY versus PRACTICE) was included in the Discussion Document as a source of different sets of research questions. This dichotomy was a critical point in the group discussion. Some participants supported the separation between theory and practice (identifying them, respectively, as RESEARCH and INNOVATION) in order to allow to understand their specific constraints and so as to allow links to be built between them in a non-naïve way. Other participants expressed a need to consider theory and practice (without identifying them with research and innovation) as being in a true dialectical relationship that also included research problems generated by practical needs.

Examples were offered of relevant research questions raised by the links between theory and practice (e.g., the role of the teacher, considered as central in the teaching–learning process; the design and analysis of long-term processes; the analysis of the constraints on teachers that prevent them from being innovative, and ways to cope with these constraints). This contrast is related to either implicit or explicit assumptions about *paradigms.* Provocative questions were posed: Are the relationships between research and innovation research questions? If so, who is in charge of coping with them? Teachers? Researchers? Both in cooperation? Other people? Is this a general question? Is it dependent on the given paradigm?

Criteria for Judging Questions

Another point concerned *criteria*. Do we need criteria to judge the validity of specific research questions? Two contrasting positions emerged in the discussion: We need criteria, and it is better not to have criteria.

Communication and Relevance Internationally

Another point concerned the problems of communication and relevance with respect to the international community. This point had been raised by Silver as one among several in his presentation, but it was transformed by the group into a major issue to be discussed. That was a partial shift from the assigned task, as the problem of internationalization was not a specific topic for the group, but is a general problem of research in mathematics education. Not only research questions, but also objects, aims, results and criteria could and should be submitted to discussion by the international community.

A first problem concerns communication (of specific questions) because of the difficulty of giving all the relevant information concerning contextual features of a given research, and so on. This problem can be solved pragmatically by publishing papers with a suitable length to allow these assumptions to be shared with the readers. A second major problem concerns relevance: The way the international community influences the choice of local research questions, the relevance of local research for the international community, and the relevance of theoretical international research for local communities. Two complementary positions emerged: On the one hand, from the experience of the past the international community may be supposed to be willing to conduct only certain kinds of discussions (related to content or to theoretical issues and avoiding questions that are too context-bound). On the other hand, several participants underlined the richness of having discussion from different perspectives in order to identify potential controversies or points of consensus. The interest of the international community in such discussion is obvious, as recognizing differences can lead to a better understanding. Consider the following metaphor from zoology: The existence of kangaroos in Australia enlarges the European knowledge of mammals, even if it makes no sense to plan the importation of kangaroos to Europe. The importance of very specific (hence context-bound) questions was also underlined by the analogy with some 'good ideas' that have been developed in science (according to history) in very specific contexts, before becoming universals. So, whether or not we have a problem of communication, we need to make explicit and accept differences in contexts, assumptions, and paradigms.

A Proposal

Because so many problems need to be addressed by the international community of researchers in mathematics education, a final suggestion of this working group was the proposal of establishing an acknowledged community of researchers in

ALAN J. BISHOP

RESEARCH, EFFECTIVENESS, AND THE PRACTITIONERS' WORLD

1. INTRODUCTION

The ICMI Study Conference on Research was a watershed event with a great deal of significant interaction between the participants. It was an energizing and involving experience, but as one of my tasks was to 'summarize' at the end of the conference, I tried to take a more objective stance during my involvement. I reported in my summary that I could detect certain emphases in the discourses together with some important silences. Here are some of them:

Emphases	*Silences*
analyses	syntheses
critiques	consensus-building
talking to ourselves	awareness of other audiences
political arguing (to persuade)	researched arguing (to convince)
individual cases	over-arching structures
local theory	global theory
well-articulated differences	well-articulated similarities
disagreements	agreements

It seems as if the researcher's training encourages one to analyze, to look for holes in arguments, to offer alternative viewpoints, to challenge and so forth. Or the pattern could reflect the fact that the idea of the conference itself and of this ICMI Study was seen as a challenge to the participants' authority. Certainly the 'politics of knowledge' was alive and well in all its manifestations.

So what is the concern of this chapter? Am I seeking just a nice, warm, collaborative engagement, and feeling that it is a pity that we seem to disagree so much? I have to admit that with so much conflict in the world today I do wish that there was more obvious peaceful collaboration. I also believe that with the whole idea of research into education under attack from certain ignorant politicians and bureaucrats, those who engage in research should at least collaborate more, and should spend less time 'attacking' each other.

My real concern, however, is with what I see as researchers' difficulties of relating ideas from research with the practice of teaching and learning mathematics. In the discussion of this ICMI Study at the Eighth International Congress on Mathematical Education in Seville, many people spoke about the dangers of

Sierpinska, A. and Kilpatrick, J. Mathematics Education as a Research Domain: A Search for Identity, 33–45.