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cédric villani

translated by malcolm debevoise

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Cédric Villani

Translated by
Malcolm DeBevoise

Illustrations by
Étienne Lécroart

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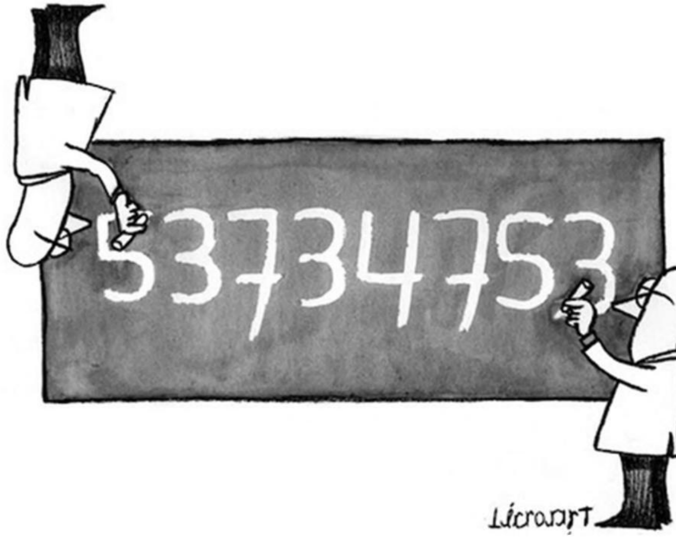
Mathematics, Science, and Poetry



Mathematics, whatever else it may be, is a science.* There are those who like to say, for a variety of reasons, that mathematics stands

* Rather than speak of *les mathématiques*, as is customarily done in French, I decided to adopt the singular form, *la mathématique*, when I realized that I could give no justification for the plural, unless it is an archaism recalling the Platonist classification of arts and sciences. Moreover, there is an ancient tradition of using the singular—carried on in our own time by Bourbaki, for example—that insists on the unity of a discipline whose branches, though they are quite varied, rest on common principles. I have nonetheless retained the plural noun and verb in Léopold Sédar Senghor's fine phrase, "Les mathématiques sont la poésie

apart, but I am one of those who say that mathematics is first and foremost a science. Like all the sciences, it seeks to describe the world, to understand the world, to act on the world. Describing, understanding, acting—thus the holy trinity of the sciences.

By themselves, these three things do not suffice to uniquely characterize scientific investigation, for they are found in other activities as well. One must also take into account certain fundamental principles that are common to mathematics and the other sciences.

The first of these principles is a priori skepticism. In science, only a logical chain of reasoning can lead us to believe something to be true, not appeal to some higher authority, whether a person, or a sacred text, or something else. One must not believe anything unless one has been convinced by a rigorous and coherent argument.

The second principle is peer review, the submission of the results of one's research to the judgment of a community of experts. A result is true not because somebody asserts it to be true, but because one's colleagues have unanimously approved the reasoning adduced in support of it. This review process is seldom straightforward. Yet in spite of disagreement, controversy, and occasional errors of judgment, scientists have always insisted upon the sharing of information and validation by qualified referees.

Third, and finally, there is the principle that no one's word counts for more than anyone else's. Only exactitude, the precision of the arguments advanced, and the conviction they inspire can lead to general agreement. In practice, of course, one more readily trusts the opinion of a respected scientist than an unknown amateur. If an article claiming to prove a difficult theorem in algebra appears under the name of Jean-Pierre Serre,[1] for example, one is naturally

des sciences," from which the present book takes its title. [In English, *mathematics* is plural in form but singular in construction, and so the problem does not arise, or at least not in so stark a form.—Trans.]

inclined to give it greater credence than if it comes from a mathematician no one has heard of. But it is important to keep in mind that here we are dealing with a human failing, something that falls short of the scientific ideal. In principle, the famous mathematician and the obscure mathematician should be considered as equals; and in fact it sometimes happens that an obscure mathematician working alone, or very nearly so, solves an important problem that has long stymied the greatest experts (this was the case only a few years ago with Yitang Zhang, and in the late 1970s with Roger Apéry; both were about sixty years old when they made discoveries that were to bring them worldwide fame).[2]

Skepticism, reasoned argument, sharing of results, peer review, collegial respect—all these things are found in mathematics, and this is why it can be said to be a science. Indeed, they are taken to extremes in mathematics. In mathematics, one is authorized to believe something only once it has been given a complete demonstration. A mathematician does not say, "Given this, you can imagine that by analogy," but rather, "I am going to prove this to you, down to the very last detail, and the logical force of my reasoning will compel your conviction." Here again, of course, we are talking about an ideal that is not humanly attainable: every proof contains small gaps, small omissions, but in principle an argument can be thoroughly verified, step by step, on the basis of the original article. Some proofs are hundreds of pages long; some take years to be verified.

Now let us consider the class of conjectures, which is to say statements that are thought to be true even though for the moment no proof can be given. Every day conjectures are finally proven (or disproven); every day new conjectures are advanced. But some are more memorable than others; Goldbach's conjecture and Collatz's conjecture (the Syracuse problem) are among the most famous of them. Mathematicians agree that Riemann's hypothesis is *the* most famous of all. It has been verified for ten thousand billion values, without a single counterexample having been encountered; but in the eyes of mathematicians, ten thousand billion corroborating instances, even a hundred thousand billion, do not constitute a proof! No other field of knowledge is so demanding.

Allow me now to qualify, for a third time, what I have just said. On the one hand, in all mathematical applications, one constantly takes the liberty of relying on statements that have not been rigorously demonstrated, but that are thought to be true because they are supported by a combination of reasoning and experiment. On the other hand, mathematicians themselves are often inclined to believe in a conjecture if it has been verified to a very large extent: even though they do not consider it to be established, they feel sure that it must be true, and this often is enough for them to feel justified in acting as though it had, in fact, been demonstrated. And while it is indeed deductive reasoning that decides whether a mathematical truth can be considered to be established, it is also true that mathematicians constantly make use of inductive reasoning and thought experiments in order to catch a first tentative glimpse of the results they seek to prove. Even when all this is taken into account, however, there is no denying that mathematics, more than any other science, enforces exceedingly stringent requirements in the way of rigorous proof.

One might go so far as to see mathematics as the quintessence of science, not in the sense that it is superior to the other sciences (even if Einstein once claimed as much),[3] but in the sense that the respect it shows for the principles of scientific reasoning is unsurpassed. In mathematical arguments, rigor enjoys a pre-eminence that would be considered unreasonable in other disciplines. Note, too, that mathematics is purely conceptual. There is no going back and forth, as in the other sciences, between mental ideas and our experience of the world; the mathematician is confined solely to ideas in working out the details of his demonstration. Mathematical ideas may be inspired by reality, of course, and the search for a proof may lead the mathematician to reflect deeply upon the implications of experimental results, but the reasoning itself belongs solely to the conceptual realm.

Mathematics is also an extraordinarily effective science. As everyone knows, most of the major scientific and technological achievements of our time contain some amount, small or large as the case may be, of mathematics. Taken together, these shares measure the power of pure reasoning to exert dominion over matter and the real world—a power so great that some feel reality itself must be, at bottom, an abstract mathematical construction!

For this reason, if mathematics were an art, and if one were to try to identify the art most similar to it, one might well think of design in its many aspects. For in design one encounters the same ambiguity as in mathematics, the same duality—or dialectic—between, on the one hand, harmony, abstraction, and aesthetic appeal, and, on the other hand, the obligation to satisfy a practical purpose. If the design for a table is both elegant and soundly conceived, the table that is made from it will be, as one would hope, handsome, solid, and useful. It is the same in mathematics: a result must be at once original, beautiful, and capable of being applied if its power is to be fully appreciated. We know this from everyday experience, whether we are listening to the weather forecast, or printing out a travel itinerary, or relying on machine translation to read something in another language. All sorts of things that are part of our daily lives would not exist without mathematics. Mathematics is all around us, like tables and chairs, everywhere to be found, in nature no less than in technology; but when it does its job well, we scarcely notice it. In order to send a text message or use an Internet search engine, we do not need to know the underlying mathematical principles, any more than we need to know the principles of electromagnetism, electronics, materials science, and so forth.

And what if mathematics were a literary genre? In that case it would certainly be poetry. This is what Senghor instinctively perceived in contemplating the sybilline language of a series of lecture topics.[4] But I was also made aware of this analogy by readers of my book *Birth of a Theorem,* in which I reproduced verbatim the conversations of professional mathematicians.[5] In literary writing, a poetic quality can arise from the appearance of foreign and unexpected elements. One may find beauty in words whose mysteriousness is the result of hearing them used in an unfamiliar context, for example, where they seem to be out of place. This is a little like what happens when you listen to a song in a foreign language that you do not understand, but in which you sense an occult and melodious force—and where the translation, by dispelling the aura of mystery, will certainly be disappointing! Indeed, Lautréamont experimented not unsuccessfully with the use of mathematical words in poetry in his *Chants de Maldoror.*

Many words that belong to everyday language have been given a special meaning by mathematicians. A ring, for example, is not a band you put on a finger; it is an algebraic structure admitting of two operations, addition and multiplication, having the properties of associativity and commutativity, and incorporating a neutral element. So, too, "spectrum" and "body" have their own definitions in mathematics, along with hundreds of other ordinary words. And when nonspecialists hear these words, which have been adapted by mathematicians to suit very particular purposes, they are apt to discover a kind of poetry in them.[6]

Setting mathematical objects in an unfamiliar context can also be done visually with shapes, as Man Ray did in photographing the collection of three-dimensional mathematical models at the Institut Henri Poincaré in Paris in the 1930s. These objects illustrate the geometric properties of mathematical equations. Ray did not understand the equations at all, but he saw a certain beauty in them; not only did he appreciate the formal elegance of their contours, their distinctive aesthetic quality, but he was fascinated by the fact that they had been made by human hands in order to represent concepts created in human brains. Intuitively he perceived their significance, though it was beyond his understanding. It is much the same when you listen to a foreign language: not only do you hear sounds that have a strange and captivating melody; you know that they have a meaning that some of your fellow human beings are capable of grasping, and this makes them all the more intriguing.

Working from the photographs he had taken, Ray went on to produce paintings of mathematical objects in which one seems to make out faces, masks.[7] These portraits, together with those of many other objects, constitute the series known as *Shakespearean Equations.* In each case the artistic result refers to something other than the original object, something that speaks to all of us, not merely to a small circle of initiates.

Let me conclude these preliminary observations by citing another author, a great poetical novelist and an excellent mathematician by the name of Charles Dodgson, better known as Lewis Carroll. Many people were surprised to learn that he wrote children's books in addition to works of logic. Austere and conservative by temperament, Dodgson very seldom permitted himself to give any hint in his public life of the richness of his imagination. Nevertheless I am certain that, to his way of thinking, everything was connected: the Alice tales are filled with mathematical concepts, logical puzzles, nonsense poems, portmanteau words, and neologisms constructed from carefully devised rules.[8] His logical work is likewise remarkable for its inventiveness and, in places, something very much like magic. Owing to his singular artistry he was able to give mathematics an unmistakably poetic cast.