

Morality and Mathematics

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Contents

Acknowledgments		ix
Introduction		
	0.1 Science and Value	1
	0.2 The Status of the Question	4
	0.3 Overview of the Book	7
1.	Realism, Ontology, and Objectivity	13
	1.1 Individuating Areas	13
	1.2 Bare-Bones Realism	15
	1.3 Faithfulness	20
	1.4 Knowledge	23
	1.5 Ontology	24
	1.6 Objectivity	27
	1.7 Conclusions	33
2.	Self-Evidence, Proof, and Disagreement	35
	2.1 A Priori Justification	36
	2.2 Axioms and Proofs	37
	2.3 Self-Evidence	40
	2.4 Plausibility and Disagreement	45
	2.5 Extent of Disagreement	49
	2.6 Philosophical Corruption	55
	2.7 The Concept of Set	57
	2.8 Error Theory	59
	2.9 Philosophy Everywhere	62
3.	Observation and Indispensability	66
	3.1 Indispensability	67
	3.2 The Quine–Putnam Thesis	68
	3.3 Harman's Thesis	70
	3.4 Instrumentalism and Modality	76
	3.5 Indispensable Mathematics and Metalogic	80
	3.6 Perception	84
	3.7 Justification and Explanation	88
	3.8 Recreational Mathematics	91
	3.9 A Priori/A Posteriori Revisited	95

viii CONTENTS

97
98
102
104
108
111
114
119
121
122
125
130
133
135
138
143
146
152
153
156
156
159
162
164
166
168
172
176
177
180
185
205

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Introduction

Philosophy is arguably the most inclusive discipline. Its subjects include everything from art to consciousness, from morality to mathematics. Moreover, it approaches its subjects from historical, conceptual, and scientific angles. Its breadth is not coincidental. Many take the defining aim of philosophy to be "to understand," as Wilfrid Sellars puts it, "how things in the broadest possible sense of the term hang together in the broadest possible sense of the term" [2007/1962, 369].

Despite its breadth, philosophy has become highly specialized. Corresponding to each of its myriad subjects are subdisciplines. Corresponding to morality is the subdiscipline of ethics, and corresponding to mathematics is the subdiscipline of the philosophy of mathematics. These subdisciplines, in turn, branch into sub-subdisciplines, including normative ethics, applied ethics, metaethics, and moral psychology in the one case, and mathematical logic, set theory, mathematical epistemology, and mathematical ontology in the other. To each sub-subdiscipline corresponds an enormous, often difficult, literature.

Specialization facilitates a kind of progress in philosophy. Problems are understood in depth, and theoretical options are developed with ever more sophistication. Such progress resembles Thomas Kuhn's normal science (Kuhn [1962]), minus the agreed-upon paradigm. But it also threatens progress toward the defining aim to which Sellars alludes. There is little hope of understanding "how things hang together" absent serious engagement between philosophy's diverse subfields—just as there is little hope of this absent serious engagement between philosophy and other fields.

0.1 Science and Value

Consider the question of *realism*. To what extent are the subjects of our thought and talk real? We all have a sense of it, prior to philosophical indoctrination. It is the question of whether the subjects of our thought and talk are "out there in the world" existing "independent of us." We are inclined to

2 INTRODUCTION

be realists about some areas while being anti-realists about others. For example, a common *naturalist* position among philosophers and scientists combines realism about the sciences with anti-realism about value. Naturalists, in the relevant sense, believe in independent facts about gene expression, plate tectonics, and quantum mechanics but do not believe in independent facts about what is morally good for us, what we epistemically ought to believe, or how prudentially we should live. Sean Carroll summarizes the naturalist's position on morality, in particular, as follows.

There are not...moral truths...existing independently of human invention... but there are real human beings with complex sets of preferences. What we call "morality" is an outgrowth of the interplay of those preferences with the world around us, and in particular with other human beings. The project of moral philosophy is to make sense of our preferences, to try to make them logically consistent, to reconcile them with the preferences of others and the realities of our environments, and to discover how to fulfill them most efficiently. [2010a, quotation marks removed]

Suppose we were to ask a naturalist why she takes different positions toward science and value. What would she say? Probably something like the following. First, facts about genes, the lithosphere, and electrons are implied by our best theories of the observable world, and those facts have been confirmed by observation and experiment. Second, knowledgeable individuals tend to agree on such facts, and, when there is a disagreement, there is a method—experiment and observation—by which to resolve it. Finally, we have at least a sketch of how human beings could acquire the knowledge of such facts that we take ourselves to have acquired. Facts about genes and so forth make causal marks on the world, marks to which our nervous systems respond.

By contrast, so-called "moral facts" would be different in all of these ways. Alleged facts about what is morally good or bad, right or wrong, obligatory or forbidden are not implied by any recognizably scientific theory. They are subject to endless controversy, even among people who agree on the non-moral facts, and who are otherwise intellectual peers. And there is no apparent method by which to resolve such disagreements. Finally, nobody has any idea how human beings could be reliable detectors of independent moral facts. Knowledge of such facts would be a mysterious extra kind of knowledge, over and above our knowledge of the natural world.

I have been speaking of the naturalist's attitude toward the empirical sciences, like physics and genetics. But a typical empirical scientific

theory, rigorously formulated, presupposes pure mathematical facts as well. It presupposes, if only implicitly, whatever pure mathematical theories govern the mathematical entities to which it appeals. For example, Newton's law of universal gravitation presupposes real analysis, since the axioms of real analysis govern the numbers over which Newton's law quantifies. (A typical empirical theory is also closed under *logical consequence*—that is, if P is in the theory, and Q follows from P, then Q is in the theory—and has implications for how the world *would have been different* had initial conditions been different. In other words, a typical empirical theory presupposes logical and modal facts as well.) If a naturalist like Carroll were to declare that he is realist about, say, the standard model of particle physics, but *not* about mathematics, then it would not even be apparent what he meant.

So, an empirical scientific realist would seem to need to be a mathematical realist as well. She would seem to need to believe in independent facts about numbers, functions, and so forth, in addition to believing in such facts about genes, particles, and so on. As Hilary Putnam puts it,

[Q]uantification over mathematical entities is indispensable for science... but this commits us to...the [independent] existence of the mathematical entities [that satisfy our theories]. This type of argument stems, of course, from Quine, who has for years stressed both the indispensability of quantification over mathematical entities and the intellectual dishonesty of denying the existence of what one daily presupposes. [1971, 347]

However, unlike the contrast between independent empirical facts and independent moral facts, the contrast between independent mathematical (or, indeed, logical and modal) facts and independent moral facts is less straightforward. Even if mathematical facts are *implied* by well-confirmed scientific theories, it seems wrong to say that mathematical facts themselves have been confirmed. Was Riemannian geometry confirmed as a pure mathematical theory when general relativity was? That would seem to imply, falsely, that Euclidean geometry was disconfirmed when general relativity was. Similarly, while it can indeed appear that mathematics generates convergence, and that there is a method by which to resolve any remaining disagreements, this is questionable on inspection. Mathematical proofs proceed from axioms. So, what they really show is that *if* the axioms are true, then so too is the theorem proved—at least assuming that there is agreement over the logic used. But moral claims admit of "proof" in this sense too. Gather together some claims from which the others follow and call

4 INTRODUCTION

them "axioms." What matters is how mathematical axioms compare to alleged "moral axioms." Is there disagreement over them? Do we have a method by which to resolve it? This is less clear. Finally, a longstanding objection to mathematical realism, famously pressed in Benacerraf [1973], is that it would be mysterious how humans could be reliable detectors of independent mathematical facts. We certainly do not interact with the likes of numbers and metric tensors!

Mathematics would, thus, appear to be a problem for the naturalist. Indeed, the outspoken naturalist, Alex Rosenberg, remarks, "[t]he criticism...that...I take seriously focuses on...our knowledge of mathematics—this is a serious problem for all naturalistic epistemologies" [2018]. On the one hand, it is not even apparent what it could mean to be a "realist" about our empirical scientific theories, while being an anti-realist about mathematics. On the other hand, there may be no principled ground on which to be a realist about mathematics and an anti-realist about value. Whether naturalism, as that position is commonly understood, makes sense would thus appear to depend on whether one can be a mathematical realist and a moral anti-realist.

0.2 The Status of the Question

Can one be? The question has long interested philosophers. Plato (*Republic*, Book VII) closely associated mathematical knowledge with knowledge of the Good (Burnyeat [2000]), and the British rationalists belabored an analogy between simple mathematical and moral propositions (Clarke [2010/1705, 12]). Some philosophers have suggested that moral realism and mathematical realism "stand or fall together." Putnam begins a book with the declaration:

[A]rguments for "antirealism" in ethics are virtually identical with arguments for antirealism in the philosophy of mathematics; yet philosophers who resist those arguments in the latter case often capitulate in the former.

[2004, 1]

Putnam's remarks are characteristic of work on the issue. Despite their sweeping character, he does not defend them. The problem is specialization. Ethics and the philosophy of mathematics are such different subjects, and

¹ For more on this, see Section 3.5.

philosophy has become so specialized, that nobody really knows whether one can be a mathematical realist and a moral anti-realist. The "debate" over the relative standing of moral and mathematical realism has been mostly limited to trading impressions.² Most of them point in the opposite direction, as the following quotations illustrate.

A few philosophers claimed that we have a moral sense that perceives the moral rightness or wrongness of things.... This theory might be worth taking seriously if morality were like mathematics. Mathematicians all agree that we know with certainty a large number of mathematical truths. Since experiment and observation could never be the source of such certainty, we...must have some other way of knowing mathematical truths—a mathematical sense that directly perceives them. For this argument to work in ethics, there would have to be little or no ethical disagreement to begin with. Since many moral disagreements seem intractable even among experts, the hypothesis that we are equipped to know moral truths directly is very difficult to sustain. [Rosenberg 2015]

[M]athematics begins with a small number of shared, self-evident assumptions, while morality begins with a large number of inter-connected assumptions...all of which sound reasonable to the assumption-maker and precious few of which are truly self-evident. (In other words, moral epistemology is *coherentist* rather than *foundationalist*.)

[Greene 2013, 184–5, italics in original]

No, there is no such thing as a universal morality, and it is somewhat surprising that people are still asking this question in the 21st century. [I]f by "universal" we mean that morality is…like mathematical theorems, or perhaps like the laws of logic, then forget it….[M]orality isn't even in the ballpark. [Pigliucci 2018]

In explaining the observations that support a physical theory, scientists typically appeal to mathematical principles. On the other hand, one never seems to need to appeal in this way to moral principles. Since an observation is evidence for what best explains it...there is indirect observational evidence for mathematics. There does not seem to be observational evidence...for basic moral principles. [Harman 1977, 9–10]

² There are exceptions, though no one has treated the matter in detail. See Brown [Forthcoming], Franklin [2014], Gill [2007], Kaspar [2015], Lear [1983], Lillehammer [2007], Parfit [2011] Scanlon [2014], and Wright [1994] for somewhat more sustained discussions of the comparison. See Leibowitz and Sinclair [2016] for a recent attempt to help rectify the situation.

6 INTRODUCTION

In the case of mathematics, what is central is the contrast between practices or beliefs which develop because that is the way things are, and those that do not. The calculating rules developed as they did because [they] reflect mathematical truth. The functions of...morality, however, are to be understood in terms of well-being, and there seems no reason to think that had human nature involved, say, different motivations then different practices would not have emerged. [Crisp 2006, 17]

Such one-off comparisons often betray serious misunderstandings. For instance, Peter Singer writes,

[Some moral realists] argued that there was a parallel in the way we know or could immediately grasp basic truths of mathematics.... This argument suffered a blow when it was shown that the self evidence of basic truths of mathematics could be explained in a different and more parsimonious way, by seeing mathematics as a system of tautologies, the basic elements of which are true by virtue of the meanings of the terms used. On this view, now widely, if not universally, accepted, no special intuition is required to establish that one plus one equals two -- this is a logical truth, true by virtue of the meanings given to the integers.... So the idea that intuition provides some substantive kind of knowledge of right and wrong lost its only analogue. [1994, 8]

First, 1 + 1 = 2 is not a logical truth (assuming that we mean first-order logic by "logic"). A countermodel is one in which the plus function maps 1 onto itself and to 3. Second, I am not aware of any contemporary advocate of the view that mathematics is a system of tautologies. Some logical positivists did suggest this. But their views were almost universally jettisoned after Kurt Godel proved the incompleteness theorems, and they were commonly ridiculed before that. Finally, far from being widely accepted, the notion of truth in virtue of meaning has been widely repudiated (Quine [1951b]). We may fix what proposition a sentence expresses. However, as Boghossian [1997 and 2003] emphasizes, we do not thereby fix whether the proposition expressed is true. Indeed, the idea of truth *in virtue of* meaning is dubiously coherent.³

³ Compare James Franklin, commenting on the same quotation from Singer: "That view is not universally accepted, nor widely accepted, nor indeed accepted at all by any living philosopher of mathematics" [2014, 198]. Singer appears to acknowledge some limitations of his remarks in a 2018 AI Alignment Podcast interview. For a contrary perspective on truth in virtue of meaning, see Russell [2011].

So, suggestions like the above are suggestive. But, as it stands, that is all they are. In order to see whether moral realism and mathematical realism stand or fall together, or whether ethics and the philosophy of mathematics have anything else to teach one another, we need to dig deeper. We need to bring ethics and the philosophy of mathematics into meaningful contact.

0.3 Overview of the Book

In this book, I explore arguments for and against moral realism and mathematical realism, how they interact, and what they can tell us about areas of philosophical interest more generally. I argue that our mathematical beliefs have no better claim to being self-evident or provable than our moral beliefs, contra the quotations from Rosenberg, Greene, and Pigliucci above. Nor do our mathematical beliefs have better claim to being empirically justified than our moral beliefs, contra the quotation from Harman. It is also incorrect that reflection on the "genealogy" of our moral beliefs establishes a lack of parity between the cases, contra the quotation from Crisp. In general, if one is a moral anti-realist on the basis of epistemological considerations, then one ought to be a mathematical anti-realist as well. And, yet, moral realism and mathematical realism do not stand or fall together, contra the quotation from Putnam. Moral questions—or the practical ones stake in moral debate—are objective in a sense that mathematical questions are not. But the sense in which they are objective can only be explained by assuming practical anti-realism. One upshot of the discussion is that the concepts of realism and objectivity, which are widely identified, are actually in tension.⁴

The book should be of interest to both ethicists and philosophers of mathematics. First, it shows that anyone who is a moral anti-realist on the basis of epistemological considerations ought to be a mathematical anti-realist too. Second, it raises problems for mathematical realism that have not been adequately explored. For example, it suggest that, in important respects, our mathematical beliefs are comparably contentious and contingent as our moral beliefs. Finally, the book reveals a special connection between the subjects of morality and mathematics. By comparing the subjects in detail, the correct philosophical account of each comes into focus.

⁴ The sense of "objectivity" in question is similar to that of Field [1998a], and is opposed to relativism in the sense of Barton [2016] and Hare [1997]. See Section 1.6.

The book may also be of general interest. It concludes with a more encompassing account of areas of philosophical interest. There are those that are more like morality, such as normative epistemology and prudential reasoning, and those that are more like mathematics, such as modal metaphysics and (non-normative) logic. It is argued that, while we ought to be realists about the latter areas, they fail to be objective in just the sense that mathematics does. And while we ought to be anti-realists about the former areas, they are objective in the sense that mathematics is not. Along the way, key topics of general interest are broached, including: self-evidence and proof, the epistemological significance of disagreement, the philosophy/ science comparison, metaphysical possibility, the fact/value dichotomy, and deflationary conceptions of philosophy.

The structure of the book is as follows. In Chapter 1 I explicate (in Carnap's sense) the concept of realism, and distinguish it from related concepts with which it is often conflated. I show that, correctly conceived, realism has no ontological implications. One can be a realist without believing in any new entities. I also show that common objections to moral and mathematical realism fallaciously assume otherwise. One upshot of the discussion is that it is no response to Paul Benacerraf's epistemological challenge, mentioned above, to claim that there are no special mathematical entities with which to "get in touch." I conclude with a distinction between realism and objectivity, a distinction which is central to Chapter 6. Very roughly, objective questions are those which only admit of a single answer. By contrast, in a disagreement over a non-objective question, we can both be right. I use the Parallel Postulate, understood as a claim of pure geometry, as a paradigm of a claim that fails to be objective, even if mathematical realism is true. Conversely, I explain how realism about claims of a kind may be false even though they are objective in a sense that the Parallel Postulate is not.

In Chapters 2 and 3 I discuss how our mathematical and moral beliefs might be (defeasibly) justified, realistically construed, whether a priori or a posteriori. By "our mathematical and moral beliefs" I mean the range of mathematical and moral beliefs that we actually have, from trivialities of arithmetic to canonical theorems of set theory, from banalities such as "burning babies just for the fun of it is morally wrong" to egalitarian theses about gender and race. I depart here from much of the literature comparing morality and mathematics, both contemporary and historical, which has tended to focus on rudimentary claims of arithmetic and geometry.

In Chapter 2 I argue that our mathematical beliefs have no better claim to being a priori justified than our moral beliefs. In particular, they have no

better claim to being self-evident, provable, plausible, "analytic," or even initially credible than our moral beliefs, despite widespread allegations to the contrary. I consider the objection that pervasive and persistent moral disagreement betrays a lack of parity between the cases, and argue that there is no important sense in which there is more moral disagreement than mathematical disagreement, or in which moral disagreement is less tractable than mathematical disagreement. That is, there is no such sense which should lead us to conclude that our mathematical beliefs have better claim to being (defeasibly) a priori justified than our moral beliefs, realistically construed. A common argument to the contrary simply confuses logic—what is true if the axioms are—with mathematics (though I sketch a way in which one could also make a parity argument in the case of metalogic, the theory of what follows from what). I conclude with the suggestion that the extent of disagreement in an area, in any familiar sense, may be of little epistemological consequence—contrary to what is widely assumed.

Having argued that our mathematical beliefs have no better claim to being a priori justified than our moral beliefs, in Chapter 3 I argue that they also have no better claim to being a posteriori—that is, empirically—justified than our moral beliefs. I focus on Harman's influential argument to the contrary. Harman argues that since the contents of our mathematical beliefs are implied by our best empirical scientific theories, while the contents of our moral beliefs are not, only the former are empirically justified. I show that, on the contrary, Harman's reasons to think that the contents of our moral beliefs fail to be implied by our best empirical scientific theories serve equally to show that the contents of our mathematical beliefs do too, realistically construed. I then formulate a better argument for a lack of parity between the cases, in terms of indispensability. I argue that while the "necessity" of mathematics is no bar to developing a mathematics-free alternative to empirical science, contra an objection of Timothy Williamson, the contents of our arithmetic beliefs, realistically and even objectively construed, do seem to be indispensable to metalogic—the theory of what follows from what. But this would still only show that a subset of our mathematical beliefs have better claim to being empirically justified than any of our moral beliefs. And I argue that it does not even show that. Surprisingly, however, the range of moral beliefs that we have may be empirically justified, albeit in

 $^{^5}$ The relevant kind of analyticity is sometimes called "epistemic analyticity" and must be distinguished from the idea of truth in virtue of meaning mentioned by Singer in the quotation in Section 0.2.

a different way. Unlike mathematics, there may be no ground on which to rule out so-called "moral perceptions" as being on an epistemological par with ordinary perceptions ascribing high-level descriptive properties. I conclude with the prospect that there may be no principled distinction between intuition and perception, and, hence, between a priori and a posteriori justification.

Having shown that our mathematical beliefs have no better claim to being (defeasibly) justified than our moral beliefs, in Chapter 4 I consider attempts to undermine the latter by appeal to their genealogy—that is, Genealogical Debunking Arguments. I argue that, as standardly formulated, such arguments misunderstand the epistemological significance of explanatory indispensability. Debunkers observe that whether the proposition that P is implied by some explanation of our coming to believe that P is predictive of its having epistemically desirable qualities when the fact that P would be causally efficacious if it obtained. The problem is that these things are independent when the fact that P would be causally inert, and Genealogical Debunking Arguments assume otherwise. For example, when P would be causally inert, then whether the proposition that P is implied by some explanation of our coming to believe that P is independent of whether our belief that P is safe (that is, roughly, whether we could have easily had a false belief as to whether P), sensitive (that is, roughly, whether had it been that ~P, we would not still have believed that P), and (objectively) probable. I formulate a principle, which I call "Modal Security," which constitutes a criterion of adequacy for debunking arguments. It says that if such arguments are to undermine, rather than rebut, our targeted beliefs, they must give us reason to doubt their safety or sensitivity. But this is something that they do not do. Even if Modal Security is false, however, I argue that Genealogical Debunking Arguments have little force absent an account of the epistemically important quality that they are supposed to threaten. I conclude that the real problem to which Genealogical Debunking Arguments point is an application of the Benacerraf-Field challenge. The challenge is to explain the reliability of our moral beliefs, realistically construed. However, this challenge has nothing to do with whether the contents of our moral beliefs are implied by some explanation of our coming to have them.

In Chapter 5, I consider the Benacerraf-Field challenge, or what I call the "reliability challenge," in detail. After substantially clarifying the dialectic, I consider different ways of understanding the challenge. I begin with Benacerraf's preferred way, and then turn to improvements on it. I argue that none satisfies two key constraints which have been placed on the challenge. I then turn to more promising analyses, in terms of variations of the truths and variations of our beliefs. The best version of the former is the challenge to show that our beliefs are sensitive, in the above sense. This challenge is widely supposed to admit of an evolutionary answer in the mathematical case, but not in the moral. I argue that, on the contrary, the sensitivity challenge may admit of an evolutionary answer in the moral case, and not in the mathematical. But this is only because the sensitivity challenge is trivial to meet when the truths in question ascribe supervenient properties of concrete things, and impossible to meet when they do not. So this is an inadequate formulation of the challenge. This leaves analyses in terms of the variation of our beliefs. I argue that the best version of these is the challenge to show that our beliefs are *safe* in the aforementioned sense. Understanding the reliability challenge as the challenge to show that our beliefs are safe explains the otherwise mysterious conviction that, whatever its costs, the view that I will call "mathematical pluralism" at least affords an answer to the reliability challenge. Understanding the reliability challenge in this way also illuminates the epistemic significance of genealogy and disagreement. I conclude that whether the reliability challenge is equally pressing in the moral and mathematical cases depends on whether "realist pluralism"—or what I henceforth simply call *pluralism*—is equally viable in the two areas.

The rough idea to pluralism about an area, F, is that any F-like theory that we might have adopted is true of the entities which it is about, independent of human minds and languages.

In Chapter 6 I show that, while standard formulations of pluralism are dubiously intelligible, the view can be refined, and the resulting theory answers the reliability challenge for F-realism, *qua* the challenge to show that our F-beliefs are safe. It does so by giving up on the *objectivity* of the truths (in the sense of Chapter 1), but not on their mind-and-language independence. However, there is an essential difference between the mathematical and moral cases. Assuming mathematical pluralism, mathematical—as opposed to logical—questions get deflated. They become verbal in the sense in which the Parallel Postulate question is, understood as a question of pure mathematics. By contrast, assuming moral pluralism, all the pressing questions remain. If we call those questions *practical*, then we can frame the point as a radicalization of Moore's Open Question Argument. Practical questions remain open even when the facts, *including the evaluative facts*, come "cheaply." This means that mathematics and morality, insofar as it is practical, do differ, but the concept of realism alone is too

crude a concept to do justice to the difference. Although practical *realism* is false, practical questions are *objective* in a paradigmatic respect. Conversely, while mathematical realism is true, mathematical questions fail to be. One upshot of the discussion is that the concept of objectivity, not realism, has methodological ramifications. Another is that the concepts of realism and objectivity (in one important sense of "objectivity"), which have been widely identified, do not only bifurcate. They are in tension.

I conclude by rehearsing key themes of the book and sketching their broader significance. I suggest a general partition of areas of philosophical interest into those which are more like mathematics and those which are more like morality. In the former category are questions of modality (counterfactual possibility), grounding, nature (essence), (non-normative) logic, and mereology. In the latter are questions of (normative) epistemology, political philosophy, aesthetics, and prudential reasoning. I argue that the former questions are like the question of whether the Parallel Postulate is true, qua a pure mathematical conjecture. They are verbal—but not because they are about words. They are verbal because reality is so rich as to witness any answer to them we might give. I illustrate this conclusion with questions of modality. I argue that, just as there are different concepts of geometrical point and line, all equally satisfied, there are different concepts of how the world could have been different. While it is, say, metaphysically impossible that you could have had different parents, it is logically possible that you could have, and there is nothing more "real" about metaphysical than logical possibility. In general, while typical questions of modal metaphysics are not about "possible," they might as well be. All we learn in answering them is how we happen to be using modal words, rather than learning what modal reality contains. By contrast, evaluative—or, more carefully, practical—questions are immune to deflation in this way. But the reason that they are is that they do not answer to the facts. So, their objectivity is not compromised if the facts are abundant. I conclude that the objective questions in the neighborhood of questions of modality, grounding, nature, and so on are practical questions as well. Practical philosophy should, therefore, take center stage.

Realism, Ontology, and Objectivity

This book is about arguments for and against moral realism and mathematical realism, how they interact, and what they can tell us about areas of philosophical interest more generally. But before I turn to those arguments, I need to say what "realism" about an area, in the pertinent sense, is supposed to mean. Of course, "realism" is a technical term, and we can define it how we like. But certain theses have been central to the debate over moral and mathematical realism. In this chapter I articulate a core notion of realism about an area, F, and explain its application to morality and mathematics. I then discuss several important theses that are independent of, though often conflated with, realism, one of which will be central to Chapter 6.

What follows is neither a conceptual analysis of the term of art "realism," nor an arbitrary stipulation for how to use the word. It is closer to an explication in the sense of Carnap [1950b, 3]. My aim is to locate a reasonably precise concept in the neighborhood of those that have been invoked in metaethics and the philosophy of mathematics which can serve as a useful point of departure for comparisons between the two areas.

1.1 Individuating Areas

Intuitively, if F is an area of inquiry, such as morality or mathematics, then F-realism is the view that typical F-sentences are true or false, independent of us, and that some substantive ones are true, interpreted at face value. So, mathematical realism is the view that some such sentences as "2 is prime" or "there are inaccessible cardinals" are true, and say what they seem to say, and similarly for moral realism. But this is very rough. What does F-realism come to, more exactly?

A preliminary question is: how are we individuating areas of inquiry? In the cases of concern, we can think of areas as individuated by their predicates—"is good," "is bad," "is a reason to," and so on in the case of morality, and " \in ," "<," "is a number," and so forth in the case of mathematics—where predicates, in

Evidently, F-realism should at least entail that typical F-sentences are (determinately) true or false.⁴ The "typical" qualifier allows that some F-sentences are neither true nor false thanks to vagueness or indeterminacy. But one is not an F-realist if one does not even believe that, barring vagueness and similar phenomena, F-sentences are apt for truth. Thus, F-realism at least entails:

[F-Aptness] Typical F-sentences are true or false.

F-Aptness implies that austere forms of noncognitivism, such as A. J. Ayer's *emotivism*, according to which moral sentences are just used to express emotions, and (a common reading of) David Hilbert's *formalism*, according to which (nonfinitary) mathematical sentences are merely used to make moves in a game, are forms of anti-realism.⁵ By contrast, it does *not* imply that sophisticated incarnations of noncognitivism which incorporate a deflationary theory of truth are too.⁶ Following Railton [2006, 216, n. 6], I will call noncognitivist views *nonfactualist*.

How do we rule out sophisticated incarnations of nonfactualism from counting as realist? Blackburn [1990] suggests that, contra realism, sophisticated nonfactualists about morality hold that our (token) moral judgments are not explained with reference to their subject matter. But that is the *point* of at least one influential brand of "non-naturalist" moral realism (Dworkin [1996], Enoch [2011], Nagel [1986], Parfit [2011], Scanlon [2014])—the most uncompromising version of the view to which nonfactualism is supposedly opposed. Thomas Nagel writes,

[I]t begs the question to assume that ... explanatory necessity is the test of reality for values.... To assume that only what has to be included in the best causal theory of the world is to assume that there are no irreducible normative truths. [1986, 144]

It might be objected that if our moral beliefs were not "explained" by their subject matter—whether causally or in some other to-be-specified way—it would have to be a fluke that they were ever true (Street [2016]). In Chapter 4 I argue at length that this worry is confused. However, even if it

⁴ I will mean determinate truth by "truth," for those who distinguish truth from determinate truth.

⁵ See Ayer [1936, ch. VI] and Hilbert [1983/1926], respectively.

⁶ See Horwich [1998] for a defense of deflationism about truth.

Index

A Priori/A Posteriori Distinction 95–6, 178 Agency 166–8 Analytic naturalism 164	British Rationalists 4 Brouwer, L. E. J. 18
Analytic 57–9 Axiom 43, 177–8	Cantor's Continuum Hypothesis 30, 127–8, 182–3
Axiom of Choice 42–3, 46–7, 151, 158–60, 165–6, 177–8	Cardinality Comparability Principle (CCP) 85-6
Axiom of Constructibility 43-4	Carnap 174
Axiom of Extensionality 40-2	Carroll, Sean <u>1–3</u> , 63–4
Axiom of Foundation 36-58, 147	Casual Theory of Knowledge 112-13, 131
Axiom of Infinity/Cantor's Axiom 42	Casually Inert Properties 72–5
Axiom of Least Upper Bound	Casullo, Albert 87, 145-6
Axiom 48, 56	Causation 70-6, 112-13, 130-3
Axiom of Pairing 40–2	Clarke, Samuel 36, 41
Axiom of Powerset 43	Classical Logic 49n.19, 50-1
Axiom of Projective Determinacy 39-40	Cohen, G. A. 149
Axiom of Replacement 42	Cohen, Paul 61
Axiom of Successor 48	Coincidence 97-114, 119-20, 131
Ayer, A. J. <u>16</u>	Colyvan, Mark 93
	Completeness Theorem 20, 21n.18, 51n.23,
Baker, Alan 90-1, 93	52–3, 160
Balaguer, Mark 110, 157, <u>158n.4</u> , 162–3	Consistency 38–40, 160–1
Banach-Tarski Paradox 46-7	Constructivism 18, 168
Baras, Dan 116–17	Contingency 146–52
Bealer, George 123	Cornell Realism 31
Bedke, Matt 68-70, 134	Counterfactual Persistence 133-4
Bell, John 53–4, 150	Crisp, Roger <u>6</u> , 136–7
Benacerraf-Field Challenge/Benacerraf	
Problem/Reliability Challenge/The	Darwin, Charles 103-4, 108-9, 127, 136,
Access Problem 26, 98, 122, 134, 153,	138–43
156-9, 164-5	David Hilbert 16
Benacerraf, Paul 4, 21, 39, 122, 130, 157	Debunkers' Thesis 103, 105-6, 109,
Berkeley, Bishop 100-1	111–14, 133
Blackburn, Simon 15, 166	Deflation <u>16</u> , 32, 167, 170, 182
Boghossian, Peter 6	Determinacy 127-8
Boolos, George <u>41n.8</u> , 43, 80–1 Boyd, Richard 31–2	Disagreement 2, 49–55, 148–9, 151–2, 174, 177–8