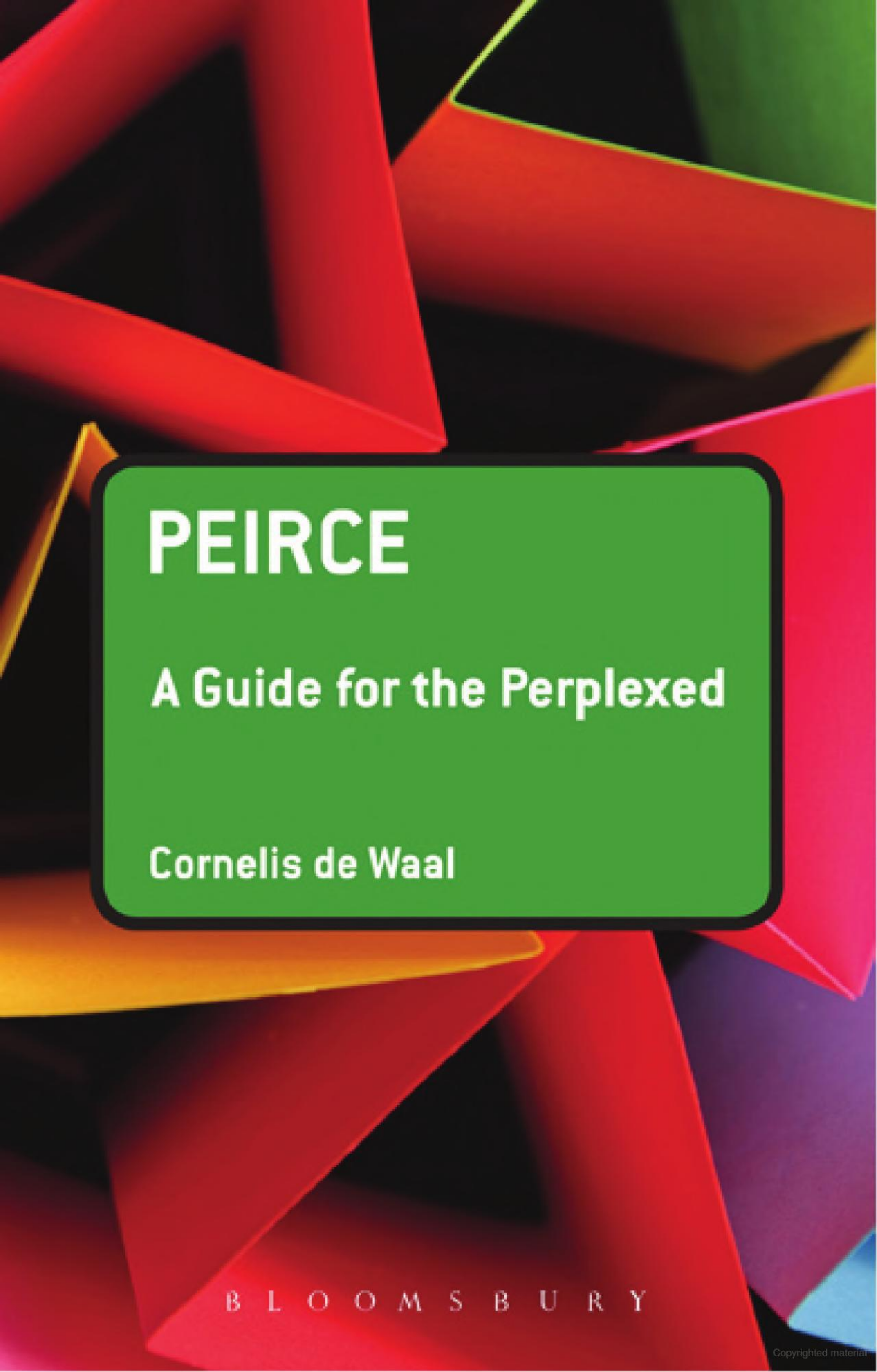

PEIRCE

A Guide for the Perplexed

Cornelis de Waal

BLOOMSBURY

Bloomsbury Academic
An imprint of Bloomsbury Publishing Plc

50 Bedford Square
London
WC1B 3DP
UK

175 Fifth Avenue
New York
NY 10010
USA

www.bloomsbury.com

First published 2013

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library.

ISBN: HB: 978-1-8470-6515-5
PB: 978-1-8470-6516-2
e-ISBN: 978-1-4411-9862-4

Library of Congress Cataloging-in-Publication Data
De Waal, Cornelis.
Peirce: a guide for the perplexed/Cornelis de Waal. – First [edition].
pages cm – (Guides for the perplexed)
Includes bibliographical references and index.
ISBN 978-1-84706-515-5 (hardback) –
ISBN 978-1-84706-516-2 (paperback) –
ISBN 978-1-4411-9862-4 (ebook pdf) (print)
1. Peirce, Charles S. (Charles Sanders), 1839–1914. I. Title.
B945.P44D385 2013
191–dc23
2012020371

Typeset by Deanta Global Publishing Services, Chennai, India

CONTENTS

ABBREVIATIONS

CN volume: page. *Charles Sanders Peirce: Contributions to* The Nation. 4 Vols. Kenneth L. Ketner and James E. Cook, eds Lubbock, 1975–87.

CP volume: paragraph. *The Collected Papers of Charles Sanders Peirce.* 8 Vols. Vols. 1–6, ed. Charles Hartshorne and Paul Weiss. Vols. 7–8, ed. Arthur W. Burks. Cambridge, Mass., 1931–58.

EP volume: page. *The Essential Peirce: Selected Philosophical Writings.* 2 Vols. Vol. 1. ed. Nathan Houser and Christian Kloesel. Vol. 2, ed. Peirce Edition Project. Bloomington, 1992–98.

HPPLS volume: page. *Historical Perspectives on Peirce's Logic of Science.* 2 Vols. ed. Carolyn Eisele. The Hague, 1985.

NEM volume: page. *The New Elements of Mathematics*, 4 Vols. in 5. ed. Carolyn Eisele. The Hague, 1976.

PM *Philosophy of Mathematics: Selected Writings.* ed. Matthew Moore Bloomington, 2010.

R followed by Robin catalogue and sheet number. Manuscripts held in the Houghton Library of Harvard University, as identified by Richard Robin, *Annotated Catalogue of the Papers of Charles S. Peirce.* Amherst, 1967, and in Richard Robin, "The Peirce papers: a supplementary catalogue," *Transactions of the C. S. Peirce Society* 7 (1971): 37–57.

RLT *Reasoning and the Logic of Things: The Cambridge Conferences Lectures of 1898*, ed. Kenneth L. Ketner. Cambridge, Mass., 1992.

SS *Semiotic and Significs: The Correspondence between Charles S. Peirce and Victoria Lady Welby*, ed. Charles S. Hardwick. Bloomington, 1977.

W volume: page. *The Writings of Charles S. Peirce*, ed. The Peirce Edition Project, 7 Vols. to date. Bloomington, 1982–2010.

CHAPTER ONE

Life and work

In the early morning of 29 December 1914, a young graduate student from Harvard and a local farmer with a droopy moustache sped on a horse-drawn sled to the Port Jervis train station. They carried with them roughly a thousand books and two heavy crates of manuscripts that belonged to the American philosopher Charles Sanders Peirce (pronounced "purse") who had died that spring. The two crates of manuscripts would establish Peirce as one of the great Western philosophers. During his life, Peirce was highly regarded as a scientist and as a logician, but not too much was known of his philosophy, as most of it had remained unpublished.

Charles Peirce was born in Cambridge, Massachusetts, on 10 September 1839, as the second son of the renowned mathematician and astronomer Benjamin Peirce. Charles Peirce (hereafter referred to as 'Peirce' the focus of this book) was far from a bookish philosopher and the scope of his work is staggering. He did pioneering work on the magnitude of stars and the form of the Milky Way. He worked extensively determining the exact shape of the earth, designing instruments, and improving methodologies. He invented a new map projection that gave a world map with a minimum distortion of the distance between any two points. He was a pioneer in mathematical logic and mathematical economy, did important work on Shakespearean pronunciation, engaged in experimental psychology, wrote several books on logic and mathematics (none of which were published), gave lectures on the history of science, developed a bleaching process for wood pulp, wrote on spelling reform, made calculations for a suspension bridge over the Hudson river, and was the first to use a wavelength of light to determine the exact length of the meter. Almost as an aside, in a short letter to his former student Alan Marquand, Peirce invented the electronic switching-circuit computer—until then computing machines had been wholly mechanical (W5:421–23). However, none of these accomplishments really helped Peirce, who died in abject poverty and almost completely forgotten in a small town called Milford, Pennsylvania, on 19 April 1914. Peirce was survived only by his second wife (whose identity is still a mystery) and by a disarray of over a hundred thousand manuscript pages. The American philosopher Josiah Royce, who was deeply indebted to Peirce's thought, worked hard to raise money for Peirce's papers and library, and for the less than impressive sum of five hundred dollars the books and papers went to Harvard, first by sled and then by train. Though the process of getting Peirce's unpublished writings into print is slow and not without controversy, it is already undeniable that he is a philosopher of great magnitude whose writings are bound to significantly alter the philosophical landscape.

This book aims to guide the reader through Peirce's philosophy. There are various ways of doing this. One can discuss it chronologically, carefully tracing the important steps he takes during the six decades he is working on philosophical and other issues.[1] Such an approach has great advantages. It will show the external and internal strains that cause pivotal shifts in his position, which leads to a better understanding of what he does and why. But as Peirce is active in so many areas it is also a complicated story, and a story that depends heavily on a good understanding of the main currents of thought in his time. Peirce does not write in a vacuum. He is keenly aware of what is going on in mathematics and in the sciences, and he makes extensive use of it.

Alternatively, one can highlight certain issues, for instance those where Peirce is most innovative or most influential. The problem with such a "greatest hits" approach is that it fails to show the systematic character of his work. A third approach is to focus on its systematic character and discuss Peirce's contributions in the framework of it. This is the approach taken in this book. Peirce spends an inordinate amount of time classifying the sciences and positioning philosophy among them. Choosing Peirce's classification of the sciences, which includes an ordered classification of philosophical activities, and structuring the discussion of his work around it, has the further advantage that one can make rather detailed detours without losing sight of the whole.

1.1 The birth of a polymath

Peirce is one of a handful in the history of thought who can truly be called a universal intellect. Robert Crease calls him "a prolific and perpetually overextended polymath," and that pretty well sums him up.[2] He is deeply involved in the main currents of thought (mathematics, logic, experimental science), most of which are in rapid transition, and he makes significant contributions to a great variety of areas. Some have called Peirce the American Aristotle, others the American Leibniz,[3] and it would certainly be no less appropriate to call him the American Leonardo, after that most famous of polymaths Leonardo da Vinci.

Typically, the making of a polymath begins at childhood, and that's true here as well. Peirce's father was a Harvard mathematician and astronomer who played a key role in the establishment of a scientific community within the US.[4] He was involved in the creation of the National Academy of Sciences and the Smithsonian Institution, and from 1867 to 1874 he was in charge of the US Coast Survey, which at the time was America's premier scientific institution. Because of this, and because his father was a polymath of sorts as well, Peirce is already at a young age exposed to the workings of science. As he later reminisces: "all the leading men of science, particularly astronomers and physicists, resorted to our house; so that I was brought up in an atmosphere of science. But my father was a broad man and we were intimate with literary people too" (SS:113).

Benjamin Peirce saw early on that young Charles was gifted and he took an active role in his early education. Benjamin was an unconventional teacher who taught his students by inspiring them rather than by carefully guiding them through proofs and to the solutions of problems. He was known to throw his proofs and solutions rapidly onto the blackboard, preferring quick and elegant solutions, and speedily erasing what he had written the moment he ran out of space. The common opinion was that the rings on Peirce's ladder stood too far apart and that though he ascended easily most others fell through. Those who fell through, however, still spoke highly of him and were grateful to have been his student. About the education he receives from his father, Peirce later remarks: "He very seldom could be entrapped into disclosing to me any theorem or rule of arithmetic. He would give an example; but the rest I must think out for myself" (R619:5). To help him with the latter, Peirce continues, "[my father] took great pains to teach me concentration of mind and to keep my attention upon the strain for a long time. From time to time he would put me to the test by keeping me playing rapid games of double-dummy from ten in the evening until sunrise, and sharply criticizing every error" (id.). About the extent of his father's influence, Peirce later writes: "He educated me, and if I do anything it will be his work" (R1608:2).

Hence, from early on Peirce is put into the habit of thinking things out for himself. Though he is an avid reader with a solid knowledge of the history of science and philosophy, and kept up with contemporary work, he retains this habit of thinking things through in his own way, which contributes greatly to his originality as a thinker.

This habit of thinking things out for himself gets a major boost when at the age of twelve his uncle Charles Henry Peirce helped him set up a chemistry laboratory at home. Charles Henry had been a student and assistant of Eben Horsford who had introduced Justus von Liebig's experimental method of teaching at Harvard (Horsford had studied with Liebig in Germany). Rejecting the purely theoretical way chemistry was being taught, Liebig gave each student a series of bottles marked with the letters of the alphabet. The student was asked to analyze the contents of each bottle, using as the sole guide an introductory textbook in qualitative analysis. Over the years, the number of bottles in Liebig's course had grown to a hundred and it took the average student about a year to complete the exercise. It was on this model that Peirce's home laboratory was set up, together with a copy of his uncle's translation of Stöckhardt's *Principles of Chemistry*.

Though Liebig's method of teaching was strictly an exercise in chemical analysis, it could be applied to experimental science more broadly. Eben Horsford picked up on this. He used the method not only in his own teaching, but when he founded the Lawrence Scientific School at Harvard—a school Peirce graduates from in 1863—he modeled the entire school after Liebig's method. In 1869, the impact of Liebig's method widened even further when

Charles Eliot—also a student of Horsford and Peirce's chemistry teacher at Lawrence Scientific—becomes President of Harvard. Eliot remains president for 40 years, making Harvard the first American university to be solidly grounded in the principles of experimental science. Briefly, already at the age of twelve, while experimenting with Liebig's bottles, Peirce is deeply immersing himself in the experimental method. This happens at a time when that method itself and the science it generates are also still in their infancy. Thus, a brilliant mind that still possesses the openness of youth finds before him a fertile land that lies mostly untilled. Moreover, the Liebig method is a very practical way of learning chemical analysis, one where the difference in the contents of the bottles is determined by the practical consequences of the various operations performed upon them. As we will see in the chapters that follow, this too leaves its mark on how Peirce comes to see not only science but also philosophy.

Also at the age of twelve, Peirce reads Richard Whately's *Elements of Logic,* a work that revitalized the study of logic in the English-speaking world.[5] Peirce finds the book in his older brother's room and promptly devours it. Later he repeatedly says that from then on logic was his strongest passion. For instance, when working as a scientist, Peirce retains a strong focus on methodology, making it his first priority to penetrate into the logic of things. As with chemistry, Peirce's introduction to logic also comes when the discipline is in the process of a dramatic transformation. It is around this time that the British mathematician George Boole develops an algebra for logic, giving logic a mathematical grounding that not only frees it from the restraints of Aristotelian syllogisms but that also opens the door for extensive new research. Peirce, who thanks to his father already had an affinity for mathematics, comes to play an important role in this. Although Whately's logic is predominantly Aristotelian, one can also discern a strong influence of John Locke. Whately rejected, for instance, the problematic notion of "abstract ideas," arguing instead that we think in signs.[6] Hence, we can find in Whately some of the early seeds of Peirce's semeiotics (Chapter 5).

Benjamin Peirce did not share his son's fascination for logic. In fact he had a very low opinion of logic, preferring instead "to draw directly upon the geometrical instinct" (echoes of this return in Peirce's logical graphs; Section 4.7).[7] Benjamin Peirce also had a low opinion of the reasoning of philosophers more generally, and time and time again he would force his son to "recognize the extremely loose reasoning common to the philosophers" (CP2.9). After Whately's logic, Peirce's first readings in philosophy are Friedrich Schiller's *Aesthetic Letters* and Immanuel Kant's *Critique of Pure Reason*. It was Schiller who introduces Peirce to Kant, and he begins reading Kant's first *Critique* shortly before his seventeenth birthday. Peirce spends roughly three years studying the first *Critique*, a process during which his father proves very influential. As Peirce puts it: "[the *Critique of Pure Reason*] was sort of a Bible to me; and if my father had not exposed the weaknesses of some of its arguments, I do not know to what lengths my worship of it might not have gone" (R619:10f). Notwithstanding the sobering influence of his father, Kant continues to have a far-reaching and profound influence on Peirce's thought.

Peirce goes to Harvard at sixteen. At Harvard the habit his father instilled in him—that of seeking his own way intellectually—works against him, and he performs rather poorly. In 1858, he joins a local expedition of the Coast Survey, which is not uncommon at the time for scientifically inclined students. It is there that Peirce finds his stride; it proves the beginning of a 30-year career as a scientist. In July 1861, Peirce received his first official appointment as a lowly paid computer, but he quickly moves up. In a little over a decade he is in charge of gravitational research and is promoted to the Survey's highest rank, that of Assistant to the Superintendant. Also in 1861, Peirce enters Harvard's Lawrence Scientific School to study chemistry. Two years later he graduates summa cum laude.

1.2 An outsider

Being found a genius also has its darker side. From early on Peirce is constantly told he is a genius and he is treated as such, and this significantly affects how he comes to see himself. Especially during the first half of his life it leaves a strong mark on his relationships with others, though other personality traits also contribute to this. Although Peirce considers himself a genius, and spends much time studying what he called "great men," he typically explains his own success as a thinker in terms of his great power of concentration and a dogged, pedestrian persistence which he jokingly calls Peirce-istance, or Peirce-everance.

George Whalley, the editor of the works of Coleridge, once remarked that what sets the genius apart is "not the sheer quantity of learning . . . but the incandescence, the opulence, the extravagant gratuitousness, the rapidity of mind."[8] Peirce's close friend William James makes a similar observation about Peirce when he characterizes Peirce's 1903 Lowell lectures as "flashes of brilliant light relieved against Cimmerian darkness."[9] Ralph Waldo Emerson, one of the literary figures that frequented the young Peirce's home, points out another aspect of genius that aptly applies to Peirce: "Genius is always sufficiently the enemy of genius by over-influence."[10] Although Ian Hacking overstates his case when he describes Peirce as a wild man who began almost everything and finished almost nothing,[11] Emerson's observation is much of the reason why the systematic philosopher never completed a book about his philosophy—he is constantly moving in new directions, never satisfied with what he had written.

Overall Peirce provides a painful example of a great thinker with a failed career. Apart from a brief stint as a lecturer in logic at Johns Hopkins (1879–84), he never holds a university position. An initially brilliant career at the Coast Survey comes to a sudden and graceless end in 1891, after roughly 30 years of service. Peirce works the last third of his life as an independent scholar, which forces him to constantly struggle for money. During this period, he writes mostly for a living, gives lectures, and does occasional free-lance work in a variety of fields. In 1887, while he is still working for the Coast Survey, he moves to Milford, Pennsylvania, a small resort town not far from the Port Jervis train station, from which it is only a few hours to New York City. Just outside Milford, he purchases a small farmhouse with quite a bit of land. The house becomes an obsession, and at the time of his death it had grown into a 25-room mansion.

The story of Peirce's life is complicated and one that still needs to be told.[12] There are many theories on why he fell from grace. He had a difficult personality. He had powerful enemies, including Harvard President Charles Eliot and Simon Newcomb (the latter became America's premier scientist). He was considered a deeply immoral man and a bad role model for students when he married his mistress, a mysterious French woman with whom he had been living openly, only two days after he divorced his first wife (even though she left him seven years before). Irrespective of his personality traits, his enemies, and the moral reprobation, the mere fact that during the last third of his life he lives in relative isolation and is not connected to a university is by itself enough to make him an outsider. Fortunately, he also has a few good friends. The latter include Harvard philosopher Josiah Royce, who is deeply influenced by Peirce and arranges for his library and papers to come to Harvard.

1.3 The Peirce papers

Peirce thought with his pen, he thought often, and he seldom threw anything away. The result is that upon his death he left behind an enormous mass of manuscripts. Estimates vary, but it is typically conjectured that there are over a hundred thousand manuscript pages preserved in Harvard's Houghton Library, with substantial deposits elsewhere. The history of the manuscripts may even be more complicated and controversial than Peirce's life.[13] From the start, the aim was to organize the manuscripts and extract from them material suitable for publication. This task proved overwhelming, not just because of the sheer volume of the papers, but also because of their disorganized state, Peirce's propensity to digress and leave things unfinished, and his constant reworking of issues. Martin Heidegger once described his own thoughts as *Holtzwege*, after the countless trails found in well-traveled woods, often so faint it is unclear whether they even are trails, and many leading nowhere.[14] More recently, Vincent Colapietro described Peirce's writing as a one-man jam session—it is restlessly experimental, improvisational, and prone to digression.[15] Both are apt descriptions, and it makes the task of any editor no easy one. Often there are endless variations on a theme, and frequently there is no clear winner as different strands have different things to offer. Peirce once described himself as having the persistence of a wasp in a bottle, and this shows in his writings.

Initially, Josiah Royce took charge of the manuscripts, but about a year into the project he suddenly passed away himself. Royce's graduate student W. Fergus Kernan did much of the initial sorting, but soon left to fight in World War I. After that the manuscripts fell into disarray. At the end of the 1920s, when Charles Hartshorne and Paul Weiss began working on what was to become their six-volume *Collected Papers*, they found that someone had gathered everything in a few large piles. The sheer quantity of the material and their disorganized state made it impossible to extract from them six volumes that would do justice to Peirce's thought. Because so few manuscripts were dated, the two editors decided on a thematic approach, following Peirce's classification of the sciences. And because they had so little space, they typically limited themselves to what they thought was the best text on a certain subject. This meant that often only parts of documents were included. The edition brings a large portion of Peirce's philosophical work together (published as well as unpublished), but it does so in a manner that gives the impression of an undisciplined thinker who was prone to contradict himself without noticing it. This greatly affected the reception of Peirce's work. Another negative consequence is that with the *Collected Papers* published, the manuscripts were thought to be of no more scholarly value and quite a number of them were given away as mementos.

In the late 1950s, Arthur Burks edited two more volumes and Max Fisch was enlisted to write an intellectual biography that was to form the capstone to the edition. Fisch quickly discovered that a systematic study of Peirce based on the *Collected Papers* was impossible, and a new effort ensued to organize the manuscripts. In 1967, this resulted in Richard Robin's *Annotated Catalogue of the Papers of Charles S. Peirce*, which also relied for its organizing principle on Peirce's classification of the sciences. The catalogue became the basis of a microfilm edition, so that by the end of the 1960s the bulk of the material held at Harvard was more widely accessible. The next large-scale editions of Peirce's works are Carolyn Eisele's *New Elements of Mathematics* (5 vols in 4) and the four-volume *Contributions to The Nation*, edited by Kenneth Ketner and James Cook. Both editions appeared in the 1970s. The first contains Peirce's extensive work in mathematics; the second contains the many book reviews he wrote for *The Nation*. Fisch, however, realized that to understand Peirce's philosophy we must be able to follow the trajectory of his thought. This resulted in a project on a much grander scale: *The Writings of Charles S. Peirce: A Chronological Edition*. This edition, which is projected to span 30 volumes, depends heavily on a far-reaching reorganization of the manuscripts in which the papers held at Harvard and elsewhere are ordered chronologically. This edition, which is still a selective edition (publishing everything would require at least a hundred volumes), is not limited to Peirce's philosophical writings—as if they can neatly be separated from the rest—but covers also the work he did in the sciences, in mathematics, and in other areas.

1.4 Classifying the sciences

In good nineteenth-century fashion, Peirce spends much time and effort devising a classification of the sciences. However, before discussing his classification we should see what he means by classification and by science. Peirce does not aim for some abstract classification in which any conceivable science has its preordained pigeonhole, but he aims more modestly for a concrete classification of the sciences insofar as they are "the actual living occupation of an actual living group of men" (R1334:13). Put differently, his classification is an empirical one that is based on what is taking place in terms of living scientific activity, and in this sense it is very similar to botanical and zoological classifications. Such natural classifications can be contrasted with artificial classifications where the criteria for inclusion are determined beforehand.

So what types of scientific activity are taking place? For our purpose here it suffices to say that when Peirce speaks of a science he means "life devoted to the pursuit of truth according to the best known methods on the part of a group of men who understand one another's ideas and works as no outsider can" (R1334:14). With the latter he means that their studies are so closely allied "that any one could take up the problem of any other after some months of special preparation" (R1334:15), and that they understand each other's work to the point of being thoroughly conversant about it with one another. It is further important to note that Peirce's interpretation of science is a very broad one. It includes any endeavor where one is devoted to the pursuit of truth, whether this is the homicide detective searching for a killer, the historian looking for the identity of Jack the Ripper, the geneticist seeking to uncover the sequence of DNA, or the astrophysicist who wants to understand the nature of black holes. Often what unites a group of scientists is a familiarity with certain theories, a shared language, or a skill in the use of certain instruments or in making particular sorts of observations; in brief, it's a division according to methods, ideas, and instruments (CD:5379). What distinguishes, say, the specialist in optics from the astronomer, is that the former is intimately familiar with the principles on which the latter's instruments are based. The astronomer lacks the conversancy in optical theory that the specialist in optics has, while the specialist in optics lacks the astronomer's skill of using telescopes to extract knowledge from the heavens. As this classification is based on scientific practices and the communicability of ideas, its boundaries will be vague and open to revision when practices change or when future inquiry leads to new areas of cooperation. Moreover, as with the evolution of biological species, the classes are not defined in terms of some ideal, suggesting that it is our task to bring us closer to that ideal.

Peirce's classification of the sciences is thus one according to actual scientific *practices,* and not one according to the *objects* of scientific knowledge, whether they are actual or merely possible. Peirce thereby rejects the standard account on which science is defined as systematized knowledge. Moreover, as we shall see in Chapter 6, he further rejects the idea that science can be defined in terms of a specific method, the so-called scientific method.

So how does Peirce classify the sciences? His first division is between the sciences of discovery (or the heuretic sciences), the sciences of review, and the practical sciences. For Peirce, *the sciences of discovery* exemplify science in its purest sense. Their aim is the acquisition of positive knowledge solely for the sake of gaining knowledge. *The sciences of review* seek to draw together the fragmented discoveries made in the heuretic sciences and make them available to a wider audience. It is here also that we find broader reflections upon and critical assessments of the work done in the narrowly focused heuretic sciences. Peirce's classification of the sciences belongs to the sciences of review, as do similar enterprises of August Comte and Herbert Spencer. *The practical sciences,* finally, seek to meet some human need. A good example is civil engineering, which uses the findings of the heuristic science of analytical mechanics for some practical application, like the construction of a skyscraper or suspension bridge. This third area is by far the largest and most people who call themselves scientists fall in it.

Peirce's interest is in the heuretic sciences and it is important to keep this in mind, otherwise one might mistakenly conclude that, for Peirce, all attempts to apply knowledge are suspect and that we should only search for disinterested knowledge. What Peirce *is* objecting to is to let practical considerations of purpose affect how knowledge is acquired in the *heuretic* sciences. I return to this in Chapter 6.

The first division in the heuretic sciences, or the sciences of discovery, is between mathematics and what Peirce calls, following Comte, the positive sciences. The positive sciences seek to affirm or deny, in a categorical proposition, something of some subject, such as, "cows have four stomachs," "the mutual forces of action and reaction between two bodies are equal but opposite and collinear," "Mount Everest is the tallest mountain on Earth," and "beriberi is not caused by an infectious agent." In Peirce's view, mathematics has a very different aim. In mathematics, we make no positive assertions of fact. It is purely the study of hypothetical or conditional propositions. The mathematician, Peirce explains, "makes no external observations, nor asserts anything as a real fact. When the mathematician deals with facts, they become for him mere 'hypotheses'; for with their truth he refuses to concern himself" (CP3.428). By making this move, Peirce rejects the view of Comte and positivists and empiricists more generally, who consider mathematics the most general and most fundamental of the positive sciences. Peirce's conception of mathematics, as well as its relation to the positive sciences, is discussed in Chapter 2.

Peirce next divides the positive sciences into philosophy and the special sciences. The special sciences are those that require special equipment or familiarity with certain theories, terminology, or methods. Typical examples are particle physics, microbiology, and linguistics. Peirce divides the special sciences in two parallel classes: the physical sciences and the psychical sciences. Philosophy, in contrast, requires no specialized equipment or background knowledge. It is nothing but "a more attentive scrutiny and comparison of the facts of everyday life" (EP2:146). In principle anyone can do it. Philosophy is subdivided in phenomenology, the normative sciences (esthetics, ethics, and logic), and metaphysics. The aim of the first is "to draw up an inventory of appearances without going into any investigation of their truth" (CP2.120); that of the second to study how these phenomena relate to certain ends (traditionally these are beauty, goodness, and truth); that of the third to develop a *Weltanschauung* that can form the basis for the special sciences. According to Peirce, we cannot avoid having a metaphysics; we can only fail to make it explicit. Note that the kind of metaphysics Peirce has in mind here is a *scientific* metaphysics. Peirce's metaphysics is discussed in Chapters 8 and 9.

The above division gives the following sequence of the sciences of discovery. First we have a division between mathematics and the positive sciences. The latter are further divided into phenomenology, esthetics, ethics, logic, and metaphysics, after which comes the special sciences parallel divided into the physical and the psychical sciences. Each of the positive sciences depends on those that precede it for its grounding principles while providing the latter with fresh material to contemplate. The relation is thus not one-directional. For instance, though logic is a more basic science than mechanics, Peirce observes that from Plato and Aristotle onward logic made no significant progress until around 1590, when Galileo developed the science of dynamics. And he further adds that, "it was the study of dynamics, more than anything else, which gradually taught men to reason better on all subjects" (R447:5). Moreover all the positive sciences can use mathematics.

Before continuing, a few words should be said on Peirce's ethics of terminology. Peirce is often berated for his penchant for complicated neologisms, and not infrequently he coins words to never use them again. This is true, for instance with *heurospude, taxospude,* and *prattospude*, which denote respectively the sciences of discovery, the sciences of review, and the practical sciences (R1334:25). But those who berate him do not give him enough credit. Peirce observes that biology and chemistry made their significant advances only after they developed a clear nomenclature, and he is keenly aware that many philosophical problems are caused, or their solutions hampered, by poor terminology and a continual and often implicit redefining of terms. Peirce, who is himself trained as a chemist, envisions for philosophy a terminology similar to chemistry, with an almost modular construction and a system of prefixes and suffixes. He also maintains that once a term is introduced one should stay as close as possible to the meaning that was then given to it.

CHAPTER TWO

Mathematics and philosophy

In the opening chapter, we saw that when classifying the sciences of discovery Peirce sets mathematics apart from the other sciences, which he calls the positive sciences. The aim of the positive sciences is to increase our knowledge of the actual universe. Mathematics, as Peirce envisions it, is not confined in this way. Its aim is not to study how things are, have been, or will be but to study purely hypothetical states of things, and for that it does not matter how far they stray from the world we experience. As noted, by separating mathematics from the positive sciences, Peirce departs from Comte and others who consider mathematics the most abstract and the most basic of the positive sciences.

In part what causes Peirce's break with Comte is the transformation of mathematics that takes place in the nineteenth century. The old view of mathematics as the science that describes nature in its most general terms—that is, only insofar as things can be counted or measured—is no longer tenable. For instance, non-Euclidean geometries that allow us to work with spaces very different from the physical space we are accustomed to, and consistent algebras that include things like the square root of minus one, cause mathematics to depart radically from the world of sense and consequently also from our intuitions (for Peirce our intuitions are products of biological evolution). The result is that at the close of the nineteenth century mathematics is a very different discipline than what it was at the beginning of that century.

Peirce is well versed in mathematics. As we saw in Chapter 1, his father used his mathematical skills to persistently attack the poor reasoning of the philosophers to whom the young Peirce took a liking. This made a deep impression and during the course of his life Peirce continues to play close attention to mathematics, so much so that in the 1880s James Joseph Sylvester, the great mathematician of the day, called him "a far greater mathematician than his father."[1] Although during his life little of Peirce's work in mathematics is published, a wealth of material survives among his unpublished papers, including two completed book manuscripts.[2] A project Peirce is long engaged in is to publish a textbook, *The New Elements,* that incorporates the recent transformation of mathematics, and that is to replace Euclid's *Elements,* which after two millennia is still the standard mathematics textbook.

In the current chapter, we look at Peirce's views on mathematics, as some understanding of Peirce's views on mathematics is crucial for an adequate understanding of his philosophy. Philosophers often look at mathematics as the model of good reasoning. This is very evident in Spinoza's *Ethics*, which even mimics Euclid's *Elements* in its structure of definitions, axioms, and theorems. Peirce agrees that mathematical reasoning is crucial for philosophy—and his classification of the sciences shows as much—but he also observes that philosophers often have no adequate understanding of what mathematical reasoning consists in, and that most are oblivious of the recent changes that had taken place in mathematics and their philosophic repercussions.

Philosophy, Peirce writes, "requires exact thought, and all exact thought is mathematical thought" (R438:3). Consequently, Peirce seeks to introduce mathematical exactitude into philosophy in part as a means of reducing error:

> All danger of error in philosophy will be reduced to a minimum by treating the problems as mathematically as possible, that is, by constructing some sort of a diagram representing that which is supposed to be open to the observation of every scientific intelligence, and thereupon mathematically,—that is, intuitionally,—deducing the consequences of that hypothesis. (R787:7)

The latter part of this quotation gives some insight in what Peirce takes mathematical reasoning to be. It involves the construction and observation of and experimentation upon diagrams. This notion and 2B, on line segment AB equals the sum of two right angles (right angles are formed when a line intersects another perpendicularly). Next extending line BC beyond C, we can see immediately that this repeats itself where BC intersects the line that runs through C. Next we observe that because BC is a straight line too, the angle right adjacent to C must again be equal to B. We can then go through the exact same routine for angle A by extending AC and showing that the angle left adjacent to C is equal to A (Figure 2.1c). This completes our proof that the sum of the angles A, B, and C is equivalent to the sum of two right angles (Figure 2.1d).

A few observations can be made. First, as is clear from the example, the construction goes well beyond what is explicitly expressed in the concept we started off with. Second, the argument does not proceed syllogistically, that is, by substituting concepts for concepts, as in: "All men are mortal"; "Socrates is a man," hence "Socrates is mortal." Instead, to use Kant's terminology, we *construct* a concept; that is, we exhibit a priori the intuition that corresponds to the concept, which is something we can do because of our already existing intuition of space and time. In the process, we create a *single* object that is representative of all possible pure intuitions relating to that concept. This allows Kant to maintain that the conclusions that are being drawn are necessary and not merely contingent as with empirical arguments. Third, this construction is essential to the argument. It is not merely an auxiliary tool to facilitate our mathematical reasoning, but it forms the very core of that reasoning. In fact, Kant denies that we can reach conclusions like the one above through a chain of syllogisms. Fourth, Kant's use of pure intuition allows him to ignore those elements of the created objects that are of an empirical nature, such as the color or the thickness of the lines, or the lines drawn not being perfectly straight, up to the point that the conclusion may be visibly false for the actual figure drawn (which typically happens when we make a quick sketch on a napkin). Fifth, mathematical reasoning, though a priori, allows us to gain new insights that carry us beyond the concepts we began with. In Kant's terminology, mathematics is thus both a priori and synthetic. This means that, for Kant, mathematics is a science of discovery.

Though geometry best exhibits this type of reasoning, Kant extends it to other areas of mathematics as well. But that is also where it ends. Most importantly, Kant believes that mathematical reasoning is not applicable within philosophy, including logic. In philosophy, we should limit ourselves to the type of discursive reasoning described above. Mathematics realizes its concepts within pure intuition, but this also means that its application is limited to pure intuition. Philosophy can do neither. It cannot realize its concepts within pure intuition, as it depends for its insights on what is given to us through the senses, and it cannot limit the application of its concepts to the realm of pure intuition without giving up on its mission of gaining knowledge of the empirical world. For Kant, the divide is radical. In fact, the difference between the two types of reasoning is so great, and what they try to establish so different, that at least in mathematics and philosophy they cannot be combined.[5] Whereas the propensity of philosophers to use mathematical reasoning only leads to extravagance and error, applying the discursive method of philosophy to mathematical problems avoids error, but only because it is perfectly vacuous. The application of discursive reasoning in mathematics can never go beyond a purely analytic explication of concepts, and thus can never yield any new knowledge.

Though Peirce finds much in Kant that he likes, he disagrees with Kant at crucial junctions, concluding not just that mathematical reasoning can be used in philosophy, but that it is even indispensible for it—one cannot do philosophy without engaging in mathematical reasoning. Hence, it is with a chapter on Peirce's conception of mathematics that we begin.

2.2 The exact study of ideal states of things

As noted, mathematics was long considered the science that gives us the most abstract description of the world, as it only studied things insofar as we can measure or count them. Because of this, mathematics was traditionally defined as the science of quantity—continuous quantities in the case of geometry and discrete quantities in the case of arithmetic. The developments in the nineteenth century made this view of mathematics as the metrics of nature no longer tenable. This had a liberating effect. It freed mathematicians from the requirement that their products must reflect the natural world. This liberation caused Georg Cantor, with whose work Peirce is very familiar, to proclaim that freedom is the essence of mathematics:

"Mathematics is perfectly free in its development and is subject only to the obvious consideration that its concepts must be free from contradictions in themselves, as well as definitely and orderly related by means of definitions to the previously existing and established concepts."[6] The natural world no longer constrained mathematics.

In his *Linear Associative Algebra,* Peirce's father defined mathematics as "the science which draws necessary conclusions."[7] This definition forms the basis of Peirce's own conception of mathematics. According to Peirce, his father's definition makes mathematics the study of purely hypothetical states of things: "Since it is impossible to draw necessary conclusions except from perfect knowledge, and no knowledge of the real world can be perfect, it follows that, according to this definition mathematics must exclusively relate to the substance of hypotheses" (PM:7). For Peirce, these hypotheses are mere mental creations, and he even goes as far as to state that except for their precision, clearness, and consistency, they are not unlike dreams (R17:7).

Benjamin Peirce concluded from his definition that neither the discovery of laws nor the framing of theories properly belongs to mathematics.[8] At this point Peirce departs from his father, maintaining that mathematics includes *both* the framing of theories *and* the deduction of their consequences. In accordance with his natural classification of the sciences (Section 1.4), Peirce argues that we should look at mathematics as the living enterprise mathematicians are actually engaged in. If we look at mathematics this way, Peirce continues, we naturally include "everything that is an indispensible part of the mathematician's business; and therefore we must include the *formulation* of his hypotheses as well as the tracing out of their consequences" (PM:91). This means that, for Charles Peirce, mathematics deals with far more than drawing necessary conclusions.

The best way to examine Peirce's broader conception of mathematics is to see how it connects with the positive sciences, as it is there that mathematicians find their raw material. In Peirce's view, pure mathematics is ultimately a product of applied mathematics. This is how Peirce sketches the typical trajectory: "the business of the mathematician lies with exact ideas, or hypotheses, which he first frames upon the suggestion of some practical problem, then traces out their consequences, and ultimately generalizes" (R188:2). On this view, when physicists, meteorologists, or economists are faced with a complicated problem they call the mathematician for help.

The mathematician then seeks to construct a state of things that is far simpler than the complex reality that is being investigated, while ensuring that this simplification does not affect the practical answer that is being sought. In this way mathematicians provide scientists with a skeleton models—or hypotheses, as Peirce calls them—that can be studied instead of the phenomena themselves in all their fortuitous detail. Since the hypothesis we want the mathematician to consider should be one that is well suited for mathematical treatment, Peirce argues that framing the hypothesis should fall under the purview of mathematics rather than the empirical science the hypothesis is meant to serve. As Peirce puts it:

> The results of experience have to be simplified, generalized, and severed from fact so as to be perfect ideas before they are suited to mathematical use. They have, in short, to be adapted to the powers of mathematics and of the mathematician. It is only the mathematician who knows what these powers are; and consequently the framing of the mathematical hypotheses must be performed by the mathematician. (R17:6f)

Put briefly, framing such hypotheses does not require more detailed empirical work, but calls for mathematical imagination; that is, "the power of distinctly picturing to ourselves intricate configurations" (R252:20).

As for the nature of the hypothesis arrived at, Peirce observes the following:

> The hypothesis . . . must have such a degree of definiteness as to permit formal deductions. . . . In other respects, the less definite the hypothesis is, the better. Thus, it would be a hindrance rather than a help to suppose a geometrical figure to have any particular color. Finally, the hypothesis of the mathematician is always of an intricate kind, so that all the relations involved cannot be seen at a glance. (NEM3:749)

Once the hypothesis is framed, the mathematician may generalize it to such a degree that it loses all connection with the practical problem that occasioned it. The development of non-Euclidean geometries and the use of imaginary numbers in arithmetic are clear examples of this.

Now what constitutes the powers of a mathematician? Peirce distinguishes three of them: imagination, concentration, and generalization. From these he extracts what he calls the duty of the mathematician, which is threefold:

> 1st, acting upon some suggestion, generally a practical one, he has to frame a supposition of an ideal state of things;
> 2nd, he has to study that ideal state of things, and find out what would be true in such a case;
> 3rd, he has to generalize upon that ideal state of things, and consider other ideal states of things differing in definite respects from the first. (NEM2:10)

In particular the power of generalization, which Peirce thinks "chiefly constitutes a mathematician" (R278a:91), is a skill difficult to attain. Peirce's emphasis on imagination, concentration, and generalization draws the attention away from the popular belief that it is the business of mathematicians to provide proofs.

Having explained how mathematical models come to be and having provided some insight into the mathematical mind-set, we can now characterize mathematics, as does Peirce, as "the exact study of ideal states of things" (NEM2:10).[9] That is to say, the practical motives that spurred the inquiry have been removed and all energy is directed to a study of the models themselves, irrespective of any relation they might have to anything external to them, and irrespective of any motives the inquirer might have other than studying the models entirely for their own sake.

For Peirce, the business of the mathematician thus consists of three parts: framing ideal states of things that are inspired by practical problems, determining what is true for these ideal states of things, and finally, studying them wholly in their own right without any reference or concern for what spurred their construction. In the next section, we examine what Peirce means by mathematical reasoning.

2.3 Mathematical reasoning

Like Kant, Peirce maintains that mathematical reasoning is diagrammatic. Having framed his hypothesis in general terms, the

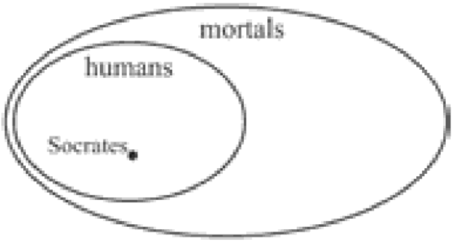

FIGURE 2.2 *Venn Diagram.*

being present in both premises. Instead of the above syllogism we could draw a Venn diagram. Here again we immediately *see* that given what we accept we cannot avoid concluding that Socrates is mortal.

In accordance with his discussion of mathematical reasoning, Peirce distinguishes two types of deduction, corollarial and theorematic, and he considers the discovery of this distinction his "first real discovery about mathematical procedure" (NEM4:49). A corollarial deduction "represents the conditions of the conclusion in a diagram and finds from the observation of this diagram, as it is, the truth of the conclusion" (CP2.267). The syllogism about Socrates is a good example. No one who sees it can reasonably doubt the conclusion. Theorematic deduction, in contrast, "is one which, having represented the conditions of the conclusion in a diagram, performs an ingenious experiment upon the diagram, and by the observation of the diagram, so modified, ascertains the truth

FIGURE 2.3 *The rabbit that is also a duck: Joseph Jastrow's duck-rabbit picture.*[10]

of the conclusion" (id.). The proof given before that the sum of the three angles of a triangle equals two right angles, is an example of theorematic deduction. Since in theorematic deduction free acts are performed upon a diagram, the same conclusion can often be reached in multiple ways. In corollarial deduction there is no such variety—one can only see what is there or fail to do so. Though merely looking at a diagram can yield surprise, as when we discover that a picture of a rabbit is also a picture of a duck, theorematic deduction is the true source of surprise in mathematical reasoning.

An important mathematical operation, one that furnishes much of the material for the hypothetical states of things studied by mathematicians, is abstraction. Peirce distinguishes two operations of thought to which the term abstraction is generally applied. First, there is the situation where we concentrate our attention on one feature of something to the neglect of others. We do this when in buying a couch we focus on its color, while neglecting its size, style, comfort, etc. Peirce calls this prescissive abstraction or *prescission* (CP4.235). Thus, in geometry we *prescind* shape from color (CP5.449). Peirce distinguishes prescission from what he calls hypostatic abstraction. By this he means the creation of an *ens rationis,* or object of reason, from nonsubstantive thought (id.). In hypostatic abstraction, a thought about a subject is made itself a subject of thought, and thus it can become an independent subject of discourse (CP5.534). We do this when we move from the adjective "virtuous" to the noun "virtue" and then proceed to develop a theory of virtues. For Peirce, the construction of objects of thought through hypostatic abstraction is essential to mathematics:

> In order to get an inkling—though a very slight one—of the importance of this operation in mathematics, it will suffice to remember that a collection is an hypostatic abstraction, or *ens rationis*, that multitude is the hypostatic abstraction derived from a predicate of a collection, and that a cardinal number is an abstraction attached to a multitude. So an ordinal number is an abstraction attached to a place, which in its turn is a hypostatic abstraction from a relative character of a unit of a series, itself an abstraction again. (CP5.534)

In brief, in hypostatic abstraction we extract a certain aspect from a hypothetical state of things and make it an independent object

INDEX