# PHYSICS FOR THE INQUIRING MIND

## The Methods, Nature, and Philosophy of Physical Science

# PHYSICS FOR THE INQUIRING MIND

## THE METHODS, NATURE, AND PHILOSOPHY OF PHYSICAL SCIENCE

BY

ERIC M. ROGERS

PRINCETON, NEW JERSEY

PRINCETON UNIVERSITY PRESS

LONDON: OXFORD UNIVERSITY PRESS

London: Oxford University Press


Publication of this book has been aided by grants
from The Rockefeller Foundation,
The Alfred P. Sloan Foundation, and The
Whitney Darrow Publication Reserve Fund
of Princeton University Press

Printed in the United States of America

Second Printing 1961
Third Printing 1961
Fourth Printing 1962
Fifth Printing 1963
Sixth Printing 1964
Seventh Printing 1965
Eighth Printing 1966

PUBLISHER'S NOTE TO THE EIGHTH PRINTING:
In this printing there has been a
significant rearrangement and revision
of the material in Chapter 30, beginning
on page 450 and continuing to the end
of the chapter. Various other minor
corrections have been made running
throughout the text.

Ninth Printing 1968
Tenth Printing 1970
Eleventh Printing 1973
Twelfth Printing 1977

# CONTENTS

# PART ONE

## MATTER, MOTION, AND FORCE

"Give me matter and motion, and I will construct the universe."

—René Descartes (1640)

". . . from the phenomena of motions to investigate the forces of nature, and then from these forces to demonstrate the other phenomena; . . . the motions of the planets, the comets, the moon and the sea. . . ."

—Isaac Newton (1686)

"No one must think that Newton's great creation can be overthrown by [Relativity] or any other theory. His clear and wide ideas will forever retain their significance as the foundation on which our modern conceptions of physics have been built."

—Albert Einstein (1948)

# PRELIMINARY PROBLEMS LEADING TO CHAPTER 1

A wise explorer reviews his maps before he starts on the expedition. You would be wise to review your present knowledge and prejudices before this chapter offers you new knowledge. The problems below are not intended to discomfort you by asking for answers before you are prepared. They are only intended to clear the ground for discussion. Some ask you to check minor matters of vocabulary. Others raise major questions that will appear again and again through the course.

1. (a) "I shall try an experiment. . . ." Suppose you have just made such a remark with your present knowledge and views. Write a note of a few lines to show what you would mean by it.
   (b) Write a similar note for, "I have a theory that. . . ."
   (c) Write a similar note for, "I shall treat this scientifically."

(At this stage, before you begin the course, we do not expect you to know all the answers to questions like this. Here you are asked to describe your present views. Later you may change them.)

2. Look up the word "logical" in a good dictionary; then write short answers to the following:
   (a) State in your own words the proper meaning of the word "logical."
   (b) State in your own words the colloquial or slang use of the word.
   (c) What word(s) could be used aptly for the meaning in (b), leaving "logical" for its important use in science, philosophy, etc.?
   (d) Do you consider algebra logical? Give reason(s) for your answer.
3. Look up the word "data." Then write short answers to the following:
   (a) What is its origin?
   (b) Which of the following statements do you consider correct language and which incorrect? (Where incorrect, mention reason.)
      (i) These data were obtained by my partner.
      (ii) This data was obtained by my partner.
      (iii) This set of data was obtained by my partner.
4. (a) What is the plural of the word "apparatus"?
   (b) What does "phenomenon" mean?
   (c) What is the plural of phenomenon?
5. (The following questions ask for written answers. Try to make them short. Some may require considerable thought. Consult dictionaries if you like. It is hoped that you will enjoy finding answers to these questions. If you have fun puzzling these out, your education will gain; if you do them with a feeling of headache, it will lose. So you are advised to treat these rather lightly, yet fairly seriously.)

   "The sun rose in the east this morning, yesterday morning, the morning before that, and for many mornings before that." This is a statement of observations. Scientists and others make statements like the following: "*I expect* the sun will rise in the east tomorrow morning."

   A number of other statements with some differences in wording may be made, and eight of these are shown below. With each of the statements:

   (a) Write a short explanation of its meaning, paying special attention to the part played by the words *in italics*. (For example, in the version above, where the word "*expect*" is used, your answer might run thus: "Because the sun has risen in the east so regularly in the past, I *look for* the same thing tomorrow with some confidence, and I shall make my everyday plans on this basis, because I have noticed that most such natural events continue to repeat themselves uniformly." Note, however, the dangers of this last view for an insect, hatched in the summer, anticipating an endless series of warm mornings, and completely ignorant of the snowstorm which will end its life.)
   (b) For each case say whether or not you consider the statement a wise or safe one for a scientist to use. (In other words, do you consider the statement scientific or superstitious, safe or risky, "right" or "wrong"?)
   (c) Give a brief reason for each answer in (b).

*Statements:*

A. I *predict* the sun will rise in the east tomorrow morning.
B. I *deduce* the sun will rise in the east tomorrow morning.
C. I conclude from *inductive* reasoning that the sun will rise in the east tomorrow morning.
D. I *believe* the sun will rise in the east tomorrow morning.
E. I *know* that the sun will rise in the east tomorrow morning.
F. I consider it *highly probable* that the sun will rise in the east tomorrow morning.
G. These observations lead to a law which *proves* that the sun will rise in the east tomorrow morning.
H. Investigations show that the world is a solid spinning body and the Principle of Conservation of Angular Momentum proves that it will continue to spin thus, and therefore proves that sunrise, which is due to this spin, will continue to occur in this same way each day.

6. Look up the verb "infer."
   (a) What is the proper meaning? (This is the one to use in science.)
   (b) What is the colloquial use? Give a better verb to replace it.
   (c) The word "infer" is used correctly in one of the statements below and incorrectly, or poorly, in the other. Explain what you think "infer" is intended to mean in each of the passages. Say which passage has the correct use and suggest a substitute for "infer" in the other one.
      (i) "Are you trying to infer, by your remarks, that my uncle is a fool?"
      (ii) "From his behavior, I infer that your uncle is a fool."

# CHAPTER 1 · GRAVITY, A FIELD OF PHYSICS

"What distinguishes the language of science from language as we ordinarily understand the word? How is it that scientific language is international? . . . The super-national character of scientific concepts and scientific language is due to the fact that they have been set up by the best brains of all countries and all times. In solitude and yet in cooperative effort as regards the final effect, they created the spiritual tools for the technical revolutions which have transformed the life of mankind in the last centuries. Their system of concepts has served as a guide in the bewildering chaos of perceptions, so that we learned to grasp general truths from particular observations."

—A. EINSTEIN, *Out of My Later Years*

*Introduction*

In this book, and the course that goes with it, we shall study the nature and methods of physical science. We shall do that by studying some parts of physics thoroughly and leaving out other parts to gain time for discussion. In the samples we study, you will learn many scientific facts and principles, some useful for life in general, others important groundwork for discussions in the course. To gain much from the course, you need to learn this "subject matter" thoroughly. In itself it may seem unimportant—such factual knowledge is easily forgotten,[1] and we are concerned with a more general understanding which will be of lasting value to you as an educated person—but we shall use the factual knowledge as a means to more important ends. The better your grasp of that factual knowledge, the greater your insight into the science behind it. And this course is concerned with the ways and work of science and scientists.

To begin by discussing scientific methods or the structure of science would be like arguing about a foreign country before you have visited it. So we shall plunge at once into a sample of physics—gravity and falling bodies—and later discuss the general ideas involved.

*What to Do about Footnotes*

You are advised to read a chapter straight through first, omitting the footnotes. Then reread carefully, studying both text and footnotes. Some of the footnotes are trivial, but many contain important comments relevant to the work of the course. They are not minor details put there with a twinge of conscience to avoid their being omitted altogether. They are moved out to make the text more continuous for a first reading. Often the footnotes wander off on a side issue and would distract attention if placed in the main text. Yet this developing of new threads itself shows the complex texture of scientific work; so at a second reading you should include the footnotes.

*Falling Bodies*

Watch a falling stone and reflect on man's knowledge of falling objects. What knowledge have we? How did we obtain it? How is it codified into laws that are clearly remembered and easily used? What use is it? Why do we value scientific knowledge in the form of laws? Try the following experiment before you read further. Take two stones (or books or coins) of different sizes. Feel how much heavier the larger one is. Imagine how much faster it will fall if the two are released together. You might well expect them to fall with speeds proportional to their weights: a two-ounce stone twice as fast as a one-ounce stone. Now hold them high and release them together. . . . Which are you going to believe: what you saw, what you expected, or "what the book says"?

People must have noticed thousands of years ago that most things fall faster and faster—and that some do not. Yet they did not bother to find out carefully just *how* things fall. Why should primitive people want to find out how or why? If they speculated at all about causes or explanations, they were easily led by superstitious fear to ideas of good and evil spirits. We can imagine how such people living a dangerous life would classify most normal occurrences as "good" and many unusual ones as "bad"—today we use "natural" as a term of praise and "unnatural" with a flavor of dislike.

This liking for the usual seems wise: a haphazard unregulated world would be an insecure one to live

[1] Once learned, it is easily relearned if needed later. Much of the difficulty of learning a piece of physics lies in understanding its background. When you understand what physics is driving at, the rules or calculations will seem sensible and easy.

in. Children emerge from the sheltered life of a baby into a hard unrelenting world where brick walls make bruises, hot stoves make blisters. They want a secure well-ordered world, bound by definite rules, so they are glad to have its quirky behavior "explained" by reassuring statements. The pattern of seeking security in order, which we find in growing children today, probably applied to the slower growing-up of primitive savages into civilized men. As civilization developed, the great thinkers codified the world—inanimate nature and living things and even the thoughts of man—into sets of rules and reasons. Why they did this is a difficult question. Perhaps some were acting as priests and teachers for their simpler brethren. Perhaps others were driven by childish curiosity—again a need to know definitely, born of a sense of insecurity. Still others may have been inspired by some deeper senses of curiosity and enjoyment of thinking—senses rooted in intellectual delight rather than fear—and these men might be called true philosophers and scientists.

You yourself in growing up run through many stages of knowledge, from superstitious nonsense to scientific sense. What stage have you reached in the simple matter of knowledge of falling objects? Check your present knowledge by actually watching some things fall. Take two different stones (or coins) and let them fall, starting together. Then start them again together, this time throwing both outward horizontally (Fig. 1-1). Then throw one outward

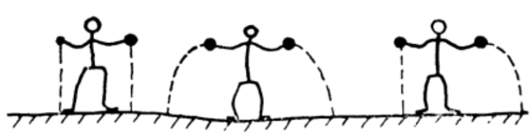


FIG. 1-1.

and at the same instant release the other to fall vertically. Watch these motions again and again. See how much information about nature you can extract from such trials. If this seems a childish waste of time, consider the following comments:

(i) This *is* experimenting. All science is built with information from direct experiments like yours.

(ii) To physicists the experiment of dropping light and heavy stones together is not just a fable of history; it shows an amazing simple fact that is a delight to see again and again. The physicist who does not enjoy watching a dime and a quarter drop together has no heart.

(iii) In the observed behavior of falling objects and projectiles lies the germ of a great scientific notion: the idea of *fields of force*, which plays an essential part in the development of modern mechanics in the theory of relativity.

(iv) And here is the practical taunt: if you use all your ingenuity and only household apparatus to try every relevant experiment you can think of, you will still miss some of the possible discoveries; this field of investigation is so wide and so rich that a neighbor with similar apparatus will find out something you have missed.

Mankind, of course, did not gather knowledge this way. Men did not say, "We will go into the laboratory and do experiments." The experimenting was done in daily life as they learned trades or developed new machines. You have been doing experiments of a sort all your life. When you were a baby, your bathtub and toys were the apparatus of your first physics laboratory. You made good use of them in learning about the real world; but rather poor use in extracting organized scientific knowledge. For instance, did your toys teach you what you have now learned by experimenting on falling objects?

Out of man's growing-up came some knowledge and some prejudices. Out of the secret traditions of craftsmen came organized knowledge of nature, taught with authority and preserved in prized books. That was the beginning of reliable science. If you experimented on falling objects you should have extracted some scientific knowledge. You found that the small stone and the big one, released together, fall together.[2] So do lumps of lead, gold, iron, glass, etc., of many sizes. From such experiments we infer a simple general rule: *the motion of free fall is universally the same, independent of size and material.* This is a remarkable, simple fact which people find surprising—in fact, some will not believe it when they are told it,[3] but yet are reluctant to try a simple experiment.[4]

[2] Yes; if you did not try the experiment, you now know the result of at least part of it. This is true of a book like this: by reading ahead you can find the answers to questions you are asked to solve. When you work on a crossword puzzle you would feel foolish to solve it by looking at the answers. In reading a detective story, is it much fun to turn to the end at once? Here you lose more still if you skip: you not only spoil the puzzle, but you lose a sense of the reality of science; you damage your own education. It is still not too late. If you have not tried the experiments, try them now. Drop a dime and a quarter together, and watch them fall. You are watching a great piece of simplicity in the structure of nature.

[3] Notice your own reaction to this statement: "A heavy boy and a light boy start coasting down a hill together on equal bicycles. In a short run they will reach the bottom together." The statement is based on the same general behavior of nature. See a demonstration. In a *long* run they gain high speeds and air resistance makes a difference.

cussion into components that have an absolute yes or no.

Aristotle's logic was safe as far as it went; modern logicians regard it as restricted and unfruitful but "true."[6] The damage to your thinking and mine comes from centuries of medieval scholarship drawing blindly and insistently on his writings—"the ingrown, argumentative, book-learned, world-ignorant atmosphere of medieval university learning." That medieval Aristotelian tradition is built into today's language and thought, and people often mistakenly require an absolute yes or no. For example, people trained to think they must choose between complete success and complete failure are heartbroken when they find they cannot attain the impossible goal of complete success. We are all in danger: students in college, athletes in contests, men and women in their careers, older people reviewing their life—all face terrible discouragement or worse if they demand absolute success as the only alternative to failure. Fortunately, many of us achieve a wiser balance; we stop judging ourselves by an absolute yes or no and enjoy our own measure of success. We then find the conflicting mixture of our record easier to live with.[7]

In science, where simple logic once seemed so safe, we are now more careful. Asked whether a beam of light is a wave, we no longer assume there is an absolute yes or no. We have to say that in some respects it is a wave and in others it is not. We are more cautious about our wording. Remembering that our modern scientific theory is more a way of regarding and understanding nature than a true portrait of it, we change our question, "*Is* it a wave?" to "Does it *behave as* a wave?" And then we can answer, "In some circumstances it does, in other circumstances it does not." Where an Aristotelian would say an electron must be either inside a certain box or not inside it, we have to say we would rather regard it as both! If you find such cautiousness irritating and paradoxical, remember two things: first, you have been brought up in the Aristotelian tradition (and perhaps you would be wise to question its strong authority); second, physicists themselves shared your dismay when experiments first forced some changes of view on them, but they would rather be true to experiment than loyal to a formality of logic.

### *Aristotle and Authority*

Aristotle's chief interests lay in philosophy and logic, but he also wrote scientific treatises, summing up the knowledge available in his day, some 2000 years ago. His works on Biology were good because they were primarily descriptive. In his works on Physics he was too much concerned with laying down the law and then arguing "logically" from it. He and his followers wanted to explain *why* things happen and they did not always bother to observe *what* happens or *how* things happen. Aristotle explained why things fall quite simply: they seek their *natural place*, on the ground. In describing how things fall, he made statements such as these: ". . . just as the downward movement of a mass of lead or gold or of any other body endowed with weight is quicker in proportion to its size. . . ." ". . . a body is heavier than another which in equal bulk moves down more quickly. . . ." He was a very able man, discussing as a philosopher the *why* of falling, and he probably had in mind a more general survey of falling bodies, knowing that stones do fall faster than feathers, blocks of wood faster than sawdust. In the course of a long fall air-friction brings a falling body to a steady speed, and he probably referred to that.[8] But later generations of thinkers and teachers who used his books took his statements baldly and taught that "bodies fall with speeds proportional to their weights."

The philosophers of the Middle Ages grew more and more concerned with argument and disdained experimental tests. Most of the earlier writings on geometry and algebra had been lost and experimental physics had to wait until they were found and translated. For centuries, right on through the "dark ages," the authority of Aristotle's writings

[6] Roughly speaking, Aristotelian logic deals with classes of things, and its arguments can be carried out by machines, e.g., "electronic brains" in which "yes" or "no" is signified by an electron stream being switched "on" or "off." Modern logic deals with *relations* (such as ". . . larger than . . . ," ". . . better than . . .") as well as classes (such as "dogs," "mammals") and, nowadays, with implicational relations between complete propositions. Its arguments, too, can probably be carried out by machines, though that may be more difficult to arrange. But a machine cannot criticize the system of logic that it is asked to administer. Only man still thinks he can do that, making judgments of value.

For descriptions of machines see the following numbers of *Scientific American*:

Vol. 183, No. 5, "Simple Simon" (a small mechanical brain); Vol. 180, No. 4, "Mathematical Machines" (a detailed account of electronic calculators); Vol. 182, No. 5, "An Imitation of Life" (mechanical animals that learn); Vol. 185, No. 3, "Logic Machines"; Vol. 192, No. 4, "Man Viewed as a Machine" (excellent article by a philosopher); Vol. 197, No. 3, a complete issue on self-regulating machines.

[7] For a fuller discussion, see Ch. I of *People in Quandaries* by Wendell Johnson (Harper and Bros., New York, 1946).

[8] A denser body (or a bigger one) has to fall farther before approaching its limiting speed; and then that speed is much greater.

remained supreme, in a misinterpreted form at that. Simple people, like children, love security more than freedom; they will worship authority blindly, and swallow its teaching whole. You may smile at this and say, "We are civilized. We don't behave like that." But you may presently ask, "Why doesn't this book give us the facts and tell us the right laws, so that we can learn reliable science quickly?"; and that would be *your* demand for simple authority and easy security! We now condemn "Aristotelian dogmatism" as unscientific, yet there are still people who would rather argue from a book than go out and find what really does happen. The modern scientist is realistic; he tries experiments and abides by what he gets, even if it is not what he expected.

*Logic and Modern Science*

Wholesale appeal to Aristotle's logic may restrict our intellectual outlook, and medieval wrangling with it certainly hampered science; but logic itself is an essential tool of all good science. We have to reason inductively, as Aristotle did, from experiments to simple rules. Then we often assume such rules hold generally and reason deductively from them to predictions and explanations. Some of our reasoning is done in the shorthand logic of algebra, some of it follows the rules of formal logic, and some of it is argued more loosely.

In extracting scientific rules from old laws we trust the "Uniformity of Nature": we trust that what happens on Friday and Saturday will also happen on Sunday; or that a simple rule which holds for several different spiral springs will hold for other springs.[9] Above all, we rely on the agreement of other observers. That is what makes the difference between dreams and hallucinations on one hand and science on the other. Dreams belong to each of us alone, but scientific observations are common to many observers. In fact, scientists often refuse to accept a discovery until other experimenters have confirmed it.

Scientists do more than assume that nature is simple, that there are rules to be found; they also assume that they can apply logic to nature's ways. There lies the essential distinction that enabled science to emerge from superstition: a growing belief that *nature is reasonable.* As science grows, mathematics and simple logic play an essential part as faithful servants. The modern scientist puts them to more use than ever, but he goes back to nature for experimental checks. In a sense, the ideal scientist has his head in the clouds of speculation, his arms wielding the tools of mathematics and logic, and his feet on the ground of experimental fact.

[9] The obvious condition, "all other circumstances remain the same," is often difficult to maintain, and we blame many an exception to the Uniformity of Nature on some failure in that respect. Magnetic experiments in towns that have streetcars may give different results on Sundays, when fewer cars are running.

*Greeks to Galileo*

> "In studying the science of the past, students very easily make the mistake of thinking that people who lived in earlier times were rather more stupid than they are now."—I. BERNARD COHEN

Aristotle's authority grew and lasted until the 17th century when the Italian scientist Galileo attacked it with open ridicule. Meanwhile, many people must have privately doubted the Aristotelian views on gravity and motion. In the 14th century a group of philosophers in Paris revolted against traditional mechanics and devised a much more sensible scheme which was handed down and spread to Italy and influenced Galileo two centuries later. They talked of *accelerated motion,* and even of *uniform acceleration* (under archaic names), and they endowed moving objects with "impetus," meaning motion or momentum of their own, to carry them along without needing a force.

Galileo ($\sim$ 1600) was a great scientist. He started science advancing to a new level where critical thinking and imagination join with an experimental attitude—a partnership of theory and experiment. He gathered the available knowledge and ideas, subjected them to ruthless examination by thinking, experimenting, and arguing; and then taught and wrote what he believed to be true. He lost his temper with the Aristotelians when they disliked his teaching and disdained his telescope; and he wrote a scathing attack on their whole system of science, setting forth his own realistic mechanics instead. He cleared away cobwebs of muddled thinking and built his scheme on real experiment—not always his own experiments, more often those of earlier workers whose results he collected.

*Thought-Experiments*

In his books and lectures, Galileo often reasoned by drawing on common sense, quoting "thought-experiments." For example, he discussed the breaking-strength of ropes in this manner: suppose a rope 1 inch in diameter can just support 3 tons. Then a rope of double diameter, 2 inches, has four times the cross-section area ($\pi r^2$) and therefore four times as many fibers. Therefore, the rope of double diameter has four times the strength—it should support 12 tons. In general, STRENGTH must increase as DIAMETER². Galileo gave this argument and extended it to wooden beams, pillars, and bones of animals.[10] Some thought-experiments deal with simplified or idealized conditions, such as an object falling *in a vacuum.*[11]

### *Ideal Rules for Free Fall*

Galileo realized that air resistance had entangled the Aristotelians. He pointed out that dense objects for which air resistance is relatively unimportant fall almost together. He wrote: ". . . the variation of speed in air between balls of gold, lead, copper, porphyry, and other heavy materials is so slight that in a fall of one hundred cubits a ball of gold would surely not outstrip one of copper by as much as four fingers. Having observed this, I came to the conclusion that, in a medium totally devoid of all resistance, all bodies would fall with the same speed."[12] By guessing what would happen in the imaginary case of objects falling freely in a vacuum, Galileo extracted ideal rules:

(1) All falling bodies fall with the same motion; started together, they fall together.

(2) The motion is one "with constant acceleration": the body gains speed at a steady rate; it gains the same addition of speed in each successive second.

Having guessed the rules for the ideal case, he could test them in real experiments by making allowances for the complications of friction.

### *Galileo's ? Experiment: Myth and Symbol*

There is a fable that Galileo gave a great demonstration of dropping a light object and a heavy one from the top of the leaning tower of Pisa.[13] (Some say he dropped a steel ball and a wooden one, others say a 1-pound iron ball and a 100-pound iron ball.) There is no record of such a public performance, and Galileo certainly would not have used it to show his ideal rule. He knew that the wooden ball would be left far behind the iron one, but he said that a taller tower would be needed to show a difference for two unequal iron balls. He certainly tried rough experiments as a youth and knew as you do what does happen, but he did not suddenly turn the course of science with one fabulous experiment. He did accelerate the growth of real physics by refuting the Aristotelians' silly dogmatic statements. And he did start science on a new kind of growth by applying his simplifying imagination to experimental knowledge. These, and not the leaning tower, made him a landmark in scientific history. Many a myth is attached to great figures in history—stories about cherry trees, burning cakes, etc. Though scholars delight in debunking these anecdotes, they also use some of them to show how the people of the great man's day thought of him. The leaning tower story is not even credited with that advantage. Yet we might use it, quite apart from Galileo and the growth of science, as a symbol of a simple experiment. In your own experiment with unequal stones, they fell almost together, and not, as some people

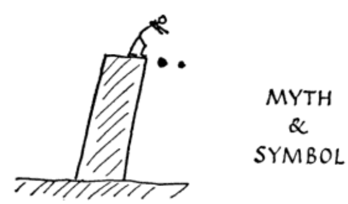



FIG. 1-3.

expect, the heavy one much faster. We shall use this Myth & Symbol in our course as a reminder of two things: the need for direct experiment, and a surprising, simple, important fact about gravity.

### *Honest Experimenting vs. Authority*

Your own experiments did not show that all things fall together; they did not even show that large and small stones fall *exactly* together; and if in obedience to book or teacher you said, "They fall exactly together," you were cheating yourself of honest science. Small stones lag slightly behind big ones—the difference growing more noticeable the farther they fall. Nor is it simply a matter of different sizes: a wooden ball and a steel ball of the same size do not fall exactly together.

Once you accept Galileo's view that air resistance obscures a simple story, you can interpret your own observations easily—though that still leaves air re-

[10] See problems in Chapter 5.

[11] The Aristotelians had argued themselves into believing a vacuum to be impossible, so they cut themselves off from Galileo's satisfying simplification.

[12] From *Dialogues Concerning Two New Sciences*, by Galileo Galilei, English translation by H. Crew and H. de Salvio, Northwestern University Press, p. 72.

[13] Pisa. The leaning tower is a charming little building in a friendly Italian town. It is a round tower of white marble, built beside the cathedral. It began to lean as it was built, and it now has a remarkable tilt, about 5° from the vertical. The visitor who climbs its winding stair or walks around one of its open slanting balconies has strange sensations of shifting gravity. The tower was built long before Galileo's day, and he must have tried using it for some experiments. In his lifetime a pro-Aristotelian used the tower, to demonstrate *unequal* fall.

sistance to be investigated. Or you might pretend you had never heard Galileo's view, and proceed towards it yourself through a series of experiments with denser and denser objects. Finding the motion more and more nearly the same for larger or denser bodies, you might guess the rule for the ideal case. To examine the blame against air resistance, you might try streamlining, or reducing, using some object such as a sheet of paper.

*Galileo's Guess: Newton's Crucial Experiment*

Galileo could only *decrease* air resistance. He could not remove it completely, so he had to argue from real observations with less and less resistance to an ideal case with none. This intellectual jump, from real observations to an ideal case, was his great contribution. Then, looking back, he could "explain" the differences in real experiments by blaming air resistance. He could even study air resistance, codify its behavior, and learn how to make allowances for it. Not long after his time, air pumps were developed which enabled people to try free-fall experiments in a vacuum. Newton pumped the air out of a tall glass pipe and released a feather and a gold coin at the top. Even this extreme pair fell together. *There* was a crucial test of Galileo's guess.

*Scientific Explanations*

When we "explain" the differences of fall by air resistance, the term "explain" means, as so often in science, to point out a likeness between the thing under investigation and something else already known. We are saying essentially, "You know about wind resistance, when you move a thing along in the air. Well, the falling bodies experience wind resistance which depends in some way on their bulkiness. A wooden ball and a lead ball of the same size moving at the same speed would suffer the same air resistance—how could the air know or mind what is inside?—but the lead ball weighs more, is pulled harder by gravity, *so the air resistance matters less to it in comparison with the pull of the Earth.*"[14]

*Further Investigation*

The explanation leads to a whole new line of enquiry: wind resistance, fluid friction, streamlining—with applications to ballistics and airplane design—new science from more accurate study of some simple rule of behavior, from a study of its failures.

You could extend your series of experiments in the other direction, making more and more resistance, first with air, then with water, and find things of importance in the design of ships and planes. For simple experiments with fluid friction, try dropping small balls in water instead of in air. Balls of different sizes do not fall together. Moreover they fail to move any faster after a while in a long fall. Each ball seems to reach a fixed speed and then move steadily down at that speed. What is happening then? Investigations might lead you to Stokes' Law for fluid friction on a moving ball, a law which plays a vital part in measuring the electric charge of a single electron. If you investigated still smaller falling bodies, specks of dust or drops of mist, you would discover surprising *irregularities* in their fall, and these in turn could lead to useful information in atomic physics.

Galileo's experimenting and thinking, which you have been repeating, led to a simple rule that applies accurately to objects falling in a vacuum. For things falling in air, it applies with limited precision. In other words, the simple statement "all freely falling bodies fall together" is an artificial extract distilled by scientists out of the real happenings of nature. This is a good scientific procedure: first to extract a general rule, under simplified or restricted conditions, secondly to look for modifications or exceptions and then to use them to polish up the rule and to extend our knowledge to new things. In the case of falling bodies we can now test the extracted rule by dropping things in a vacuum. Ask to see Newton's "guinea and feather" experiment. In many cases in physics, however, we have to be content with knowing that our rule is an extracted simplification, believing in it as a sort of ideal statement, with only indirect evidence to justify our full belief.

*Restricting the Number of Variables*

Apart from ignoring air resistance, we have restricted our study of falling bodies in another way: we have concentrated attention on just one aspect of them, their comparative rate of fall. We have not observed what noise they make as they fall, or watched how they spin, or looked for temperature changes, etc. By narrowing our interests, temporarily, we have better hopes of finding a simple guiding rule. Again this is good scientific procedure. In many investigations we not only concentrate on a few aspects but even arrange to hold other aspects

[14] At this stage, the explanation ends in unsupported dogmatism that might be "straight out of Aristotle." Wait for studies of mass, force, and motion to make it good science.

constant so that they do not muddle the investigation. In physics we nearly always try to limit our investigation to one pair of variables at a time. For example, we compress a sample of air and measure its VOLUME at various PRESSURES, while we *keep the temperature constant*. Or we warm up the gas and measure the PRESSURE at various TEMPERATURES, while we *keep the volume constant*. From these experiments we can extract two useful "gas laws" that can be combined into one grand law. If we did not make restrictions but let TEMPERATURE and PRESSURE and VOLUME change during our experiments, we could still discover the grand law but our measurements would seem mixed and complicated—it would be harder to see the simple relationship connecting them. But other sciences such as biology and psychology, following the successful example of physics, have found this method very dangerous. While restricting attention to one aspect of growth or behavior, the investigator may lose sight of the body or mind as a whole. In attempts to apply the methods of natural sciences to social sciences such as economics this danger is even more severe.

### *Why Do Things Fall?*

Aristotle was concerned with the answer to "Why?" Why *do* things fall? What is *your* answer to the question? If you say, "because of gravitation or gravity," are you not just taking refuge behind a long word? These words come from the Latin for heavy or weighty. You are saying, "things fall because they are heavy." Why, then, are things heavy? If you reply "because of gravity," you are talking in a circle. If you answer, "because the Earth pulls them," the next question is, "How do you know the Earth goes on pulling them *when they are falling*?" Any attempt to demonstrate this with a weighing machine during fall leads to disaster. You may have to answer, "I know the Earth pulls them because they fall"—and there you are back at the beginning. Argument like this can reduce a young physicist to tears. In fact, physics does not explain gravitation; it cannot state a cause for it, though it can tell you some useful things about it. The Theory of General Relativity offers to let you look at gravitation in a new light but still states no ultimate cause. We may say that things fall because the Earth pulls them, but when we wish to explain why the Earth pulls things all we can really say is, "Well it just *does*. Nature *is* like that."[15] This is disappointing to people who hope that science will explain everything, but we now consider such questions of ultimate cause outside the scope of science. They are in the province of philosophy and religion. Modern science asks *what?* and *how?* not the primary *why?* Scientists often explain why an event occurs, and you will be asked "why . . . ?" in this course; but that does not mean giving a first cause or ultimate explanation. It only means relating the event to other behavior already agreed to in our scientific knowledge. Science can give considerable reassurance and understanding by linking together seemingly different things. For example, while science can never tell us what electricity *is*, it can tell us that the boom of thunder and the crack of a man-made electric spark are much the same, thus removing one piece of fearful superstition.

Aristotle's explanation of falling was: "The natural place of things is on the ground, therefore they try to seek that place." People today call that a silly explanation. Yet it is in a way similar to our present attitude. He was just saying, "Things *do* fall. That's *natural*." He carried his scheme too far however. He explained why clouds float upward by saying that *their* natural place is up in the sky and thus he missed some simple discoveries of buoyancy.[16] Aristotle was much concerned with stating the "natural place" and "natural path" for things, and he distinguished between "natural motion" (of falling bodies) and "violent motion" (of projectiles). He might have produced good science of force and motion except for a mistake of applying common knowledge of horses pulling carts to all motion. If the horse exerts a constant pull, the cart keeps going with constant speed. This probably suggested Aristotle's general view that a constant force is needed to keep a body moving steadily; a larger force maintains larger speed in proportion. This is a sensible explanation for pulling things against an adjustable resisting force, but it is misleading for falling bodies and projectiles. In all cases it forgets the resisting force is there and prevents our seeing what happens when there is no resistance.

To explain the motion of projectiles, the Greeks

---

[15] Parents often give answers such as, "Well it just *is*" or "because it *is*" to children's questions. Such answers are not so foolish as they sound. For a child they provide the reply that is really needed at that stage, an assurance that everything is normal, that the matter asked about is a part of a consistent world. When a child asks "Why is the grass green?" he does not want to have a lecture on chlorophyll. He merely wants to be reassured that it is o.k. for the grass to be green.

[16] Buoyancy affects falling objects. When a thing falls in water its effective weight is lessened by buoyancy, and this makes falling in water quite different for different objects. Even air buoyancy has some effect, trivial for cannon balls, overwhelming for balloons.

vant information piece by piece. The other follows a hunch, like Newton, and, like Newton, abandons it at once when it comes into conflict with observed facts. From time to time the philosophers of science emphasize the merits of one or the other, and write as if one or the other were the true method of science. There is no one method of science. The unity of science resides in the nature of the result, the unity of theory with practice. Each type of detection has its use, and the best detective is one who combines both methods, letting his hunch lead him to test hypotheses and keeping alert to new facts while doing so."[20]

And here is an overall view, from a leading American physicist, P. W. Bridgman:

"I like to say that there is no scientific method as such, but that the most vital feature of the scientist's procedure has been merely to do his utmost with his mind, *no holds barred.*"[21]

### *Accelerated Motion: Inductive and Deductive Treatment*

Much of the *early* growth of science was made by induction; general laws were inferred from the knowledge gained in crafts and trades. In a simple way we have treated falling bodies inductively, inferring from many observations a general statement that all bodies falling freely in a vacuum fall together. When Galileo studied the details of this falling motion, he probably used a mixture of two approaches. He was good at making guesses, and he used geometry and reasoning powerfully.

We shall now follow the second method, deduction, in our study. We shall start by *assuming* a likely rule, and then we shall make a test comparing its consequences with real falling motion.

We choose guess (3) above and *assume that a falling body gains speed steadily, gaining equal amounts of speed in equal stretches of time.* We can express this more conveniently if we give a definite meaning to the word *acceleration*, so that we can say "the acceleration is constant." Therefore, we give the name ACCELERATION to

$$\frac{\text{GAIN OF SPEED}}{\text{TIME TAKEN}} \text{ or RATE-OF-CHANGE OF SPEED}$$

In making this definition of acceleration, we are really *choosing* the thing (GAIN OF SPEED)/(TIME TAKEN) to work with, and then giving it a name. We are not discovering some true meaning which the word acceleration possessed all along! We make this choice and assign it a name because it turns out to be useful in describing nature easily.

We shall start using the grander word *velocity* instead of *speed*, and presently we shall make a distinction between their meanings. Since we shall often deal with changing things, we want a short way of writing "change of . . ." or "gain of . . . ." We choose the symbol Δ, a capital Greek letter D, pronounced "delta." It was originally used to stand for the *d* of "difference." Then our definition[22] of acceleration states that:

$$\text{ACCELERATION} = \frac{\text{GAIN OF VELOCITY}}{\text{TIME TAKEN}} = \frac{\text{CHANGE OF VELOCITY}}{\text{CHANGE OF TIME-OF-DAY}}$$

$$a = \frac{\Delta v}{\Delta t}$$

where $a$, $v$, and $t$ are obvious shorthand.

### *Deductive Treatment of Motion with Constant Acceleration*

Now we express our assumption about falling bodies in this new terminology. We are *assuming* that:

$\frac{\Delta v}{\Delta t}$ is *constant*, for bodies freely falling (in vacuum). This states a huge assumption regarding real nature. Is it true? Is $\Delta v/\Delta t$ constant? To test this directly we should need an accelerometer to measure the acceleration of a body, $\Delta v/\Delta t$, at each stage of its fall. Such instruments are manufactured, but they are complicated gadgets which would not provide convincing proofs at this stage. Instead, we follow Galileo's example and ask mathematics, the logical machine, to grind out a consequence of our assumption, and then we test the consequence by experiment. The machine tells us that:

IF the acceleration $a$ ($= \Delta v/\Delta t$) is constant, and $s$ is the distance travelled in time $t$ with this constant acceleration, THEN

$s = \frac{1}{2}at^2$, if the motion starts from rest

$s = v_0t + \frac{1}{2}at^2$, if the motion starts with velocity $v_0$ at the instant $t = 0$, when the clock is started.

(The logical argument of this *IF . . . THEN . . .*

[20] *Science for the Citizen* (Allen and Unwin, London, 1938), p. 747.

[21] "New Vistas for Intelligence" in *Physical Science and Human Values*, ed. E. P. Wigner (Princeton, 1947), p. 144.

[22] In calculus, VELOCITY, $v$, at an instant, is defined by $v = \frac{ds}{dt}$, and ACCELERATION, $a$, at an instant, is $\frac{dv}{dt}$ or $\frac{d^2s}{dt^2}$.

is given in Appendix A of this chapter.) In these relations, $\frac{1}{2}a$ is a constant number, since we are assuming $a$ is constant; so, for motion starting from rest,

DISTANCE = (constant number) (TIME)$^2$
or DISTANCE increases in direct proportion to TIME$^2$
or DISTANCE varies directly as TIME$^2$
or DISTANCE $\propto$ TIME$^2$, this being shorthand for any of the versions above.

For example, if a body moving with fixed acceleration falls so far in one second from rest, then it will fall four times as far in two seconds from rest, nine times in three seconds, and so on.

★ PROBLEM 7. A CHART OF ACCELERATED MOTION

(a) Suppose a beetle crawls home with a motion for which it is true that *distance* $\propto$ *time*$^2$. Starting from rest he travels $\frac{1}{4}$ of an inch in the first second. How far will he travel in 2 seconds from his start? in 3 secs? in 4, 5, 6 secs?

(b) Draw a line across a sheet of paper; mark a starting-point near one end, and mark a rough scale of inches on it. Make marks to show the beetle's position at the end of each second.

★ PROBLEM 8. A SIMPLER RULE

Galileo announced the relation $s \propto t^2$ for uniformly accelerated motion, (where $s$ is the *total* distance travelled in total time $t$ from rest); but he stated another simple rule for such motion, relating the *distances* $d_1, d_2, \ldots$ *covered during 1 second,* in successive one-second intervals: (that is, the distance travelled in the first second, the distance travelled during the next period of one second, &c.) *Look for such a rule in the example of Problem 7, and state it. (Hint*: Calculate $d_1 = s_1 - 0$, $d_2 = s_2 - s_1, \ldots$ and look for some rule relating these one-second distances.)

★ PROBLEM 9. SCIENTIFIC THINKING

(a) You might have foreseen the rule of Problem 8, by common-sense thinking about accelerated motion, without using special algebra or studying an actual example. Why? (*Hint*: the distance travelled in any period of one second is a measure of . . ? . . in that period.)

(b) Is the rule of Problem 8 restricted (like $s \propto t^2$) to motion starting from rest when $t = 0$, or does it apply to any motion with constant acceleration?

★ PROBLEM 10. ANALYZING MOTIONS

Here are the records of four cyclists, moving with different motions. They all passed a post P at the instant the clock was started. Their distances from P after 1 second, 2, 3, 4, 5 seconds were as follows:

| *Time from Start* | *1 sec* | *2 sec* | *3 sec* | *4 sec* | *5 sec* | |
|---|---|---|---|---|---|---|
| Cyclist A | 1.8 | 7.2 | 16.2 | 28.8 | 45.0 | feet |
| Cyclist B | 1.8 | 3.6 | 5.4 | 7.2 | 9.0 | feet |
| Cyclist C | 1.8 | 5.2 | 10.2 | 16.8 | 25.0 | feet |
| Cyclist D | 1.8 | 14.4 | 48.6 | 115.2* | 225.0* | feet |

* These are distances Cyclist D would have travelled if he could have continued his motion.

(a) Try analyzing each of these motions, *looking for constant acceleration*, not by asking if $s \propto t^2$ but in the light of answers to Problems 8 and 9 above.

(b) Where the motion does not have constant acceleration describe its general nature if you can.

*Experimental Investigations*

The converse can be shown to be true. IF the distance $s$ varies directly as $t^2$, THEN the acceleration is constant.[23] That gives us a relation to test in investigating real motions. We can arrange a clock to beat equal intervals of time, and measure the distances travelled from rest by a falling body, in total times with proportions 1:2:3: . . . If the total distances run in the proportions 1:4:9: . . . and so on, we may infer a fixed acceleration. Or, as in one form of laboratory experiment, we can measure the time $t$ for various total distances $s$, and test the relation $s =$ (constant number) ($t^2$) by arithmetic, or by graph-plotting.

Over three centuries ago Galileo used this method, though he had neither a modern clock nor graph-plotting analysis. Galileo was one of the first to suggest an accurate pendulum clock, but he probably never made one. All he used to measure time was a large tank of water with a spout from which water ran into a cup. He estimated times by weighing the water that ran out—a crude method yet accurate enough to test his law. However, free fall from reasonable heights takes very little time—the experiment was too difficult with Galileo's apparatus.[24] So he "diluted" gravity by using a ball rolling down a sloping plank. He measured the times taken to roll distances such as 1 foot, 2 feet, etc., from rest.

On the basis of rough experimenting and sturdy guessing, Galileo decided that a ball rolls down a sloping plank with *constant acceleration.* Believing that this would be true for *any* slope, and arguing from one slope to greater slopes and greater still, he expected it to hold for a vertical plank, that is for free fall.[25] The idea of constant acceleration had

[23] By calculus: if $s = kt^2$, then velocity $\frac{ds}{dt} = 2\,kt$; and acceleration $\frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = 2k$, which is constant.

[24] Galileo's apparatus was rough. He used it to illustrate his argument rather than to measure acceleration.

[25] He convinced himself that the speed acquired by a body sliding down a frictionless incline depends only on the height, $h$, not on the length of slope, $L$. If so, a body falling freely through a vertical height, $h$, would acquire the same speed, since this would be like a vertical incline. Then he could argue safely from his experiments to vertical fall.

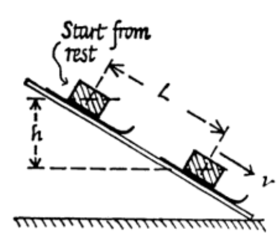



FIG. 1-5.

been suggested by earlier scientists—who were scorned for it. Galileo did his best to minimize friction, which threatened to complicate matters—though we now know that constant friction would not spoil the simple relationship. His results were rough, but seemed to convince him that his guess was right. It was the simplest kind of accelerated motion he could imagine, and he was probably influenced by the general faith, which has inspired scientists from the Greeks to Einstein, that nature is simple.

Later experiments, with improved apparatus, confirmed Galileo's conclusion: the motion *is* one with constant acceleration, i.e., with $\Delta v/\Delta t =$ constant, in all the following cases:

for a ball or wheel rolling down a straight inclined plank;
for a body sliding down a smooth inclined plank, or a truck with wheels running down it;
for free fall.

Yet each such test has only shown that the acceleration is constant for that one set of apparatus, on that one occasion and within the limits of accuracy of that experiment. If as scientists we want to believe in a general rule inferred from these experiments, if we want to codify nature's behavior in a simple "law" as a starting point for new deductions, then we need a great body of consistent testimony as a foundation for our inference. The more the better, in quantity and variety, and no witness is unwelcome. If any experiment contradicts this general story—and some do—it thereby offers a searching test. "The exception proves the rule" is a fine scientific proverb—though often misunderstood—if "proves" means "tests" (as in "proving-grounds" for artillery, the "proving" of bank accounts). If "proves" had the modern common meaning of "shows it to be right" the proverb would be nonsense.[26] Exceptions do *not* show that the rule is correct. Exceptions do put a rule to fine tests and show its limitations. They raise the question "What is to blame?" and they lead either to limitations of the rule or to greater care in experimenting. Either way, the rule emerges more clearly established.

*Experimental Tests in Lecture and Laboratory*

Therefore you should see and make some tests of accelerated motion yourself. Not only will these make you feel that the experimental basis of science is more real, but they will enable you to add your assurance to the accumulated body of testimony. Galileo made little more than wise guesses; others have added careful measurements, and you should add your measurements and judgment.

[26] The original legal meaning is amusing, but irrelevant here: "the quoting of an exception makes it clear that the rule exists."

*Demonstration Experiment*

We let a small truck run down a long sloping track, and make measurements to estimate its acceleration. It is not easy to measure speeds in a lecture experiment but rough estimates will suffice to show how the acceleration is derived.

We measure the truck's speed at some station, A, early in the run, and again at B farther down the track. The difference between these speeds gives us the gain of speed $\Delta v$. The time taken for this gain, $\Delta t$, is the time taken by the truck to travel from A to B. Then the acceleration is $\Delta v/\Delta t$. To measure $\Delta t$ we equip the truck with a thin mast, M, and measure with a stopclock the time taken for M to travel from A to B.

To estimate the truck's speed at A we have to time it over a short run in the region of A. We might install a short billboard there, with its mid-point at

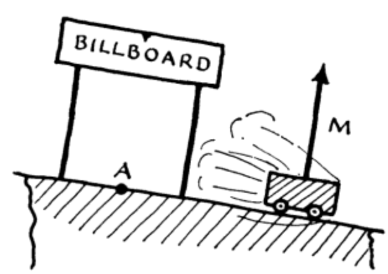



FIG. 1-6.

A, as in Fig. 1-6, and measure the time taken by the mast to run the length of the billboard. But human errors are inconveniently big for such short timings, so it is better to install the billboard on the moving truck and time its transit past A with the help of an electric eye (photocell). Fig. 1-7 shows a good arrangement. A lamp sends a beam of light across the track into the electric eye, where it produces a tiny electric current. The current is amplified and used to run an electromagnet. The electromagnet keeps an electric clock switched off. When the light is obstructed, the electromagnet releases the clock and lets it run. The truck carries a long strip of cardboard which obstructs the light while the truck carries it past. Thus the clock runs while the truck is passing the electric eye at A, and records the time

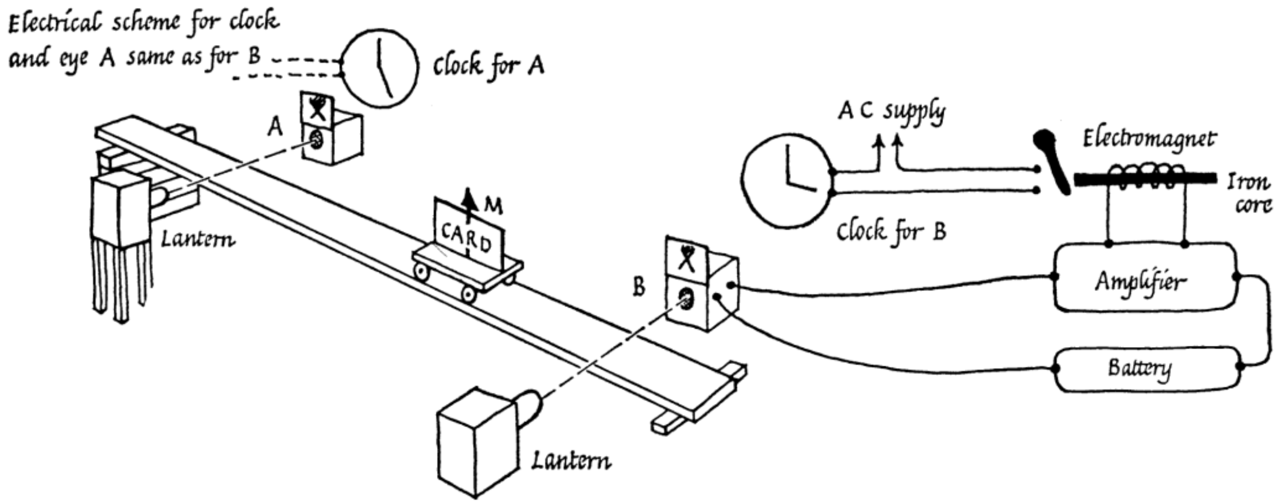



FIG. 1-7. EXPERIMENTAL ARRANGEMENT: measuring acceleration of model truck running down hill, with two electric eyes and three electric clocks. (Clock C for total travel-time not shown, operated by hand.)

taken for the card-length to travel past. In this time, the truck must travel the card-length. Then

CARD LENGTH/OBSTRUCTION TIME

gives the truck's speed.

We need three clocks, one to measure the total time between the two stations A and B where the speeds are estimated, one to measure obstruction time for the card passing A, and another for similar measurements at B. The following problem illustrates the calculation of the acceleration.

PROBLEM 11

Suppose the card on the truck is 2 feet long, and the obstruction at A takes 0.30 seconds. What is the truck's speed when running past A? If the obstruction at B takes 0.10 seconds, what is the truck's speed there? What is the gain of speed, $\Delta v$? If the truck takes 2.0 seconds to travel from A to B, what is its acceleration?

Nothing is said in Problem 11 about starting from rest. The truck is already moving when it passes A, and we can start it with any shove we like before that. So we can repeat the experiment with a variety of starting speeds. We can even start the car with an uphill shove so that it is moving backwards when it first passes A; but then we must be careful about + and − signs. The measurements tell us the acceleration whatever the starting speed. Whether the acceleration is the *same* for different starting speeds is a question about nature. To answer that you must see the real experiment.

*In laboratory*, you may experiment with a wheel rolling down sloping rails. You cannot easily measure the acceleration directly, or the (increasing) velocity. Instead you should measure DISTANCE TRAVELLED and TIME TAKEN, from rest, and then test whether they fit with the relation

DISTANCE varies directly as $(\text{TIME})^2$.

When you have collected reliable measurements, you should make the test both by arithmetic and by graph plotting.

*Example of an Accelerated Motion Experiment*

Meanwhile we shall proceed with a fictitious case of constant acceleration. Suppose measurements on a moving body gave the following results:

TABLE 1

| DISTANCE TRAVELLED *from starting point* *(feet)* | TIME TAKEN FOR TRAVEL *(seconds)* | | | |
|---|---|---|---|---|
| 0 | 0 | | | |
| 2 | 5.1 | 5.4 | 5.0 | 5.3 |
| 8 | 10.1 | 10.3 | 9.6 | 10.4 |
| 18 | 15.6 | 15.0 | 15.9 | 15.5 |

These measurements are too few and too poorly spaced for a good test, but will suffice for illustration. The four measurements 5.1, 5.4, 5.0, 5.3 are the result of four attempts to time the motion for 2 ft from start. Averaging is likely to remove some chance errors—though some errors may remain, such as the effect of impatient stopping of the watch too early. So we average these; we add them, and divide by 4:

$$\text{average time} = \frac{(5.1 + 5.4 + 5.0 + 5.3)}{4} = \frac{20.8}{4} = 5.2 \text{ secs}$$

Treating the other timings similarly we can make this table.[27]

TABLE 2

| DISTANCE TRAVELLED *from starting point* *(feet)* | AVERAGE OF TIMINGS TIME OF TRAVEL *(secs)* |
|---|---|
| 0 | 0 |
| 2 | 5.2 |
| 8 | 10.1 |
| 18 | 15.5 |

A glance at these numbers tells us that the times do not increase in proportion to the distances. Plotting the values on Graph (a) tells us the same thing. The graph shows clearly that the body is covering ground faster and faster, i.e., accelerating. It does not tell us whether the acceleration is constant.[28] To test that we plot a different graph, which will give a straight line *if* the acceleration is constant. We get a hint of what to plot by *assuming* constant acceleration and deducing DISTANCE $\propto$ TIME$^2$, which suggests we should plot DISTANCE against (TIME)$^2$. We make Table (3).

[27] A sensible experimenter in a real laboratory would save trouble by combining the two tables. He would leave a spare column for "average time" in his first table. If he foresaw the need for Table 3 he would leave another column for TIME$^2$. Even if he foresaw no need, an experienced experimenter would leave some blank columns, and blank lines below 18, for possible later use.

[28] We can make an indirect test by drawing tangents to the graph. See next section.

TABLE 3

| DISTANCE TRAVELLED *from starting point,* $s$ *(feet)* | AVERAGE OF TIMINGS, TIME OF TRAVEL $t$ *(secs)* | (TIME OF TRAVEL)$^2$ $t^2$ *(secs)*$^2$ |
|---|---|---|
| 0 | 0 | 0 |
| 2 | 5.2 | 27 |
| 8 | 10.1 | 102 |
| 18 | 15.5 | 240 |

Then we plot Graph (b). To see whether the acceleration is constant, we draw a "best" straight line through the origin. We deliberately draw it straight, as a test, but we try to make it pass "as near as possible to as many as possible" of the plotted points. In this example, the points lie close to a straight line. If we think their displacements from the line are genuinely accountable by the incompetence of our apparatus, then we say that *so far as we can tell* from our measurements, the motion may well have constant acceleration.

### *Very Honest Graphing: Showing Likely Experimental Errors*

If we wish to be more outspoken about our experimental uncertainties, we may spread each plotted point out into a patch to exhibit uncertainties of timing and distance measurement. Graph (c) in Fig. 1-10 shows this, with black points given by measurements surrounded by grey uncertainty patches. The timing is more risky than the distance measurement, so each patch is wider than it is tall.

Since we do not know how big our errors *are* but only how big they are *likely* to be, each patch should extend an indefinite distance out from its point; but we should show that the outer regions represent very unlikely errors. This might be done by shading

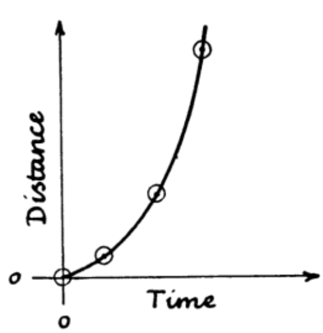



FIG. 1-8. GRAPH (a)

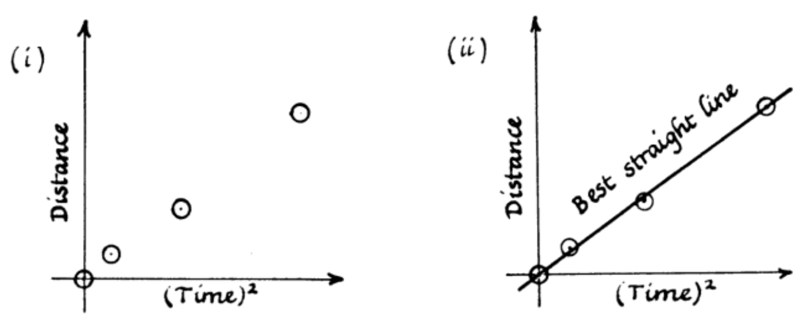



FIG. 1-9. GRAPH (b)

*Units for Acceleration*

Return to the definition of acceleration to look for its units directly;

$$a = \frac{\Delta v, \text{ measured in velocity-units, e.g., feet/second}}{\Delta t, \text{ measured in time-units, e.g., seconds}}$$

$$= \text{acceleration measured in acceleration-units, e.g., } \frac{\text{ft/sec}}{\text{sec}}.$$

Thus we expect to measure acceleration in units such as $\frac{\text{ft/sec}}{\text{sec}}$, which we write ft/sec/sec or ft/sec$^2$.

*The Use of "per" in Science*

The word "per" is of great use in science. We started using it above to mean "divided by" or "for each . . . ," as it does in ordinary arithmetic. Later we shall concentrate on a different aspect of its meaning, when it is used for ratio or proportion.

In arithmetic we divide 10 cents by 5 and get 2 cents. Or we divide 10 sheep by 5 sheep, and get 2 flocks. We feel doubtful about dividing 10 sheep by 5 cents—we object that they are different kinds of thing. But sometimes we do divide one kind of thing by another; such as 10 cents divided by 5 boys, which gives a pocket-money proportion of 2 cents per boy. Again, 60 cents divided by a dozen oranges gives a PRICE of 5 cents per orange. In science we often make divisions like these, and we preserve the truth by preserving the units as well as the number in the answer. If a beetle crawls steadily 10 feet in 2 hours, we can say "10 feet divided by 2 hours, or 10 feet/2 hours gives *5 feet per hour*." The answer shows the *distance* it crawls in *each hour*, but the statement does not restrict the beetle to one hour's travel. It applies to ¼ hour, ½ hour, 1½ hours, perhaps 2½ hours. But it also applies to very short time intervals; the beetle can still have a speed of *5 feet per hour* during a few seconds. We can, in imagination, shorten the time interval more and more, and still picture the beetle moving *5 feet per hour*. In the limit, we speak of the beetle having a *speed of 5 feet per hour at some particular instant.* This is a new idea, speed *at an instant* of time, at a certain mark on the clock. We can no longer divide a distance by a time—zero divided by zero is meaningless—yet a speedometer can register 5 feet per hour at an instant. True, a real beetle moves unevenly, but we can easily imagine an ideal one moving smoothly. Then the unit "one foot per hour" is no longer the result of division, but a thing-of-itself, a unit of rate; and the speed, 5 feet per hour, is a rate, *a limiting value*, glimpsed at an instant.

Mathematical limits appear in physics as well as calculus—which *is* the algebra of calculating limits. To understand the essential idea of a *limit* look at the sum to many terms of the series: 1, ½, ¼, ⅛, 1/16, . . . . The sum of the first two terms is 1½; of three terms 1¾; of ten terms 1 511/512; &c. However far you go, the sum is never quite 2, but you can get as near as you like to 2 by taking enough terms. (Notice that the sum always falls short of 2 by just the last term that is included; so you can make that failure as small as you like.) So we say 2 is the *limit* of the sum of many terms. You met a limit in tangent-slope, the limit of the slope of a chord through two points on a graph.

Until this century, physics dealt with many smooth ratios, such as speed, density, illumination. But now, much as we find a real beetle's speed uneven, we find many physical quantities jumpy or chunky; we cannot reduce them smoothly to limiting values. As an obvious example consider the ratio mass/volume, which we call density. We can divide the mass of a large chunk of aluminum by its volume, or the mass of a small chunk by *its* volume, obtaining the same density. But if we try to push our determination of density to the limit of smaller and smaller samples we are stopped when we meet a single atom. What ratios in physics can be pushed to the mathematical limit? What things are not "atomic"? This is a question worth watching, to which we shall return at the very end of the course.

At present, you should take "per," or the sign / used for it, to mean "divided by" or "for each," but you should think about letting it take its place in the idea of a ratio.

*Scientific Units*

In ordinary life, we measure speeds in *feet per second* or in *miles per hour*, and engineers often use these units. We express accelerations in *feet/second per second*, or sometimes in stranger units such as *miles/hour per second.* But scientists all over the world have agreed to use the metric system of units in their measurements, and we shall use one version of this, the Meter-Kilogram-Second system. In this "MKS" system, lengths and distances are measured in meters instead of feet, masses of stuff in kilograms instead of pounds, and times in seconds. A meter is almost 10% longer than a yard, its exact length being defined by a bar of fireproof metal which is carefully preserved, with copies in standardizing laboratories throughout the world. A kilogram is roughly 2.2

Table of Units and Abbreviations

| | Ordinary system used by householders and engineers (FPS system) | Metric system used by scientists: MKS system used in this course | Metric system used by scientists: CGS system (in common scientific use; not used in this course) |
|---|---|---|---|
| Length | foot (ft) | meter (m.) | centimeter (cm) |
| Mass | pound (lb) | kilogram (kg) | gram (gm) |
| Time | second (sec) | second (sec) | second (sec) |

Conversion Factors

1 foot = 12 inches

1 meter = 100 centimeters
= 1000 millimeters

1 inch = 2.540 centimeters = 0.02540 meters

1 foot = 0.3048 meter

1 meter = 39.37 inches
≈ 1.1 yards

1 pound = 454 grams = 0.454 kilogram

1 kilogram ≈ 2.2 pounds

pounds, 10% more than 2 pounds. It is defined by a standard lump of fireproof metal. A meter is subdivided into 100 centimeters, each about a fingerbreadth, and a kilogram is subdivided into 1000 grams, each about 1/28 ounce. Though many science courses use centimeters and grams, we shall follow the new fashion and use meters and kilograms, to make it easier to understand electrical units such as amps and volts. Scientists write m. as an abbreviation for meter or meters, but as this is easily confused with an algebra symbol $m$ for mass, it is better to write it in full as meter(s). We write kg as an abbreviation for kilograms.

The gram was originally made of such size that one cubic centimeter of water weighs one gram. This gives the density of water, mass/volume, the useful value 1.00 gram per cubic centimeter (useful, but misleading because it can be left out so harmlessly). The density of water is *not* 1.00 kilogram per cubic meter. Nor is it 1 pound per cubic foot. If you make a hollow box with internal dimensions 1 ft × 1 ft × 1 ft, you will find it holds 62.4 pounds of water. The density of water is therefore:

62.4 pounds per cubic foot,
or 1.00 grams per cubic centimeter,
or . . ? . . kilograms per cubic meter.

In our scientific MKS system we measure speeds in *meters/second*, accelerations in *meters/second per second*. The acceleration in the example above, 0.076 *feet/sec per sec*, is the same as about

0.076 × 0.3 *meters/sec per sec*

since each foot is about 0.3 meter.

*Acceleration of Free Fall*

For free fall, the acceleration can be measured. To show that the acceleration is constant as a body falls faster and faster is difficult, though of course it can be done with modern timing apparatus, some of which can measure to one-millionth of a second. If we *assume* the acceleration is constant, then it is fairly easy to measure its value by timing free fall for one known distance from rest and using the relation $s = \frac{1}{2}at^2$. This leads to $a = 2s/t^2$. As a reminder that we are dealing with a characteristic constant acceleration "due to gravity," we label this particular acceleration "$g$" and write $g = 2s/t^2$. Using experimental values of $s$ and $t$ we can compute $g$. However, air friction limits the accuracy; it is difficult to make sure that we start the timing just when the falling body starts from rest, and the time of fall itself is a very short one; so such measurements do not give an accurate value of $g$. Yet we need to know $g$ accurately for a number of uses in physics. Could we possibly eliminate the effects of friction? And could we lump together many falls, say several thousand, and measure the total time for the whole bunch to obtain the time for one fall with greater accuracy? These look like hopeless ambitions. Yet they can be achieved in a simple, easy experiment which Galileo foreshadowed, and which you will meet.

Measurements give a value about 9.8 meters/sec$^2$ for $g$, or 32.2 ft/sec$^2$. For ordinary calculations, 32 ft/sec$^2$ will suffice: accurate within 1%.

At the Equator, $g$ is slightly smaller; and at the North Pole $g$ is slightly greater.

*Force and Acceleration*

We think of a falling body as being pulled down by a force which we call its weight. To hold a body suspended we must support its full weight. If we cut the suspending cord we imagine the weight still acting, now unopposed by our supporting pull. If we suppose the body's weight remains constant while the body is falling, we may picture this constant force "causing" the constant acceleration of free fall. Trucks running down a slope have a smaller acceleration, a fraction of $g$; but only a fraction of their weight is available to pull them down along the slope. Later you will find what this fraction is. It depends on the slope of the hill. If you knew this fraction, you could follow Galileo in comparing downhill FORCE and downhill ACCELERATION. What kind of relation would you expect[29] to find between the force and the acceleration? You can see how early experimenters like Galileo could guess at it by studying falling and rolling bodies. That relation, to be discussed soon, is a very important piece of physics, a basic relation governing the motion of stars and the action of atoms, one of obvious importance in engineering.

While looking forward to discussing force and acceleration, we will end on a note of doubt. How do you know the weight of a body pulls it while it is falling freely? When you sit on a chair you feel the supporting force of the chair, and you believe you feel your own weight. But if you jump out of a window, do you feel your weight while you are falling? Suppose you jump out of a window with a lump of metal in your hand and try to weigh the lump as you fall. To make the temporary laboratory more comfortable, for a time, suppose you and the lump and the weighing apparatus are enclosed in a vast box which has been dropped from a tower and is falling freely. Suppose the box has no windows. When you release the lead lump inside the box, will it fall to the floor? If you think about this, you will see that gravity will seem to have disappeared. Can you possibly tell whether gravity has really disappeared or whether your laboratory is accelerating downwards? If you cannot tell the difference, *is* there any difference? Discussion of these questions would lead you towards the Theory of Relativity.

[29] Do we mean "expect" or "hope"? If *expect*, on what basis? If *hope*, is this scientific or not?

PROBLEMS FOR CHAPTER 1

1-6. These are at the beginning of the chapter.
7-11. These are in the text of Chapter 1.

★ 12. METHODS

Write a short note distinguishing between inductive and deductive methods.

★ 13. YOUR PRESENT VIEWS

Write a short note ($\frac{1}{2}$ page to 2 pages) saying what *you* think are (or should be) the parts played by *experiment* and *theory* in a science like Physics. (*Note*: At this stage of the course we do not expect you to know all the answers to questions like this. Later you should know more of them. So we ask you now just to write some general comments stating *your present views*. Please do not extract some complicated statements from a book.

14. EXPLANATIONS

(a) How would Aristotelians explain the rising of a helium balloon?
(b) How would modern scientists explain it?

15. DENSITIES

(a) Look up the relative densities of gold, silver, aluminum, brass, stone, iron, and wood in reference tables (often at the end of physics books).
(b) Why did Newton use a gold guinea?

16. SCIENTIFIC WRITING

(a) Write a short essay, (half a page at most), giving your answers to the following questions:
  (i) Do you consider it good scientific writing to use long words wherever possible?
  (ii) Why do you suppose people who are trying to imitate a scientist tend to use long words?
  (iii) Do you consider it good scientific writing to avoid long technical words?
(b) Rewrite the following passage, replacing long words by suitable shorter ones wherever you can: "Henderson conducted considerable experimentation concerning the relationship between superficial area and electrical charge of aqueous solutions atomized into numerous spherical particles of microscopic dimensions. He theorized that the phenomenon of electrification was attributable to friction."
(c) Rewrite the following passage replacing a word by its technical equivalent wherever you feel that the change would make the passage more scientific: "When the r.p.m. of the fan is pepped up, the atoms of air whiz down the tube at a great rate of speed; and when they hit the thermometer its mercury rises and registers more degrees of heat."

17. Do you agree with Bernard Cohen's remark on page 8? Is it a mistake? Discuss briefly.

18. In the discussion of mass it is stated that ". . . gravitational pulls are exactly proportional to the amounts of stuff being pulled." On what piece of experimental knowledge is this statement based?

19. Suppose on a certain (fictitious) island it is a custom for each member of a family to give a small present to every other member of his family, and a present to him-

self as well, on New Year's Day. Suppose this custom is followed in every family, and that each present costs the same amount of money.

(a) How will the total expenditure of any one family be related to the number of members in it? (Find this empirically, that is, by trial and error, if you like.)
(b) Sketch a graph showing *total cost* plotted upward against *number of members in the family*.
(c) What graph do you suggest plotting, using these things, to obtain a straight line?

### ★ 20. IMPORTANT PUZZLE

When a ball is thrown vertically upward, it continues up until it reaches a certain point, then falls down again. At that highest point it stops momentarily and is not moving up or down.

(a) Is it accelerating at that point?
(b) Give reasons for your answer to (a). (*Hint*: See Problem 21.)
(c) Devise an experiment (given any apparatus you need) to find out whether it *is* accelerating at that point.

### ★ 21. PROBLEM TO HELP SOLVE PUZZLE

A man leans out of a window high above the ground, and throws a ball vertically up. The ball rises till it is about 30 feet above the man, then falls. (See Fig. 1-13.)

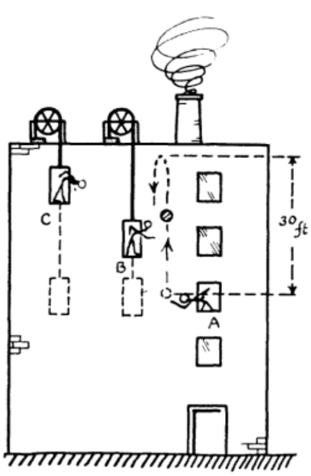



FIG. 1-13. PROBLEM 21

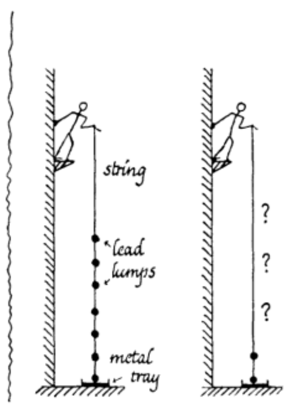



FIG. 1-14. PROBLEM 22

(a) Give a short description of the motion of the ball, as seen by the man, A.
(b) At the instant that the man throws the ball, an elevator running up the outside of the building is passing the window with the same upward speed that the man gives the ball. The elevator continues upward with constant speed, carrying an observer, B, who watches the ball. Describe the motion of the ball as seen by B (who forgets that he is moving, and thinks that all the motion he observes belongs to the ball).
(c) Another elevator runs beside the first carrying an observer, C, steadily upward, with smaller constant speed. C is just passing the window when the man throws the ball. Describe C's observations of the motion of the ball.
(d) In the light of your answers to (a), (b), (c), comment on the puzzle of Problem 20. (Note that A, B, C all agree that the ball has the usual decelerated and accelerated motions, but they disagree in one respect.)

### ★ 22. DEMONSTRATING CONSTANT ACCELERATION

A lecturer wishing to demonstrate the constant acceleration of free fall drops a chain of lead lumps down a stairwell and asks his audience to listen to the sounds of them hitting a metal tray at the bottom. He makes one such chain by tying quarter-pound lumps of lead to a light string every foot along the string. Then holding the string so that the lowest lump is just on the ground, he has lumps 1 ft, 2, 3, 4 ft, and so on, from the ground. When he releases the string the lumps hit the ground with a tattoo of *increasing frequency*.

(a) What does this tell the audience about the motion of falling bodies?
(b) The lecturer wishes to test for constant acceleration by arranging the lumps *unevenly* on the string in such a way that if the acceleration is constant the audience will hear an *evenly spaced tattoo*. He ties one lump to the bottom of the string on the ground, the next 1 ft above the ground. Where should he tie the next five lumps? (See Fig. 1-14.)

### 23. HISTORY

Read Galileo's description of his own experiment on accelerated motion (available in Magie's *Source Book in Physics*, New York, 1935) and write a short account of it. Indicate the apparatus he used and the results he got.

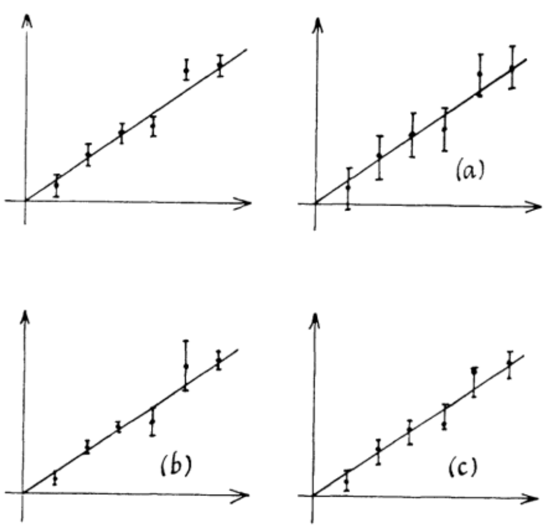



FIG. 1-15. PROBLEM 24

### ★ 24. ERROR-BOXES ON GRAPH

A student, S, making an experimental investigation finds when he plots a graph of his measurements that his points do not lie on a straight line as he hoped. However, they are fairly near a "best straight line," and he knows his measurements are liable to some small errors. So he draws lines as "error-boxes" through each point on his graph. He finds that even the error-boxes miss his best line in several cases. He is, therefore, tempted to change his error-boxes (a) by making all of them taller or (b) by making *some* of them taller or (c) by sliding them up or down through each point until they hit the line.

Is any of the changes (a) or (b) or (c) a wise one for a scientist to make? Discuss with student S what he is really saying about his experiment in each case, (a), (b), (c).

★ 25. MKS UNITS

(a) Make a rough estimate of your height in meters.
(b) What do you weigh in kilograms?
(c) What is the width of this page in meters?
(d) What is the thickness of a page of this book in meters?
(e) Explain how you made the estimate asked for in (d) without using any special micrometer gauge.
(f) Atomic scientists find molecules and atoms so small that they like to use a much smaller unit than a meter for measuring them. They use an "Ångstrom Unit," which is one ten-billionth of a meter or $10^{-10}$ meter. What is the thickness of this page, in Å.U.?
(g) Most atoms are a few Å.U. in diameter, say 3 Å.U. How many atoms thick (roughly) is this page?

★ 26. DENSITIES IN MKS UNITS (Learn these values)

(a) What is the density of water in *kilograms/cubic meter?*
(b) Show the reasoning by which the answer to (a) can be obtained from the data in Chapter 1.
(c) Lead has a "specific gravity" of about 10. This means its density is 10 times that of water. What is the density of lead on the MKS system of units?
(d) Olive oil has a specific gravity of about 0.8. What does this mean?
(e) What is the density of olive oil in the MKS system?
(f) The specific gravity of mercury is 13.6. The atmosphere presses on each square inch of table, chair, our bodies, walls, . . . etc. with a force that can balance a column of mercury of cross-section 1 sq. inch and height about 30 inches. That is the "height of the barometer" in which mercury with a vacuum above it inside balances atmospheric pressure outside. What would be the height of a water barometer?

★ 27. A USEFUL CONVERSION FACTOR

Show that 60 miles/hour = 88 ft/sec.

28. A SPECIMEN ACCELERATED MOTION

The motor of a certain elevator gives it an upward acceleration of 150 ft/min/sec. The elevator starts from rest, accelerates thus for 2 secs, then continues steadily with constant speed.

(a) Explain what this statement of acceleration means.
(b) What is the final speed after 2 secs?
(c) Calculate the speed after 0 sec, 0.5 sec, 1 sec, 1.5 secs, 2, 3, 4, 5 secs. Sketch a rough graph showing speed (upward) against time from start (along), for the first 5 seconds.
(d) How far has the elevator risen 1 second from the start? How far has it risen 2 secs, 3 secs, 4 secs, from the start? Sketch a rough graph of distance against time.

★ 29. CALCULUS STATEMENTS

In this question, $v$ is a symbol for speed or velocity; $a$ is a symbol for acceleration, $t$ for time.

(a) What does $\Delta v$ mean?
(b) What does the statement "$\Delta v/\Delta t$ = constant" mean?
(c) What does the statement "$\Delta a/\Delta t$ = constant" mean? (Make an intelligent guess.)

30. FORMAL LOGIC*

Here is an example of a syllogism, a type of perfect deduction—too restricted to be much use in science but an important part of classical logic.

(1) All dogs have 4 legs. (the "major premise," a generalization)
(2) Fido is a dog. (the "minor premise")
(3) ∴ Fido has 4 legs. (the conclusion)

These three steps involve three "terms":

(a) 4-legged creatures (the major term, a large class)
(b) dogs (the middle term, a smaller class)
(c) Fido (the minor term, a member of a class)

The argument holds true if:
(c) falls wholly within the class (b) and (b) falls wholly within the class (a). Then, (c) must fall within the class (a). A corresponding argument can be carried out if (c) falls wholly within (b) but (b) falls wholly *outside* (a). Then (c) must fall *outside* (a).

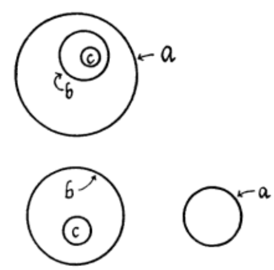



FIG. 1-16.

The inference or conclusion drawn in a "syllogism" may be untrue

(i) because the major or minor premise is not true,
(ii) because the reasoning is untrue, e.g., (b) falls partly within (a),
(iii) because there is confusion of language, e.g., ambiguous terms.

In each of the following examples there is something that makes the conclusion untrue or at least unjustified. Point out the defect in each case.

(A) All plums are vegetables.
This is a plum.
∴ This is a vegetable.
(B) All poisons are harmful.
Sugar is a poison.
∴ Sugar is harmful.
(C) All dogs are animals.
An antelope is an animal.
∴ An antelope is a dog.
(D) All salts dissolve in water.
George is an old salt.
∴ George dissolves in water.
(E) Caesar Augustus was a Roman Emperor.
Julius Caesar was a Roman general.
∴ Julius Caesar was the uncle of Augustus.

* Drawn from "Clear Thinking" by R. W. Jepson (Longmans, Green and Co., London, 1936).

ments on falling bodies that distances and timings agreed closely with the relation $s \propto t^2$, then you could say that they agree with the relation predicted for constant acceleration. You could say that falling bodies seem to move with constant acceleration. In experiments on balls rolling down a plank, Galileo found that distances and timings fitted fairly well with the relation $s \propto t^2$. So they agreed with his prediction for constant acceleration.

Notice that the experiments do not prove the formula is the right one for constant acceleration. The formula itself is necessarily, logically, true for any motion which does have fixed acceleration. Experiments only show that the rolling motion, in agreeing with the formula (probably) has constant acceleration. When we compare experimental data with the formula we can discover something about nature.

Arriving at the formula involved the following stages:

*Definition* of acceleration: We invented it, chose a name, then used it.

*Decision* to think about motion with constant acceleration. This is one of the many choices we might have to try for real falling bodies. But, once made, the decision enables us to proceed with algebra. In making this decision we are not discovering anything about nature.

*Algebra*: A logical sausage-making machine. Mathematics cannot manufacture scientific facts, though it may help us to discover them.

*Common-sense assumption* that the proper $\bar{v}$ to use is $(v_0 + v)/2$. The risk in this can be avoided by Galileo's geometry (Problem 1), or by a calculus investigation, which would justify it for fixed acceleration.

*Algebra again*

*Result*: A useful relationship, deduced from our assumptions, useful in experimental tests.

(4) $v^2 = v_0^2 + 2\,as$ [This is a form which we shall not need for a long time yet. This section may be postponed till it is needed.]

We can use further algebra, a few more turns of the sausage machine, to change the formulas to other forms. We already have three relations:

(1) involving $v$, $v_0$, $a$, $t$, but not distance, $s$;
(2) involving $s$, $v$, $v_0$, $t$, but not acceleration, $a$;
(3) involving $s$, $v_0$, $a$, $t$, but not final speed, $v$.

Later we shall want a relation expressing $v$ in terms of $v_0$, $a$, $s$, but not involving the time $t$ explicitly. Since we want it without $t$, we obtain it from any two of the earlier relations by eliminating $t$. For example, we can use (1) and (3). Then $v = v_0 + at$ gives $t = \dfrac{(v - v_0)}{a}$ and we substitute this in $s = v_0 t + \tfrac{1}{2}at^2$.

Then: $$s = v_0\left[\frac{(v - v_0)}{a}\right] + \tfrac{1}{2}\,a\left[\frac{(v - v_0)}{a}\right]^2$$

Will this lead to the formula (4) quoted above? Yes, if you use courage and algebra. You will have to square and cross-multiply and rearrange and simplify. The work will be clumsy and messy, but the final expression for $v^2$ will be $v_0^2 + 2\,as$. Try it, if you like.

The professional mathematician has a strong poetic sense of form in his own language of mathematics and he would consider the method above horribly clumsy. He would say, "Here is a more elegant derivation . . ." and would produce the answer quickly and neatly. Non-mathematicians who see him do this are mystified by his superior knowledge, and may be annoyed by the magical atmosphere. The real story is a sordid one. The mathematician is quite human, and feels his way in several trials, like any other explorer—though in simple problems his exploring may have all been done before and stored in his mind as "mathematical common sense." When he has found the answer by *any* method, clumsy or not, he may try working *backwards* from it to find a neat method of deriving it, like a mountaineer seeking a better path. There is no sin in this, but then he often forgets to tell the layman about the previous work, and startles him by producing the elegant method out of his hat. Let us try such an analytical search, thinking aloud as we go. The answer we want is $v^2 = v_0^2 + 2\,as$, so far obtained by algebraic drudgery. Try to undo it. Does it look as if it could be twisted or changed easily by algebra? Does it simplify or split up in any obvious way? No. Then we must push it around. Try shifting something across the $=$. Then we can have $v^2 - v_0^2 = 2\,as$. Is *this* easily attacked by algebra? Yes, the left hand side is an old friend, with factors $(v + v_0)\,(v - v_0)$. We could manufacture it from those factors if we could obtain them separately from somewhere. Where have we seen $(v + v_0)$ before? In the relation (2), $s = \tfrac{1}{2}(v + v_0)t$. Then $v + v_0 = 2s/t$. Where have we seen $(v - v_0)$? In the definition of acceleration, which we wrote $a = (v - v_0)/t$. Therefore, $(v - v_0) = at$. Now we want $v^2 - v_0^2$, which we can get by multiplying $(v + v_0)$ and $(v - v_0)$. We do this, using $(v + v_0) = 2\,s/t$ and $(v - v_0) = at$.

$$(v + v_0)\,(v - v_0) = (2\,s/t)\,(at)$$

$\therefore\ v^2 - v_0{}^2 = 2\,as$, which leads to the form we want. Now, having found the method by analysis, we erase the details of our search and start afresh, thus:

To derive $v^2 = v_0{}^2 + 2\,as$ by an elegant method, start with the definition of acceleration,

$$a = (v - v_0)/t,$$

and with the formula for distance travelled in terms of average speed, $s = \frac{1}{2}\,(v + v_0)t$, and just multiply these two equations together, obtaining $a \cdot s = \frac{1}{2}\,(v^2 - v_0{}^2)$ which reduces to

$$\underline{v^2 = v_0{}^2 + 2\,as}$$

Here, then, are four relations between $v$, $v_0$, $a$, $s$, and $t$.

$$v = v_0 + at \qquad s = \tfrac{1}{2}\,(v + v_0)\,t \qquad s = v_0t + \tfrac{1}{2}\,at^2$$

$$v^2 = v_0{}^2 + 2\,as$$

They provide a quick way of calculating the value of any one of these quantities, given the values of three others.

### *Algebra Yields* Net *Distance*

The numerical values must be given appropriate + and − signs. For example, if the initial velocity is 6 ft/sec eastward and the acceleration 2 ft/sec/sec eastward, we can say $v_0 = +6$ and $a = +2$. However, if $v_0$ is 6 ft/sec eastward but the acceleration is in the opposite direction, 2 ft/sec/sec westward, then one of them must have a *minus* value. If we say $v_0 = +6$ we must say $a = -2$, using + signs for eastward velocities, accelerations and travel-distances, and − signs for westward ones. Then $s$ is the *net* distance travelled in time $t$, not the arithmetic sum of westward and eastward travels. This is because in calculating each part of the trip the algebra will give + sign to eastward travels and − sign to westward ones and in adding up these + and − parts to find $s$ the algebra will give the net difference. With $v_0 = +6$ and $a = -2$ the motion is decelerated: slower and slower forward for 3 secs, then at rest, then faster and faster backward. In 5 seconds it will show a path like Fig. 1-17, with 9 ft forward travel, then 4 ft backward, giving a net travel 5 ft.

Algebra gives:

$$s = v_0t + \tfrac{1}{2}\,at^2 = (+6)(5) + \tfrac{1}{2}\,(-2)(5)^2$$
$$= 30 - 25 = 5\text{ ft.}$$

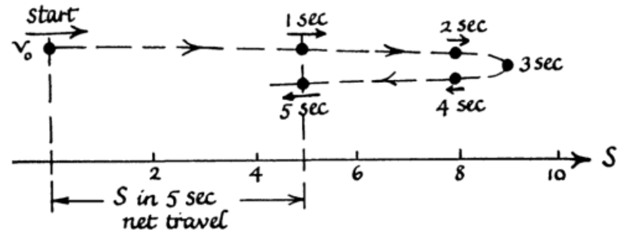



FIG. 1-17. S IS NET DISTANCE

Thus $s$ always gives the *net* distance from start to finish.

These useful relations are tools, not vital pieces of science. They are absolutely true for motion with constant acceleration, and they are not reliable for other motions. Only experiment can tell us where they apply in the real world.

#### PROBLEMS FOR APPENDIX A

#### ★ A-1. NON CALCULUS PROOF

Galileo, lacking the help of calculus and preferring geometry to algebra, dealt with uniformly accelerated motion as follows: Imagine a graph with time plotted along and velocity of a moving body plotted upwards. *If* the body has constant acceleration, its velocity must increase *steadily* as time goes on. The graph must be a straight line. It will not necessarily pass through the origin, but will start at the initial velocity, $v_0$ when time is zero, and run up to some value $v$ at time $t$.

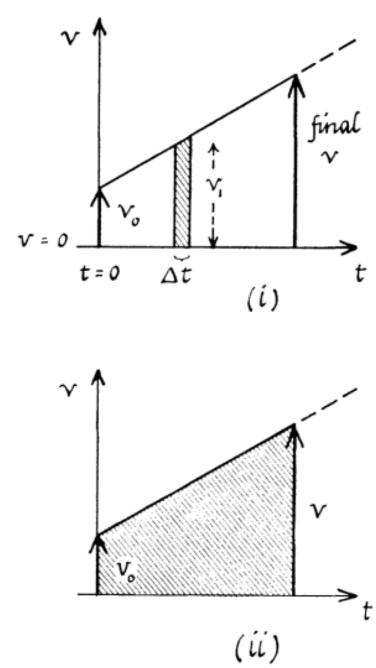



FIG. 1-18. GALILEO'S PROOF

Now consider what happens in some very short interval of time $\Delta t$, when the velocity is, say, $v_1$. (Of course $v$ is increasing, but we can take $v_1$ as the average during short $\Delta t$.) Then the body moves a distance $[(v_1) \cdot (\Delta t)]$ in that time. But on the graph $[(v_1) \cdot (\Delta t)]$ is the [*height* · *width*] of the small pillar resting on $\Delta t$ and running up to the graph-line. It is the *area* of that pillar, shaded in sketch (i).

Therefore, the *total* distance covered is given by the total area of all such pillars—i.e., the shaded area in sketch (ii).

(a) If in sketch (ii) the heights of this patch at its edges are $v_0$ and $v$ as marked, and the base is time $t$, what expression gives the *area*? (Outline your geometrical argument briefly.)

(b) If the heights at the edges are $v_0$ and $v_0 + at$ (which follows from the definition of acceleration), what expression gives the area? (Outline your argument briefly.)

(c) Write the results of (a) and (b) as expressions for $s$ the distance covered by the body in time $t$.

(d) Now suppose the acceleration is *not* constant but starts with a smaller value, rising to a greater one, so that the velocity still changes from $v_0$ to $v$ in time $t$, but *not* steadily. (i) Sketch the new graph picture. (ii) Will the expressions from (a) and (b) apply now? (iii) What weakness in the earlier algebraic discussion in Appendix A has now been removed?

## ★ A-2. CALCULUS PROOF

In the limit, velocity, $v$, is rate-of-change of distance, $ds/dt$, and acceleration, $a$, is rate-of-change of velocity $\frac{dv}{dt}$ or $\frac{d}{dt}\left(\frac{ds}{dt}\right)$ or $\frac{d^2s}{dt^2}$ · Show that if $a$ is constant, each of the following is true:

(i) $dv/dt = a$ integrates to $v = v_0 + at$
(where $v_0$ is a constant, the value of $v$ at time $t = 0$)

(ii) $v = v_0 + at$ integrates to $s = v_0t + \frac{1}{2}at^2$
(*Hint*: remember $v = ds/dt$.)

(iii) $dv/dt = a$ integrates to $v^2 = v_0^2 + 2as$
(*Hint*: try multiplying both sides by $v$.)

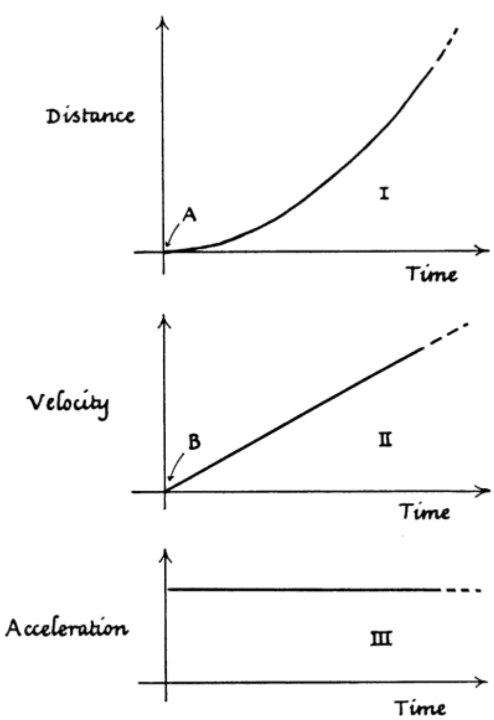



FIG. 1-19. PROBLEM A-3, parts a, b, and c

## A-3. GRAPHS OF MOTION

Fig. 1-19 shows an arrangement of three time-graphs for the motion of an object along a straight track. Graph I shows *distance* plotted against *time*; graph II *velocity* against *time*; graph III *acceleration* against *time*. They are drawn with matching time-scales. The graphs sketched relate to an object moving with constant acceleration, starting at $s = 0$ (shown by A) and velocity $v = 0$ (shown by B) at $t = 0$. In graphs for more complicated motions, all three lines may be curved.

(a) In the general case of any motion, one or more of the graphs can be derived from another of the three by tangent slopes. Which one(s)? Explain why.

(b) In the general case, one or more of the graphs can be derived from another of the three by measuring areas under the curve. Which one(s)? Explain why.

(c) A motorcycle policeman starts from rest, accelerates 15 ft/sec² for 6 secs; runs at constant velocity for 10 secs; then skids to a stop in 4 secs, with constant deceleration. Sketch a trio of graphs I, II, III, for his motion.

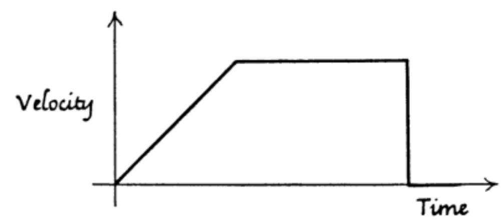



FIG. 1-20. PROBLEM A-3, part d

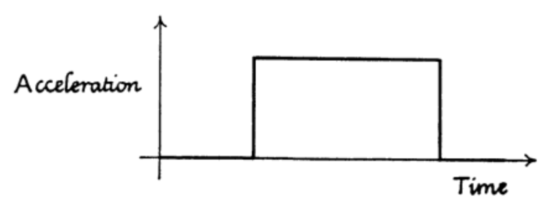



FIG. 1-21. PROBLEM A-3, part e

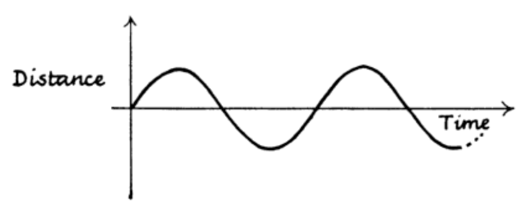



FIG. 1-22. PROBLEM A-3, part f

(d) Fig. 1-20 shows graph II for the motion of a car. Copy it and add sketches of graphs I and III.

(e) Fig. 1-21 shows graph III for the motion of a truck. Copy it and add sketches of graphs I and II.

(f) Fig. 1-22 shows graph I for the motion of the bob of a long pendulum along its almost-straight path. Copy it and add sketches of graphs II and III. (*Difficult*: Deserves careful guessing.)

## APPENDIX B · "g"

### *Measurement of "g"*

We have glibly announced the value of "g" as 9.8 meters/sec² (or 32 ft/sec²), but this came from laboratory measurements. You will use it for simple calculations concerning falling bodies, and for important calculations of forces when you treat "g" as gravitational field-strength. "g" is such a useful quantity that you should see its value measured before you use it. You could make a very rough estimate with a stone and a stopwatch and a meter-stick.

PROBLEM B-1. ROUGH MEASUREMENT OF "g"

An experimenter drops a big stone from a 14th-story window and finds it takes "just over" 3 seconds to reach the ground. If the window is 150 ft from the ground,
(a) Make an estimate of "g."
(b) Taking 150 ft to be about 46 meters, estimate "g" in meters/sec².

A better measurement can be made with an electric clock, as illustrated in Fig. 1-23, and you should see some such demonstration. For very accurate measurements you must wait for the promised scheme which avoids friction and takes a group of falls.

PROBLEM B-2. MORE ACCURATE MEASUREMENT OF "g"

A metal ball is allowed to fall from ceiling to floor. At the ceiling it is held against two metal pins so that it makes an electrical connection which *prevents* the electric clock from starting. The ball is released abruptly, and the clock starts.

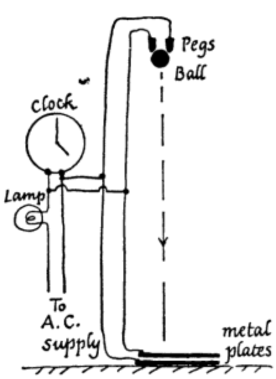



FIG. 1-23. MEASURING "g"

As it reaches the floor, the ball pushes two light metal plates together, making another electrical connection which stops the clock. In an actual experiment, the height of the fall was 7.00 meters from ceiling pegs to floor contacts, and the clock recorded a time of 1.20 secs.
(a) Estimate the value of "g," using these data.
(b) Say what assumptions you made in (a) concerning the type of motion; the apparatus; the conduct of the experiment. (Give details; avoid prim generalities such as "apparatus accurate" or "avoided personal error.")

### *Values of "g" in various localities*

"g" has been measured very precisely at a few standard laboratories. Comparative measurements have then provided accurate values of "g" at many places all over the world.

| | New York | Equator | Pole |
|---|---|---|---|
| Value in meters/sec/sec | 9.80267 | 9.780 | 9.832 |
| Value in feet/sec/sec | 32.16 | 32.09 | 32.26 |

For ordinary calculations, in problems or in experiment-design, you should use the rough values $g = 9.8$ meters/sec/sec, $g = 32$ ft/sec/sec.

### *Arithmetical Problems on Free Fall: Dissected Problems*

When you know the value of "g," you can make simple calculations about dropping stones, arrows shot at monkeys, etc. Such calculations are occasionally used by physicists in designing apparatus or in dealing with some experiment, but they are not important physics. Elementary textbooks and examinations make much of them "because they make accelerated motion clearer." Students trained to solve them mechanically may gain little but a damaging prejudice that "physics consists of putting numbers in the formulas." We wish to avoid that foolish picture of science, and we would not give you such problems in this course except for two reasons: (1) You may meet similar calculations, that are important, in atomic physics; (2) They will show you something important about the place of mathematics in physics. For these two reasons you should work through Problems B-3, 4, 5, and 6. Even so, if earlier studies have made you a convinced formula-monger you had better omit these problems unless you are prepared to start with an open mind.

Problems B-3 to B-6 have been dissected. You should answer them step by step, on question sheets reproduced from the small ones printed here. This scheme—which you will meet several times in the course—is intended to give you preliminary help and teaching towards later problems to be done on your own. Note that this insulting simplicity is meant to help you with the mathematics but not to save you from thinking out the physics for yourself. As you work such problems you should stop to notice that you are learning a method of solving them, but you should then concentrate on the physical results that emerge.

The problems here are intended to be answered on typewritten copies of these sheets. Work through the problems on the enlarged copies, filling in the blanks, (______), that are left for answers.

PROBLEMS ON ACCELERATED MOTION NAME________________

Work through these problems, filling in the blanks which are left for answers. Doing that will show you how to solve such problems and will, it is hoped, give you a pleasant understanding of the use of mathematics in physics. You will see that mathematics is a faithful servant, but sometimes a slightly dumb one, carrying out instructions relentlessly.

PROBLEM B-3. A rock falls from rest, with constant acceleration 32 ft/sec per sec, downward. (GIVEN: The acceleration is constant, and air friction is negligible, in this case.)

(a) What will be its velocity after 3 seconds of fall?

(b) How far will it fall in 3 seconds?

A. ARITHMETICAL METHOD.

(a) Acceleration 32 ft/sec per sec means that the rock gains in velocity by ________ . ________ (units) in each second.

∴ In 3 secs of fall it gains in velocity. . . . . . . . . ________ ft/sec

∴ Since it starts from rest, its final velocity is. . . . ________ ft/sec

(b) The velocity grows from ________ ft/sec (at start) to. . . ________ ft/sec

∴ Average velocity is $\frac{1}{2}$(________ + ________) or. . . ________ ft/sec

∴ Distance travelled, with this average velocity, in 3 sec is (________) · (________) or. . ________ ft

B. ALGEBRAIC METHOD.

(a) The acceleration, $a = 32$ ft/sec/sec     Time, $t = 3$ secs

Initial velocity, $v_o = 0$

Substituting these values in $v = v_o + at$

we have: final v = ________ + ________ = ________ ft/sec

(b) Substituting the values above in $s = v_o t + \frac{1}{2}at^2$

we have: s = ________ + $\frac{1}{2}$(________) = ________ ft

(NOTE: In using algebra, always state the "formula" clearly first, as above. Also state the values you are going to substitute, attaching units to them. For example, "t = 3 secs", not "t = 3".)



PROBLEM B-4. A ball is thrown downward, and released with velocity 10 ft/sec, to fall freely at the instant when the clock is started.

(a) What will be its velocity after 3 seconds of fall?

(b) How far will it fall in 3 seconds?

A. ARITHMETICAL METHOD.

(a) Acceleration 32 ft/sec per sec means that the ball gains in velocity by ________ ________ (units) in each second.

∴ in 3 secs of fall it gains in velocity . . . . . . . . . . ________ ft/sec

∴ since it starts with velocity 10 ft/sec downward, its final velocity will be ________ ft/sec

(b) The velocity grows from ________ ft/sec at start to the final value of ________ ft/sec

∴ average velocity is $\frac{(\_\_\_\_\_\_)+(\_\_\_\_\_\_)}{2}$ or . . ________ ft/sec

Distance ball would travel in 3 sec with this average velocity is. . . . ________ ft

B. ALGEBRAIC METHOD.

(a) The acceleration, $a = 32$ ft/sec/sec, downward

Initial velocity, $v_o = 10$ ft/sec, downward     Time of travel, $t = 3$ secs

Substituting these values in $v = v_o + at$, we have

Final velocity, v = ________ + ________ = ________ ft/sec

(b) Substituting the values above in $s = v_o t + \frac{1}{2}at^2$, we have

Distance s = ________ + $\frac{1}{2}$(________) = ________ ft

EASY PROBLEMS ON FREE FALL MOTION* (Neglect air resistance)

* In working problems on accelerated motion, you will find it pays to organize your information clearly, like a good engineer. A table like this is worth making. Write your data in the table, with ? where you seek information, and X where you neither have it nor want it. (This specimen shows the data and question in Problem B-10(a).) Then you can see which algebraic relation will be useful. (In this example it must be the one that does not contain s.)

| | |
|---|---|
| $v$ | ? |
| $v_0$ | −5 ft/sec |
| $a$ | + 32 ft/sec/sec |
| $s$ | X |
| $t$ | 2 sec |
| $\bar{v}$ | X |

B-8. A helicopter, remaining still above the ground, drops a small mailbag. When the bag has fallen for 2 seconds:
(a) What is its speed?
(b) How far has it fallen?

★ B-9. FREE FALL FROM MOVING OBJECT
A helicopter, falling steadily 5 ft/sec without acceleration, releases a small mailbag. After 2 seconds:
(a) What is the speed of the bag?
(b) How far has it fallen?
(c) How far is it below the helicopter?

★ B-10. A helicopter, rising steadily 5 ft/sec, releases a small mailbag. After 2 seconds:
(a) What is the speed of the bag?
(b) How far has it fallen?
(c) How far is it below the helicopter?

★ B-11. FREE FALL FROM MOVING OBJECT
What common property is shown by the answers to Problems 8, 9, 10?

★ B-12. IMPORTANT PROBLEM (Answer needed for later problems)
A man standing on a shelf 4 ft above the floor steps off and falls to the floor.
(a) How long does he take to fall?
(b) What is his speed just before landing?

★ B-13. CAR BRAKES
A certain car with smooth tires on a wet road can have an acceleration of 1/5 of "$g$" but not more. (To accelerate, the car must be pushed by some real, external, agent. The agent is the road, pushing the car by friction. With these tires, friction can provide up to $g/5$ acceleration but, if asked to provide more, the wheels begin to slip and friction falls to an even lower value, giving a smaller acceleration.)
(a) What speed will the car gain in 4 secs, with this maximum acceleration?
(b) How far can it travel from rest in 4 secs?

★ B-14. CAR BRAKES AND SAFETY
A car with good brakes but smooth tires on a wet road can have a deceleration of 1/5 of "$g$" but not more (see Problem B-13). Discuss the stopping of this car by answering the following questions:
(a) Driving at 30 miles/hour (= 44 ft/sec) the driver takes 1 sec to react to danger, decide to stop and get the brakes working; then he makes the brakes give maximum deceleration.
  (i) How far does he travel in the 1 second before braking?
  (ii) How much time do the brakes then take to reduce the speed from 30 miles/hour to zero?
  (iii) How far does the car travel in the braking time?
  (iv) How far does the car travel in the *total* time from seeing the danger until stopped?
(b) If the car is travelling twice as fast, 60 miles/hour, how far does it travel in the *total* time, as in (iv) above?
(c) The car is travelling 30 miles/hour and the driver (after 1 second of thought, etc.) jams the brakes on so that the tires skid, commanding less friction, giving a deceleration of only $g/8$. How far does the car travel in coming to rest? (Sliding friction in a skid is unable to provide such a large maximum force as non-slip friction.)
(d) With new tires on dry concrete, the car has maximum deceleration $g/2$. (Friction of rubber on concrete can do much better than that, but many a brake mechanism can not.) Again calculate the total distance of stopping, from 30 mi/hr.

B-15. "$g$" IN A MOVING LABORATORY
A portable timing apparatus can now be made to time free fall of a few feet from rest accurately enough to give a value of "$g$" reliable within 1% or better. Suppose such an apparatus gave $g = 32$ feet/sec². What would you expect it to give:
(a) If used in a railroad train running smoothly at fixed speed along a level track? (Think what happens when you drop something, say an orange, in a moving train.)
(b) In an elevator moving downward at constant speed? (*Hint*: Think . . .)
(c) In an elevator falling freely after its cable has broken?
(d) In elevator accelerating downward 16 ft/sec²? (Make a bold guess.)
(e) In an elevator accelerating up 16 ft/sec²?

MORE SIMPLE PROBLEMS ON FREE FALL

B-16. How long would it take a freely falling body to fall 400 feet from rest?

B-17. A ball is thrown upward with speed 80 ft/sec. How high will it rise?

B-18. An explorer discovers a deep crevasse in a rocky mountain. He drops a stone into it and 4 seconds later he hears the sound of the stone hitting the bottom of the crevasse.
(a) Estimate the depth.
(b) Comment on the accuracy of this method.

B-19. A stone thrown vertically upward with initial velocity 40 ft/sec takes 1 second to reach a bird.
(a) What is the vertical height of the bird above the thrower?
(b) A time of 1.5 seconds gives the same answer for the bird's height. Give a physical reason for this duplicity.

PROBLEMS ON APPARATUS OF PROBLEM B-2

B-20. Why is the lamp (or some other resistance) necessary in the arrangement sketched in Problem B-2?

B-21. In the experiment of Problem B-2, the following troubles may occur:
(a) The clock may lag a few tenths of a second in starting.
(b) The clock may lag a few tenths of a second in stopping.
(c) The pegs at the top, being compressed when the ball is held there, may give the ball a small downward shove when it is released.
(d) Air friction may have an appreciable effect.
  (i) For each of the troubles (a)-(d), say whether it, operating alone, would make the estimated value of "$g$" too big or too small; and give a brief reason for your answer.
  (ii) What would happen if (a) and (b) operated together, about equally?
  (iii) Suggest experiments to test for each trouble, (a)-(d). Describe them with sketches where possible.

# CHAPTER 2 · PROJECTILES: GEOMETRICAL ADDITION: VECTORS

"What hopes and fears does the scientific method imply for mankind? I do not think that this is the right way to put that question. Whatever this tool in the hand of man will produce depends entirely on the nature of the goals alive in this mankind. Once these goals exist, the scientific method furnishes means to realize them. Yet it cannot furnish the very goals. The scientific method itself would not have led anywhere, it would not even have been born without a passionate striving for clear understanding."

—A. Einstein, *Out of My Later Years*

*Experiments*

This chapter might start with crisp statements of simple rules of projectile motion. Or you could consult a modern textbook on "Ballistics, the science of projectiles," which would give you profuse information and more abstruse rules. The text would mention ancient prejudices only to sneer at them, and tell you that Galileo's simple rules are of little use in modern gunnery. But with such a start you would miss a share of the delight of the great experimenters. Instead, please start with your own experiments.

Throw stones or coins outwards and watch their motion. Try this with a variety of objects ranging from a heavy stone to a crumpled sheet of paper. Try releasing two stones simultaneously, dropping one to fall freely downward, projecting the other horizontally. Make any other investigations and comparisons that occur to you; and try to extract simple rules or generalizations.

Watch a stone or baseball follow a curved path. Labelling this curve a "parabola" is neither true nor helpful at this stage. But it is good science to note that the curve is almost symmetrical, like (a) in Fig. 2-1, and unlike (b) or (c). This suggests that

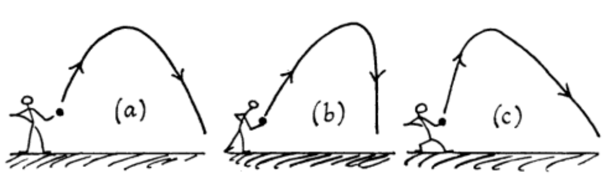



Fig. 2-1. Projectile Paths?

the motion in the downward half somehow matches the motion in the upward half. Perhaps the upward motion and downward motion take equal times—a suggestion to be investigated directly.

A careful experimenter trying a series of materials such as lead, stone, wood, cork, paper finds that for the later members of that series (b) is nearer the truth than (a).

As late as the 16th century, people believed the traditional statement that heavier things fall faster in proportion to their weight. And their beliefs about the path of a projectile were stranger still. It was said to be made up of three parts (see Fig. 2-2): (A) the violent motion (straight out unaffected by gravity)[1]; (B) the "mixed motion"; (C) the "natural motion" (where the bullet falls splosh on the victim below). You may see from your


Fig. 2-2. Medieval Idea of Projectile Path

own experiments with a ball of crumpled paper how such an idea arose, and you can see why it was foolish to apply it to dense and slow-moving cannon balls. Air resistance and gravity were making a confusing mixture. Galileo got rid of air resistance by thinking out what would happen if it were negligible. Cannon balls of his day moved so slowly that air resistance did matter very little, and thus his rules might have helped artillery men to hit their mark. As usual, practical men took little notice of scientists' suggestions for a long time; and by the

[1] If you feel inclined to jeer at this ancient picture, you should, as Lloyd Taylor suggested, ask your friends what path the bullet from a modern rifle takes when it first leaves the muzzle. Does it travel straight ahead, or does it begin to fall at once?

time Galileo's theory was taken up by gunners it had long been rendered useless by higher speeds. Meanwhile Newton and others had produced more useful theory which included air resistance. By now, three centuries later, projectiles move so fast that air resistance modifies their path tremendously. Fig. 2-3 shows paths for a large high speed projectile, (a) the "ideal" path without air resistance, as Galileo would have sketched it, (b) the actual path in air for the same elevation and muzzle velocity. Modern ballistics involves much more mathematics and even requires electronic-brain calculators to cope with the details of real problems. These are matters of engineering or applied mathematics which do not help our study of the growth of mechanics. Here we shall keep to the simple case of negligible air resistance.

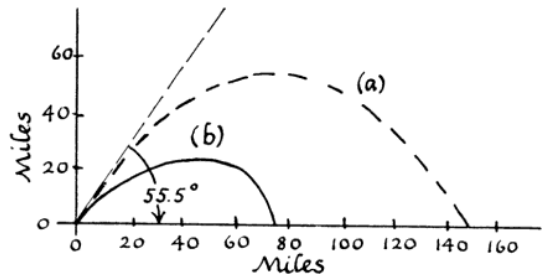



Fig. 2-3. Path of a Projectile
Curves (a) and (b) show paths of a projectile shot with initial velocity 1 mile/second in a direction making 55.5° with horizontal. (From *Science for the Citizen* by Lancelot Hogben; Allen and Unwin, London.)

Galileo tried to separate the up and down (vertical) motion of a projectile from its horizontal motion. Experiment vouches for this treatment by showing that these two motions are independent. Try this yourself. Throw one stone out horizontally and at the same moment release another to fall vertically. They both hit the floor at the same instant. Stone B moving in a curve has to fall the same *vertical* distance to reach the floor as stone A falling vertically. They take the same time. Do A and B keep abreast at intermediate stages of their fall? You need not place special observers to sight them at various levels. Instead, you can move the floor up to catch them earlier and repeat the experiment. Or, more easily, you can move the starting point down nearer to the floor. If A and B arrive at the same instant whatever height they start from, you can say fairly that they keep abreast all the way down. Notice how a series of experiments can be used to replace a difficult complex of simultaneous observations. In trusting our inference from such a set of experiments, we assume the "Uniformity of Nature."

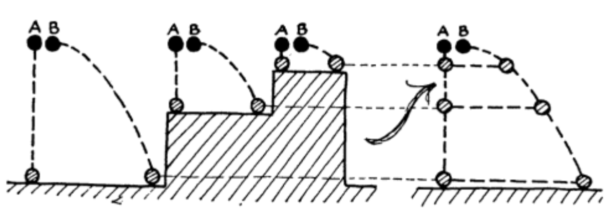



Fig. 2-4. Experimental Comparison of Motions: inferring the general result that falling stone and projected stone keep level all the way.

*Demonstration Experiments*

*(1) Vertical and horizontal motions independent.* Fig. 2-5 shows a simple demonstration experiment in which two metal balls are released by a small spring gun to fall like A and B. You should watch this experiment carefully, and ask to see it repeated with a different height.

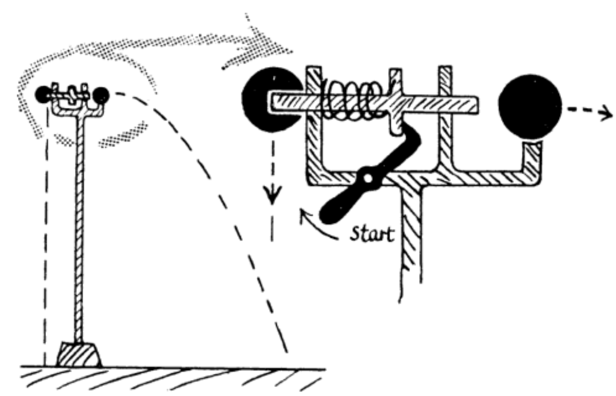



Fig. 2-5. Demonstration Experiment.
The spring gun releases one ball to fall freely at the instant that it projects another ball horizontally. A latch releases the gun's piston. The piston, driven by a compressed spring, hits the second ball, which is resting loosely on a support. The first ball has a hole in it which accommodates the other end of the piston until the piston is released; then the first ball is left behind to fall freely.

*(2) Horizontal motion unchanging.* A projectile moves *vertically* with the acceleration of gravity quite independently of its horizontal motion. How does its *horizontal* motion behave? The symmetrical path of a stone or ball suggests that it does not move slower and slower horizontally, or the path would be more like Fig. 2-1b. Galileo, revolting against the medieval view that any motion needs a force to keep it going—gravity, or demon or rush of air—suggested that the horizontal motion just continues unchanged, since there is no pull like gravity to increase or decrease it. You will see in a later

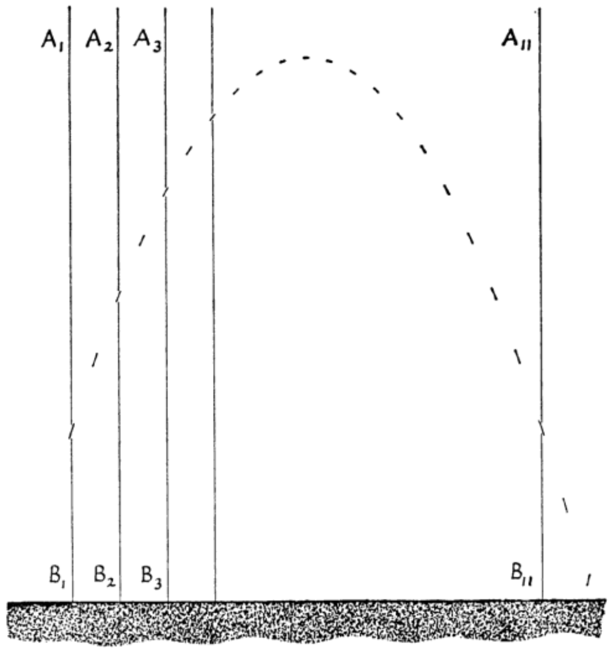



FIG. 2-6. PROJECTILE PATH: photographed with regularly spaced flashes of light (after F. A. Saunders.) Photo by A. Dockrill, University of Michigan.

chapter how he arrived at this by a theoretical argument. For the moment, we can make a direct test.[2] Fig. 2-6 shows a photograph of a ball thrown into the air and illuminated by a series of short flashes of light, evenly spaced in time. Measure the picture for yourself drawing in lines like $A_1B_1$, $A_2B_2$, $A_3B_3$. You will find that the lines are evenly spaced: $A_1A_2 = A_2A_3 = \ldots$ etc. Therefore the ball moved across steadily, moving neither faster nor slower horizontally while it rose vertically slower and slower then fell faster and faster. Once thrown, it kept its horizontal motion unaltered.

Galileo recognized this property of moving things and handed it on to Newton. For many centuries before him most scientists had insisted that steady mo-

[2] You should see a demonstration of this. One beautiful form uses a stream of water drops squirted from a pulsating jet and illuminated by flashes of light which are repeated at the same rate as the jet's pulses. You may see this effect in a movie film of a moving cart-wheel when the time between one frame of the film and the next is just sufficient for the wheel to turn through one spoke-angle; then the spokes "all move on one" between frames and thus seem to be at rest in the picture. The wheel then seems to skid along without rotating. If the real wheel is made to move 10% faster (or the camera is slowed) the wheel in the picture will seem to turn, but only at about 1/10 of its true speed. Though this is a nuisance in films, intermittent, or "stroboscopic," illumination is often used in physics or engineering to "freeze" or slow down the rapid motion of a series of similar things—wheel spokes or water drops. Or it can be used to study a single vibrating object which is repeating a motion rapidly (e.g., a bell, or a violin string). Fig. 2-7 shows the arrangement for water drops. Water is fed from a tank to a small glass nozzle through a rubber pipe which is squeezed by an electromagnet. The magnet, run by an alternating current, squeezes the pipe 120 times a second (twice per cycle of the A.C.) making the stream emerge in drops at a steady rate, 120 a second. The stream is shadowed on a screen by light from a small lantern. With steady illumination, the stream looks continuous. But when a spinning shutter is interposed, the flashes getting through show up individual drops. The shutter, a disc with a slit in it, can be spun by a synchronous motor run on the same A.C. supply. Then the flashes are synchronized with the drops and the pattern stays still. A rectangular grid of wires can be shadowed as well, for measurements.

As a simpler demonstration, balls or water drops can be projected in front of a blackboard and the curve of their path sketched and analyzed. Or you can try your own experiment, rolling a ball with diluted gravity on a slanting table. See Fig. 2-8.

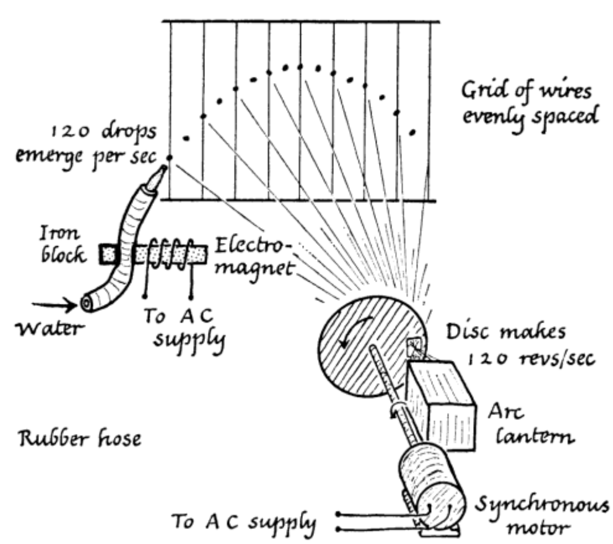



FIG. 2-7. STROBOSCOPIC ILLUMINATION OF A STREAM OF WATER DROPS. The drops emerge regularly, 120 a second, from the pulsed water jet and are illuminated by flashes of light, 120 a second.

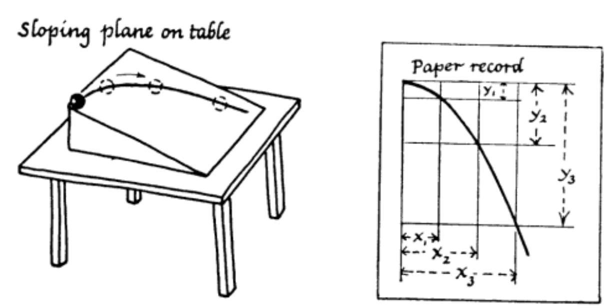



FIG. 2-8. DEMONSTRATION AND ANALYSIS of projectile motion with diluted gravity. A ball rolls across and down a sloping plane, on carbon paper to mark its path. Analysis: On the paper record, draw lines with $x_2 = 2x_1$, $x_3 = 3x_1$, and so on. Measure $y_1$, $y_2$, etc., and find out whether $y_2 = 2^2y_1$, $y_3 = 3^2y_1$, and so on.

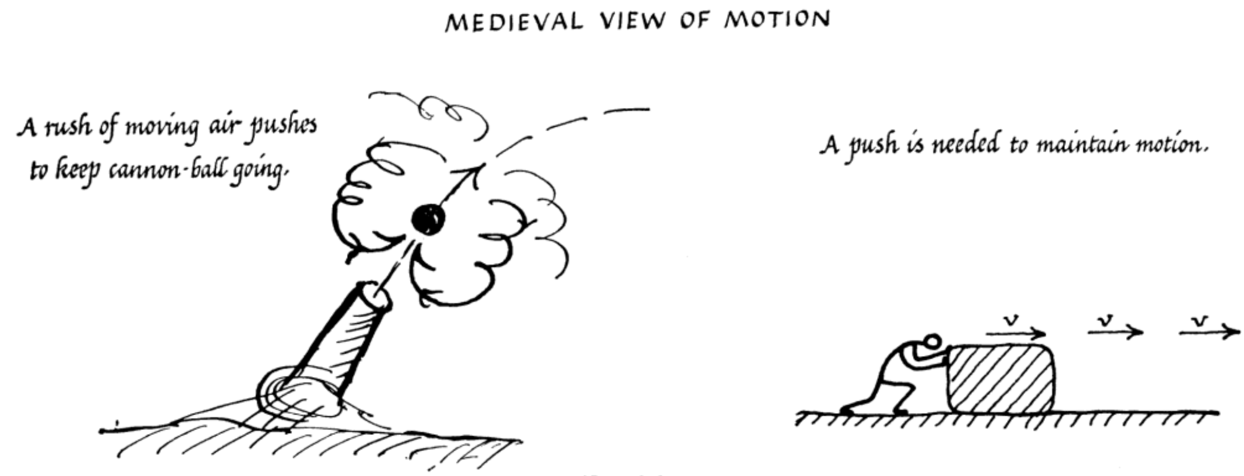



FIG. 2-9.

tion requires a force to keep it going. That ancient idea appeals to common sense today. To keep a box moving along the floor you have to shove; a car running along the level uses gasoline for its engine and the engine somehow provides a steady shoving force. "If you leave a moving thing alone," said the ancients, "it will come to a stop." But to Galileo and Newton, a rough floor and blowing wind do not leave a moving body alone; they exert forces opposing the motion (what we call friction-forces or air resistance). A massive cannon ball moving slowly experiences only trivial resistance; it *is* almost being left alone, so far as horizontal motion is concerned, and it keeps that motion. Hence the new view of a moving body, that it possesses something intrinsic in its motion that keeps it going, unless it is opposed. This something was called "impetus" by some 14th-century thinkers in Paris and Oxford. Their writings reached and influenced Leonardo da Vinci by 1500 and Galileo by 1600—if printing had been available, modern views on motion might have spread three centuries before Galileo. Impetus is a useful name for this quality of a moving body, with a comfortable feeling of "driving ahead" in our modern vocabulary. Later we shall change the name to "momentum," with a more precise meaning. Note that neither word explains anything; at best they are suggestive labels, reminders that a moving body carries its own motion with it and needs no maintaining push. Both are Latin words meaning motion; a Latin dictionary gives them different flavors.

Watching the motion of a cannon ball, Galileo said the gun gives it impetus *which it retains.* The horizontal part of this impetus remains unaltered. The vertical part is changed by the pull of gravity

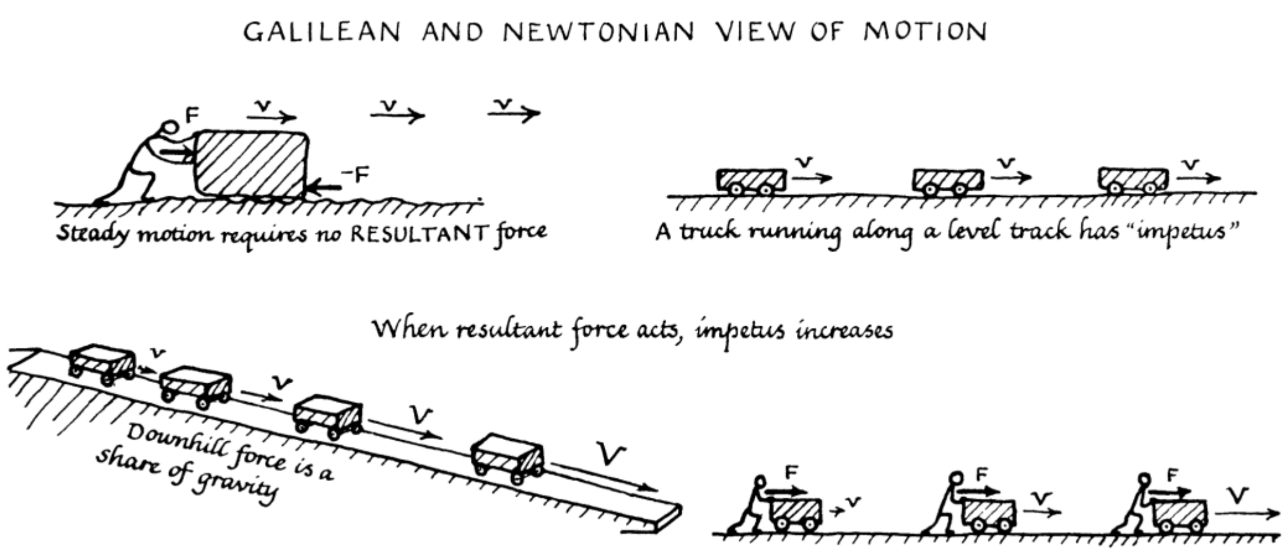



FIG. 2-10.

Experiment soon shows us this will not work unless the separate journeys to be added are straight ahead in the same direction. Then we see 4 ft due North and 3 ft due North do make a total trip of 7 ft due North as in Fig. 2-15. (And, therefore a

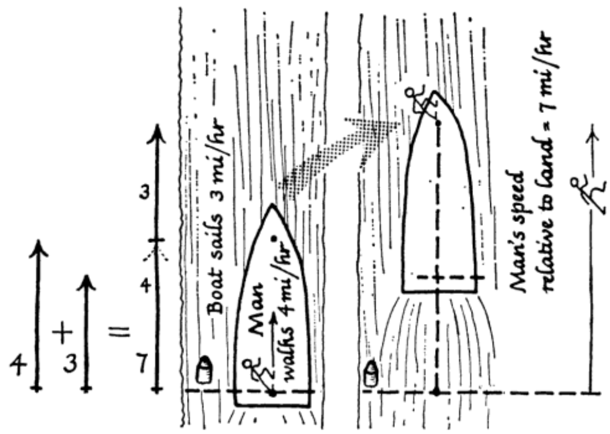



FIG. 2-15. ADDING MOTION IN SAME DIRECTION

speed of 4 ft/sec and a speed of 3 ft/sec both due North do make a total speed of 7 ft/sec due North. And 4 miles/hr plus 3 miles/hr both in the same direction do make a total speed of 7 miles/hr.)

However, if the directions are different simple arithmetic does not work. A trip of 4 ft due East added to 3 ft due North does not make a trip of 7 ft. Nor does a speed 4 miles/hr due East plus a speed 3 miles/hr due North make a speed of 7 miles/hr in any direction. To fit the facts of the world, we have to use another kind of addition, which we call *geometrical addition.* Common sense—in this case simple knowledge accumulated in crawling, walking, driving, sailing, etc.—suggests how geometrical adding should be done. Suppose you wish to add trips of 4 ft to the East and 3 ft Northward, to find the *single trip that would carry you from the starting point to the destination.* Though it seems childish, try this for yourself. Stand facing North with your feet together. Then try to make both these trips, i.e., step four paces to the right and three paces forward at the same time. You could try this by doing one trip with each foot; sideways with your right foot and forward with your left foot, simultaneously; but

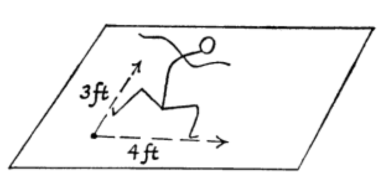



FIG. 2-16. TRYING TO ADD TWO MOTIONS IN DIFFERENT DIRECTIONS

the result is uncomfortable (Fig. 2-16). Instead you had better take one trip first, then the other, thus: move 4 paces to the right *then* 3 paces forward (Fig. 2-17). Or you can take them in the other

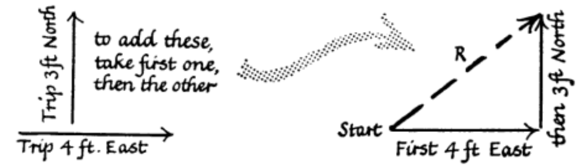



FIG. 2-17. ADDING MOTIONS

order, and arrive at the same destination. If you could somehow make the two trips simultaneously you should reach the same end-point. In fact this can be done if you have a rug which can be drawn across the floor by an electric motor. Then have the motor drag the rug with you on it (or a toy, as in Fig. 2-18) 4 paces to the right while you move

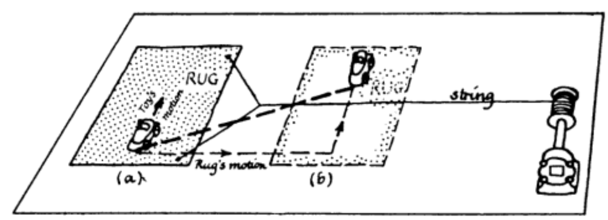



FIG. 2-18. ADDING MOTIONS. The toy crawls along the rug while an electric motor pulls the rug across the floor. The toy has a diagonal motion over the floor.

3 paces forward at the same time. On the rug—relative to the rug—you only move 3 paces forward. From a bird's eye view you make both journeys simultaneously and reach the same destination as if you made first one journey then the other. What single trip could replace these two, whether they are taken simultaneously or separately, and get you to the same destination? The simple single trip is along the straight line from starting point to finish. This


FIG. 2-19. ADDING PERPENDICULAR TRIPS

is called the *resultant* of the two trips. If the trips are drawn to scale on paper, as in Fig. 2-19, then the single trip which would replace them (if they are taken separately) is trip **R**. If the trips are not at right angles, a similar scale drawing will work, as in Fig. 2-20. If the trips are taken simultaneously—as when a plane flies in a wind—we can

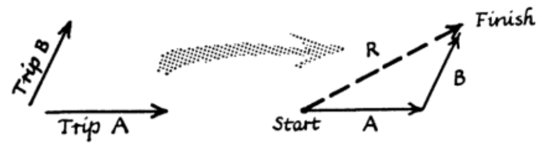



FIG. 2-20. ADDING TRIPS

still pretend to take first one then the other, and arrive at the resultant **R**, as in Fig. 2-21.

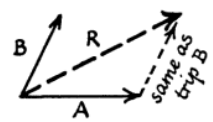



FIG. 2-21. ADDING TRIPS

We find the resultant by taking first one trip then the other, as in Fig. 2-22a or Fig. 2-22b. Com-

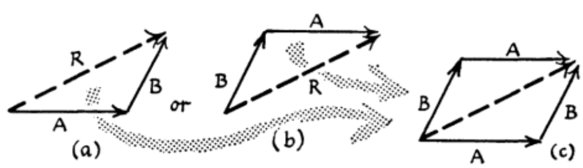



FIG. 2-22. ADDING TRIPS

bining these figures in Fig. 2-22c, we see that the resultant is given by the diagonal of the parallelogram whose sides are the original trips.

This system is obviously right for adding trips: we are assured by common sense, drawing on ex-

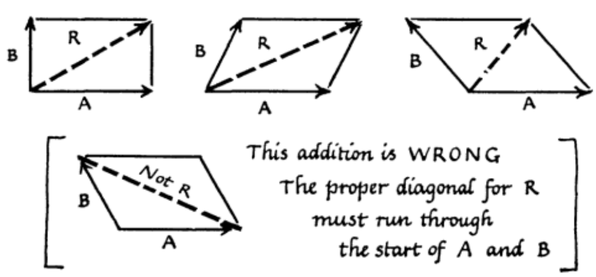



FIG. 2-23. EXAMPLES OF PARALLELOGRAM ADDITION

perience ranging from nursery exploration to complex navigation.

The system can be reversed, and the trip **R** split into components **A** and **B**. They are one possible pair that would combine to make **R**. There are an infinite number of such pairs, each adding to the same **R**.

PROBLEM 6

(i) Sketch (a) in Fig. 2-24 shows a trip **R** split into two components, $\mathbf{A}_1$ and $\mathbf{B}_1$; and (b) shows the same **R** split into a different pair, $\mathbf{A}_2$ and $\mathbf{B}_2$. Copy these sketches, and add several more, all showing the same **R** split into different components, $\mathbf{A}_3$, $\mathbf{B}_3$; $\mathbf{A}_4$, $\mathbf{B}_4$; etc.

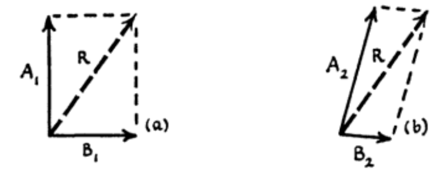



FIG. 2-24. PROBLEM 6. The vector $R$ may be split up into components $A_1$ and $B_1$, or into components $A_2$ and $B_2$, or into other pairs of components. The components need not make 90° with each other.

(ii) Show that we can assign component **A** *any direction and any size*, and still find a **B** to fit, so that **A** and **B** add up to **R**. (This is equivalent to subtracting vectors, **R—A**, useful in later physics.)

### *Velocity and Speed*

The *direction* of a motion is just as important as its size. We now need a name for the idea of a *definite speed* associated with a *definite direction*. We call this *velocity*.[4] Velocity then has two qualities: size (= speed) and direction. Do velocities add by the geometrical system? Or, as a scientist would say, are velocities "vectors"?

### *Vectors: Definition*

*Vectors are those things which are added by the geometrical system.* They are called "vectors," because we can draw[5] a line to represent them, showing both their size (to some scale) and their direction.

RULE FOR ADDING TWO VECTORS

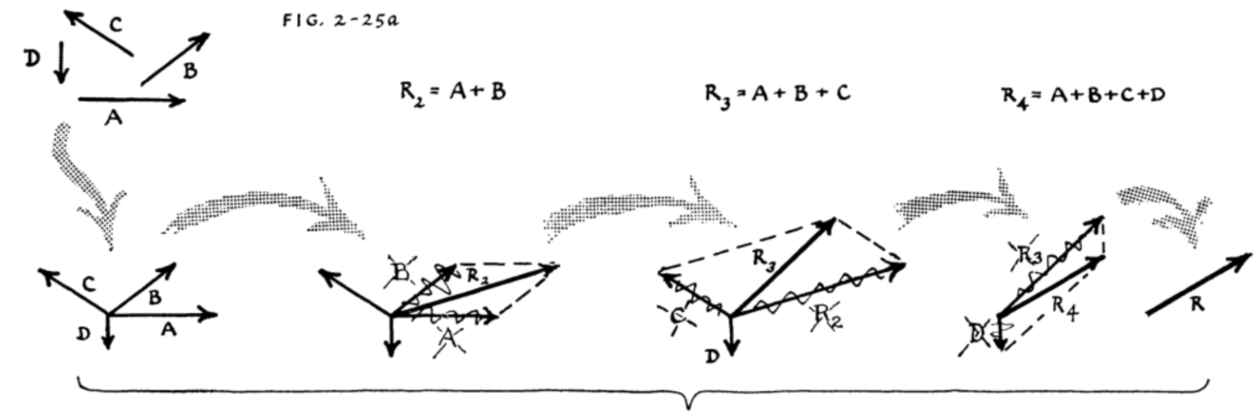
The following rule describes geometrical addition. Our definition of vectors makes it automatically true for vectors.

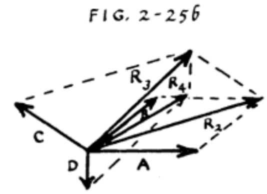
Geometrical addition: *To add two vectors, choose a suitable scale, and draw them to scale starting from the same point. Complete the parallelogram. Then, on the same scale, their resultant is represented by the diagonal from the starting-point to the opposite corner.*

In this, the *resultant* of a set of vectors is defined as *that single vector which can replace, or has the same physical effect as, the original vectors taken together.*

[4] In ordinary language, speed and velocity mean the same thing: how fast an object is moving. In physics, it is useful to reserve the name velocity for speed-in-a-particular-direction, which is a vector. From now on, we shall use speed to mean just rate of covering distance along some path whether straight or crooked—a worm's measure of progress. A speed is specified by a number with a unit, such as 15 miles/hour. A velocity needs a number with a unit *and* a direction to specify it, e.g., 15 miles/hour Northward.

[5] Vector and vehicle come from the Latin verb meaning to carry or convey.



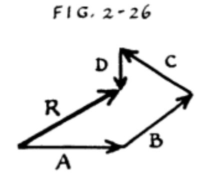




FIG. 2-25. ADDING VECTORS BY THE PARALLELOGRAM CONSTRUCTION.
(a) the details of the process (b) the result

FIG. 2-26. ADDING VECTORS TAIL-TO-HEAD

Just as vectors **A** and **B** add to give resultant $\mathbf{R}_2$ in Fig. 2-25 so we can add vectors **A** and **B** and **C** by adding **C** to $\mathbf{R}_2$ to get $\mathbf{R}_3$. Further addition of vector **D** would give $\mathbf{R}_4$ and so on. Or, more simply, any set of vectors can be added tail-to-head as in Fig. 2-26 (which is only a simplification of Fig. 2-25b), and their resultant is shown by the single vector joining start to finish.

What things are vectors? That is, which things in science do add geometrically by the parallelogram construction? Trips, or to give them a more official name "directed distances" or "displacements," are vectors. If trips are vectors, we need only divide by the time taken to travel them to see that velocities are vectors too. If we use as vectors the length travelled in unit time, then these vectors, which add geometrically as trips, themselves represent velocities. As an extension of this, we see that accelerations are vectors too.[6] We shall find other vectors, other things that can be measured with instruments and which obey geometrical addition. At the moment an important question arises: are forces vectors, i.e., do they obey geometrical addition? This cannot be answered by thinking about it.[7] It is not obvious. It needs experimental investigation. See Chapter 3.

[6] *Trips* are vectors. *Velocities* are *trips per hour*, say. Therefore velocities are vectors. Therefore *changes of velocity* (which are themselves each a velocity gained or lost) are vectors. *Accelerations* are *changes of velocity per hour*, say. Therefore *accelerations* are vectors.

[7] Unless we are prepared to *define* forces as things which add geometrically and then take the consequences of our definition in our later development of mechanics!

### *Scalars*

Things which are not vectors but have only size, without any direction attached, are called *scalars*; for example, volume, speed, temperature. There are other things which are neither vectors nor scalars: vague things such as kindness, and some definite ones, some of them "super-vectors" called tensors. The stresses in a strained solid provide an example of tensors: pressure perpendicular to any sample face and shearing forces along it. More complicated examples appear in the mathematical theory of Relativity. For example, we shall treat momentum, $m\mathbf{v}$, as a vector with three components, $m\mathbf{v}_x$, $m\mathbf{v}_y$, $m\mathbf{v}_z$; and we shall treat kinetic energy as a scalar. Einstein, taking an overall view of space-time, would lump momentum and kinetic energy into a "four-vector" with four components, three for momentum, one for kinetic energy.

### *Addition of Many Vectors*

Two vectors are added by the parallelogram method. At the top of Fig. 2-27, **A** ✚ **B** 〓 **R** (the heavy ✚ and 〓 referring to geometrical addition). We can work back from this definition to the crude "first one trip then the other" method of adding, as in Fig. 2-27. This tail-to-head method is the easiest way of adding several vectors. If we wish to add vectors **A**, **B**, **C**, **D**, we could add them by applying the parallelogram construction again and again—getting the resultant of **A** ✚ **B**, adding the latter to **C**, adding the new resultant to **D**. But the drawing is tedious, and if we perform all the

FIG. 2-27. "TAIL-TO-HEAD" ADDITION. Adding two vectors by parallelogram method is equivalent to "tail-to-head" addition.
Starting with parallelogram addition, we can omit part of the drawing and still obtain *R*.
We can economize still further and draw only a triangle, and we are back to our first discussion of trips, where we added them by taking first one trip and then the other. This leads to an easy *rule for adding vectors*:
DRAW ONE OF THEM FIRST.
THEN DRAW THE SECOND, STARTING IT WHERE THE FIRST ONE ENDED—that is, draw them one after the other, "tail-to-head."
THEN DRAW THE LINE JOINING START TO FINISH, AND THAT REPRESENTS THE RESULTANT, *R*.

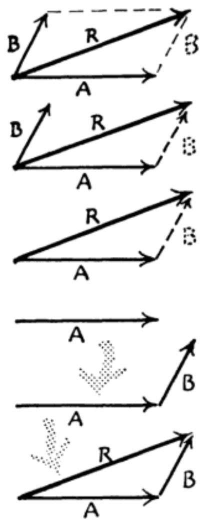
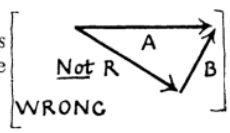



FIG. 2-28. BEWARE of adding vectors "*head*-to-head." That gives quite the wrong answer, not their resultant.




stages on one diagram it is a gorgeous mess (Fig. 2-29, I). Instead, we add **A** and **B** by the tail-to-head method, then add **C** to their resultant, tail to head, then add **D**. We can omit the intermediate resultants, and *find the main resultant* **R** *by joining the start of the first vector to the end of the last* (Fig. 2-29, II).

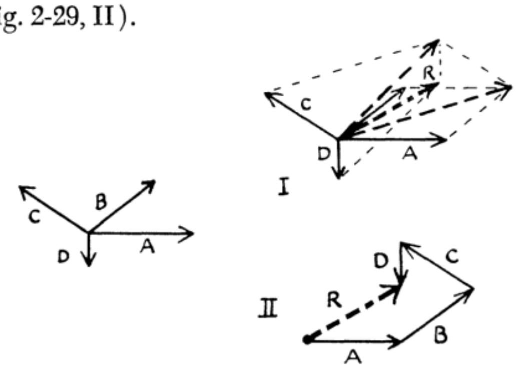



FIG. 2-29. ADDING A SET OF VECTORS:
(I) by the piecemeal parallelogram method.
(II) by the tail-to-head polygon method.

*Drawing Parallel Lines*

To transfer a vector from one place on a sheet of paper to another, we must draw a line in the new place with the same length and the same direction as the old line; so it must be parallel to the old line. There are geometrical methods and machines for drawing a line parallel to another line. If you are not familiar with at least one good method, ask for instructions. Difficult drawing of angles is quite unnecessary.

Fig. 2-30 shows one easy method using a ruler and a book cover (or any rectangle or triangle). To

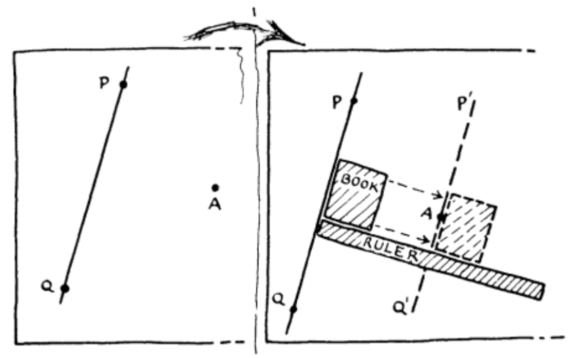



FIG. 2-30. AN EASY WAY TO DRAW PARALLELS. To draw a line P′Q′ through a given point A and parallel to a given line PQ, set the edge of a book along PQ; then slide the book along the ruler until its edge passes through A; then draw P′Q′ along the edge through A.

transfer from line PQ to a parallel line through point A, place one edge of the book on PQ. Place the ruler along the other edge of the book. Hold the ruler fixed, and slide the book along it till its first edge passes through A. Draw along that edge the required line through A.

★ PROBLEM 7. COMPOUNDING VELOCITIES

A ship surrounded by fog is pointed due North and sailing, as the navigator thinks, 4 ft/sec due North in still water. Actually it is in a current moving 4 ft/sec due East. If the fog disperses and the navigator can observe nearby islands, in what direction will he find he is really moving? How fast?

★ PROBLEM 8. CALCULATING A RESULTANT

A ship sails "northward 4 ft/sec" in a fog, as in Problem 7. It is really moving in water that is flowing eastward 3 ft/sec. What is its speed relative to land?

★ PROBLEM 9. NAVIGATION

A navigator trying to follow a narrow channel through reefs, has to sail in a fog.

(i) He knows the channel runs North-east, and that the ocean current carries him eastward 10 ft/sec. His propeller carries him ahead 10 ft/sec. In what direction should he steer, by his compass? (*Hint*: Sketch the known current-vector. From its starting point sketch the direction of the resultant; and at its end point fit in the engine-vector. Complete the parallelogram.)

(ii) Suppose the channel runs due North, the current is 10 ft/sec eastward and his propeller speed 20 ft/sec. Draw a diagram to show the direction in which he should steer by the compass.

(iii) Suppose the channel runs due North and the current is 10 ft/sec eastward. Prove that he cannot follow the channel unless his propeller speed is greater than 10 ft/sec.

### *Does the Order in Which the Vectors Are Added Affect the Resultant?*

In adding vectors tail-to-head one after another, we might choose them in a different order—**A**, **D**, **C**, **B**, . . . instead of **A**, **B**, **C**, **D**, . . . , say—making quite a different pattern. *Will this give the same resultant?* The problems below explore this question.

★ PROBLEM 10

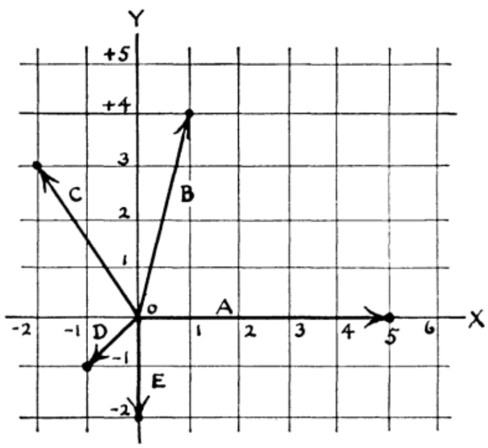



FIG. 2-31. VECTORS FOR PROBLEM 10.
(Copy this diagram on a larger scale, making each square 1 inch by 1 inch.)

Fig. 2-31 shows a set of vectors, **A, B, C, D, E,** all starting from one point, O. Add vectors by the "polygon method" of drawing them tail-to-head following the instructions below. The sketch shown here is too small for accurate drawing and measurement, so you should first reproduce the sketch on a larger scale on a sheet of graph paper. Expand the squares ruled lightly on the sketch to one-inch squares. Then, starting with **A** already drawn, add **B**, then **C**, then **D**, then **E**, tail-to-head. For this you will have to transfer **B, C, D, E,** by some parallel ruling method. (Either use the information given by the graph grid of Fig. 2-31, or use the method of Fig. 2-30.) Mark the result. Measure and record its size. To record the direction of the resultant you could either measure some angle or find its slope. Try both, as follows:

(a) Measure and record the angle between the resultant and the original vector, **A**.

(b) Draw a pair of perpendicular axes, OX and OY, with OX along vector **A**. Then drop a perpendicular *h* from the end of the resultant on to OX (you need not draw this carefully. Just measure *h* without drawing). Measure height *h*, and the base *b* which the perpendicular cuts off on OX. Then calculate the fraction

$$\frac{\text{height, } h}{\text{base, } b},$$ which is called the slope of the line **R**.

This enables you to specify **R** as a vector of size . ? . and direction having slope . ? .

★ PROBLEM 11. UNIQUE RESULTANT

*Is the resultant different if the vectors are added in a different order?* Repeat Problem 10 on a new sheet of graph paper, starting with vector **A** as before but then adding the rest in a new order, **B**, **E**, **D**, **C**. Record the size of the resultant and its direction.

PROBLEM 12: ARGUMENT CONCERNING VECTOR ADDITION

Think of the vectors, **A**, **B**, **C**, **D**, **E** in Problem 10 as navigated trips to be taken one after the other. Think of the axes OX, OY, as compass directions, East, North. Then one trip, say **B**, carries us a certain amount Northward and a

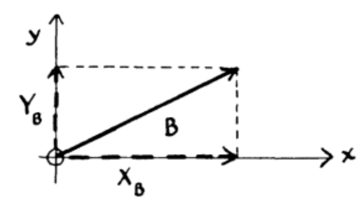



FIG. 2-32a. RESOLVING A VECTOR.
A vector $B$ may be resolved into a pair of perpendicular "components," $X_B$ and $Y_B$, which can replace it. In Problem 12 the two directions $x$ and $y$ are taken to be East and North.

certain amount Eastward. We may say that trip **B** gives us so much "northing" and so much "easting." In fact we are thinking of **B** as split into components, a northward one and an eastward. This is called "resolving" **B** into North and East components.

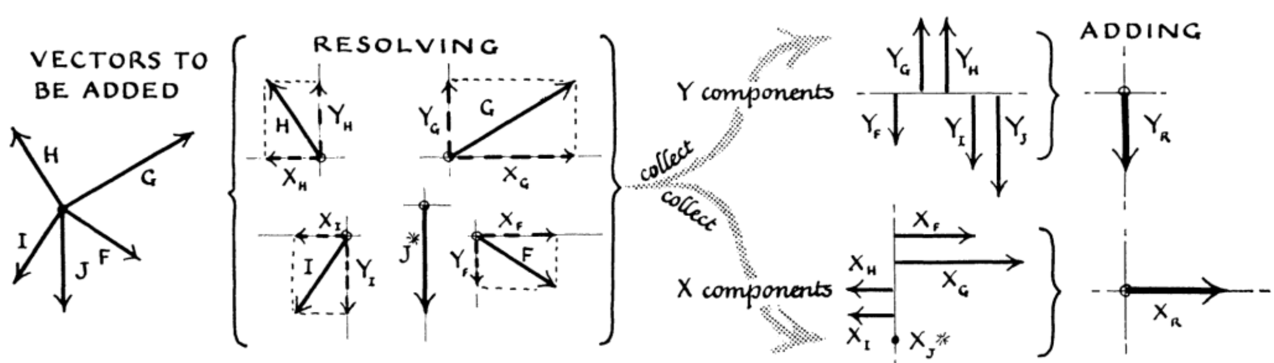



FIG. 2-32b. ILLUSTRATION OF PROBLEM 12.
A set of vectors, $F, G, H, I, J$, resolved into components in directions $x$ (East) and $y$ (North). These components are then added to give the components of the resultant $R$.
* Note that the vertical (southward) vector $J$ has zero $X$-component. Its $Y$-component is, of course, the full vector $J$ itself.

As the stone moves along its path, both these statements are true at each stage; so we say

$$x = 10t$$
$$y = 16t^2$$

To find a single equation that describes the path, we ask, "What relation between $x$ and $y$ makes both requirements above true at each stage in the path?" If we choose any point on the path, its $x$ and $y$ values must satisfy the two equations above, for the appropriate value of $t$. That value of $t$ must be the same in both the equations—it is the time when the stone reaches that chosen point. Therefore we can get rid of $t$ by making one equation yield an expression for $t$ which can be substituted in the other equation; thus:

$x = 10t$ gives $t = x/10$, and we can use $x/10$ for $t$ in $y = 16t^2$, which then becomes
$y = 16\,(x/10)^2$ or $y = (16/100)x^2$.

The equation of the path is then $y = (0.16)x^2$.

More generally, if the stone is thrown horizontally with initial velocity $v_H$ feet/second, and falls with vertical acceleration $g$ feet/second per second,

$$x = v_H t \text{ and } y = \tfrac{1}{2} g t^2$$

$$\therefore \quad y = \frac{1}{2} g \left[ \frac{x}{v_H} \right]^2 = \frac{1}{2} \left[ \frac{g}{v_H{}^2} \right] x^2$$

$\therefore \quad y = (constant)\, x^2$ since $\tfrac{1}{2}\, g/v_H{}^2$ is constant.

*This is the equation of a parabola.*[8]

You can plot beautiful parabolas on graph-paper by starting with an equation like this. Try plotting the graph given by $y = (\tfrac{1}{2})x^2$ on paper marked in inch squares, taking $x = -4$ inches, $x = -3, -2, -1, 0, 1, 2$, etc. Try to match this curve with a real projectile. Put the paper with the sketched curve on a sloping table and experiment with a rolling ball. Or hold the paper upright and throw a small object up in front of it.

### *Projectile from Tilted Gun*

If the projectile is not thrown out horizontally but starts upward in some slanting direction, its path is still a parabola. Algebraically this can be shown by starting with $s = v_0 t + \tfrac{1}{2} g t^2$ instead of $s = \tfrac{1}{2} g t^2$. Or, we appeal to the visible symmetry of the curved path and say that the decreasing upward motion to the top must match the increasing downward motion from the top, and so we draw the complete path from the horizontally projected case. This is only good guessing but experiment confirms it. Or we may argue thus: on the downward part of the path from the vertex, O, the stone cannot know whether it started at O or earlier or later. So a stone started on this part of the path, say at A by being thrown outward and downward must therefore follow the same path as one thrown horizontally from some earlier vertex, O. (See Fig. 2-38.) Similarly for one thrown upward at B.

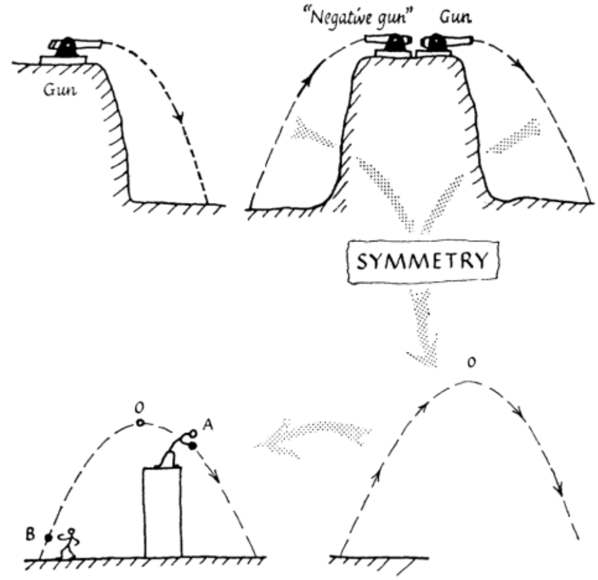



Fig. 2-38. Projectile Motion Up and Down. Symmetry of path suggests that motion up to a "negative gun" on a hill, is similar to motion down from a gun on the hill. These two motions join to give a full parabola. Then a projectile *started* along this parabola, at any point $A$ should follow the same path as if it started (earlier) at the vertex, O. Symmetry extends this argument to the whole parabola.

This implies an extension of the idea of independence of motions. The *vertical component* of the starting motion also continues *unchanged*, while the accelerated motion of falling is added to it. This constant vertical motion is responsible for the distance $v_0 t$ in the relation $s = v_0 t + \tfrac{1}{2} g t^2$. Then we might lump together the two constant motions, the vertical and horizontal parts of the initial throw, and say that *the initial slanting motion given by the thrower remains unchanging in flight*, while the gain of vertical falling motion responsible for $\tfrac{1}{2} g t^2$ is added to it. Thus a stone thrown as in Fig. 2-39 may be regarded as having two motions, its initial motion continuing along the line AB, and free fall measured from successive positions on AB.

[8] Originally described as one of the shapes made by slicing a cone, a parabola is often defined now as a curve whose graph-equation is of the form
$y = (\text{constant})\, x^2$, or $y \propto x^2$.
A piece of algebraic geometry shows that the algebraic and geometrical definitions are equivalent.

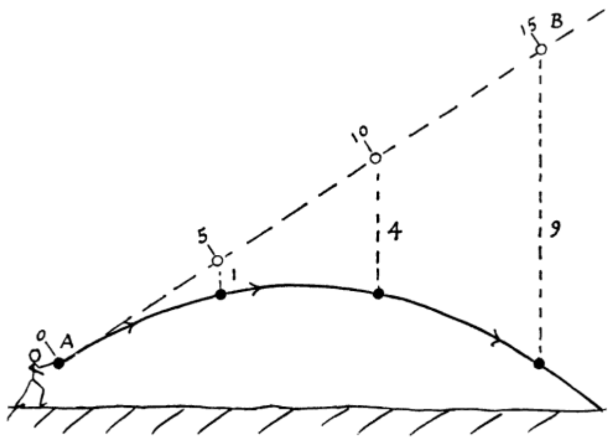



Fig. 2-39. Analysis. The motion of any projectile may be regarded as being made up of an unchanging motion along the initial direction and a free fall from that line.

We can demonstrate this by the "monkey and gun" experiment. Suppose a hunter, ignoring gravity, sights through his rifle barrel on a monkey hanging by one arm from a tree. If the hunter fires, the bullet will miss the monkey because of its falling motion, as in Fig. 2-40. Now suppose the monkey

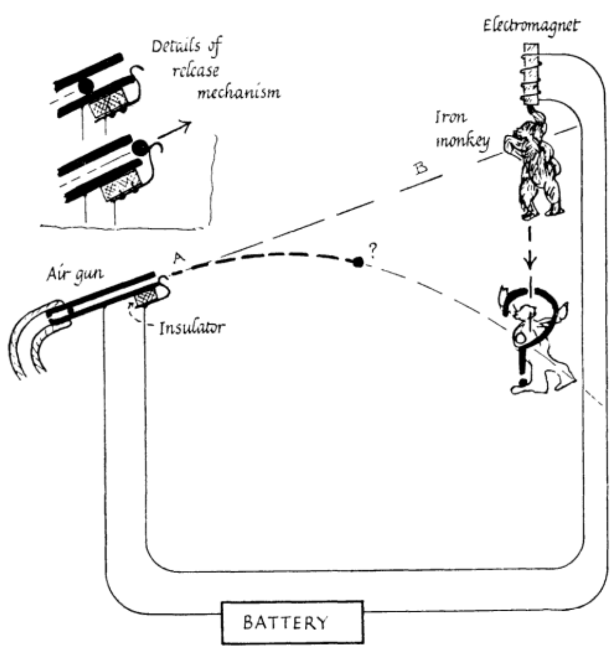



Fig. 2-40. The Monkey and Gun Experiment. At the instant the bullet emerges from the gun, it breaks a contact and allows the electromagnet to release the "monkey." The electrical connection is maintained by a small spring touching the metal gun barrel until the emerging bullet moves it.

watches the gun and lets go at the instant the bullet leaves the gun, when he sees the flash of the gun. From this instant, both monkey and bullet are accelerated downwards by gravity; the monkey falls from rest, the bullet—according to our recent view—falls from the line AB of its "undisturbed path." What will happen? This can be demonstrated by using an iron monkey released by an electromagnet which is switched off when the bullet trips a switch at the muzzle of an air-gun aimed at the monkey.

Such experiments confirm our guess that vertical fall is quite independent of the initial motion, which continues unchanged. Any projectile drops freely from its starting line, from the very beginning. It falls 1, 4, 9, 16, . . . feet in 1, 2, 3, 4, . . . quarter-seconds from start. If the starting line slants upwards, the projectile's actual path rises at first and then falls when the accelerated rate of free fall has beaten the steady rate of rise due to the initial motion. (See Fig. 2-41. Note that this path is a parabola.)

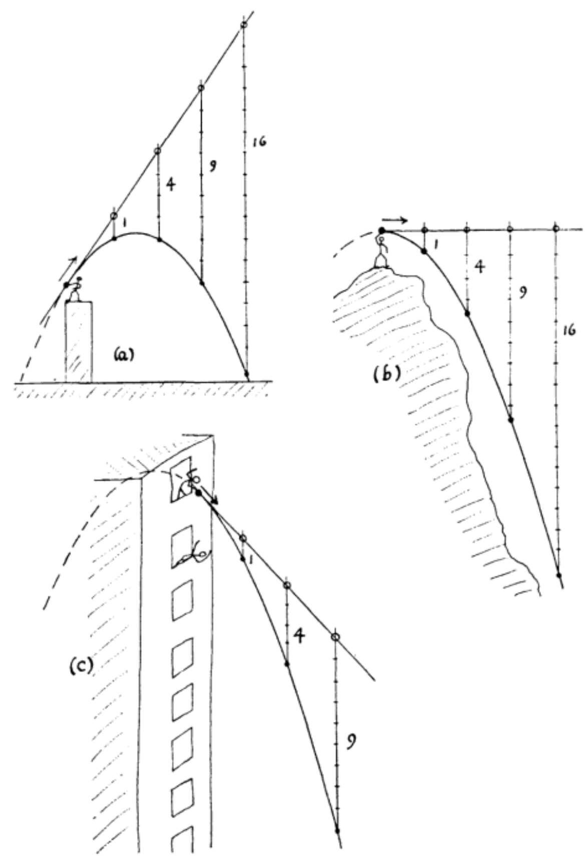



Fig. 2-41. Free Fall of a Projectile. However it is started, a projectile falls with the same "free fall" from its original starting-line as an object released from rest. The accelerated motion of fall is independent of both the vertical and horizontal components of the initial motion.

Notice how our discussion has torn the problem of projectile motion to pieces, leaving it easier to deal with, ready for further studies by experts in ballistics. We have not so much set forth new information as made existing knowledge easier to use.

When the projectiles are speedy electrons (or, in other cases, charged atoms) pulled by electric and magnetic fields instead of gravity, we assume that similar "rules" apply and use our measurements of the curved path to obtain information about electric charge and mass and speed. We use that information in turn—still assuming the same rules for projectile behavior—to predict the effects of fields on particles moving at other speeds. Then we are becoming literally electronic engineers, designing television tubes and other radio devices; and we are becoming atomic scientists, bending streams of electrons or atoms to our bombarding uses or sorting light atoms from heavy ones by differences of their projectile paths.

## PROBLEMS FOR CHAPTER 2

1-16. These are in the text of Chapter 2.

### ★ 17. POLICE WORK

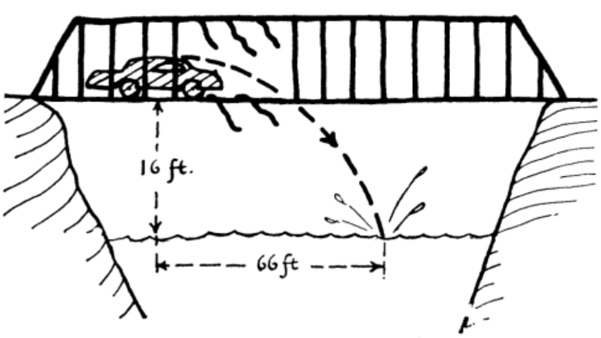



Fig. 2-42. Problem 17

A lunatic, driving too fast on a bridge, skids, crashes through the railings along the side of the bridge, and lands (man + car) in the river 16 feet below the level of the roadway of the bridge. (*Note:* 16 feet is the vertical distance between bridge and river.) The police find that the car is not vertically below the break in the railings, but is 66 ft beyond it horizontally.

(a) Estimate the speed before the crash.
(b) Say whether this is probably an overestimate or an underestimate; and say why.
(c) State clearly the properties of falling bodies that you assumed in making the calculation of (a).

### ★ 18. ELECTRON STREAM

An electron moving 6 million meters/sec (which is quite slow, as electrons go) along a horizontal path, runs into a region where a vertical electric field gives it a downward acceleration of

40,000,000,000,000 m./sec/sec or $4 \times 10^{13}$ m./sec/sec.

This region extends for 0.30 meter, in the direction of the original path; so the electron travels along in a region of no field, then for 0.30 meter (horizontally) of vertical field, then out into a region of no field again.

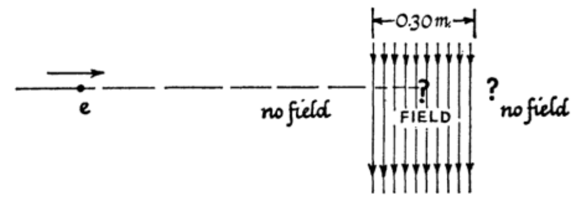



Fig. 2-43. Problem 18

(a) Do you expect the vertical acceleration to affect the horizontal motion?
(b) Calculate the time taken by the electron to travel across the field-region.
(c) Calculate the distance it will fall in the field. (This is the distance an experimenter would measure to investigate the electron's behavior.)
(d) Calculate the electron's vertical velocity at the instant it emerges from the field.
(e) Predict the path of the electron, and draw a rough sketch showing the path before, through and after the field.
(f) Why is it unnecessary to take gravity into account in this question? (It *does* act on the electron.)

### 19. RANGE OF PROJECTILE (A problem using algebra and trigonometry.)

(a) An ancient gun projects a cannon ball at an elevation of 45° with speed 141.4 feet/sec.
  (i) Split this velocity into horizontal and vertical components.
  (ii) Calculate the time from the start of the ball till it reaches the ground again.
  (iii) Calculate the range.
(b) An ancient gun projects a cannon ball with velocity $v_0$ in a direction making angle $A$ with the horizontal.
  (i) Resolve $v_0$ into horizontal and vertical components.
  (ii) Calculate the time taken by the vertical motion, from the start of the ball till it reaches the ground again.
  (iii) Calculate the horizontal distance the ball travels (i.e., its range).
(c) Show, by trig. or calculus, that range is maximum for $A = 45°$, for a given $v_0$.

(Remember that $2 \sin x \cos x = \sin 2x$.)

### ★ 20. MEASURE HOW FAST YOU CAN THROW A BALL

A scientist wants to find out how fast he can throw a baseball. He throws it out horizontally at shoulder height, 4 ft above the ground. It lands on the ground 20 ft away from his feet.

(a) What was the ball's original speed? (See Problem 17.)
(b) Apart from any formula for accelerated motion, an important general principle concerning projectiles (formulated by Galileo) has to be used in calculating the answer to (a). What is it?
(c) Instead of throwing the ball, the scientist runs along at the speed calculated in (a) above, carrying the ball at shoulder height. While running, he releases the ball so that it can fall. Describe carefully the path of the falling ball:
  (i) as seen by a stationary observer,
  (ii) as experienced by the running scientist.

21. An automobile travelling 96 ft/sec (over 65 miles/hour) along a horizontal mountain road failed to make a corner and crashed into a snowdrift 144 ft (vertically) below.

(a) How long did the car take to fall?
(b) How far (horizontally) did it land from the place it left the road?
(c) What was its acceleration when half way down?
(d) Describe the angle of the tunnel that it made in the snowdrift on landing.

22. A man holds a rifle 9 ft above the level ground and aims it horizontally.

(a) How long is it from the instant of firing until the bullet hits the ground?
(b) If the cartridge is ejected horizontally to the side, just as the bullet leaves the barrel, when will the cartridge hit the ground?
(c) Would the man be able to shoot farther (with this aim) on the Moon?
(d) Give a clear reason for your answer to (c).

★ 23. A man inside a large elevator throws a ball straight out from him horizontally with speed about 10 ft/sec. In each of the cases (a), (b), etc., sketch the path of the ball as observed by the man in the elevator.

(a) The elevator is moving downward with constant velocity 10 ft/sec.
(b) The elevator is accelerating steadily with downward acceleration 32 ft/sec/sec.
(c) The elevator is accelerating steadily with downward acceleration 10 ft/sec/sec.
(d) The elevator is accelerating with downward acceleration 64 ft/sec/sec (suitable machinery being used to achieve this).

# CHAPTER 3 · FORCES AS VECTORS

Brute force, unsupported by wisdom, falls of its own weight.

—Horace, *Odes*, III, 4

Forces are pushes and pulls: things you feel when they act on you, things that stretch springs, things that make moving bodies accelerate. We shall measure forces with spring balances. As these instruments are commonly graduated in pounds or in kilograms, we shall use those units for force at present. Later we shall change to more proper units.

Engineers are much concerned with adding forces in bridges—cranes, buildings, machinery—or with subtracting them to find the remaining force needed to hold some system balanced. We can show that forces are vectors, i.e., that they obey geometrical addition. The vector treatment of balanced forces is called "Statics." It is a bulky but dull part of physics, and most texts spend a lot of space teaching tricks for solving engineering statics problems. We shall give only a few examples, and even they might be better omitted to give time for more study of force and motion.

First we must have some assurance that forces *are* vectors. To say that they *must* be vectors because they have size and direction is risky. That does not make sure they add geometrically. Though it looks plausible—especially to people who deal with ropes on ships or tents—we ought to test it directly. You should see the demonstration described below.

*Demonstration Experiment*

Fig. 3-1 shows a large contraption set up in front of a blackboard. O is a metal ring pulled by two ropes, OA, OB, with spring balances A and B to measure the pulls. The ropes must exert considerable pulls to hold O in the position shown because it is pulled the opposite way by a large spring S, which is anchored to the wall at its other end. The rope-pulls are together sufficient to stretch the spring and hold O in its present position. The position of the ring O is marked, and the lines of the ropes, OA, OB, are marked. The balances A and B are read to give the pulls $\mathbf{F}_A$ and $\mathbf{F}_B$.

The resultant of these two pulls is found by drawing, assuming geometrical addition of forces. For

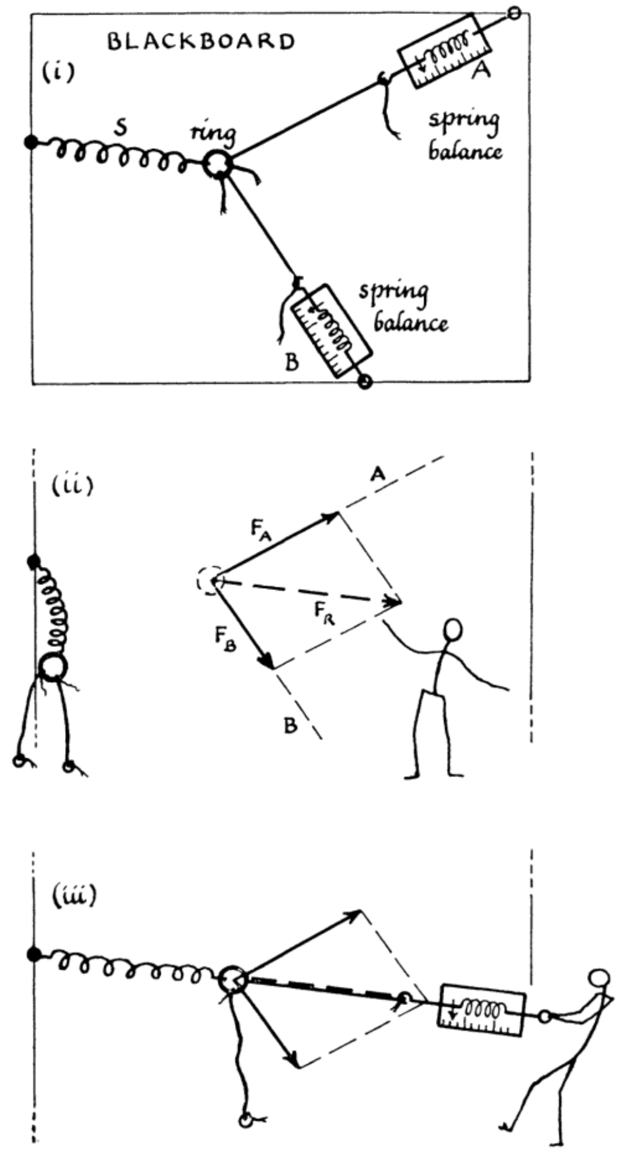



Fig. 3-1. Demonstration Test.
Are forces vectors? (i.e. do forces add by geometrical addition?)

(i) Two ropes exert measured pulls on a ring O, pulling it out to a marked position against the pull of spring S.

(ii) The ropes and spring are removed, and the predicted resultant of the pulls $F_A$ and $F_B$ is obtained by geometrical addition.

(iii) Then the prediction is tested by measuring the *actual* force needed to pull the ring out to its marked position with a single rope.

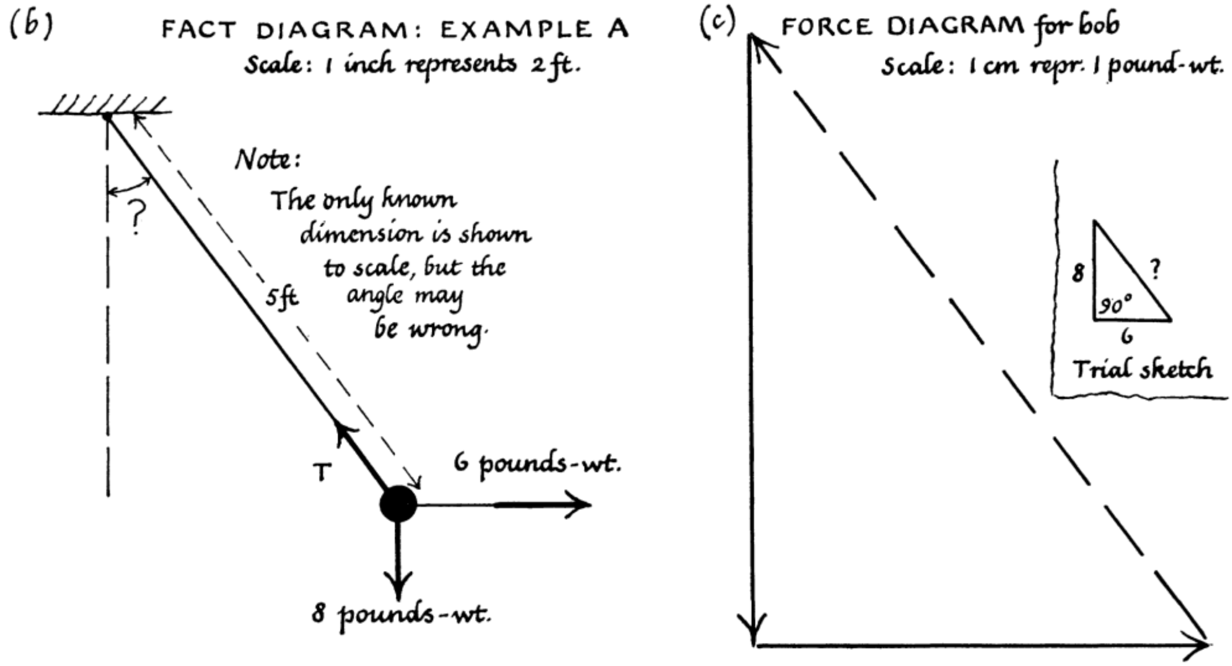



FIG. 3-5b,c. DIAGRAMS FOR EXAMPLE A.

We start by drawing a vector we know all about: the vertical pull of 8 pounds. We represent this by an 8-centimeter line **AB** drawn vertically, with an arrow head to show it is downwards.* We then add the other vector we know all about, the 6-pound horizontal pull, representing it by a line **BC**, 6 centimeters long. The third line for force must close the triangle since the resultant is zero. Therefore the third force must be in the line **CA**. Measuring this on a carefully drawn diagram we find it is 10 centimeters long, representing 10 pounds tension in the slanting string.

Or we can in this case look at a rough sketch and use Pythagoras and say the length is $\sqrt{8^2 + 6^2}$ or $\sqrt{100}$ or 10 centimeters. The direction makes an angle with the vertical whose slope (tangent) is ¾. Therefore from trig. tables, or by measurement, the angle is about 37°. Transferring to the actual pendulum we then say: The string tension must be 10 pounds and the string must make an angle 37° with the vertical.

* The points A, B, C are not labelled on Fig. 3-5c. Mark them.

*Example B*

A pendulum consisting of a 10-pound bob on a 5-foot string has its bob pulled aside by a horizontal pull **F**. If the bob is thus displaced 3 feet sideways, what is the size of the force? The fact diagram and the stages of the force diagram are shown in Fig. 3-6. In the force diagram we start by drawing the only force we know all about, the 10-pound downward pull of the bob's weight, **AC**. Then we try to add to it the horizontal pull, tail-to-head, but as we do not yet know the size of that pull we do not know how long to draw its line. However, we do know that when we add the string's pull to the other two forces the diagram must be a closed triangle (if the bob is in equilibrium). So the string's pull must start where **F** ends and finish at A. Also the string's pull must be in the direction of the string itself. (Can you visualize a string pulling in any direction but along itself?) So we transfer the string direction from the fact-diagram to the force-diagram and draw a line through A parallel to the string. This slanting line gives the third side of the force triangle, **BA**, for string tension. The corner B lies on the slanting line and on the horizontal

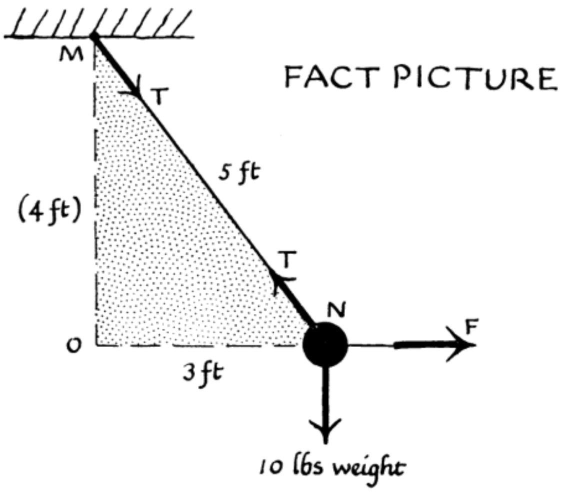
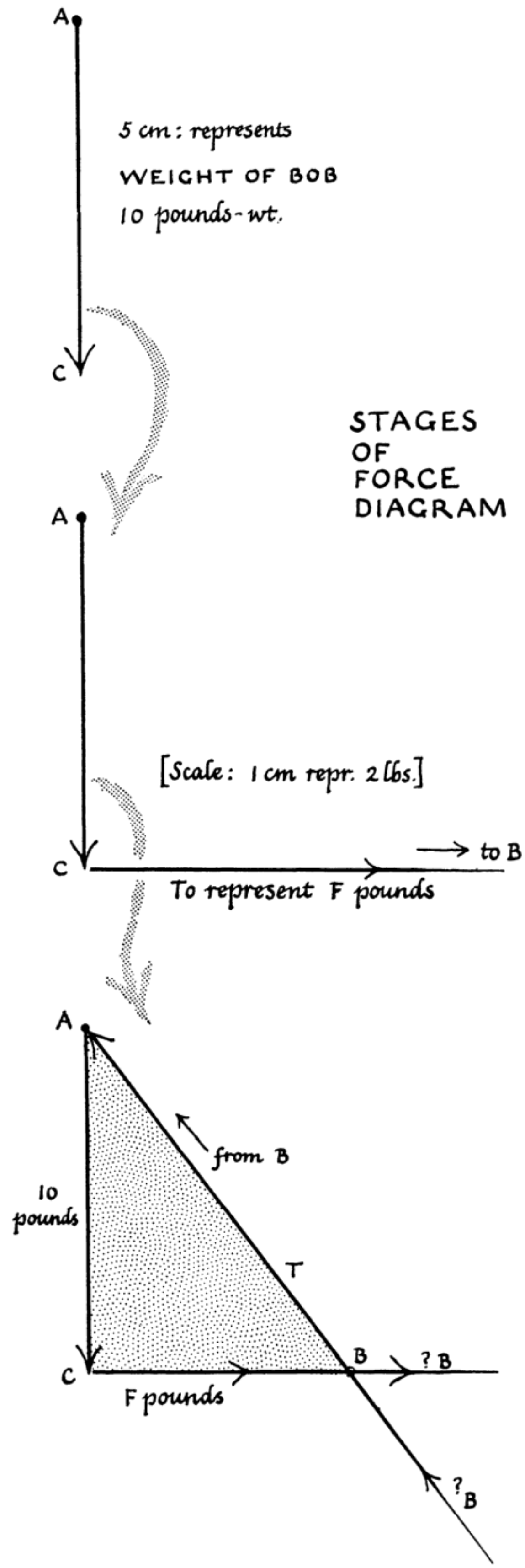



Fig. 3-6. Example B.
These show how a force-diagram is built up from information shown in the fact picture. Since a triangle can be specified completely by two angles and a side, the force-diagram can be constructed in this case.




line; so it must be the place where these two lines intersect. Now that we know where B is, we know the size of **F**, and, incidentally, the string tension also. We can find the size by careful drawing and measurement.

Or, in this case where the data provide easy geometry, we can calculate **F** from rough sketches, arguing thus: The sides of the force triangle ABC are parallel to the sides of the triangle MNO in the fact picture. Therefore[2] these triangles are similar.

(*Note*: Pythagoras' Theorem tells us that OM = 4 feet.)

$$\therefore \quad \frac{\text{F pounds}}{\text{10 pounds}} \text{ in the force triangle}$$

$$= \frac{\text{3 feet}}{\text{4 feet}} \text{ in the fact picture.}$$

$$\therefore \quad \text{F} = (\text{10 pounds})(3/4)$$

$$\therefore \quad \text{Horizontal Pull, F} = 7.5 \text{ pounds}$$

$$\text{Similarly, } \frac{\text{T pounds}}{\text{10 pounds}} = \frac{\text{5 feet}}{\text{4 feet}}$$

$$\therefore \quad \text{String tension, T} = 12.5 \text{ pounds}$$

[2] If you are not familiar with the properties of similar triangles, review them in a geometry book, or ask for instruction. You will need to use them confidently.

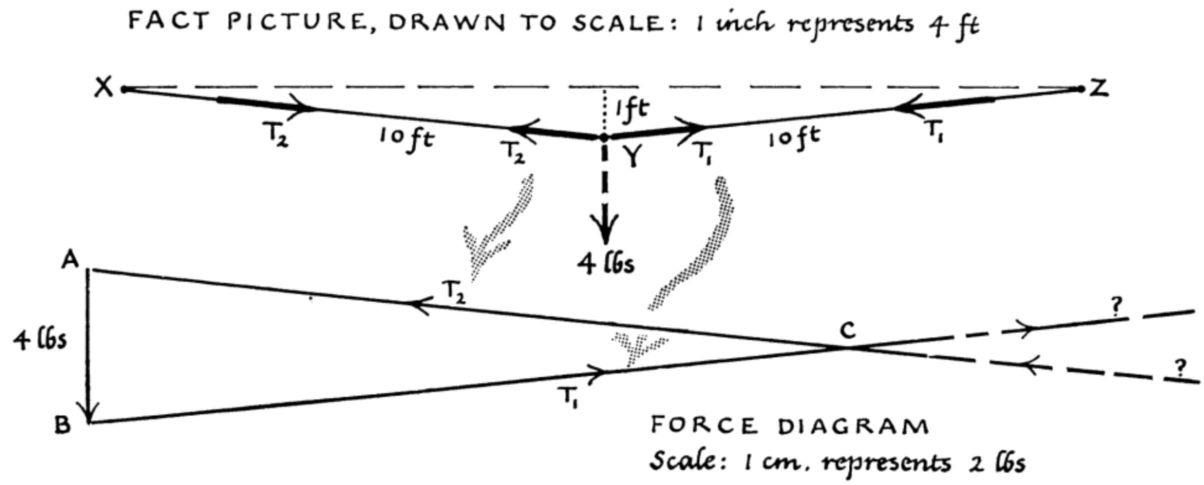



FIG. 3-9. DIAGRAMS DRAWN TO SCALE FOR EXAMPLE C

*Example C*

A 20-foot telephone wire is strung loosely between two supports. A 4-pound bird perches on the

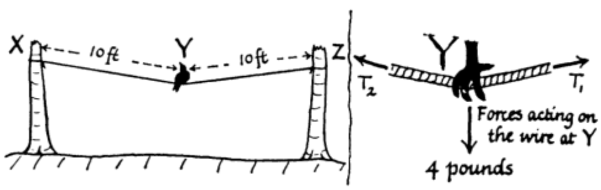



FIG. 3-8. EXAMPLE C

mid-point of the wire. The wire is thus pulled into a shallow V. Its mid-point is then 1 foot below the level of the end-supports. *Calculate the tension in the wire.* (This may seem a trivial artificial problem, like so many mechanics problems in statics, but it is a very serious matter for telephone and power companies. As the answer to this problem suggests, birds and ice may produce huge tensions in wires and overstretch or break them.)

We draw a force diagram for the small central bit of wire, Y, where the bird perches.[3] Three forces act on it, the weight of the bird downward and the slanting pulls of the wire tensions $\mathbf{T}_1$ and $\mathbf{T}_2$. Call the angle between wire and horizontal E.

To draw the force diagram for Y (shown in Fig. 3-10) we start with the fully known weight of the bird, drawn as a downward vertical vector, **AB**, 2 centimeters long to represent 4 pounds weight.

[3] How do we know that it is wise to choose that bit of wire as the victim to draw the diagram for, rather than half the wire, XY, or the whole wire, XYZ? To know which victim to choose is one of the "tricks" of solving mechanics problems, soon learned, not of much value in serious science.

From B we draw BC parallel to the right-hand section of wire to represent its tension, and then another vector parallel to the other section of wire. This must close the triangle, since the resultant force on Y must be zero. But we do not know how long to make the vectors for the tensions. So we draw unlimited lengths, one slanting at angle E upwards *from* B and the other slanting at angle E upwards *to* A, and thus fix C by their intersection. We now have a triangle of forces, which we could measure and interpret by the scale we use.

We could avoid measuring if we could link up the fact-picture and the force-diagram by similar triangles. The force-triangle ABC is *not* similar to the fact-triangle XYZ, but, as in most of these problems, we can find similar triangles by playing with the diagrams and adding simple construction lines. In this case we can add the broken lines in the diagrams and use the argument shown below.

The triangles WYZ and DBC are similar.
In triangle DBC, **DB** represents *half* the bird's weight, or ½(4 pounds).
In triangle WYZ, **WY** is the vertical sag of wire, given as 1 foot.

$$\therefore \ \frac{10 \text{ feet}}{1 \text{ foot}} \text{ in triangle WYZ}$$

$$= \frac{\text{T}_1 \text{ pounds}}{2 \text{ pounds}} \text{ in triangle DBC}$$

$\therefore$ Tension, $\text{T}_1 = (2 \text{ pounds})(10/1) = 20$ pounds
Similarly, $\text{T}_2 = 20$ pounds

A 4-pound bird can produce a 20-pound tension. If the wires were less slack, sagging only one inch instead of one foot, what would the tension be?

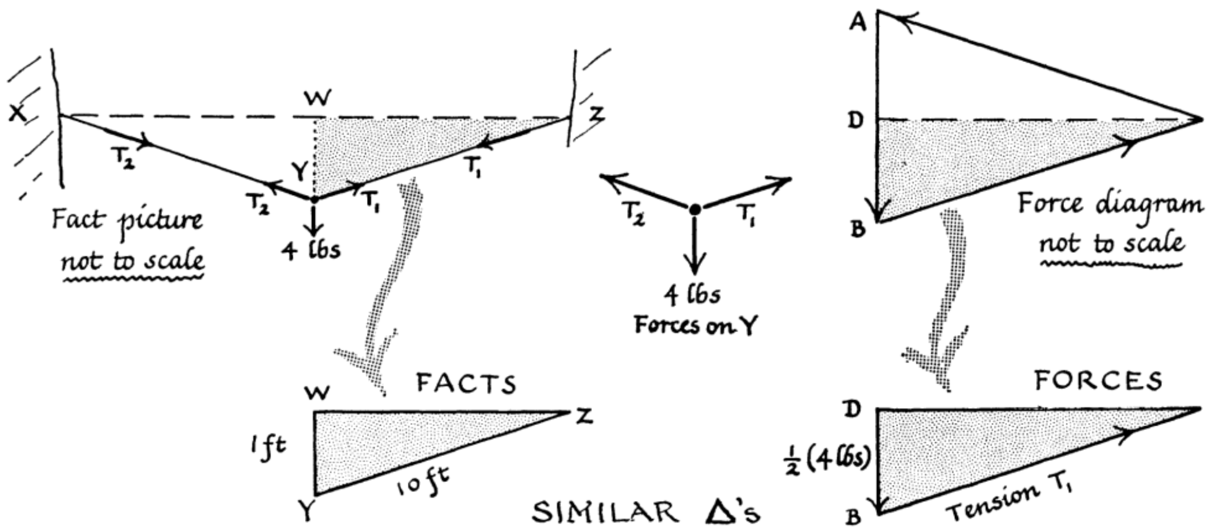



FIG. 3-10. EXAMPLE C. These sketches, not to scale, illustrate the treatment of Example C by geometrical argument with similar triangles.

PROBLEMS FOR CHAPTER 3

1. In text.

2. A pendulum consisting of a 12-pound bob on a 10-foot cord is pulled aside by a horizontal pull on the bob. The tension in the slanting cord is then 20 pounds.
(i) How big is the horizontal pull on the bob?
(ii) Describe the slant of the cord.

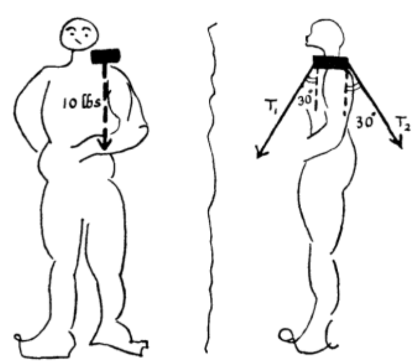



FIG. 3-11. PROBLEM 3

3. A surgeon wishes to apply a vertical force of ten pounds down on a special splint on the shoulder of his patient. He proposes to do this by pulling the splint down with a string. But the patient's shoulders and ribs get in the way, so the surgeon decides to use two strings, one slanting forward and downward, the other backward and downward, from the patient's shoulder, each string making an angle of 30° with the vertical. (See Fig. 3-11.)
(i) Calculate the tension that each string should have. (Give a large clear force diagram.)
(ii) Explain your calculation.

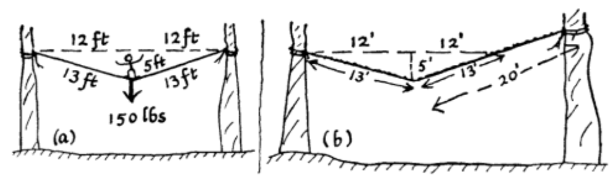



FIG. 3-12. PROBLEM 4.

4. (a) A 150-pound tight-rope walker stands on the midpoint of a 26 ft wire strung between two posts 24 ft apart. Find the tension in the wire, giving diagrams and clear explanation. (*Note*: With these dimensions, sag at middle is 5 ft. See sketch, Fig. 3-12.)

(b) Suppose extra wire is added so that the wire at one end extends farther and rises to a higher support as in sketch (b), but the angles made by the two sections of wire remain unaltered. How will the tension(s) be affected?

5. (a) Which of the following do you consider must be vectors? (A vector is a quantity which obeys the geometrical addition rule.) Force, volume, acceleration, velocity, temperature, density, kindness, humility, humidity, electric field.

(b) Write, in two lines at most, a definition of the *resultant* of a set of vectors. (Do not give a rule for finding the resultant. Give a clear *description*, showing what it *is*.)

(c) Show by sketches and a little description how the parallelogram method of adding vectors (i.e., geometrical addition) leads to the polygon method of adding vectors tail-to-head.

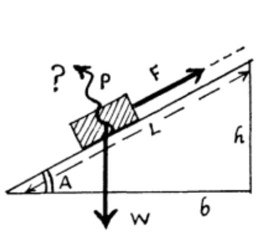



FIG. 3-13. PROBLEM 6

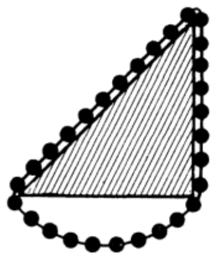


FIG. 3-14. PROBLEM 8.

★ 6. AN IMPORTANT RELATION: LOAD ON A HILL

A sled rests on a frictionless incline which makes an angle $A$ with the horizontal (or rises $h$ feet vertically for a distance $L$ feet up the slope so that $\sin A = h/L$). The sled is prevented from sliding down hill by a rope which pulls uphill with tension $F$. Gravity pulls vertically down on the sled with a force which we call its weight, $W$. (See Fig. 3-13.)

(a) Draw a sketch of the sled on the hill and add arrows to show the directions of $W$ and $F$. Add another arrow to show the direction of $P$, the push of the hill. Assume that, since the hill is frictionless, $P$ must be perpendicular to the slope. Make all these arrows sprout out from the sled.
(b) Now draw another sketch showing the vectors $W$, $P$, and $F$ all adding up to a resultant zero.
(c) If you agree that your two sketches contain triangles which are similar, use this idea to *write down the ratio $F/W$ in terms of $h$, etc.*, or in terms of the angle $A$.
(d) Now suppose the rope is cut so that $F$ disappears and the sled accelerates downhill. Without the rope there is a *resultant force* downhill of the same size as $F$ was uphill. How big is this?

7. Treat the sled problem (6) by a different method. Resolve the weight $W$ into components $F$ downhill and $P$ across the hill (i.e., perpendicular to it). Express $F$ in terms of $W$ and $h$, etc. This gives the tension the rope must have; or, if there is no rope, the downhill resultant accelerating force.

8. Shortly before Galileo's work, Stevin published an ingenious "thought experiment," arguing thus: Imagine a necklace of smooth beads hung on a triangular wedge or prism, as in Fig. 3-14. The necklace must be in equilibrium—we do not expect it to slide round and round, faster and faster, just because there are more beads on the slope. Cut off the loop that hangs freely underneath. Since that loop is symmetrical, its removal cannot spoil the equilibrium. From this, Stevin predicted that $F/W$ must $= h/L$ for a load on an incline. Try to continue and complete his argument and make that prediction. (*Hint*: Condense all the beads on the slope into one lump, all the beads in the vertical portion into another lump. Connect the lumps by a thread over a pulley.)

9. A designer wishes to incorporate a pendulum in his apparatus, with a string to pull it aside *with a pull perpendicular to the pendulum-cord; i.e., along the tangent to the arc along which the bob moves.* (See Fig. 3-15.) The pendulum cord is to be 10 feet long, its bob a 20-pound lump of iron.

(a) What pull, $P$, is needed to pull the pendulum aside by 1 ft *horizontally*? Give careful diagrams and explanations of your calculations.
(b) Repeat the calculation for the bob pulled aside 2 ft horizontally, 3 ft, 4 ft, 5 ft. . . .
(c) What can you say in general about the force $P$ needed for such deflections? (This contains the germ of the theory of swinging pendulums.)

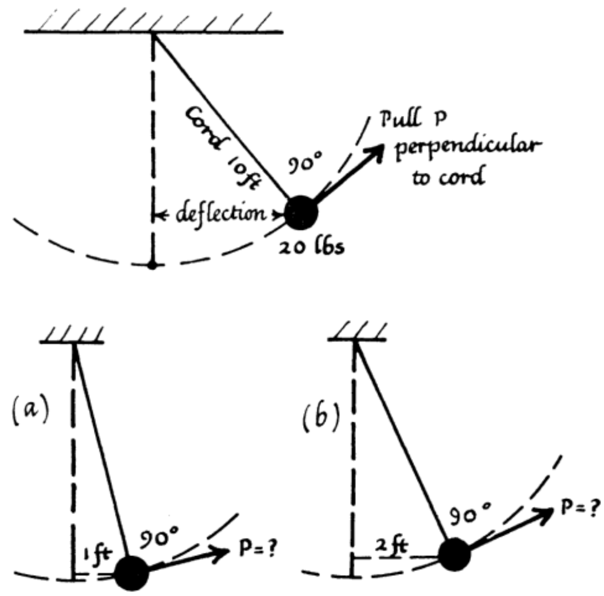



FIG. 3-15. PROBLEM 9.

ment offers a fruitful field, and almost any equipment gives you good opportunities. But you need time to bring each experiment to a close—or sometimes to extend an experiment along a new line. Thus it is good if the laboratory organization lets you extend the time for some experiments instead of requiring each to be run through in a single period. Given time, you can work on your own, needing only occasional advice from instructors—if you ask them for cookbook instructions, they will remind you, *"It's your experiment."*

### *The Objectives of Laboratory Work*

Now, with doubts and conditions of "transfer" in mind, we can review the purposes of laboratory work realistically.

You can learn "facts" of physics, from a small detail to a general law, very comfortably in lab—but very slowly. Tests have shown that information is acquired quicker and just as well in classes or by reading. It seems a waste of time and expensive equipment to use labs to teach you facts. Yet you will find yourself learning what you learn in lab with a fuller sense of understanding. In that way laboratory provides a valuable sense of understanding the information of science.

However, laboratory can provide much more important gains if it can teach you scientific ways and give you a more general understanding of science. For that you must make your experiments your own work, work that you like and regard as part of your present life and future hopes. If you work as a scientist yourself, you are on common ground with scientists, gaining an understanding of science.

So we offer laboratory work with a variety of aims: learning some physics with the thoroughness that comes from doing it yourself; gaining an appreciation of scientific techniques by using them; and, above all, gaining an understanding of science and scientists by experiencing for yourself the delights of scientific work and its sorrows, its honesty and its risks, its successes and failures, its uncertainties as well as sure results. When you come to the lab, make yourself "a scientist for a day," and you will gain understanding that will outlast all information.

restrict the word to *systematic trial and planned investigation*, in contrast with casual playing around with apparatus that gives entertainment with little promise of new knowledge. The distinction cannot be made a hard and fast one without a loss of good science, and yet you will find that the word "experimenting" develops a clear flavor as you proceed. The longer word "experimentation" has a respectable history, but it now seems to many professional scientists a childish use of a longer word to make science sound grander.

### *"Open" Experiments*

On some days in lab you will be free to find out all you can (within a region of physics) by experimenting and making inferences—an "open" lab. Though some apparatus is provided, you may get other equipment by asking for it. (Modern laboratory store-rooms have vast resources, and you should receive extra equipment if it is reasonably possible. You may have to explain what you want it for, but your plans will not be laughed at.) A good lab encourages independent experimenting and should offer good apparatus. In return you will be expected to treat apparatus with care. Scientific instruments are expensive,[6] often made with great care by skilled craftsmen; and, as agents to extend human senses and skills, they deserve your respect.

You will also meet simple harmless instruments like rulers and watches which you take for granted as good. Do not be too sure: there are crooked rulers, and shrunk ones of modern plastic, and there are rough watches in brightly polished cases. Make sure they are as good as you need, or ask for better.

Those who provide for "open" labs foresee many of your investigations and needs; but someone, you or a neighbor, will branch out in unexpected directions, sometimes very fruitfully, and such work will be welcomed and provided for. It is not necessary for you to do everything your neighbors do—methods and results can be pooled in conferences.

### *Discoveries?*

You may think of new experiments or devise new methods, but you are not likely to discover entirely new physics unknown to professional scientists: we shall not deceive you into thinking that, nor should you deceive yourself—yet you can enjoy making what is to you a new discovery, uncovering a delightful simplicity in nature, or exploring some surprising phenomenon.

### *Classical Experiments*

On other days of laboratory, you may find the field narrowed to a definite requirement, perhaps trying for yourself some famous experiment. In

[6] e.g.: A single set of electrical apparatus for one of your later experiments will cost \$50 to \$150. The jewelled pivots of its ammeters are so fine that the small weight they bear exerts a *pressure* of several tons/sq. inch at the points. Putting a meter down on the table abruptly applies many times that pressure; and that may blunt the pivots and reduce the meter to a sticky unreliable one which you do not deserve. The repair is costly, chiefly in skilled instrument-maker's time.

such cases, you will gain more if you do not regard it as routine repetition but take it as an opportunity to share the original scientist's delight.

*Reports*

As a scientist for a day, you are entitled to ask for good equipment, to work hard, extract inferences, argue with all who come, and trust your own results. In all this activity you should keep some record of your work, a diary of *what you did* and *what you observed*, with a discussion of *inferences and conclusions*. Such a record is usually called a Laboratory Report. It can be drawn out by formal requirements into a long work of futile drudgery, and that is not genuine science. Yet without some record you would miss the scientist's systematic care. Every professional scientist treasures the pencil-written notebook that is his companion as he experiments. He copies it and expands it for formal publication, and then he dreads the errors that creep in when data are copied. So you should make simple records. We hope you will value them as a diary of your work, and, like professional scientists, will keep them long after the experiment is over.

A good report need not be long. Write it at the time of the lab, and do not copy it out later in a mistaken worship of neatness. As a written workshop for data and calculations of your experiment, your report is more efficient if fairly tidy, but we should not expect it to be as clean as a workshop that is not in use. What would you think of a carpenter who was afraid to use his shop for fear of making it untidy with sawdust and shavings? Mistakes crossed out, poor measurements put in [ ], circuits redrawn with improvements, calculations tried roughly then repeated in detail, . . . are all welcome as signs of honest work. (Never erase—that looks like suppressing evidence—but cross out when you change your mind.)

As a guide: your report should serve to tell you clearly what you did, if you consult it a year later. Perhaps it should enable another student to carry the experiment through quickly, if he comes to the lab alone with no instructor but with the same apparatus and takes your record as a specimen, with your warnings of useful techniques.

A good report should contain:

(i) a full record: measurements and other observations—the actual pointer reading you saw on each meter or scale, not some number calculated from it.

(ii) analysis: calculations, graphs, etc.

(iii) comments, inferences and conclusions. These are essential, the fruitful outcome of your work.

(iv) a critical survey of the report's accuracy—something a good professional scientist would add. If other scientists want to use his results they would hardly welcome it alone like this:

In fission of Li nucleus,
energy released = 17.4 Mev

They want the scientist's own estimate of his likely error—or they may prefer to study his discussion of errors and make their own assessment too. They would want at least this:

energy released = $17.4 \pm 0.2$ Mev (see note 7)

Estimating errors and discussing them is a fine thing to do, but it takes time and skill; do not spend time on it unless you feel the need and enjoy it.

(i) *Description.* This is a timewaster and should be reduced to notes jotted while you work. Omit the obvious. (e.g., if your record says

Temperature of water . . . . . . 16.2°C

it is quite unnecessary to say in your description, "We took a thermometer and measured the temperature of the water.") It is a waste of time to list the apparatus used (unless some was of special quality or a numbered instrument that you wish to use again). On the other hand, if you took special precautions, say so (e.g., "We stirred the water carefully before taking maximum temperature"). A few lines will usually suffice for (i)—an account of what *you did*, not a copying of some official instructions that you did or did not follow.

(ii) *Record of measurements.* This is the central part of your report. Record everything that might be useful. To avoid re-copying, plan your original record to be easy to read and use. There are two standard forms used by scientists:

(a) sets of measurements in labelled columns and rows;

(b) a collection of separate measurements like this:

```
   MEASUREMENT OF THERMAL CONDUCTIVITY   --   RECORD

Temperature of water . . . . . . . . . . . .  16.2 °C

Diameter of rod   2.48  2.46 }
                  2.46  2.44 } . . . average  2.46 cm

    ∴ radius of rod  =  2.46/2 . . . . .  =  1.23 cm
      (see note 8)

    ∴ area    =  πr²  =  3.14 x 1.23²     =  4.75 cm²

Length of rod ... (and so on)
```

(iii) + (iv) *Analysis and conclusions* make half the main value of your experiment—the other half is in doing it. Be critical and ingenious, bold and imaginative. Squeeze all the inferences you can from your observations, and outline your reasoning. Where you can draw a definite conclusion, state it clearly—but avoid pious statements such as "I verified the law," "The difference is due to experimental error." (This last is a catch-all, an alibi used by amateurs.)

*Partners*

You may be asked to work in pairs, or larger groups. Most laboratories require this as a necessary economy in equipment and staff. A partner is a doubtful advantage. If you rely on him, a partner will take both apparatus and thinking away from you, to your short-term

[7] That means "17.4 is the best guess, from my measurements. I do not think the correct value *is* 17.4 + .2 or 17.4 − .2, but from my work I consider the true value may be anywhere in the region between."

[8] The *diameter* of a rod is the easy thing to measure, not the *radius*. Record the diameter. Then calculate the radius, setting that line in your record a short distance in, to show it is derived.

gain and ultimate loss. But you can gain from your partner if you treat him as a fellow scientist and critic: plan your experiment with him; criticize his techniques; watch his measurements and have him watch yours; and compare your results with his independent ones. (Use his measurements in your report as well as yours, but make their origin clear in case they prove doubtful.)

## SUGGESTED EXPERIMENTS

(This list is tentative and incomplete, to be supplemented by suggestions from other chapters and from your Instructors in laboratory.)

### *EXPERIMENT A. Falling Bodies and Projectiles (An informal, OPEN lab)*

This was mentioned in Ch. 1. If you have an opportunity for laboratory work at the very beginning of the course, try any physical experiments you like relating to falling bodies and projectiles (in air or other media). Start with old, simple, obvious ones such as dropping unequal coins: try them quickly and record, in a few lines, what you did, what you observed, and any inferences. Then proceed quickly to any more complicated investigations you can devise. Try for ingenuity and variety. This is a lighthearted stage, at which careful planning and long systematic investigations are not called for.

### *EXPERIMENT B. Investigation of Springs (OPEN lab)*

Find out anything you can about the physics of springs. (You will be provided with a ready-made spiral spring of steel wire, but you can make other springs for yourself, e.g., by winding copper wire on a rod. Other materials, and other shapes than spirals, are easily available for investigations of springy behavior.) This is a wide field, usually called "Elasticity," that allows you to push your investigations in many directions: stretching, twisting, effects of various treatments, . . . You would be wise to start with the simple "official experiment" below and then branch out on investigations of your own devising—and it is the latter that carry most "credit" in this course, and offer you most lasting benefit.

#### *"Official Experiment"*

Find out how the STRETCH of your spring depends on the LOAD hung on it. STRETCH is defined as INCREASE-OF-LENGTH, from the spring's length when unloaded to its length when carrying a given load. (Thus, STRETCH is reckoned from the original, unloaded position each time.)

Record the position of some pointer on a scale of centimeters or meters (and then compute the stretch in a later column), and the load in grams or kg. Plot a graph to exhibit your measurements. (*After* this experiment you should, at some time, consider the meaning of a Law in Science.)

### *EXPERIMENT C. A Precise Study of Accelerated Motion*

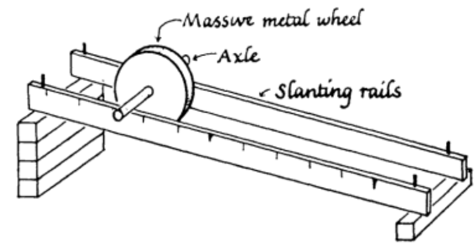



FIG. 4-1. EXPERIMENT C.

This is the "rolling wheel" investigation, mentioned in Ch. 1. Try some form of that experiment (see Fig. 4-1). Make very careful measurements, and exhibit your results by plotting graphs. Use a graph of $s$ against $t^2$ to see how closely your wheel's motion fits the simple motion described by

ACCELERATION = constant.

If possible, make a general review of accuracy, after your experiment, in conference with instructors and other experimenters. If you can decide on likely errors for your average values of times and distances, mark error-boxes around your graph points.

#### *Analysis Questions for Rolling Wheel Experiment*

When you have finished Experiment C, give your results an analytical "work-out" along the lines of the questions below. These questions—which must be modified to fit the arrangements of your apparatus—were framed for a wheel that rolled down slanting rails a meter long, taking about 25 seconds from start. The experimenter is supposed to have measured times for travels of 0.2, . . . , 0.6, 0.8, . . . meter from rest and used them for his graphs. (He also recorded the time for an intermediate distance, 0.7 meter, and he also noted the distance travelled in 15 secs from rest, but he did not plot these data on his graph; he kept them secret for use as checks in this analysis.)

(1) Did your Graph I (of $s$ against $t$) suggest that $s$ varies directly as TIME?

(2) Did your Graph II (of $s$ against $t^2$) suggest that $s$ varies directly as TIME$^2$?

(3) *Interpolation.* Assume that your graph-lines are "true" and interpolate to answer the following questions:

  (a) What is the value of $t$ for $s = 0.7$ meter,

    (i) by reading from Graph I?

    (ii) by reading from Graph II?

(iii) by your direct measurement (stored up for this check)?
(b) What is the value of $s$ for $t = 15$ secs,
(i) by reading from Graph I?
(ii) by reading from Graph II?
(iii) by your direct measurement (stored up for this check)?

(4) Why is it more accurate to use Graph II than Graph I for the interpolation asked for in (3)?

(5) (a) If $s = \frac{1}{2}at^2$ (the acceleration $a$ being constant) how would you expect $t$ for 0.8 meter to compare with $t$ for 0.2 meter?
(b) Write down the ratio of your average measurements for these two times, and calculate its value.

(6) *Calculating Acceleration from Graph-Slope.* Choose two convenient points on your second graph, far apart, read off their values of $s$ and $t^2$, record them. Calculate $a$ from them, assuming $s = \frac{1}{2}at^2$ applies.

(7) Why is the method of (6), using a graph-slope, a more accurate way of estimating $a$ than using measured values of $s$ and $t$ for one distance alone, say 0.8 meter?

(8) *Estimating Velocity by Several Methods.* Choose a suitable place in the motion, say at 0.6 meter, and find the velocity there by three methods:
(a) Use $v = at$, with your measured value of $t$ for that instance and the value of $a$ from (6).
(b) Use $v^2 = 2as$, with your chosen $s$ and the value of $a$ from (6).
(c) Use the slope of Graph I. Draw a tangent at $s = 0.6$ meter. Extend the tangent right across the paper; choose two points on the tangent, far apart; read off their coordinates and record them. Thence calculate the slope. (See the discussion in Ch. 1. Also see notes on graphs in Ch. 11.)
(d) Compare the results of methods (b) and (c). Express the resulting difference between VELOCITY given by (c) and VELOCITY given by (b) as a % of that velocity (see discussion of % differences in Ch. 11).

(9) *If the wheel were to slide without friction instead of rolling, would you expect the same acceleration, or greater or less? Why?* (This is a very important question, which deserves a guess at an answer now. The full answer involves more complicated physics. You would be wise to leave your guess without asking now whether it is wholly "right." A later chapter will supply a clear answer.)

## *EXPERIMENT D. PENDULUMS*

Narrow your field of investigation down to a *simple pendulum*, a small bob swinging to and fro on a long thread; and narrow it still more to the question, "How does the time-of-swing or *period* of a pendulum depend on each of the physical factors that might affect it?" Thus restricted, this is still a complex investigation unless you follow the good scientific practice of holding all the other factors constant while you change one chosen factor at a time.

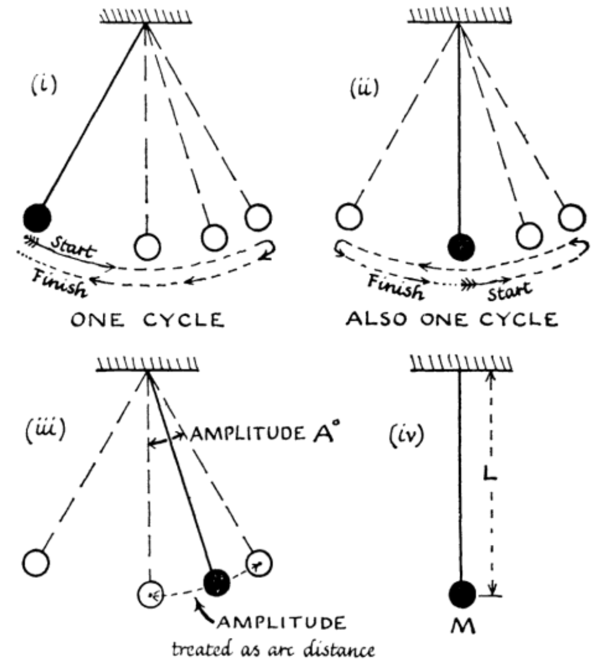



FIG. 4-2. SIMPLE PENDULUM

We define the PERIOD as the time of one "swing-swang," one complete cycle. What factors might affect the period? Obviously the LENGTH of the pendulum—most easily defined as distance from support to center of bob, but not easily, or wisely, measured *directly* in that form. We all know a longer pendulum takes more time to swing to-and-fro. But just how is period $T$ related to length $L$? Is there a simple mathematical rule? (There is. We can deduce it from other experimental knowledge, e.g., from force vectors and Newton's Laws of Motion; but here you should make an "empirical investigation"—ask a straight question of nature in your own experiments.)

What other factors might affect $T$? Mass or weight of bob; *amplitude*, or size of swing; and possibly other factors?

Start by investigating how $T$ depends on
length of pendulum, $L$
amplitude of swing, $A°$ each side of vertical
mass of bob, $M$

To avoid confusing several effects, keep two of the three, $L$, $A$, $M$, constant while you change the third, and measure $T$. Does it matter which of the three you choose to vary first? In this case it does: there is one logically correct choice. While you are considering this, make some preliminary measurements to try out techniques.

*EXPERIMENT D (0). Preliminary Measurements*

A good scientist does not expect a stream of accurate measurements to flow at once from his apparatus. He experiments on his experiment, trying out techniques, gaining skill by practice. Choose a long pendulum, 2 or 3 ft long, and make accurate timings of its period with a good stop-watch (or use a magnifying glass over the seconds hand of an ordinary watch while a partner gives you signals.) Record your measurements. Compare them with your partner's. Look at the methods being used by neighbors, and get ready to criticize.

*Discussion Group*

When you have practical experience of your apparatus, meet with other students and the instructor, as a "research council" to discuss difficulties and techniques. This is the time to suggest good tricks you have discovered, to criticize mistakes you saw neighbors making, to discuss the reliability of the equipment, and to decide on good techniques and plan the order of experiments. There is little use in such a discussion before you have done preliminary experimenting: that would lead to childish guessing or else your instructor would have to step in with cookbook directions. As in a professional research group, an adult discussion needs practical experience of the apparatus.

At this time you will find there is a good reason for investigating the effect of AMPLITUDE first, before MASS or LENGTH.

*EXPERIMENT D (1). Measurements: T vs. A*

Make careful measurements of *T* for various amplitudes such as 80°, 60°, 40°, 30°, 20°, 10°, . . . Plot a *rough* graph of *T* against *A* as you go, to guide your further measurements. If you find the graph-points use only a narrow region of your paper, you should plot another graph with one coordinate expanded, so that the blown-up graph reveals the shape better. (A blown-up graph need not have the origin on the paper—in fact its origin may be many inches off the paper.) If you run into difficulties, discuss them with your instructor—treat him as a source of good advice from another scientist, not as a short cut to "right answers."

When you have enough good measurements, plot a careful graph of *T* against *A*—with a blown-up version, too, if that seems called for.

At this stage, another conference is likely to be useful. Comparing your graph with the graphs of others, you will probably decide there is a definite relationship, but many graphs may show such large accidental errors that their form is obscured. An accurate answer to "How does *T* depend on *A*?" is essential: *the other parts of this investigation will be impossible without it.* You will need very careful measurements of *T* for a certain range of amplitudes. It will be obvious from your present graphs what that range is, and why measurements outside that range need not be very accurate.

*More accurate measurements*

With increased skill from practice and better knowledge of techniques, make the measurements needed to settle the essential question about *T* and *A* and plot them in the graph of *T* against *A*. (This sounds like a long piece of drudgery, fussing over better precision. It will take time and trouble, but the outcome is rewarding.) When you have settled the question write your answer or conclusion clearly, and use it in D(2).

*EXPERIMENT D (2). Measurements: T vs. M*

How much faster would you expect a heavy bob to pull the pendulum to and fro than a light one? Measure *T* for a fairly long pendulum, using the conclusion of your *T* vs. *A* experiments to guide your arrangements. (Of course you should not repeat your *T* vs. *A* investigation all over again for each bob in this investigation. Once that is settled it is settled, and its results can be used without repeating the investigation.)[9] Change to another bob much heavier or much lighter and repeat your measurements. Make sure that the *L*, from support to center of bob, is the same for both. (That is why we offered the cookbook suggestion of a *long* pendulum: the longer the thread the smaller the % change in *L* if you make a small mistake in changing bobs.)

*EXPERIMENT D (3). Measurements of T vs. L*

The period *T* changes greatly as *L* changes, and a good graph (or other investigation) of the relationship between them needs many sets of measurements. Here is where cooperation among the whole laboratory group is welcome. With the answers to questions D(1) and D(2) known, it is easy to arrange for a pooling of comparable measurements from every member of the group.

Each student (or pair) should measure *T* for one length, and then the group should pool their meas-

[9] We assume that the factors *A* and *M*, *L*, etc. affect *T* independently, so that changing *M* here does not affect the *T vs. A* story. That is a safe assumption in most places in physics. In some other fields of study—such as biology, psychology, economics—it would be a very dangerous assumption.

the top of the dam. The water extends for a distance of 2.93 miles behind the dam.

(a) The total weight of water held back by the dam can be found. To calculate it we must use the 2.93 miles. Why does the 2.93 miles not enter into the calculation of water *pressure* on the dam? (In other words, how can the pressure be the same as if the water extended only 1.93 miles?)

(b) The atmosphere presses on the open *outer* face of the dam. It also adds its pressure to the water pressure on the *inner* face. These two contributions subtract out when we are trying to find the forces pushing the dam over. So in the following calculations, *atmospheric pressure may be neglected.* Calculate:
 (i) The pressure at the water's open surface. (*Answer*: zero)
 (ii) The pressure at the bottom of the water.
 (iii) The average pressure over the region from the water-level down to the bottom of the water. (Use common sense.)
 (iv) The total *force* with which the water pushes on the dam. (*Hint*: *Pressure = force/area.* ∴ *force = pressure · area.* Now use the *average pressure* to calculate *force.*)

PROBLEM 5 (HARD)

A dam has been built incompetently low, so that the water-level behind it is higher than the dam, and water gushes over the top of the dam to a height 2 ft above the dam top (Fig. 4-6). The dam is 100 ft wide, 40 ft high, and the water-depth behind it is 42 ft. Repeat the calculations of Problem 4 to find the total force pushing the concrete dam. (Ignore any strange pressure-changes due to rapid motion of water, such as "Bernoulli effects.")

### *Laws of Pressure (Due to Pascal)*

We find that pressure has the following useful characteristics, in fluids at rest:[14]

(I) The PRESSURE is the same all over the bottom of a rectangular tank of liquid. More generally, the pressure is the same at all points which are at the same level in one liquid (or gas).

(II) Fluid PRESSURE on any surface is perpendicular to it. (A diver carrying a coin finds the pressure perpendicular to its surface whatever direction it faces.)

(III) At any place in a fluid, PRESSURE pushes equally in all directions. (A diver carrying a coin finds the same pressure on the coin whatever direction it faces.)

(IV) PRESSURE is transmitted without loss from one place to another throughout a fluid. (Push a piston in at one place in a hydraulic system and the pressure you exert is carried to every wall and any other pistons in the system.)

[14] When there is motion, there are complications, such as fluid friction and "Bernoulli effects." (See Ch. 9.)

(V) The DIFFERENCE IN PRESSURE between any two places in a single fluid is given by $h \cdot d$ where $h$ is the *vertical* difference of level and $d$ is the density of fluid. This leads to an easy way of measuring pressures. It is derived below.

### *Algebra and Pressure Laws I and V*

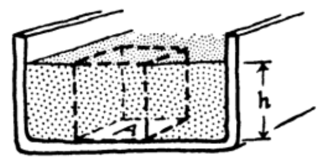



FIG. 4-7. LAW I.

FIG. 4-7. LAW I. The pressure is the same all over the bottom of a rectangular tank of liquid.

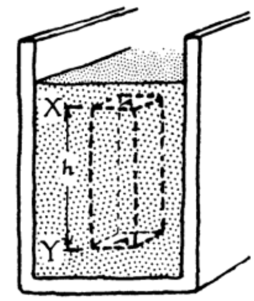



FIG. 4-8. LAW V.

FIG. 4-8. LAW V. PRESSURE DIFFERENCE between two places in a fluid is Δ(HEIGHT) · DENSITY.

(I) *The pressure is the same all over the bottom of a rectangular tank of liquid.* We can calculate the pressure on any area of bottom thus:

Choose an area $A$ sq. inches.

Find the weight of the vertical pillar of liquid which sits on $A$ (= the pull-of-the-Earth on that liquid). Then divide this WEIGHT by the AREA $A$, to find the pressure.

VOLUME of pillar = HEIGHT · AREA = $h \cdot A$

MASS of liquid in this pillar = VOLUME · (MASS/VOLUME) = VOLUME · DENSITY = $hA \cdot d$

In "bad" units (such as kg-wt.) the MASS of the pillar of liquid, in kg, tells us the WEIGHT of liquid, in kg-weight.

∴ PRESSURE, $p$
= FORCE/AREA
= (WEIGHT OF PILLAR)/(AREA OF BASE)
= $hAd/A = hd$

Thus the pressure on *any* base area is

DEPTH OF LIQUID · DENSITY

and is *independent of the area chosen.*

If we want the weight in "good" units, such as newtons, we must multiply MASS by gravitational FIELD-STRENGTH, $g$ (9.8 newtons/kilogram). Then,

PRESSURE = $hd \cdot$ (FIELD-STRENGTH, $g$)

PRESSURE on *any* base area = DEPTH OF LIQUID · DENSITY · $g$

(V) *Pressure difference between two places in a fluid is* Δ (HEIGHT) · DENSITY. To find the difference of pressure, $p_Y - p_X$, between Y and X, we imagine a rectangular box, or vertical pillar, drawn in the liquid, with base of area $A$ and height $h$ from Y to X. The fluid in this block is in equilibrium, so the resultant of all vertical forces on it must be zero. These forces are:

WEIGHT of fluid in block, $h \cdot A \cdot d$
PUSH DOWN of neighboring fluid on top, $p_X \cdot A$
PUSH UP of neighboring fluid on bottom, $p_Y \cdot A$

$$\therefore \quad p_Y \cdot A = p_X \cdot A + h \cdot A \cdot d$$
$$\therefore \quad p_Y - p_X = h \cdot d$$

In "good" (*absolute*) units

$$p_Y - p_X = hdg$$

*U-tubes for Measuring Pressure-Differences*

To measure pressures, we often use liquid in a U-tube, which need not be uniform in bore. We apply the last result, PRESSURE-DIFFERENCE $= hd$.

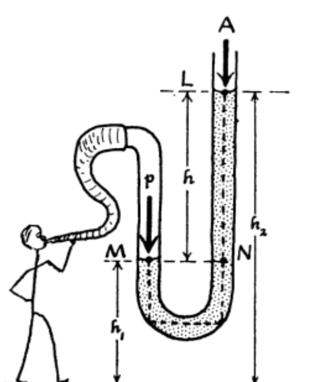



FIG. 4-9. MEASURING PRESSURE

For example, in Fig. 4-9, the man's breath exerts a pressure $p$ which we wish to measure. So the pressure at M is $p$. The pressure at N, opposite M, is also $p$. (We may argue our way down from M to the bend, then across, then up to N, getting back to the same pressure $p$ at the same level.) The pressure at L is the atmospheric pressure $A$.

But (PRESSURE at N) = (PRESSURE at L) + $(hd)$
$\therefore$ pressure, $p = A + hd$

*Units for Pressure*

When we use $hd$, we obtain pressure differences in "engineering" units such as pounds/sq. inch or kilograms/sq. meter. (This is how they are written and spoken. Strictly, their force-unit should be pounds-weight or kg-wt.)

If we then multiply by $g$, the Earth's gravitational field-strength (9.8 newtons/kg), we obtain pressure in "absolute" units such as newtons/sq. meter.

Sometimes pressures are expressed in liquid heights such as "inches-of-water," just as a mountain distance may be expressed in "hours (of climbing)."

Sometimes pressures are expressed in "atmospheres" using a standard average value for atmospheric pressure.

*EXPERIMENT E(1). Simple Pressure Measurements*

Use U-tubes with liquid to make the measurements listed below. It is difficult to measure accurately from one level to the other. It is much wiser to make two measurements, each *from the table* to the liquid level. Surface tension makes the liquid surface in each tube curve into a meniscus. Since you want a level difference, you should measure to the same part of the meniscus on both sides. Professional observers consider the bottom of the meniscus bowl the best for this—*viewed with eyes level with it.* (Do you also need the original levels, before the pressure is applied? Why?)

(i) Measure your lung-pressure in *inches-of-water*, excess over atmospheric pressure. Then calculate your lung pressure in lbs/sq. inch. Call the atmospheric pressure, which you do not yet know, $A$, simply writing $+A$ where necessary.

(ii) If you like, also measure your *minimum* lung pressure, using suction.

(iii) Measure your lung pressure in *meters-of-mercury*, excess over atmospheric pressure. Thence calculate it in (a) kilograms-wt./sq. meter; (b) newtons/sq. meter. Call the atmospheric pressure, $A$, adding it as $+A$.

(iv) Measure the excess pressure of the illuminating gas in *inches-of-water*.

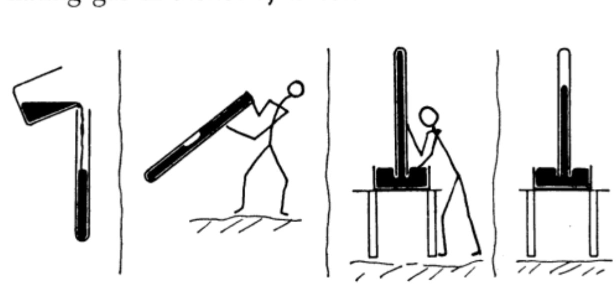


FIG. 4-10. BAROMETER

(v) Demonstration Experiment. A barometer will be set up to measure the pressure of the atmosphere at the time and place of your experiments. Record the "barometer height" in inches-of-mercury and meters-of-mercury. Calculate the atmospheric pressure in (a) pounds/sq. inch; (b) kilogram-wt./sq. meter; (c) newtons/sq. meter. (It is likely to be near an easily remembered round-number value in these units. You will need this in Ch. 25.)

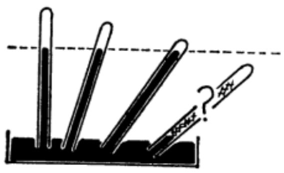


Fig. 4-11. Testing for a Good Vacuum

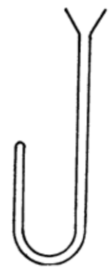


Fig. 4-12. Boyle's Law Apparatus

PROBLEM 6

In calculating air pressure from barometer height, we assume there is a vacuum at the top of the tube.

(a) Why do we expect a vacuum? Give detail of experimental procedure that makes us expect it.

(b) What practical test makes us believe there is a vacuum?

### *EXPERIMENT E(2). Boyle's Law Test (Original Form)*

(This is a simple, single test using Boyle's own arrangement.)

Robert Boyle gave an account of his experiments on the "Spring of the Air" in a paper communicated to the Royal Society of London in 1661. The quotation below is an extract from his account. With a supply of mercury, and a J-shaped glass tube, like the one in Fig. 4-12, carry out the test described by Boyle. (Record the two stages of the experiment by two sketches, with measurements marked on them.)

"We took then a long glass tube, which by a dexterous hand and the help of a lamp was in such a manner crooked at the bottom, that the part turned up was almost parallel to the rest of the tube, and the orifice of this shorter leg . . . being hermetically sealed, the length of it was divided into inches (each of which was subdivided into eight parts) by a straight list of paper, which, containing those divisions, was carefully pasted along it. Then putting in as much quicksilver as served to fill the arch or bended part of the siphon that the mercury standing in a level might reach in the one leg to the bottom of the divided paper and just to the same height or horizontal line in the other, we took care, by frequently inclining the tube, so that the air might freely pass from one leg into the other by the sides of the mercury (we took, I say, care), that the air at last included in the shorter cylinder should be of the same laxity with the rest of the air about it. [The same density and pressure as the atmosphere.] This done, we began to pour quicksilver into the longer leg of the siphon, which by its weight pressing up that in the shorter leg did by degrees strengthen the included air, and continuing this pouring in of quicksilver till the air in the shorter leg was by condensation reduced to take up but half the space it possessed . . . before, we cast our eyes upon the longer leg of the glass, on which was likewise pasted a list of paper carefully divided into inches and parts, and we observed not without delight and satisfaction that the quicksilver in that longer part of the tube was 29 inches higher than the other . . . the same air being brought to a degree of density about twice as great as that it had before, obtains a spring twice as strong as formerly."

Boyle made more extensive measurements and obtained close agreement between the pressure observed and "what that pressure should be according to the hypothesis that supposes the pressures and expansions [= volumes] to be in reciprocal proportions."

### *EXPERIMENT E(3). Boyle's Law Test with Modern Apparatus*

Use some modern form of apparatus for an accurate test of Boyle's Law for a sample of some gas over as wide a range of pressures as possible. (You would be wise to regard this as a test of your skill—you against Nature—rather than a routine verifying of a well-known law.)

The tube containing the sample of dry air (or other gas for the test) must have a uniform bore: otherwise you will be testing the taper of the tube as well as a gas law.

If the pressures are measured by a mercury column open to the atmosphere at the top, there is a useful trick for calculating the pressure of the sample. Since the atmosphere presses on the open mercury, replace it by an extra column of "imaginary mercury," thus: (1) read the open mercury level; (2) add the barometer height, to obtain a new "open level" with atmosphere allowed for; (3) then continue with any subtraction, etc. . . .

Make Boyle's test with your measurements by multiplying PRESSURE of gas, $p$, by its VOLUME, $V$. Also plot two graphs:

Graph I PRESSURE against VOLUME

Graph II (taking a hint from Boyle of what to plot for a straight line)

$$\text{PRESSURE against } \frac{1}{\text{VOLUME}}$$

PROBLEM 7

(a) If the points on Graph II fit a straight line through the origin, show that $pV$ = constant expresses the behavior of the gas.
(b) Since the gas is enclosed by a leak-proof piston, its mass, $m$, is constant. If Graph II is a straight line through the origin, what does that tell you about the density of the gas?

PROBLEM 8

To see the shape of a "Boyle's Law Graph" more clearly sketch an extended $p:V$ graph. *Assume* that Boyle's Law gives the behavior of air quite accurately over a much wider range than that of your laboratory experiment, and obtain more "data" by extrapolation as follows. Suppose your experiment ran from $\frac{1}{2}$ atm. to 2 atm. Calculate the (average) value of $pV$ for your measurements, and then calculate $V$ for, say, $\frac{1}{4}$ atm., $\frac{1}{8}$ atm., and for 4 atm., 8 atm. Plot these "data" and your measurements on a graph with a suitable scale—distinguishing carefully between *true points* from your experiment and *guesses by extrapolation*.

## *EXPERIMENT F. General Investigation of Heat-Transfer*

[These experiments, with closely defined field and some cook-book instructions, are intended to offer an OPEN field for making inferences. They can be done early in the course, as they need only a simple idea of heat (supplemented by the notes given here). They need not wait for the experiments on the measurement of heat suggested in Ch. 27.]

### *Introduction*

Work in a scientist's laboratory ranges from specific measurements or tests through carefully planned study of some new phenomenon to general, freehanded investigation of some field. This last extreme was the way in which much early science developed, and it is still useful today when a new field opens up. Scientists carry out such investigations with flexible hand-to-mouth planning as the work proceeds, with open eyes and ears for unexpected possibilities of future experimenting or hints of new knowledge. In science, "chance favors only the prepared mind,"[15] and science itself favors the alert, flexible mind.

In Experiments *F(1)-(10)* you are asked to find out all you can, from your own observations, concerning the *transfer of heat.* Apparatus is provided, some of it already set up with definite instructions. However, you should apply for any extra apparatus you need, and if time permits you should devise further experiments of your own. First, read the notes below.

[15] From the writings of Louis Pasteur, whose preparation in physics, chemistry, and good scientific thinking served him well in his brilliant researches in biology.

### *Descriptive Notes on Forms of Heat-transfer*

Heat can be transferred from one thing to another; and, besides being a nuisance in experiments, such transfer can be of great importance, e.g. in heating houses and in chemical manufacture. There are three distinct methods of transferring heat, rather like the three ways of transferring a message: by handing a note from person to person in a crowd; by a runner carrying it; by sound waves.

*Conduction.* When heat is handed on from one piece of material to the next without the material moving visibly, we call the process *conduction.* Heat is *conducted* along a poker from red-hot end to colder end; or up a silver spoon dipping in coffee.

In terms of atoms or molecules—discussed fully later—we imagine the hotter particles of material jostling their less agitated neighbors so that the molecular motion which we call heat is handed on. In liquids and gases, the process is just a progressive sharing of energy, in collisions between richer (hotter) molecules and poorer ones. In solids we picture molecular vibrations being handed on by elastic binding forces. (Sometimes modern theory treats this slow diffusion of heat through solids as a case of waves ganging together into a group that travels slowly under some quantum restriction.)

*Convection.* When a piece of hot material moves as a whole, thus carrying heat with it to another region, we call the process *convection.* Chunks of the hot material *convey* heat elsewhere. A red-hot poker carried across the room is a case of convection, if we must give it a name, but the word is usually applied to warm currents carrying heat through a fluid while colder reverse currents complete the flow. In this sense, convection occurs in liquids and gases but not in solids. Winds are convection currents on a vast scale.

When some hot water or air moves upward in such a current, people say, "hot water rises" or "hot air rises." These statements are poor science. They merely repeat the observation, in a dogmatic voice. Taken literally they are obviously untrue, but they can be expanded to become sensible. Hot coffee does not shoot up out of a cup. Hot air does not rise when all alone any more than a cork rises when all alone. On the other hand, corks do rise when released under water . . . , and there is the hint of the proper explanation. A chunk of hot water in cold water is pushed up by the presence of the denser water around it, a case of buoyancy. Hot gases are pushed up the chimney by the denser cold air outside the chimney. One current moves up and another down—often in a pattern of circulation. (Usually the hotter material moves up, but not always. Water expands, growing less dense, when heated from 4° to 10°C and on up to boiling point. But it shows an unusual behavior below 4°. As it warms from 0°C, melted ice, to 4°C it *contracts*—though very little, only 0.013%. How does that peculiarity affect a lake when its surface water is cooled by freezing winds or warmed by bright sunshine?)

*Radiation.* There is another way in which heat travels, or rather disappears in one place and reappears in another. This form of transfer occurs extremely fast, along straight lines. We call this "radiation" after the Latin "radius" for the spoke of a wheel. Though scientists use the word for anything spreading out along straight spokes, we use it here for some process that transfers warming from a glowing fire to us. This includes warming by the Sun—carried through millions of miles of vacuum—and warming by light, visible and invisible, through the vacuum of some electric lamps. So we are dealing with transfer which can take place through a vacuum, and also through glass, ice water, . . .[16] This can hardly be conduction or convection, as we have pictured them. It is not actual heat travelling, because the material it travels through remains unwarmed. As an extreme example, a lens made of ice can be used as a burning glass to focus sunshine without the ice melting. Later experimenting shows all such radiation to be electromagnetic waves, which include light. We then picture the hot source producing waves, at the expense of some of its heat, and the waves travelling till they reach a receiver, where they are stopped and heat is generated again.

### *Record and Inferences*

Write your record as you work, making very short notes of what you did, writing clear statements of what you observed. Then add conclusions, or inferences. These conclusions should be the facts that you extract from your observations, or the guesses you make by inference from them, or even generalizations.

If you use some other knowledge, from books or previous science courses, to *explain* your observations, you are reversing the logic asked for and are missing the whole point of this experimenting.

For example, suppose in some very simple experiment on another aspect of heat, your record ran: "Plunged thermometer in hot water. Saw mercury run up the tube." You might proceed to the *inference*: "*I conclude* that (or *I infer* that . . ." or "*Therefore* . . .) mercury expands when heated."[17] Or, on the other hand, you might try to explain your observation, saying, instead of any inference: "This is *because* mercury expands when heated." The two look almost the same, but the second form spoils the logic of this investigation. Please avoid such "explanations" here, even where you are sure of them, and pretend you are restricted to what you can draw from your experiment.

In general, there are great possibilities of drawing a rich variety of conclusions from some of these experiments.

### *EXPERIMENT F(1). General Experimenting*

You are provided with Bunsen burner, glass beaker, test tubes, samples of metal wire (iron and copper), glass rod, "dye" (crystals of potassium permanganate), and you may ask for other apparatus relevant to your researches. Find out all you can about the travelling or transfer of heat.

*Pre-arranged Experiments.* When you have tried informal experiments with the materials of F(1), try Experiments F(2), F(3), etc. Though these are given with some cookbook instructions you are hardly told what to look for and, still less, what kind of conclusions to hope for. In your record, make notes of what you do and what you observe. Then add clear conclusions, squeezing out all the inferences you can, even at the risk of guessing.

### *EXPERIMENT F(2). Experiments with Water*

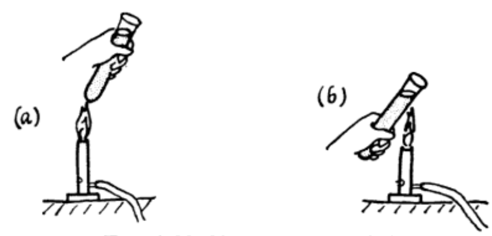



FIG. 4-13. EXPERIMENT F(2)

Heat some cold water in a pyrex test tube. To mark any currents in the water, drop a crystal of "dye" (potassium permanganate) into the water and let it fall to the bottom without stirring. It will leave little color; but if there is any circulation it will color the stream and show it. Perform two experiments, in each case *holding the tube with bare fingers at one end* and heating it with a Bunsen flame at the other end.

(a) Hold the tube near the top of the water, but *not above the water level.* Heat with flame at the bottom of the tube as long as you can hold it. Watch the "dye."

(b) *Cool the tube carefully* and refill with cold water. When the water is still, add a crystal of "dye" without stirring. Hold the tube at the bottom, and heat with flame near the top just below the water surface. Continue as long as you can, and watch the "dye."

Record your observations. Infer.

[16] And some forms of it through other materials; e.g., infra-red radiation through hard-rubber, X-rays through cardboard or flesh, radio waves through brick walls.

[17] In fact this is not the logically safe inference! What *can* you infer? This is a good scientific puzzle. When you have guessed the right answer, you will know you are right; and you can suggest an experiment to settle the matter. And in fact the observation is not quite correct. When the thermometer is first plunged in hot water its mercury dips down momentarily, then rises. With some common-sense knowledge, *this* provides a discriminating inference.

reaching them is absorbed and gives them a small temperature-rise leading to a corresponding voltage. (The absorbing metal quickly warms up until it is losing heat by convection, etc., as fast as it is gaining it from the radiation; and its temperature-rise gives a measure of the rate-of-receiving radiation.)

Ordinary glass happens to be transparent for the visible spectrum but only a little way beyond it at each end. In the far ultra-violet and in most of the infra-red glass is BLACK. Since glass is used in the spectrum apparatus we find a sharp "cut-off" when we reach the limit of glass-transmission in the infra-red. This is a defect due to our choice of apparatus, not a real cut-off in the energy spectrum.

Make a note of the micro-voltmeter readings for various portions of the spectrum. (Remember it may have a "zero reading" due to other radiation reaching it.) Sketch a rough graph.

If time and equipment are available, experiment with various color filters (which *subtract* some colors from the spectrum) and colored spotlights (which *add* colors on a screen).

## *CONNECTIONS OF EXPERIMENTS IN THIS CHAPTER WITH OTHER CHAPTERS*

Ch. 1: Mentions open-lab investigation of falling bodies and projectiles; also precise investigation of motion down a hill. These are *Experiments A* and *C* of this chapter.

Ch. 5: Gives a discussion of Hooke's Law, which should *follow Experiment B* (Springs) of this chapter.

Ch. 10: Gives pendulum analysis and discussion of S.H.M., which should *follow Experiment D* (Pendulums) of this chapter.

Ch. 25: Assumes a knowledge of Boyle's Law from *Experiment E* of this chapter.

## *SUGGESTIONS FOR EXPERIMENTS IN OTHER CHAPTERS*

Ch. 10: Young's fringes, rough estimate of wavelength of light.

Ch. 21: Test of $F = Mv^2/R$ for motion around a circle.

Ch. 27: Calorimetry, *Experiments A-E*: simple measurements of heat.

Ch. 28: Measurements of power.

Ch. 29: "J": measuring the mechanical equivalent of heat.

Ch. 32: Electric circuits, *Experiments* A-W. Intended to give a knowledge of "Current Electricity"—from simple circuits to a diode radio tube—by laboratory work and reading, without other teaching.

Ch. 34: Mapping fields of magnets and currents.

Ch. 39: Measuring radioactive decay (depends on facilities).

Ch. 41: "Laboratory work with electrons," *Experiments A-J*. These are:

*A-G* "Magnets and Coils" (electromagnetic induction, generators, transformers, capacitors).

*H* Triode tube, used for amplifying.

*I* Electrons: measurement of $e/m$ and velocity.

*J* Oscilloscope: working and use.

# CHAPTER 5 · LAW AND ORDER AMONG STRESS AND STRAIN[1]

At this moment the King, who had been for some time busily writing in his note-book, called out "Silence!" and read out from his book, "Rule Forty-two. All persons more than a mile high to leave the court."

Everybody looked at Alice.

"I'm not a mile high," said Alice.

"You are," said the King.

"Nearly two miles high," added the Queen.

"Well, I sha'n't go, at any rate," said Alice: "besides, that's not a regular rule: you invented it just now."

"It's the oldest rule in the book," said the King.

"Then it ought to be Number One," said Alice.

The King turned pale, and shut his note-book hastily. "Consider your verdict," he said to the jury, in a low trembling voice.

—LEWIS CARROLL, *Alice in Wonderland*

What is a scientific Law? Who makes it, who obeys? Who uses it, the great thinker or the practising engineer? In this chapter we select one aspect of your work on springs, their proportional stretching, to discuss it as an example of a scientific law; and to show how engineers put it to use.

*Hooke's Discovery*

In 1676 Robert Hooke announced that he had made a discovery concerning springs. It was a simple law, accurate over a wide range, destined to play an important part in physics and engineering. Hooke was delighted with his discovery but jealous of his colleagues and anxious lest someone should steal the credit for it. The publishing of discoveries in regular scientific journals was only beginning to replace personal books and private letters; and there was still danger that when one revealed a discovery others would jump up and say, "Oh, we found that long ago." So Hooke gave his law of springs as an anagram:

c e i i i n o s s s t t u v

This was like patenting his discovery. He gave his rivals two years in which to claim *their* discoveries about springs; then he translated his puzzle: "ut tensio, sic vis," or "as the stretch, so the force."[2] He had discovered that when a spring is stretched by an increasing force the stretch varies directly as the force.

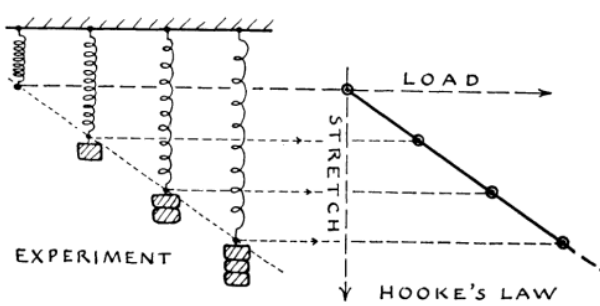



FIG. 5-1. EXPERIMENT AND GRAPH FOR SPRING
(Note: In this case the graph is plotted *downward* to match the experiment.)

As you know from your own work, this simple relation holds for a steel spring with remarkable accuracy over a wide range of stretches. It holds for springs of other materials, perhaps best of all for a spiral of quartz (pure melted sand). It would not be so surprising, or so useful, if it only applied over a narrow range of small stretches—almost any curve can be treated appropriately as a straight line for short distances. But this relation continues until the spring's stretch is several times its original length. It gives many of us, as it did Hooke, a delightful feeling of success to discover something so clear and simple about nature.

[1] You are advised to postpone the reading of this chapter until you have finished your laboratory investigation of springs.

[2] The Latin word "tensio" means stretch (extension) not tension.

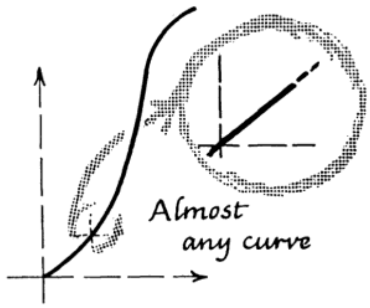



Fig. 5-2.

We meet similar Hooke's-Law behavior in many cases of stretching, compression, twisting, bending, all varieties of elastic deformation. Here are some examples:

(i) for a wire being pulled: STRETCH ∝ TENSION.

(ii) for a rod being stretched or compressed:
Δ LENGTH ∝ FORCE.

(iii) for a rod being twisted:
ANGLE OF TWIST ∝ TWISTING FORCE.

(iv) for a beam being bent: SAG OF BEAM ∝ LOAD.

(v) for a sample of solid or liquid being compressed:
CHANGE OF VOLUME ∝ APPLIED PRESSURE.

and, in general,
DEFORMATION ∝ DEFORMING FORCE.

This general rule is called "Hooke's Law" in honor of Hooke's discovery. The sketches in Fig. 5-3 show devices for investigating some examples of Hooke's Law. See them demonstrated.

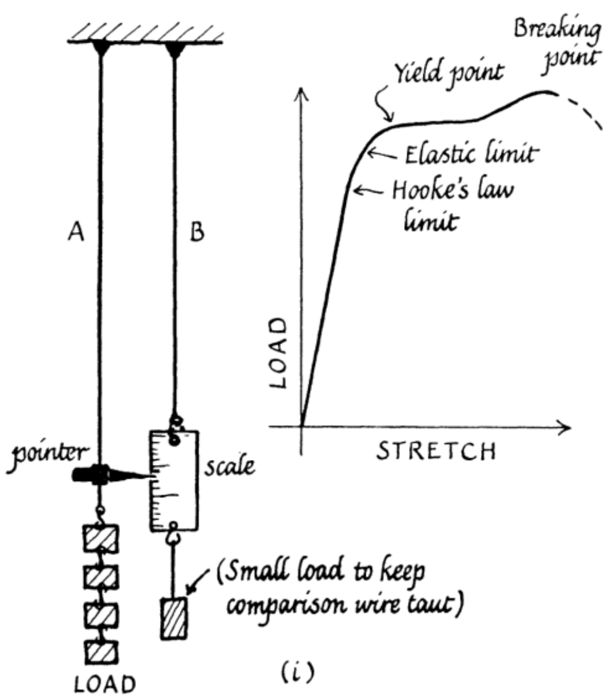



Fig. 5-3. Demonstrating Elastic Changes
Fig. 5-3 (i) Stretching a wire.

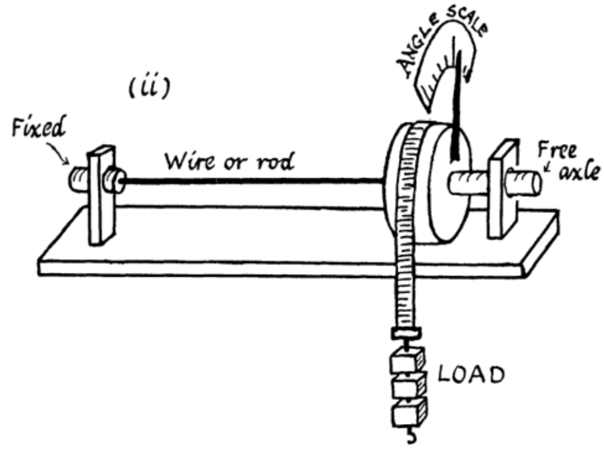



Fig. 5-3 (ii) Twisting a metal rod or wire.
The left-hand end of the specimen is clamped and cannot turn. The right-hand end is attached to the large wheel which is free to turn. Loads are hung on a tape which is wrapped around the wheel's circumference. A pointer on the wheel shows the angle of twist.

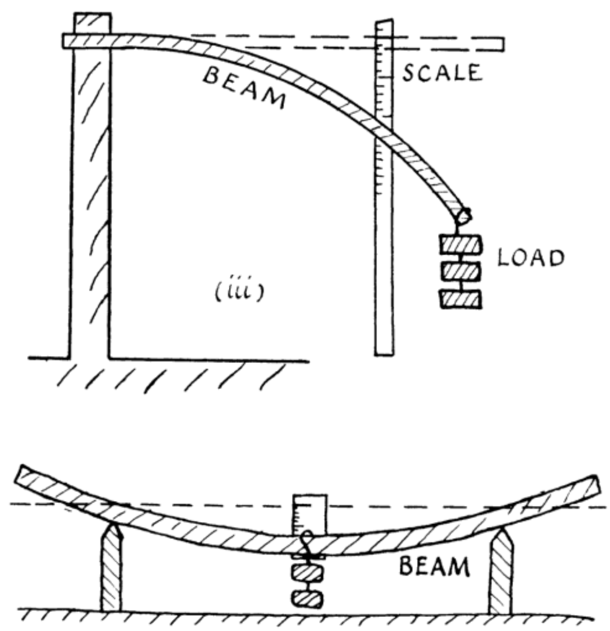



Fig. 5-3 (iii) Bending.
A wooden beam, anchored at one end, is loaded at the other end. The vertical deflection is measured near the loaded end. Or the beam may be supported near its ends and loaded in the middle. This avoids doubts about the choice of deflection for measurement.

*Scientific Laws*

When we say that wires "obey" Hooke's Law, for small stretching loads, we do not mean that either Hooke or his Law compels them to behave in this simple way. We mean they just do behave thus—as shown by experiment—and that this is an example of the general behavior which is summed up by Hooke's Law. The word "Law" is misleading. It is used in science for a relationship, or description of behavior which has been discovered, and which seems very general and appeals to us as simple and important.

Most scientific laws are first derived inductively from experiment, as in the case of Hooke's Law. Others are first deduced from some theoretical scheme: chemistry's Law of Multiple Proportions developed from atomic theory; the Law of Equipartition of energy among molecules was deduced from mathematical mechanics (and turned out to be partly inapplicable). Sometimes a different title is awarded: "principle" or "rule" or even the honest word "relation"; for example: the Principle of Conservation of Energy, the Quantum Rules, the Mass-Energy Relation $E = mc^2$. Law, principle, rule[3]—at present you may regard these as all much the same; all are summaries of what we find, or think, does happen in nature. So it is unfortunate that scientists say, ". . . obey . . . Law." Scientific laws do not command nature like policemen. Nor should we use them to "explain" the observations that suggested them—though they can throw light on *other* experiments; they come *from* experiments themselves, and can hardly be taken *to* experiments as heaven-sent causes. Laws are, rather, simple guiding threads which we have drawn from the tangled web we study, the main threads of experimental knowledge which we weave into the fabric of science. Science gets nowhere if knowledge is just a vast tangle of facts or random observations.

We take it for granted that there *are* simple laws to be found, and that they are true descriptions of nature when we find them. But modern philosophy of science warns us that we are being over-confident. It reminds us that our whole behavior in seeking law is artificial. The nature we codify is just our idea of nature. Our laws are man-made because we make assumptions to suit our hopes. Even in deriving Hooke's Law we assumed we could simply add up the weights we put on the spring to find the load. We have no possible way of proving that load 200 + load 300 makes load 500; we simply assume that as a definition of "total load." So some of the simplicity is of our own making; we do not force nature into a simple mould, but we do force our description of nature to be simple. If this cynicism irritates you, you are in good company with many physicists.

*Another View of Laws*

Once extracted, must scientific laws live a precarious existence, ready to be discredited by the discovery of exceptions or limitations? Some modern philosophers quarrel with such poor courtesy to laws and award them much more permanent privilege. They take the view that the law is there, a clear statement of possible simple behavior, with no question of its being wrong or untrue. It just states what it states, a peg on which to hang our information. The important science, they say, is the knowledge that goes with the law concerning its limitations.

In discussing Hooke's Law, "stretch varies as load," we should not ask, "Is that statement true?" but rather, "How closely do the facts fit the statement? Do many substances in many shapes 'obey' it? Does it apply over a small range of stretch or a large one?" When we find that most springs and wires obey it over a large range of stretching, we consider it a useful law, worth naming. We may picture the law itself as going on for ever, right out towards infinite stretches and back into compressions, but we have no illusion that real materials obey it over such a range. Instead we pride ourselves on a cunning knowledge (drawn of course from experience) of its limitations. We consider we know within what range of stretches it is likely to apply to, say, a steel wire, and in that range how closely experimental measurements are likely to fit it. And we keep track of special substances, such as glass and clay, that we suspect of serious deviations.

On this view, a law is rather like the permanent timetable of a railroad. A timetable just says what it says; there is no question of its being wrong (apart from foolish misprints). But how accurately real trains follow it each day is quite another question, the important one for travellers and one that may take an experienced railroad scientist to answer. Notice that this view of scientific law is not as different as it looks from our first view, that a law sums up experimental behavior. We have merely pushed the knowledge of experimental tests and limitations of the law itself into the guide-book knowledge that now goes with it. Then we must think of every good scientist as carrying an invisible "little black pocket-book" of detailed knowledge. That is what makes him an expert.

[3] There is a tendency to use "law" for great simple outcomes of experiment, "principle" for general beliefs which are built into theory, and "rule" for more earthy working statements. Perhaps there are signs of the times, reflections of changes in scientific philosophy, in the shift in popularity from proud laws in the 17th and 18th centuries to great principles in the 19th, and now hard-working unassuming rules.

The body of knowledge that we call Science will stay much the same however you regard laws, but thinking about these views may help you to see how real nature, which seems complex indeed, can be interpreted with the help of a framework of simple laws.

We take it for granted that there are simple laws to be found—whichever of the two views above we choose to favor. Extracting laws is one of the great activities of physical science, but there is imaginative thinking, too, and above all much scheming to combine laws together, hoping to find a common key or reveal new predictions. We shall return in a later chapter to a discussion of laws and concepts and theories. Meanwhile, in following this course, you should watch for laws, give each a critical welcome, and look forward to seeing for yourself the growth of science when laws are combined.[4]

### *Stretching beyond the Hooke's Law Range*

In the apparatus of Fig. 5-3(i), the stretch of a copper wire, A, several meters long is shown by a pointer which slides on a scale held by another wire B suspended from the same ceiling. Up to a certain load (many kilograms for a copper wire a millimeter in diameter), the wire "obeys" Hooke's Law, stretching a few millimeters. As the load is increased still more, the stretches are slightly greater than Hooke's Law would predict, then suddenly far greater, the wire running visibly. Finally after stretching hundreds of millimeters, perhaps 40% of its length, the wire breaks. Try this for yourself with a short copper wire, well anchored. The broken end is worth examining.

Physicists are interested in these changes and try to interpret them in terms of metal crystals distorting or slipping. There are surprises; the irregularity introduced by jamming many small crystals together makes a much stronger wire than a single crystal whose atom-layers slide easily on each other. The forces between atoms which bind crystals together are still being explored. We know that these forces change rapidly with distance, so it is surprising to find that they can produce anything as simple as Hooke's Law, even for the smallest stretches.

[4] For example: combining Hooke's Law with Newton's Second Law of Motion can produce surprising and useful predictions about the bouncing of a load hung on a spring, the vibrations of a tuning-fork and the motion of the balance-wheel in a watch, and even certain vibrations of atoms in molecules. These diverse motions and many others are shown to be linked together by a common characteristic, which you will meet later. Without the help of combined laws, the common behavior might have remained unknown, and some of the motions never put to use or fully understood.

### *Engineers and Elasticity*

To the engineer, Hooke's Law offers an easy way of allowing for elastic changes in his structures when loaded. He can calculate the sag of a bridge before he builds it, or estimate the twisting force in a ship's propeller shaft by measuring its slight twisting-distortion. For such uses, he needs to know the exact behavior of a measured sample of the material; he can argue from that to his large structure. He is also interested in properties beyond the Hooke's Law range, such as the breaking load; and these too he calculates from measurements on a sample. How do experimenters who compile engineers' reference tables remove the unwanted details of the sample? How do they reduce their measurements to a number that belongs to the material itself, not just the sample?

The descriptions and questions in the special Problems on Elasticity will show you some of the engineer's treatment, and answering the questions will enable you to think out the "rules of the game." Some of the questions are "thought experiments" drawing on common sense. Others are merely illustrations of the useful terms introduced by engineers.

"The problems on pages 82, 83, and 84 are intended to be answered on typewritten copies of these pages. Work through the problems on the enlarged copies, filling in the blanks, (———), that are left for answers."

STRAIN.

10. In dealing with wires of different lengths, we see that the ratio STRETCH/LENGTH should be the same for all wires of the same material with the same stress, though they have different lengths. Do you consider this statement risky, reasonable, probably right, right? ____________________

This useful ratio is called STRAIN. By using it we get rid of the length of the sample, and specify the state of the material itself. If we measure STRETCH in inches and LENGTH in inches, STRAIN must be measured in $\underset{\text{units}}{\underline{\hspace{3em}}}$

MODULI.

11. Engineers and physicists often want to record the elastic properties of a material in some form which will hold for a variety of shapes and sizes of sample, and for a variety of applied forces. To do this, we use:

STRESS, which is $\dfrac{\text{FORCE}}{\text{AREA to which it is applied}}$ instead of just FORCE (or load);

STRAIN, which is $\dfrac{\text{CHANGE OF LENGTH (or size)}}{\text{ORIGINAL LENGTH (or size)}}$ instead of just CHANGE OF LENGTH.

Then in the Hooke's Law region, where the simplest statement says STRETCH $\propto$ LOAD, (or LOAD/STRETCH = constant), we manufacture a much grander fraction, which, like LOAD/STRETCH, is constant. But this grander fraction does not depend on the shape or size of sample used. It is the same for all samples of a given material. To make this grander fraction we use stress and strain instead of load and stretch. And we now write Hooke's Law in the general, grand form, $\dfrac{?}{?}$ is constant.

We call such a constant a "modulus." The easier a substance is to stretch (or compress), the $\underset{\text{larger? / smaller?}}{\underline{\hspace{6em}}}$ its modulus must be.

*Elastic Moduli*

Using stress and strain, we can state Hooke's Law in general form, $\frac{\text{stress}}{\text{strain}}$ is constant; that is, the fraction

$$\frac{\text{force/area}}{\Delta\ \text{length (or size)/(original length or size)}}$$

is constant.

We call such a $\frac{\text{stress}}{\text{strain}}$ fraction a "modulus." Within the Hooke's Law range, moduli are constants characteristic of the material, different for different types of distortion but independent of the shape and size of the sample and the force applied. The bigger the force a material needs for a given distortion the bigger the modulus. So the size of the modulus indicates the *stiffness* of the material, not its ease of stretching, etc.

For *simple stretching* of a rod or wire by tension—which is what we have been discussing—the modulus given by *stress/strain* is called *Young's modulus.* It applies to compression too. Engineers use it to predict changes in bridge girders when pulled or compressed.

Bending an elastic beam stretches some fibers and compresses others, therefore Young's modulus is involved in bending. Mark a rubber tube or block of eraser with ink and try stretching it and bending it.

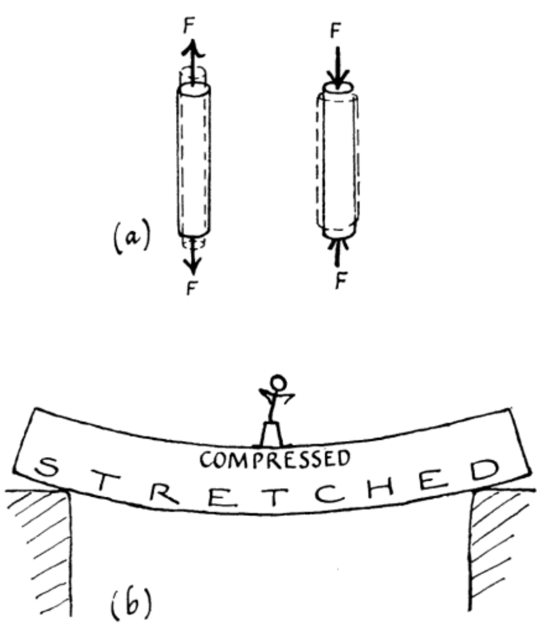



Fig. 5-4
(a) Stretching or compressing a rod or a wire
(b) Bending a beam

It is the outermost fibers of a bent beam that are greatly compressed and stretched and therefore exert great pressures and tensions to oppose the

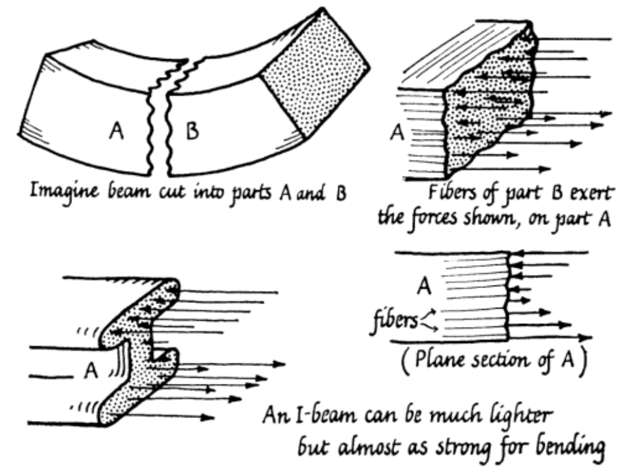



Fig. 5-5. Bent Beam.
Stretching and compressing oppose bending.

bending. The inner fibers suffer little strain, so they exert only small forces; they can be removed with little loss of strength but valuable saving of weight. That is why solid beams are scooped out into I-girders and solid rods hollowed into tubes in bicycle frames.

Other types of distortion lead to other moduli. For *pure change of size,* without change of shape (i.e., pure compression) we have the "bulk modu-

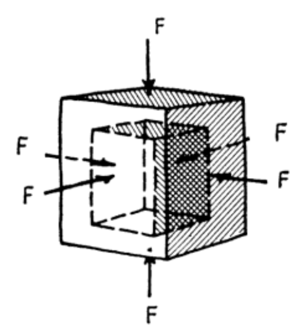



Fig. 5-6. Pure Change of Size

lus." The compression stress is easily applied by fluid pressure.

For *pure change of shape,* without change of size (shearing) we have the "rigidity modulus." Torsion or twisting of a rod involves shearing, and therefore involves the rigidity modulus. Try twisting rubber blocks or tubes marked with ink. Place a fat book on the table and push the cover, so that the pages slide. A square, pencilled on the end edges of the pages, distorts into a diamond. The book is being sheared; its shape changes while its volume stays the same. You might imagine each layer of atoms or molecules—represented by a page of the book—being urged to slide over the next layer and experiencing increasing restraining force. When a rod is twisted, fibers parallel to its axis are made to lean over: they are sheared. (See Fig. 5-7c.)

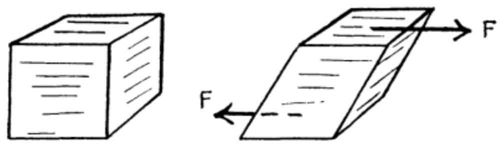



Fig. 5-7a. Shearing. As a cubical block is sheared, its square sides become diamonds.

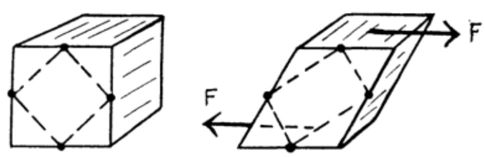



Fig. 5-7b. Shearing.
Alternative view of the same shear-distortion; slanting fibers are stretched and compressed so that a "45° square" becomes a rectangle. Try this on the top edge of a large book.

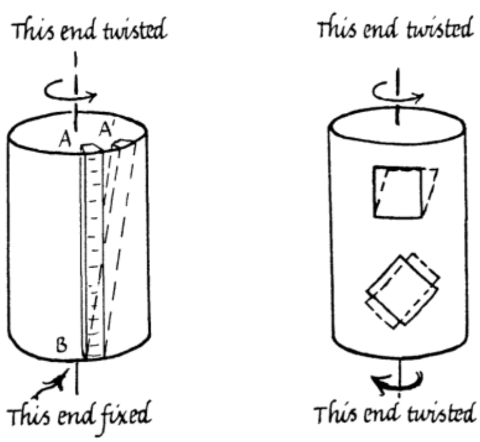



Fig. 5-7c. Torsion.
When a cylinder is twisted, a fiber AB is sheared to a slanting position A′B and squares drawn on its surface show shearing distortions.

Here again the *inner* layers of a twisted rod suffer relatively small strains, produce small opposing stresses, and so contribute little to its strength against twisting. A tube is almost as strong as a solid rod and far lighter.

*Various Strains in Various Materials*

Liquids and gases offer no permanent opposition to change of shape, so they do not have a rigidity modulus for shearing. But they are elastic for changes of volume, and have a bulk modulus for compression. Liquids obey Hooke's Law, with their small changes of volume, over a large range of pressures; but gases soon deviate from it, and another law must be sought for them. For solids, the simple changes of shearing and compression can be combined into more complicated distortions, e.g., in coiled springs or loaded machinery, and in all cases the simple Hooke's behavior holds over a wide range:

$$\frac{\text{stress (due to applied forces)}}{\text{strain (distortion)}}$$

is a constant for the material; or stress $\propto$ strain.

*Hooke's Law*

The general form of Hooke's Law, "STRESS/STRAIN is constant," applies to all materials (within limits) and to many types of distortion. The law is remarkable, and useful, not just for its simplicity but for its wide range. A closely coiled steel spring may stretch to five or ten times its original length before reaching its Hooke's Law limit. A wooden beam may be bent, or a hair-spring coiled up, through a large angle and still fit with Hooke's Law. Even a simple metal wire being stretched fits Hooke's Law over a surprising range of stretches—far beyond the tiny expansion caused by heating. Its atoms, being dragged apart against electrical attractions, seem to experience individual Hooke's Law forces.

If we plot a graph of $y$, to represent STRAIN, against $x$, to represent STRESS, Hooke's Law would be shown by a straight line through the origin. That line would represent a relation $y = kx$. The accurate statement for real materials might be a much more complicated mathematical relation, but in many cases where $y =$ (a complicated function of $x$) we can express it as a series:

$$y = A + Bx + Cx^2 + Dx^3 + \ldots$$

where $A, B, C, \ldots$ are constants. In this case, $y = 0$ when $x = 0$ (no stress applied, no strain). So $A$ must $= 0$. Now the experimental fact that Hooke's Law fits well suggests that $C, D, \ldots$ are very small. Then $y \approx Bx$, for Hooke's Law. However, when $x$ increases $x^2$ and $x^3$, etc. grow even larger in importance (since doubling $x$ makes $x^2$ 4 times as big, $x^3$ 8 times as big, etc.). So unless $C, D, \ldots$ are exactly zero, we should expect their terms to make themselves felt with big stresses. The wide range of Hooke's Law tells us that those constants are remarkably small. Yet they are there, so we must regard our great simple Hooke's Law as only a very close guess at nature. Have we discovered that simplicity, or have we manufactured it?

# CHAPTER 6 · SURFACE TENSION: DROPS AND MOLECULES

"We especially need imagination in science. It is not all mathematics, nor all logic, but it is somewhat beauty and poetry." —Maria Mitchell (*American Astronomer*, ~ 1860)

[This chapter is not an essential part of the course. It is inserted because:

(1) It provides many pretty experiments, some of them simple household ones. See the experiments first and enjoy them; then read the chapter.

(2) It shows, on a local scale, how scientists investigate a piece of nature: observing, interpreting, guessing, testing, thus increasing both useful knowledge and scientific understanding.

(3) In addition to comments on practical matters ranging from shampooing to gold-mining, it provides a measurement of the size of a single molecule, useful in our studies of atoms.]

### *Demonstration Experiments*

Start, like a scientist yourself, collecting information by watching the behavior of liquid surfaces.

I. *General Observation.* Look at the shapes of small drops:

(a) Drops forming on a dripping faucet.

(b) Pools of liquid on a table: (i) water on clean glass, (ii) water on waxed glass, (iii) mercury on glass. The sketches of Fig. 6-1(b) show the shapes roughly, but as a wise scientist you should observe the real shapes and pay little attention to pictures in books except as reminders. Does a real teardrop on a heroine's cheek have the shape shown in the story books?

(c) Raindrops are perfect spheres. Accurate direct observation is difficult, but we get indirect assurance from two sources: first, the shape and position of rainbows. These appear exactly where

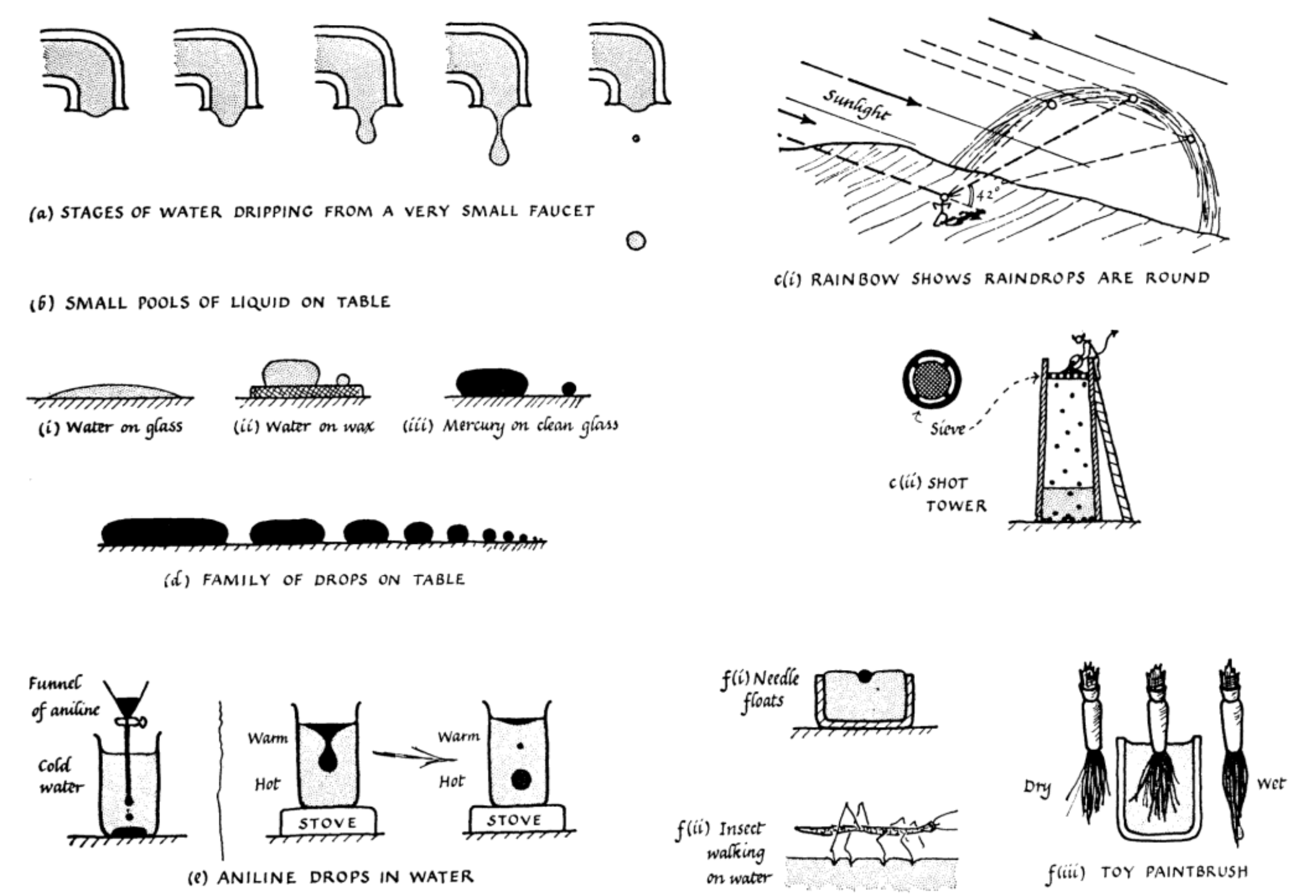



Fig. 6-1. Surface Tension

round raindrops would place them. Distorted drops would shift the bows. Second, the shape of lead shot made in an old fashioned shot tower—molten lead poured through a sieve fell in a rain of drops through air into a deep tank of water and arrived as round balls.

PROBLEM 1

A tiny raindrop resting on a woolly sleeve is spherical, but a large drop of water on a waxed floor takes a flatter shape. Why?

II. *Special Apparatus.* The next scientific move is to use instruments or apparatus to help us. With a projection lantern, observe the drops of (a) and (b) in Fig. 6-1 on an enlarged scale. (If water now seems to move too fast, try dropping viscous oil from a medicine dropper.)

(d) If you observe a whole "family" of pools and drops of different sizes, as in Fig. 6-1(d), you should be able to infer (by induction) several interesting rules. Look for properties common to most of them.

(e) Remove most of the effects of gravity by using a supporting liquid. Crude aniline, a brown poisonous liquid, is slightly denser than water. When it drips from a funnel submerged in water the drops form very slowly; a bag of aniline forms and grows deeper, then a narrow waist develops and the drop quickly breaks off, the waist turning into a smaller drop which follows.[1] If the apparatus is jogged, the hanging drop vibrates.

(f) Sometimes small objects which we expect to sink will float on the surface of water; for example, a needle, or a razor blade if it is a little greasy, and some kinds of water-beetle. Strange supporting forces seem to be available.

III. *Soap Films.* Soap bubbles show liquid surface effects on a large scale. They are "all surface and no bulk," with little weight to compete against surface forces. See the following, sketched in Fig. 6-2.

(g) A soap bubble on a funnel contracts, blowing air out against a candle flame.

(h) The "window shade." A flat soap film is formed on a wire frame whose lower edge is a movable slider. The film can be stretched by pulling the slider down by a thread which is then released.

(i) A flat soap film is formed on a square wire frame. A silk thread knotted into a loose loop is thrown on to the film. Then the film inside the loop is broken.

(j) The window-shade experiment is repeated with a movable bar at the top as well as the bottom. The upper bar is held by a small spring. A soap film is formed between the two bars. The lower bar is then pulled up and down with a thread.

(k) Two soap bubbles, unequal in size, are blown on a T-shaped tube. The blowing inlet is then closed, leaving the bubbles connected.

★ PROBLEM 2. INFERENCES

For each of the experiments (g) to (k) described above, first record your observations and then say what you can infer from them regarding soap films and their "surface tension." (*Note:* The plane figure with *maximum area* for a given perimeter is a *circle.*) Warning: An important inference from (h) will rule out the simplest interpretation of (k).

### *Extracting General Comments*

What do these experiments show about liquid surfaces? The drops forming on a faucet look as if they were supported by a rubber bag. We can make a giant artificial "drop" with a real skin of rubber, which goes through similar shapes as more and more water is poured into it; but the increasing tension of the rubber spoils the strict analogy.

Raindrops and pools of liquid on a table seem to be pulled towards round shapes, again suggesting a skin holding them together against the pull of gravity. Thinking about our observations, we may extract two general comments, vague and risky, but worth further study:

(A) The surfaces of liquids seem to behave as if held together by an elastic skin, pulling them into round shapes.

[1] Another method: Some aniline is poured into a tall glass beaker of hot water which is heated steadily at the bottom. Hot aniline is less dense than hot water, so the aniline starts at the top as a huge drop hanging in the water. When it cools it drips down through the water to the bottom where it is warmed again and rises to repeat the giant dripping motion.

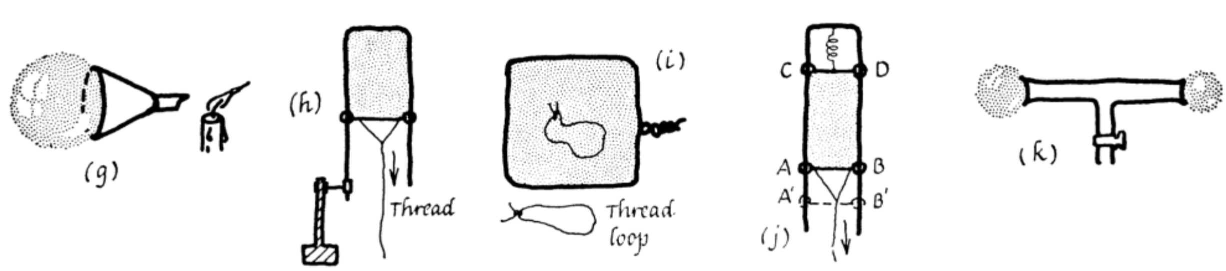



FIG. 6-2. SOAP BUBBLES

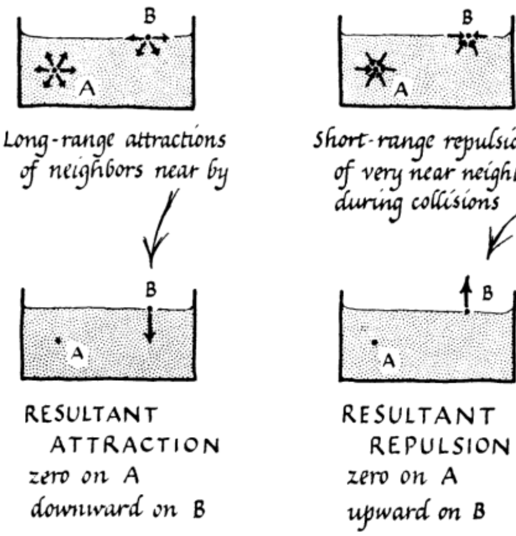



FIG. 6-5. FORCES ON MOLECULES IN A LIQUID

pulled in no particular direction; molecules on the surface will be like B, pulled *inward.* With all such "B" molecules trying to get in towards the middle of the drop, the surface will try to shrink: in fact it will *seem* to have a stretched skin. Obviously, if

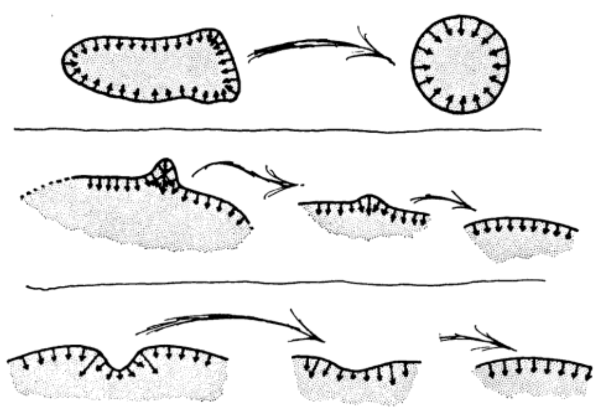


FIG. 6-6. SURFACE FORCES ON SMALL DROP OF LIQUID. The attractions of neighbors on "B" type molecules tend to pull the liquid mass into spherical shape. (Note that a sphere is the shape with minimum surface for a given volume.) If small irregularities develop on surface, surface forces tend to remove them—the full discussion of such effects is very complicated.

a pimple developed on the surface, molecular attractions would flatten it out, against jostling opposition. (A small depression, a dimple, would also be removed, though this is less obvious; molecular attractions would flatten the convex corners of its rim; Fig. 6-6.) To picture the general effect, compare a drop full of molecules with a crowd of people attracted by a street fight. Fig. 6-7b shows a bird's eye view of a crowd collecting. More and

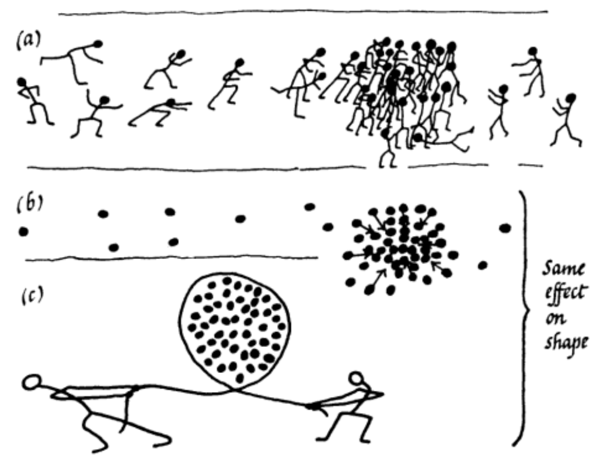



FIG. 6-7. A CROWD COLLECTS

more interested spectators arrive. Later arrivals, finding it hard to see, press in towards the middle—they are attracted by the fight, but they would push just the same if attracted by neighbors farther in. What is the effect of this inward attraction on the crowd as a whole? A fluid crowd is shoved into a round shape with minimum outside perimeter. (A circular patch has less rim than any other shape with the same total area.) A person, A, well inside the crowd gets squeezed; and if he is tall enough he sees that his unpleasant sensations are caused by the outer members, B, pushing inward. But we could make him suffer in the same way by running a great belt of rope around the crowd and pulling it tight. A belt in tension would have much the same effect on the external shape of the crowd, and its internal discomfort, as inward attractions acting on the outermost members. Using this analogy,[5] can you see how molecular attractions might produce the effects of a stretched elastic skin in tension all over a liquid surface? On this view, there is a privileged layer on liquid surfaces, a layer of outer "B" molecules, not a real skin like a rabbit's.

### *Surface-Effects versus Volume-Effects. Tragedy in a Bug's Life*

Why does this "skin" pull tiny drops into a perfect ball, in defiance of gravity, while larger pools compromise? On our molecular view—on our theory, if you like—the skin effect is due to the peculiar experience of surface molecules, B; thus its forces

[5] *Analogy,* often a help in learning, can never prove anything. Some theories are really analogies; for example, the older mechanical models of atom-structure. While we should welcome their help to our thinking and give them credit for fruitful suggestions, we should not make the mistake of thinking they must tell us "the real truth" and we should not cling to them when their usefulness is over.

should be related to the surface, and should not involve the main bulk of liquid inside. Gravity, however, pulls on all the liquid, outer layers and inner ones alike. Surface tension is a "surface-effect," weight is a "volume-effect"; and their *relative* importance will change with actual size of drop or pool. To study this contrast, pretend that surface forces increase in direct proportion to surface area, while weight, of course, increases in direct proportion to volume. Consider the change from a small drop to one ten times as big. For geometrical simplicity, pretend the drops are shaped as *cubes*:[6] a small cube, $C_1$ (Fig. 6-8), with each edge of length $a$, and a big cube, $C_2$ with edge $10a$.[7]

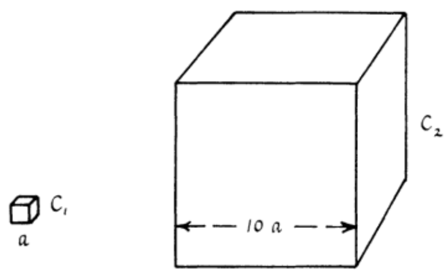



FIG. 6-8. CUBICAL "DROPS."
Illustrating surface and volume comparisons.

How do their surfaces compare? Each cube has six sides. $C_1$ has area $6a^2$; and $C_2$ has area $6(10a)^2$ or $600a^2$. The larger cube, with 10 times the linear size, has $10^2$ or 100 times the area. How do the volumes compare? They are $a^3$ and $(10a)^3$ which is $1000a^3$. The larger cube has $10^3$ or 1000 times the volume and therefore would contain 1000 times the weight of water. When we change from small cube to large, surface-effects increase only one hundredfold; but gravity-effects increase a thousandfold and thus grow tenfold in relative importance.

Actually, surface tension forces appear to tug at any *boundary* or *rim* in the surface. So they increase in direct proportion to *linear* dimensions, edge or radius, and their comparison with volume-forces is even more extreme.

For a very large pool, gravity literally outweighs surface tension effects by a huge factor: ponds are flat and a pailfull of water poured on the floor spreads out under gravity's control. For small drops surface tension has an important effect on shape. For very small drops it becomes paramount. A man diving into a lake contends with gravity-pressures. A tiny bug interviewing a raindrop finds surface-tension-forces insurmountable. Now can you see why *small* water-spiders can run about on the surface of a pond without falling in? They are safe enough: most of them are waterproof and *cannot* fall in. Even if pushed under water they bob out through the surface with skin helping them. Other small creatures with a wettable body find a drop of water a clutching prison. Some partially waterproof ones can keep above water if they are small enough, but once in, once through the terrifyingly tough "skin" which they encounter, they can never get out. To still smaller creatures, bacilli for example, surface forces are everything and weight hardly matters. Their surface is their channel of life, through which all food must come and which they must change if they wish to move. No wonder their life can be ruined by surface poisons, which cover their surface as a dye covers cloth fibers.

Imaginative thinking has carried us far beyond the experimental facts. Some of the ideas set forth are justified by further experimenting; others remain little more than wild picturing, to be suspected of romantic lying and to be used only so far as fruitful suggestions emerge.

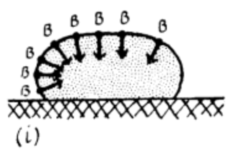
### *Molecular View of Angle of Contact*

Yet we might carry molecular pictures one stage further and discuss the way liquids hang on to solids: questions of wetting and waterproofing. Reverting to small pools on a table, and our classification by angle of contact, we picture the pool being humped together by surface forces on "B" molecules. However, at the edges where the pool meets the table the corner molecules, C, must be attracted by the table as well. How do the combined attractions tilt the surface and determine the angle of contact? Adding the attractions as vectors, we could obtain the resultant attraction, R, due to neighboring molecules of both liquid and table. The liquid surface will treat this resultant as a local "vertical" and will set itself perpendicular to it, just as the surface of a big pool takes a horizontal position perpendicular to gravity's vertical. The direction of the resultant attraction, R, determines angle of contact; but before discussing that we should give a more detailed account of the forces that mold the surface.

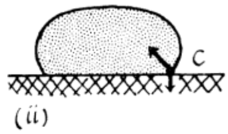
[6] Cubical drops are unreal, but lead to the same result as spheres—or any other pair of similar shapes. If you know the formulas for a sphere, surface $4\pi r^2$ and volume $(4/3)\pi r^3$, argue with them instead. The result must be general, since we have to measure surface in units like $ft^2$ and volume in $ft^3$.

[7] One of the soap film demonstrations supports this view that surface tension is independent of the main bulk of liquid. So does another simple experiment, sketched in one part of Fig. 6-1.

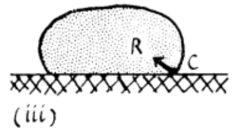
FIG. 6-9. MOLECULAR VIEW OF SURFACE TENSION AND ANGLE OF CONTACT

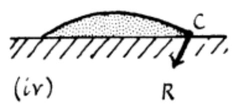



(i) Attractions of neighbors on "B" molecules in the surface make skin effects.




(ii) and (iii) Corner molecule C is attracted by neighbors in liquid, and by molecules of table: resultant attraction R.




(iv) In this case C is attracted more strongly by table than by liquid neighbors.

*Molecular Forces and Liquid Surface*

To see why the liquid surface sets itself perpendicular to the resultant attraction, R, return to the general discussion of forces on a molecule. Molecules are acted on by:

"long-range forces"
- (a) gravity
- (b) attraction of neighbors (range only a few molecular diameters)

"short-range forces"
- (c) violent repulsions during "collisions" with neighbors (range, a small fraction of molecular diameter)

"Equilibrium" is a doubtful term in a molecule's detailed life, but we can say that each molecule, in a liquid at rest, is *on the average* in equilibrium. For any surface molecule, "B", the *short-range* forces come from neighbors at each side and below; and, by symmetry, their average resultant will be perpendicular to the surface. Because it balances that short-range force, the resultant *long-range* force must have the opposite direction: it too must therefore be perpendicular to the surface. Putting the last comment the other way round, the surface must be perpendicular to the resultant long-range attraction—all the forces involved will push the surface about until it is so. (Of course, looked at in molecular detail, the surface itself would vanish into a hubbub of irregular motion, like the edge of any crowd. It is only when we view it from afar with gross human eyes that it seems so smooth; we are then taking a molecular average.) Two of these forces on a surface molecule belong with the surface and change their direction when the surface tilts. These are the short-range repulsion and the long-range attraction of neighbors. The third force, Earth's gravity, always acts vertically down. In a large pond, vertical gravity gives the defining direction, and pulls the whole surface into a horizontal plane, and that makes the other two forces then vertical also. In the case of molecules very near a solid wall or in the surface of a small curved drop, the effect of the neighbors' attractions is much greater than that of gravity. So we neglect gravity in a first attempt to explain a curved meniscus or an angle of contact. We simply say: "the surface will set itself perpendicular to the resultant long-range attraction on a surface molecule."

*Angle of Contact and Molecular Forces*

To interpret angle of contact in terms of molecular forces, consider the attractions acting on a corner molecule, C, where a liquid pool meets a solid table (see Fig. 6-11). A wedge of liquid neighbors pulls with resultant attraction, $F_1$, along the bisector of the wedge-angle—the direction suggested by symmetry. The molecules of the solid table within range of C exert a resultant pull, $F_2$, perpendicular to the table—symmetry again.

Vector addition gives the resultant of these pulls, R, and we expect the liquid surface near C to set itself perpendicular to R. This is sketched in Fig. 6-11 with $F_1$ drawn much smaller than $F_2$, showing

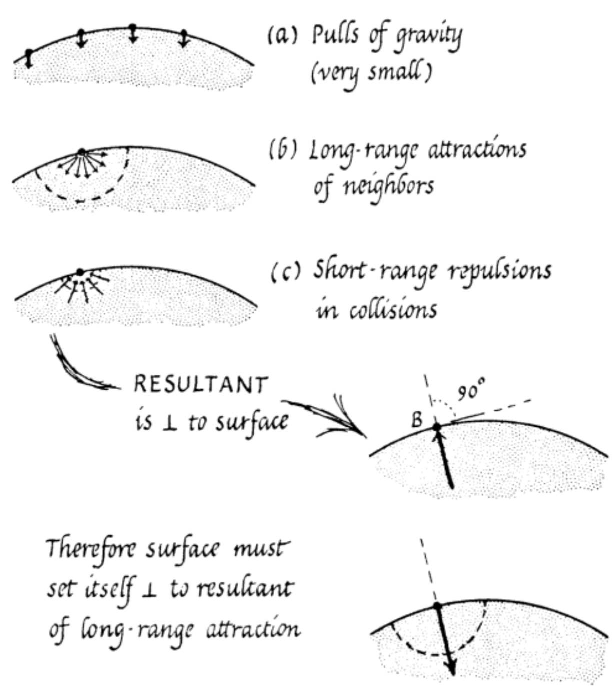



FIG. 6-10. LONG- AND SHORT-RANGE FORCES

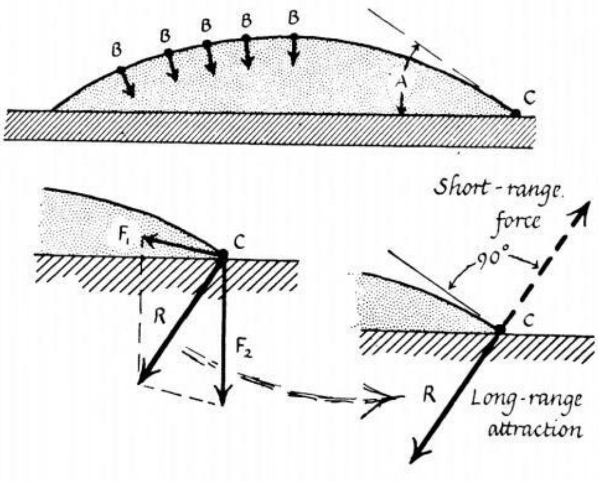



FIG. 6-11. FORCES ON A MOLECULE AT THE EDGE OF A SMALL LIQUID POOL.
The pool is on a table that attracts liquid molecules strongly.

the molecule C attracted less by its brethren than by the table. This leads to a small angle of contact, with the liquid wetting the table. We can picture the highly attractive table encouraging the liquid to spread. Thus wetting appears to be a matter of relative molecular attractions. If liquid molecules are pulled harder by neighboring solid molecules than by neighboring molecules of the liquid itself, the liquid will wet the table and spread.

On the other hand, if the liquid molecule C likes its brethren better than the table molecules, the force $F_1$ must be drawn larger than $F_2$, and the pattern swings over to Fig. 6-12 showing a large angle of contact. "Waterproofing" (non-wetting) seems to require the table molecules to exert relatively small pulls on a liquid molecule nearby compared with the pulls of liquid neighbors.

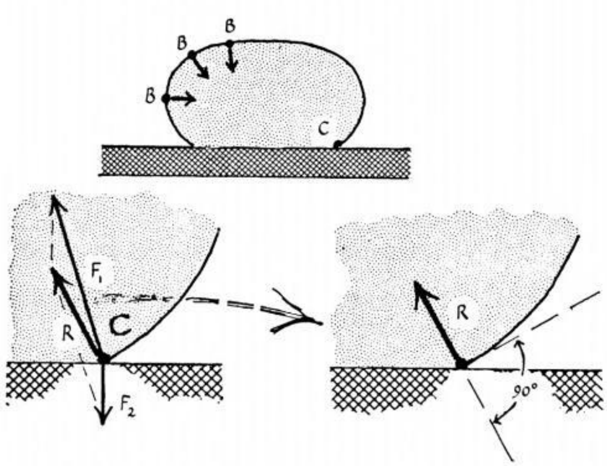



FIG. 6-12. FORCES ON A MOLECULE AT THE EDGE OF A SMALL LIQUID POOL. The pool is on a table that does not attract liquid molecules strongly.

### *Waterproofing and Wetting*

Now we have a molecular explanation of wetting and angles of contact. Explanation? Is it anything more than an interpretation in terms of a fairy story woven to fit the facts? It is not as bad as that because it uses molecular ideas which fit in well elsewhere in physics and chemistry. And it makes useful suggestions such as the following:

(1) To promote wetting (the laundryman's dream), make $F_2$ larger than $F_1$: have the liquid molecules attracted more by the solid than by their own brethren. This can be done by using middleman-molecules, which are in fact molecules of soap. This is the secret of soap and has pointed the way to new synthetic soaps.

(2) To make a large angle of contact (the waterproofer's hope), coat the textile fibres with something that has $F_2$ small compared with $F_1$. In answer to the question, "how thick a coating?" (the waterproofer's worry), footnote 4 says a very thin layer will suffice, just a few molecules thick. (When the waterproofer's purchasing department says, "how thick *is* a molecule?" our own studies later in this chapter will provide an answer.)

(3) Offered a situation where surface forces are important, a liquid with small angle of contact ($F_2$ greater than $F_1$) will try to crawl over the solid surface, even climbing upward. This is specially noticeable when liquids climb up very narrow tubes: "capillarity," a useful behavior, which we shall now discuss.

### *Capillarity*

*Demonstration experiment.* Melt a piece of glass tubing, draw it out into a very thin tube, and dip one end into ink (Fig. 6-13). The dyed water runs up, in defiance of gravity, refuting the adage "water finds its own level." Yet the apparatus used to

FIG. 6-13.

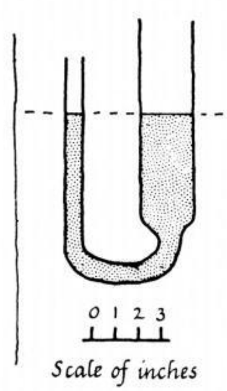



FIG. 6-14.

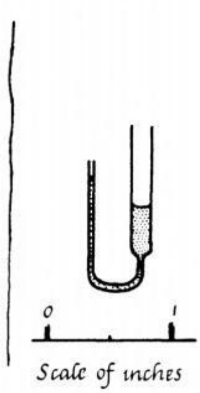



FIG. 6-15.

demonstrate that adage, a U-tube with unequal arms, shows the liquid at the same level on both sides (Fig. 6-14). Remembering the discussion of surface-effects *vs.* volume-effects, we guess that a *small-scale* apparatus would show surface tension effects more clearly. A tiny U-tube shows this (Fig. 6-15). Of course, this was already shown, slightly disguised, by dipping the thin tube in ink. The sketches of Fig. 6-16 show the gradation from

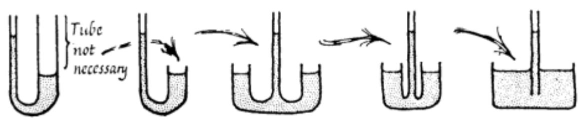



FIG. 6-16.

one experiment to the other. If liquid runs up fine tubes, it should run farther up finer tubes. Test this. (See Fig. 6-17.)

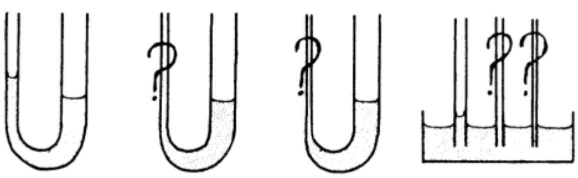


FIG. 6-17.

Since this effect of surface tension shows up in tubes "as fine as a hair" it is named after the Latin word for hair, *capilla.* Capillarity, then, is an old name for surface tension, still used, particularly for the action of liquids climbing up fine tubes. It is a nice name, but it does not *explain* liquid rise. If we say water runs up a small tube "because of *capillarity*" we are saying "because of small-tube behavior." Magnifying the meniscus (liquid surface)

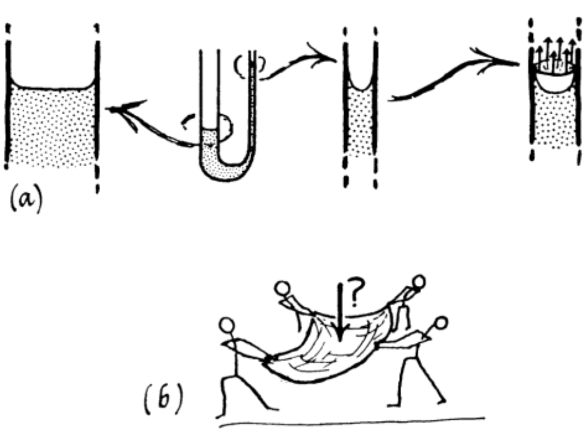



FIG. 6-18.

in each tube we see it hanging like a curved bag hitched up on the glass rather like a fireman's blanket receiving a heavy citizen. We are back at the rubber skin idea. If we measure the hitch-up forces involved, we find they are the same as those holding small drops together. We may even talk of the sagging skin supporting the weight of liquid which rises up the tube,[8] but it is much more realistic to talk of molecules climbing up the tube's inner surface to make the slanting meniscus.

The liquid does not have to have a round glass capillary tube to run up. Any narrow spaces will show capillarity. When water runs among the bristles of a paintbrush or seeps up from a bathtub into your hair, it is not filling hollow hairs but running into narrow spaces between hairs. This behavior has many uses: pulling oil up lamp wicks, water into bath towels, etc.

PROBLEM 3. (Difficult) CAPILLARITY FORMULA

Suppose you accept the view that capillary rise is determined by a pressure-difference across the meniscus. Look back at the demonstration of two soap-bubbles connected together (Fig. 6-2k). What can you infer, *simply from that demonstration,* concerning the relation between capillary rise and diameter of capillary tube?

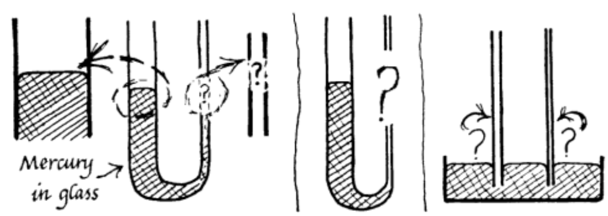



FIG. 6-19. PROBLEM 4

★ PROBLEM 4. CAPILLARITY IN A "WATERPROOF" TUBE

Suppose the liquid makes a *large* angle of contact on the tube. Fig. 6-19 shows mercury, for example, in a glass tube. The mercury meniscus in the large tube is shown but the diagrams are unfinished. Sketch all the diagrams and complete them.

---

[8] We may use this idea to derive a formula, much beloved of the problemsetter in old-fashioned examinations, for measuring the surface-tension, $T$, pulling on each inch or meter of rim: Pull up of skin = weight of liquid supported in tube.

$T$ · [rim, $2\pi r$] = [VOLUME, ($\pi r^2$) · (RISE HEIGHT)] · [DENSITY] · [FIELD STRENGTH, $g$]

Therefore, $T = \frac{1}{2}(g)$ · DENSITY · RISE HEIGHT · TUBE RADIUS. This formula is more or less right and is used in rough measurements of $T$; but this derivation is almost a swindle. There *is* no rubber skin hitched onto the glass; and the $T$ in the real formula relates to the liquid/air surface and is not a hitch-on-to-glass force. There is a curved surface, however (the meniscus); and, as in any balloon, the pressure is greater "inside" (above this meniscus) than outside. Using this pressure-difference we can both explain capillary rise and derive the formula honestly.

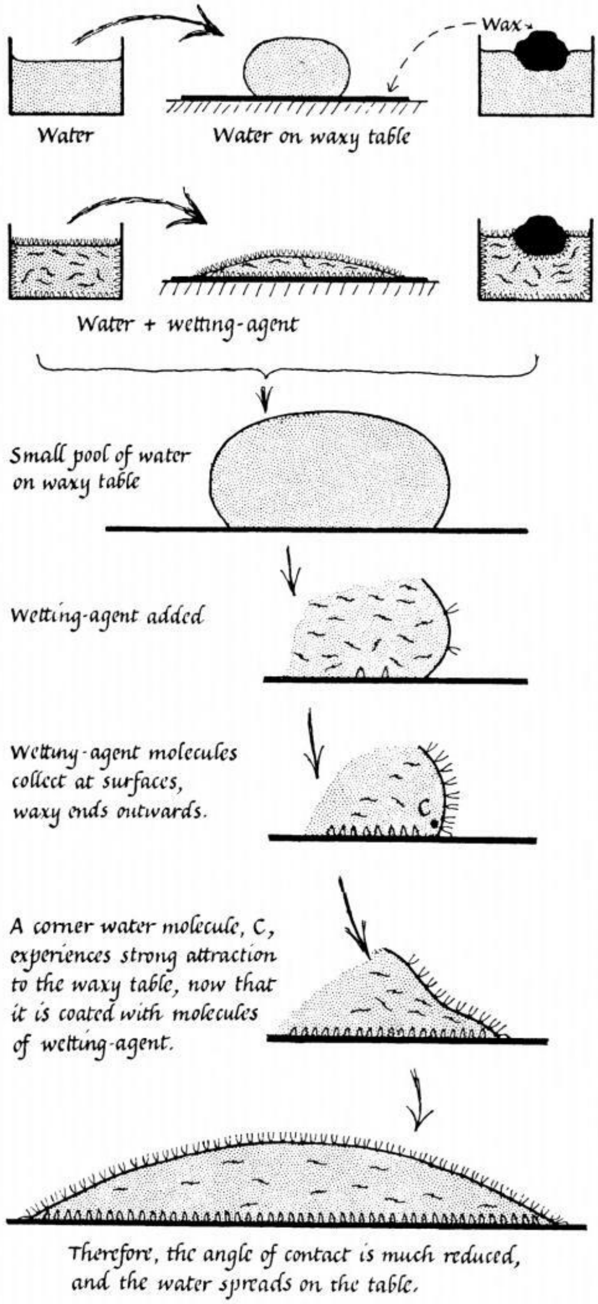



FIG. 6-21. ACTION OF WETTING-AGENT (AEROSOL O. T.)
Note: Molecules of Aerosol O. T. wetting-agent are shown utterly out of scale—far too large.
The long molecule, whose structure is sketched in the text is shown by a line with a dot at the water-liking center. At a waxy surface this molecule hitches both ends to the wax and humps its water-liking middle inward. At a free surface it humps itself with its inert ends outward.

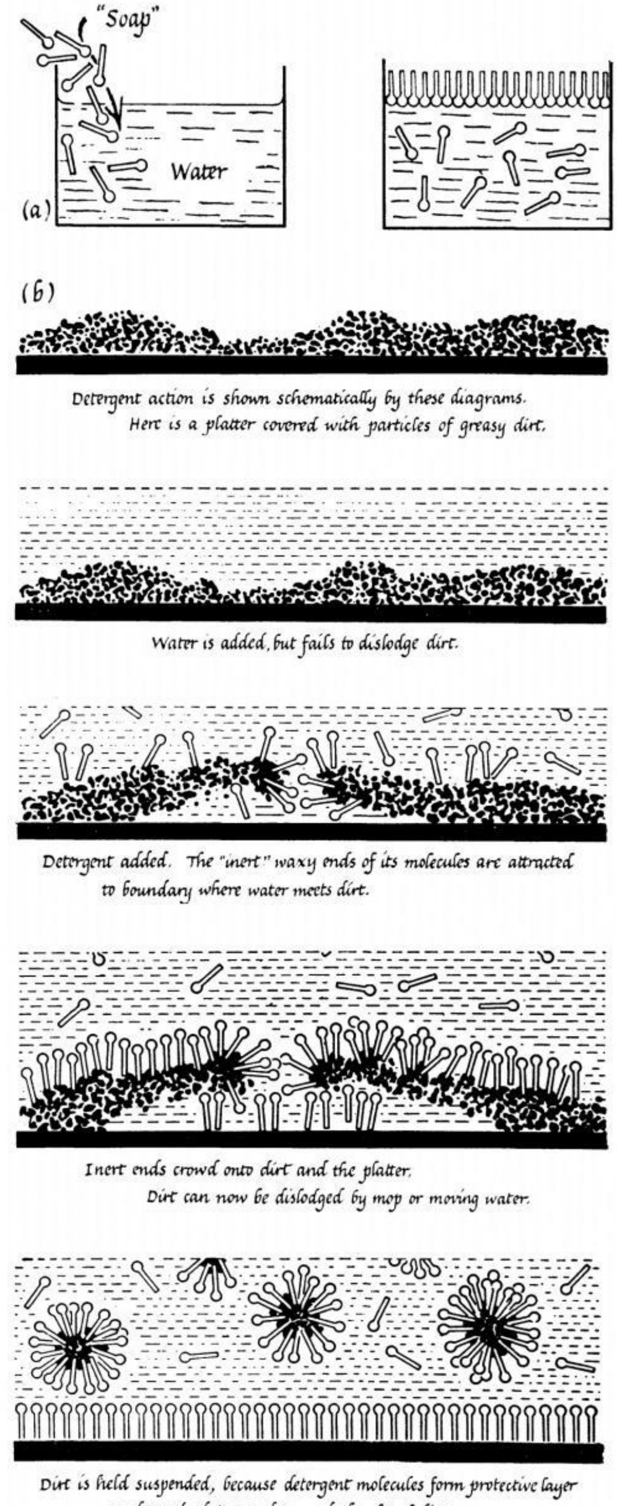



FIG. 6-22.* ACTION OF DETERGENT (SOAP OR SYNTHETIC). (a) Molecules of "soap" being added to water. The molecules are shown out of scale—much too big. These are molecules with one water-liking end and the other end waxy so that it can attach itself to grease or wax. The water-liking end is shown as a round blob. Many of the dissolved detergent molecules crowd onto the surface of the water and line up there with their water-liking ends in water and their inert ends out. They also crowd on the walls of the container and on any wax or grease. (b) The detergent (= cleaning) action of soap or modern synthetic detergent.

* From *Scientific American*, October 1951, pages 26-27.

And (ii) soap solution, an uneven mixture, provides the film with a slightly variable surface-tension which enables it to carry extra weight near the top and to pull any irregularities back to normal. A pure liquid, however, seldom forms stable bubbles or froth—beware of drinking from ponds with froth.

*Waterproofing.* To waterproof a raincoat, we make surface tension discourage water from running through the pores. This is done, without blocking the pores, by coating the fibers with wax, to give a large angle of contact with water. Then, if the pores are small, the water does not run right through but is restrained by bulgy skin surfaces. The sketches of Fig. 6-23 show water being poured on coated fibers—a magnified version of rain meeting umbrella fabric or tent canvas. The stages can be demonstrated by a model in a lantern, or the general effect can be shown by a small sieve of metal screen netting. When its wires have been waterproofed with molten paraffin wax, the sieve will hold water poured in gently. A wet finger touching the sieve underneath will break the bulging water droplets and start a deluge, like the unbelieving Boy Scout's wet head in the tent.

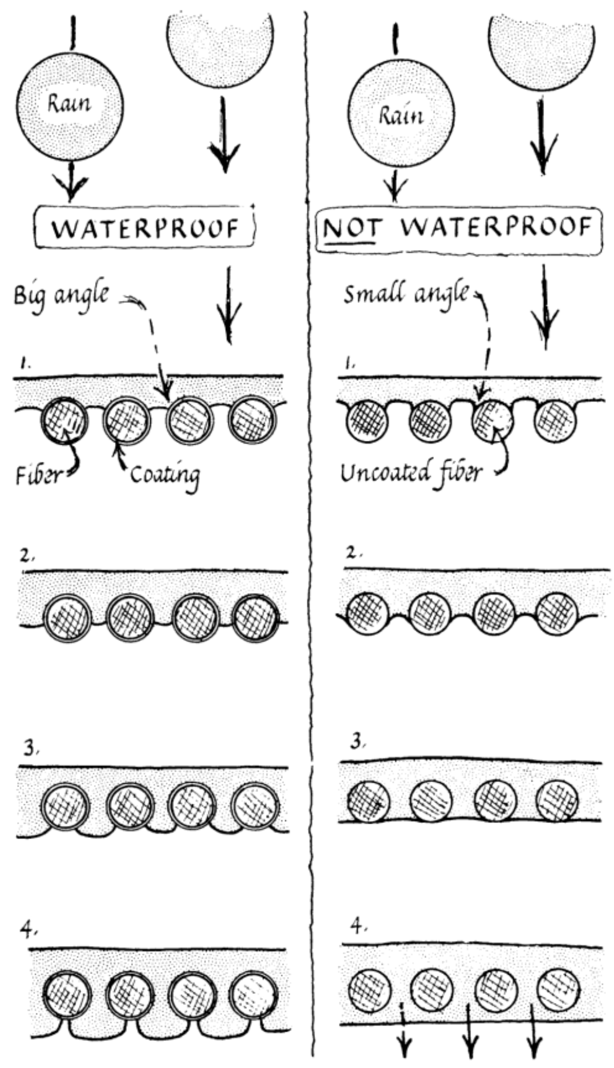



FIG. 6-23. WATERPROOFING AND WETTING.
These diagrams show the fibers of a woven fabric, in section, greatly enlarged, with water descending on them. The fabric might be umbrella fabric or tent canvas. The pores are not blocked up, but where the fibers are coated to make a large angle of contact (between water and coated fiber), the water bulges through with surface tension in opposition.

### *Surface Chemistry and Mining Miracles*

*The chemistry of angle-of-contact-changers* opens fields of technical miracles: wetting-agents to help launderers, sheep-dippers, window-cleaners; trifling additions to nose-drops to help them wriggle past the barrage of hairs in the patient's nose; waterproofing agents for raincoats and industrial filters; and differential waterproofing/wetting agents to separate valuable mineral from useless rock. In this last use, rock containing metal ore is pounded to dust in a mill, then stirred in a vat of water. A differential agent added to the water coats the particles of ore and makes them "float"[13] easily; but it lets the useless sand be wet and sink to the bottom as a sludge which is thrown away. The open air surface of the water is insufficient to collect all the waterproofed ore particles; so a froth of air bubbles is blown in the muddy mixture to carry the ore up to the top, where it is skimmed off. This "froth flotation" scheme is no impractical toy. It is a successful process, used in mining to separate a million tons of rock a day. There is much clever chemistry in finding agents that will grip the ore with a proofing coat and refuse to protect sandy rock. Some agents go further, selecting one metal in mixed ores and refusing another—cleverer chemistry still. Strange new uses of froth-flotation include separating ergot fungus from rye grain, sorting peas for canning, recovering particles of waste rubber; but the main business of separating lead, zinc, silver, etc., has grown to a vast industry in which surface-tension is the essential worker.

### *Amoebas and Surface-Tension*

How do small simple creatures living in water travel and find their food? You can get hints from crude chemical models like the wriggling camphor boat and the following demonstration of a fake mercury "amoeba" (Fig. 6-24). A small pool of

[13] To float a needle or razor blade on water, first waterproof it with a little wax or grease. A large angle of contact enables surface-tension forces to give much greater support.

mercury, resting in a watch-glass in a dish, is covered with dilute nitric acid. A crystal of potas-

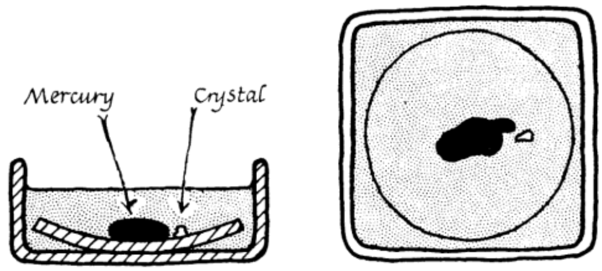



Fig. 6-24. A Mercury "Amoeba"

sium bichromate is dropped in near the mercury. The mercury's amoeba-like movements are due to changes of surface tension caused by chemical or electrical effects. A real amoeba pushes and pulls its irregular shape in a similar way, possibly using changes of surface-tension.

### *More Demonstrations*

*Changing the surface skin of water.* Here are some pretty experiments which show changes in surface-tension.

(i) A sewing needle or a tiny leaf of metal can be made to float in a dish of water. If surface-tension is lessened, the boat sinks. Try adding alcohol or soap to the water.

(ii) Sprinkle the surface of clean water with waterproof dust (soot, talc, or lycopodium seeds). The weakening of the surface skin can be shown by motion of dust. When alcohol is dropped on the surface the dust rushes away. The usual explanation is this: the alcohol provides a weak skin and the dust is pulled away by the strong skin of plain water farther out. But some people prefer to say that the alcohol molecules push the powder as they spread, with a "surface pressure." Though the two views differ, either is useful in interpreting experiments.

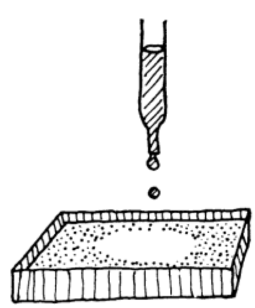
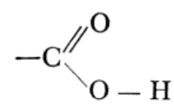


Fig. 6-25. Drops of Alcohol Fall on Water, which has been dusted to mark surface motions

(iiia) Add olive oil to a clean water surface sprinkled with dust. So little is needed that a match stick dipped in oil and then wiped dry will suffice. Even your finger rubbed in your hair will collect enough natural oil. In experiment (ii), the surface recovers from alcohol, but the effect of oil remains; hence the need for very clean grease-free apparatus in these experiments. Soap and saliva have effects like that of alcohol.

Mosquito larvae live in ponds and protrude tail-whisker breathing tubes through the surface. Oil placed on the surface gets into these tubes and kills the larva. The old explanation, that oil so weakened the skin that the larva could not hang on to breathe, is discredited.

(iiib) A tiny drop of oil placed on a huge dish of lightly powdered clean water, spreads very fast to a big round patch, then seems to have no further interest in spreading. This is observed with vegetable oils, which are "fatty acids" with one end of their molecule attracted to water, the acid end



(Mineral-oil molecules, which have both ends inert, seem to lie flat on the water surface and move around like a two-dimensional gas, spreading more casually—waxy unemployables.) In all cases, the oil film seems to exert an outward "pressure" on the surface boundaries—this seems a more realistic explanation than "weakening the skin tension of water." Nowadays we measure this outward push with delicate balances which weigh the sideways push of the oil film on a movable boom.

### *Uses of Long Oil Molecules*

*Lubrication.* In modern lubrication of high-speed bearings molecules of vegetable oil attach themselves to the metal—the metal ousting hydrogen at the acid end of the oil molecule—and the oil forms monomolecular velvet carpets whose inert outer surfaces slide comfortably on each other. (Mineral oils are added to provide inert oily rollers between these velvets.) Under extreme ill-treatment even the velvet monolayers are torn off the metal; the moving metals then grip each other ("seize") with great force at close quarters, and there is serious damage.

In a similar way lanolin grease will grip your skin and soak in, to bring it drugs or general comfort, while inert mineral oil wanders about on the surface in a greasy mess—beware the druggist who prefers mineral oil to lanolin in ointments. Again, molecules of good shoe polish grip the leather; but paraffin wax (a longer version of mineral oil) merely makes messy smears.[14] Shoe-polishing promotes the grip and aligns the chains.

*Calming of storms at sea.* The calming of rough seas by oil is no mere fable. Quite small amounts of suitable oils poured overboard will spread over a big area. As the wind tries to whip up high waves by pushing the shallow humps which start them, the oil is blown into patches whose different surface-tensions hamper the wind's effect, with a sort of surface friction. Then some of the big waves may never be formed. And big ones arriving from far away are at least discouraged from breaking into damaging whitecaps. Surface-tension plays an important part in the breaking of a crest into spray, and oil can weaken the surface and stop the breaking.

### *More Experiments*

(iv) How would you expect surface-tension to be affected by temperature rise? Try warming a dusted water surface by bringing a red hot poker near it.

(v) Sprinkle crumbs of camphor on *clean* water. Each crumb swims about irregularly. Camphor dissolves slowly in water, making a weaker skin. Each crumb is pulled forward by clean water ahead and less strongly backward by weaker camphorated water, so it sails ahead like a little boat, steered by its own irregular shape. Then try adding a little oil. This "kills" the camphor movements permanently. A pretty little experiment, but rather a childish one? By no means a useless toy. It played an important part in one of the great simple experiments of atomic physics: measurement of a molecule's size.

### *The Size of a Molecule*

Sixty years ago, Lord Rayleigh watched oil spread on water. At a time when scientists were speculating about molecular sizes, he made a very clever guess. He guessed that the thinnest sheet of oil that could cover a water surface completely would be just one molecule thick, and he set out to estimate this thickness. He pictured a spreading drop of oil as a huddle of molecules tumbling and crawling over each other till each reached the water surface and could hitch one end to water—for these oils have long chain molecules with a water-liking chemical group at one end. Once all the oil molecules are thus attached they should keep together in a monomolecular carpet, showing little tendency to spread more. With just enough oil for a given

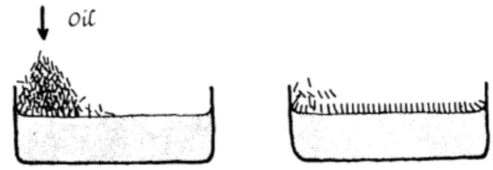



FIG. 6-26. OIL ON WATER.
A drop of oil placed on a clean water surface spreads to cover it with a layer one molecule thick. The oil molecules probably stand on end like a velvet carpet.

water surface, the layer would be one molecule thick, with the molecules packed close and upright, like the pile of velvet. With less oil, patches of open water should be revealed. With more oil, there should be excess puddles on the water[15] (as on greasy soup).

Lord Rayleigh cleaned a large dish and filled it with water—the biggest washtub he could buy, 33 inches across. He placed a weighed drop of oil on the surface and watched it spread to cover the surface completely. Then he started again with clean water and used a smaller drop, then smaller still, until he found a size that failed to cover the water completely. How did he know it failed? Dusting the surface beforehand might spoil it. He used camphor crumbs afterward, the childish toy. As long as the water surface was completely covered with oil the crumbs could find no clean water to spoil and dance around on; but when the oil drop was too small it left patches of clean water. In Problem 5 below, follow Rayleigh's calculation, using data based on his actual measurements, and find out how tall an oil molecule is.

#### PROBLEM 5. MEASURING A MOLECULE

Rayleigh placed a drop of olive oil on clean water in a large tub. For simplicity, pretend the tub was rectangular so that the water surface measured 0.55 meter by 1.00 meter. (This would have the same area as Rayleigh's round tub.)

[14] For a pleasant discussion, on which this section has drawn, see W. H. White, *A Complete Physics: Written for London Medical Students* (London, 1935), Chs. XXII and XXIII (pp. 250-269). Also see *Science News*, No. 20. May 1951.

[15] As a poor analogy, picture a herd of pigs released near a long food-trough. Just as one end of a vegetable oil molecule likes water, one end of a pig likes food. They scramble and push till every pig reaches the trough. If the herd is too big an unsatisfied crowd is left waiting (like thick drops of excess oil on water). If the numbers are just right, they form a mono-porcine line, all crowded perpendicular to the trough. If too few, they are unevenly oriented and there are vacant patches.

Camphor movements showed that the oil just sufficient to cover that water was a drop weighing 8/10 of a milligram, or 0.00000080 kilogram.

The density of the oil was 900 kg per cubic meter (0.90 times the density of water). Assume the density remained the same, even in a very thin film. (Remember that since the oil is *less* dense than water, its volume must be *greater* than the volume of the same mass of water.)

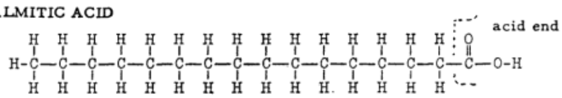
(a) Estimate the thickness of the oil film made by the drop when it spread over the water.

(b) Assume Rayleigh was correct in guessing that the film just sufficient to stop camphor movements is one molecule thick, and use the chemists' suggestion that this oil has "long" molecules one end of which is strongly attracted to water; say what you infer from (a) concerning the molecule's dimensions.

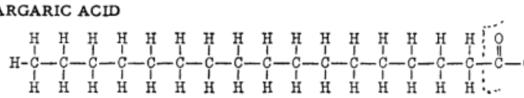
The length is minute: millions of molecules to make a line one inch long. At the time when Rayleigh made his estimate, scientists had been making rough, rash guesses at the size and mass of molecules; roundabout guesses based on gas friction, on scattering of sunlight by molecules in the sky, and on some risky electrical arguments. Here was an amazingly simple measurement, probably reliable.

The method has since been refined and extended by many, especially by Langmuir in the United States. Rayleigh's olive oil was a vague mixture of oily substances. Later workers used pure chemical compounds, often measuring several members of a "homologous series" (= chemical family). For example, Langmuir used the "fatty acids." These are derived from natural oils and fats, and they combine with sodium or potassium to make soap. They have long molecules with one end inert and the other an "acid" end which likes water. There is a whole series of such compounds, each member's molecule being larger than its predecessor's by one carbon atom and two hydrogen atoms. Long ago, chemists sketched the molecules of different members of this series with structural formulas like the three shown below.

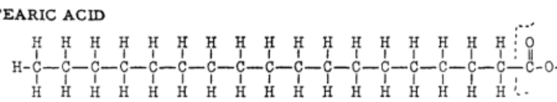
PALMITIC ACID




MARGARIC ACID



STEARIC ACID



These patterns were only guesses based on chemical evidence but they suggested long chain molecules, longer for every $-\overset{\text{H}}{\underset{\text{H}}{\text{C}}}-$ link added to the chain in changing from one family member to the next. Problem 6 is based on improvements of Rayleigh's method made by Langmuir, Adam and others.

PROBLEM 6. MEASURING MOLECULES PRECISELY

N. K. Adam used a rectangular bath 0.14 meter wide and half a meter long. The bath was filled brim-full of water and the experimental region was limited by two booms A and B across the top, about 0.4 meter apart. (See Fig. 6-27.)

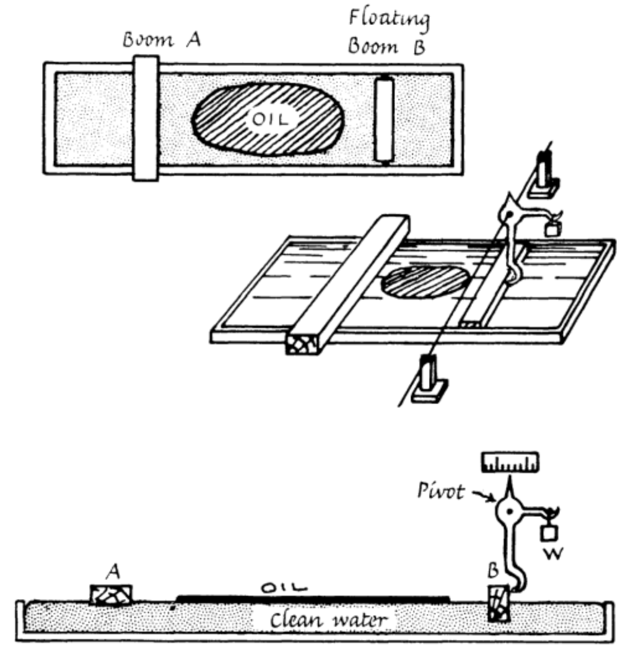



FIG. 6-27. SIMPLIFIED SKETCH OF N. K. ADAM'S APPARATUS. Oil film is confined between Booms A and B.

Boom B was free to move; it floated freely, but was prevented from drifting by a weighing-device with spring or weight to indicate any horizontal push on the boom. Boom A was placed across the bath, resting on the sides, and could be moved along by hand. The bath and the booms were waxed, so that the water level could be slightly above the top of the bath—thus booms A and B cut off a central section of the surface.

Starting with boom A far from B, Adam placed a minute measured quantity of palmitic acid on the water surface between the booms. Boom B showed no push. He then moved boom A along, crowding the oil film into smaller and smaller area, until B suddenly showed a considerable push, this suggesting that the oil molecules had been pushed together into a close-packed velvet. (In the actual experiments, the push did not suddenly rise from zero to its full value absolutely abruptly. It started at a certain area, and rose rapidly with further compression, reaching a constant value, after which still further compression probably made the "velvet" buckle. From a graph it was easy to find the point at which a considerable push developed.)

Adam added the fatty acid by dissolving it in benzene and placing a few drops of the solution on the water surface. The benzene quickly evaporated.

# CHAPTER 7 · FORCE AND MOTION: $F = M \cdot a$

"Pooh," said the Elephant's Child . . . "I'll show you."

Then he uncurled his trunk and knocked two of his dear brothers head over heels. "O Bananas!" said they, "where did you learn that trick, and what have you done to your nose?" "I got a new one from the Crocodile . . ." said the Elephant's Child.

"It looks very ugly," said his hairy uncle, the Baboon.

"It does," said the Elephant's Child. "But *it's very useful*," and he picked up his hairy uncle, the Baboon, by one hairy leg, and hove him into a hornet's nest.

—RUDYARD KIPLING, *Just So Stories*

(This is a long, hard, important chapter: ugly but very useful. It may need several readings and much careful thought; but without it you would make little of astronomy or atomic physics, which play important parts in the course.

If you find this study of motion difficult, reflect that it took mankind a long time to master it. Greek scientists had a good knowledge of the easy things in physics, levers and simple machines and floating bodies, etc., but they were muddled and foggy about motion. Much of the fog remained until three or four centuries ago. It took mankind over sixteen hundred years to reach a clear understanding of motion; you should hardly be impatient if it takes you several weeks.)

### *Force and Changing Motion*

How can a rocket propel itself in a vacuum? How do we know the charge of a single electron? How can we predict the behavior of gases by theory? How can we explore the structure of atoms with alpha-particles shot from radium? How can we predict the energy-release in nuclear fission? All these things can be done; but, to understand what happens or how scientists make the measurements, you must know the relation between *force* and *motion*. This chapter will explore that relation in detail, not for the sake of dull problems on speeding bicyclists but as a necessary foundation for almost all the most important physics, ancient as well as modern.

To the modern scientist, motion is not very interesting unless it changes. He expects steady motion to continue of its own accord; but if he sees a moving thing speed up or travel in a curve with changing direction, he considers he can gain useful information. He thinks there is force at work. Changing motions enable him to study the play of "force" in the physical world, perhaps even to make rash surmises about cause and effect. The world is full of *changing* motions: cars accelerate, cannonballs rise and fall, baseballs "curve," pendulums swing, the Moon sweeps around its orbit, planets wander across the sky in looped patterns, gas molecules reverse their motion violently when they bounce on the walls of their container, a beam of charged atoms squirted through an electric field is tugged into a parabola, a fine stream of electrons is wiggled up and down by magnetic fields in a television tube; and it would be a final marvel if even rays of light fell in a curve under gravity.[1] In this book you will study all these examples of changing motion. Each involves force, and if we are to go beyond mere cataloguing description we must know the relationship between "force"—whatever "force" may be—and changing motion. We shall call any push or pull a force, and we shall measure such forces by simple spring-balances (without assuming Hooke's Law).

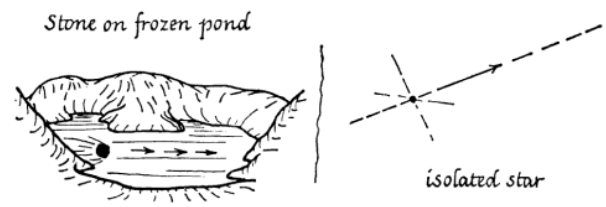



FIG. 7-1a. STEADY MOTION WITH CONSTANT VELOCITY
Fixed speed in fixed direction.

This is the time for more experiments, mostly demonstrations.

[1] Perhaps they do. How could you tell whether a beam of light is curved? How do you test whether a ruler is straight?

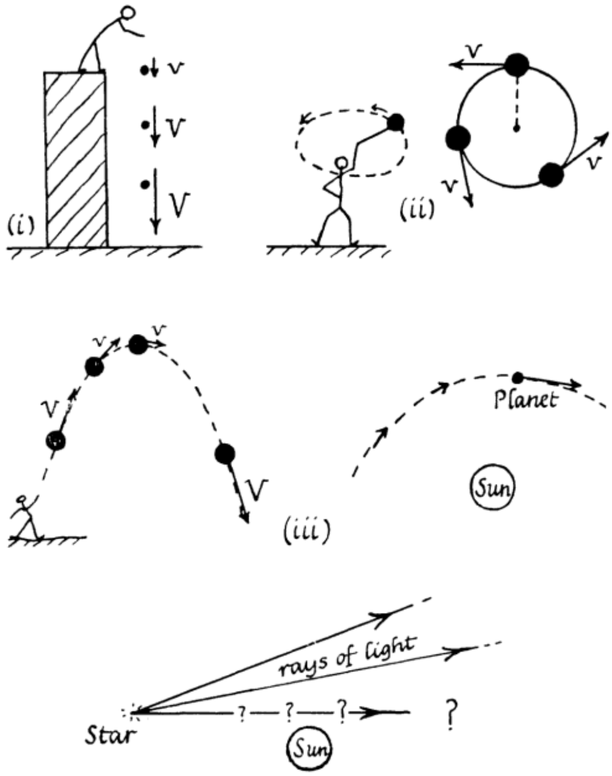



FIG. 7-1b. CHANGING MOTION
Velocity changes in size (i), direction (ii), or both (iii).

*Force and Acceleration: Anticipating the Laws*

Tie a rock to a string and hold the string. You can feel the rock pulling the string, and you say this pull comes from something pulling the rock down—the Earth or "gravity" or simply the rock's weight. That downward pull on the rock is balanced by

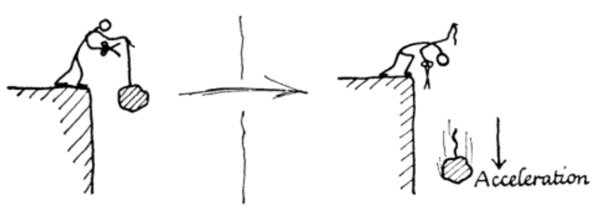



FIG. 7-2.

your upward pull. Now cut the string: the rock falls with constant acceleration. You have stopped pulling the rock *upward*, but you may assume that the same *downward* pull still acts on it, and is the only force, a constant downward force. In that case, *a constant force produces a constant acceleration.* This is the beginning of good knowledge of force and motion. Put with it Galileo's teaching that *when there is no force there is no acceleration: an object with zero RESULTANT force acting on it stays at rest or moves with constant velocity.*

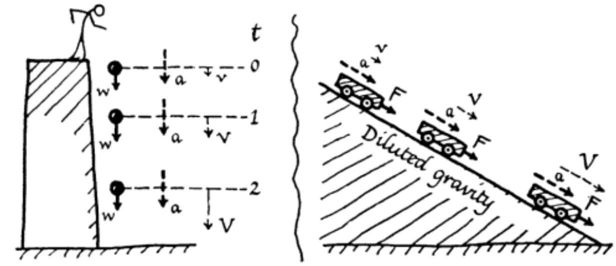



FIG. 7-3. ACCELERATION WITH CONSTANT FORCE

To investigate force and motion further, we must apply forces of different sizes, and we must try various moving objects. In this course, we shall use a primitive spring balance to measure FORCE, in

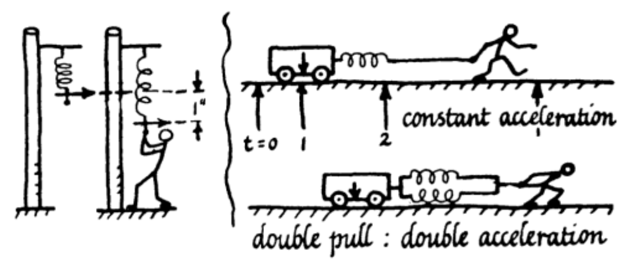



FIG. 7-4.

arbitrary units. Take a good steel spring and pull it to stretch some standard amount, say 1 inch. Call that pull "unit force," one "strang." (*Strang* is a new name for a new unit, coined for use here. Presently it will be replaced by an official unit.) Then we can apply one strang to accelerate some "victim"—a small truck or a block of ice on a level table—by pulling with the spring kept at standard stretch. It is no easy job to pull a cart along with a constant pull as it runs faster and faster. For the moment we shall pretend we can do that and look ahead to the results of such experiments. Measuring times and distances would show that *the acceleration is constant.* The victim's travel-distances in 1 second, 2, 3, . . . secs from rest would show the proportions 1:4:9: . . . . (Or from measurements of $s$ and $t$ we could calculate $2s/t^2$ and we should find it constant.) Then apply double force, 2 strangs, by a pair of identical springs side by side "in parallel," each at standard stretch. We should get double acceleration. *The acceleration increases in the same proportion as the force.*

To provide a whole range of forces, 1, 2, 3, 4, . . . strangs, make several equal[2] springs. Then accelerate the victim with 1 strang, 2, 3, . . . and we should

[2] For a discussion of the philosophy of making forces "equal" and adding them, see later in this chapter.

find accelerations in proportions 1:2:3: . . . . Then ACCELERATION increases in the same proportion as the accelerating FORCE, or $a \propto F$, for a given victim.

So far we have always pulled the same victim. Now change to different victims, different quantities

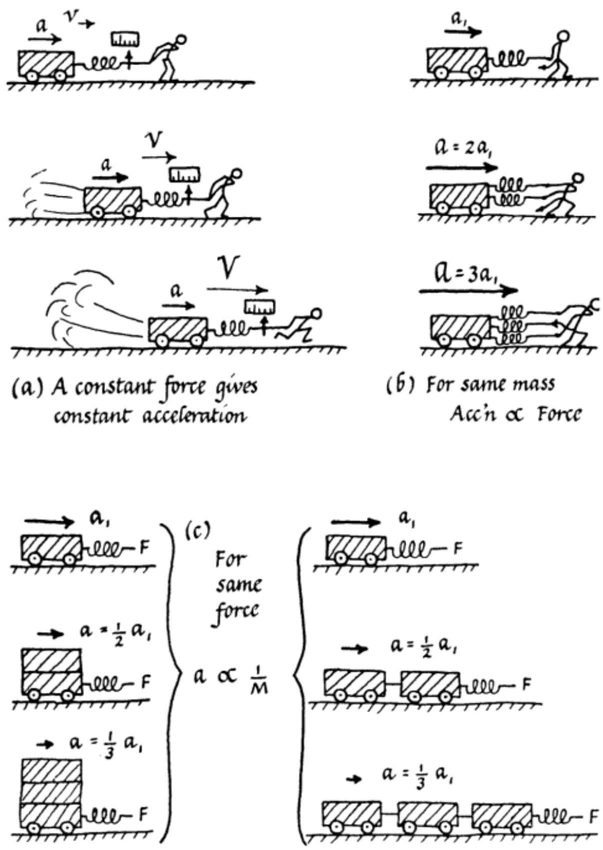



FIG. 7-5.

of moving matter, to twice and three times the MASS. Make several identical victims[3] (trucks or blocks of ice). For double mass, tie two together—or pile one on top of the other—and pull with 1 strang. Then put three victims together and pull them. With double mass, we should find half the acceleration; with triple mass, one third. The *acceleration* decreases in the proportion that the *mass* increases, or $a \propto 1/M$, where $M$ is measured by counting the number of trucks. This is a harder relationship to picture, so we ask our question another way: how must the force change if we want the *same acceleration* for different masses? With double mass, 1 strang gives half acceleration, so 2 strangs would give the original acceleration. Then, *for the same acceleration*, single mass, double mass, and triple mass need forces in proportion 1:2:3. The FORCES needed are proportional to the MASSES, $F \propto M$.

[3] For a discussion of the philosophy of making masses "equal" and adding them, see later in this chapter.

Here, by mass we mean how much stuff is to be accelerated, how many equal trucks (or blocks of ice).

*Summary*

Then we have two important relationships:

(i) ACCELERATION ∝ FORCE, for a constant mass, or FORCE ∝ ACCELERATION

(ii) FORCE ∝ MASS for a constant acceleration.

These can be combined[4] into

FORCE ∝ MASS · ACCELERATION
or FORCE = (constant) · MASS · ACCELERATION

*Newton's Second Law of Motion*

We have pretended glibly that we can apply constant forces to moving masses and measure the accelerations accurately. And we have assumed that the pull of our springs is the *only* horizontal force acting on the victim, so that it is the resultant pull. The relation we have announced,

RESULTANT FORCE ∝ MASS · ACCELERATION,

is true. It is Newton's great SECOND LAW OF MOTION (which includes his FIRST LAW, and assumes his THIRD LAW in any experimental tests). This law relating FORCE, ACCELERATION and MASS is essential in later physics. It fits experimentally the motion of all large bodies, from toy trucks and tennis balls to jet planes and planets; and we extend it, *by assumption*, to atoms, electrons, and nuclei. To understand this law clearly and use it well, you must understand its basis of experiment and definition. So it is very important for you to see good experimental tests or demonstrations. Before describing some demonstrations, we shall discuss the special case $F = 0$.

*No Force: Unchanging Motion—Newton's First Law of Motion*

If $F \propto M \cdot a$, then in the special case $F = 0$, the acceleration must be zero; the motion must continue without change. You could infer this from projectile motion: you see in the vertical acceleration the effect of Earth-pull, and you should also see in the horizontal motion the effect of any horizontal force. Apart from air friction (which is not involved in this ideal case) there is no horizontal

[4] See note 16 for the algebra of this combining. For the moment, compare this with the cost of labor for some job:
COST ∝ NUMBER OF MEN working
COST ∝ NUMBER OF HOURS worked
combine into
COST ∝ (NUMBER OF MEN) · (NUMBER OF HOURS)

force. Yet a cannon-ball continues to move forward with constant horizontal velocity. Then you can make the guess that with no force the velocity continues unchanged. In such cases the horizontal *acceleration* is zero, but the *velocity* does not have to be zero: it can maintain any constant value. So physicists say, "it takes no force to keep steady motion going." This seems absurd at first sight. It takes a large and continuous shove to keep a box

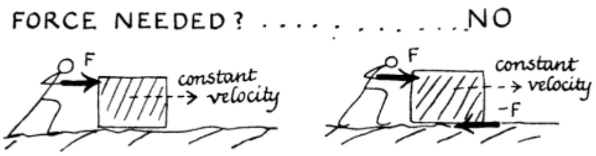



FIG. 7-6. ANCIENT QUESTION AND MODERN ANSWER

moving along a rough floor or to keep a car moving steadily on a level road. But then we are taking a limited view; we are forgetting the backward shove of the floor's friction or of air resistance. If we include that, the resultant force may well be zero. And it is zero *resultant* force that we say goes with constant velocity (see Fig. 7-6). Even in the case of the flying cannon-ball's horizontal motion we could add a contraption of springs with resultant

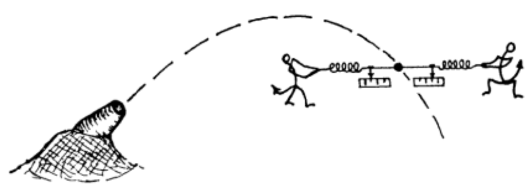


FIG. 7-7. FORCES DO NOT AFFECT MOTION, IF THEIR RESULTANT IS ZERO!

force zero, one pulling forward, the other backward, and the horizontal motion would still continue unchanging (see Fig. 7-7).

## *DEMONSTRATION EXPERIMENTS*

(Here are descriptions of some experiments. The forms you see must depend on the equipment available.)

I. *Motion of a body with "no resultant force"—the skater's dream.* It is impossible to demonstrate this honestly. We cannot provide a moving body which *obviously* has no force acting on it. We run into trouble from gravity, friction, or logic. All we can show are experiments that illustrate our line of thought, running towards the ideal case, itself an imaginary experiment extrapolated from all real ones.[5] The rule "no force: constant motion" applies whether there is friction or not. We only look for *frictionless* experiments to demonstrate it because friction is difficult to measure and allow for.

*Demonstration* I(a). *Watch a ball roll along a level table.* Unfortunately it slows and stops; we blame friction. (And, unfortunately, a rolling ball is also used as a test to find out whether the table is level; so there seems to be a danger of arguing in a circle. But you can avoid that if you experiment intelligently.)

★ PROBLEM 1. SCIENTIFIC EXPLANATION vs. DEMONS*

How do you know it is friction, not demons, that brings a rolling ball to rest? Suggest experiments to test or support your view.

* This problem, which looks like a joke at first sight, raises the whole question of the nature of scientific explanations and laws. Try to make a logical defense—but remember that an opponent defending demons could claim a variety of properties for them.

*Demonstration* I(b). *Watch a large block of "dry ice"* (solid carbon dioxide) *slide* along a level table of aluminum or plate glass. The block is kept from contact with the table by a cushion of gaseous carbon dioxide which is constantly being roasted off its bottom surface by the table. The block is cold, far colder than ordinary melting ice; and it finds the table very hot, so it evaporates to gas and skates over the table like a block of ice on a hot sidewalk.

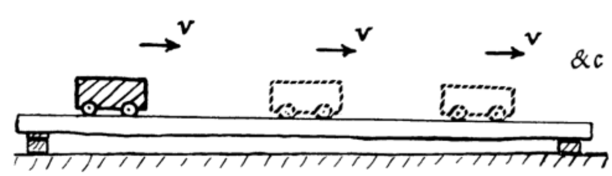



FIG. 7-8. "NEWTON'S LAW I"
Truck on friction-compensated track.

*Demonstration* I(c). *"Model railroad"* (see Fig. 7-8). Honestly admitting defeat by friction, we can make a friction-compensated railroad by tilting the track. A truck on a tilted track is pulled downhill by a fraction of the Earth-pull on it, and we can adjust the slight slope to make that small downhill

[5] Perhaps the ideal experiment is *unthinkably* difficult, requiring a single moving body infinitely isolated from all others which might disturb it. Then how could we observe its steady motion? Where would we be, and where would our mile posts be? Since it would be impossible to observe such motion if it existed, are we wise to talk about it as part of scientific knowledge? We are safer to put up with minor disturbances from friction or perturbations by gravity.

force just counteract friction. Then we start the truck with a momentary push and watch it move. This is a fake demonstration—how did we find out how much to tilt the track? Yet it is interesting to watch the truck creep along, the slope of the track being almost invisible. In fact, we believe the resultant force *is* zero. Pull of the Earth and push of the track and drag of friction combine by vectors to make zero. If the truck is started with a bigger push, it maintains its new speed all the way along. Loaded with sand or metal and given a start, it again moves steadily. Without measurements this is an unconvincing demonstration, really telling things about friction rather than about motion with no force, but we shall find the friction-compensated track useful in later experiments.

*Demonstration* I(d). We meet illustrations of straight-line travel in the paths of very fast projectiles; rifle bullets move so fast that their gravity-fall is unnoticeable in a short travel. This only shows us that the path is nearly straight; it gives no assurance about unchanging speed. Or streams of electrons (and other atomic particles) moving faster still can be shot through pinholes in a series of barriers in a long pipe (Fig. 7-9). If the pinholes are not lined

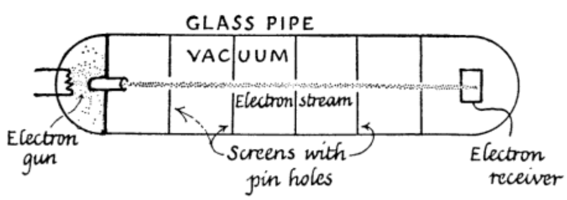



FIG. 7-9. ELECTRON STREAM TRAVELS ALONG A STRAIGHT LINE, IF LEFT ALONE

up in a straight line, the stream does not get through.[6]

Some fast atomic particles make a black track when they pass through sensitive photographic gelatine. Shot at glancing angles through photographic films they make very straight streaks. (See emulsion photographs of the tracks of electrons, protons, etc. from cosmic rays.)

*Demonstration* II. *Force and Acceleration.* The relationships suggested above (with optimistic stories about measurements with springs) form a great basic law of physics, so you should see real demonstrations. To test whether accelerations are proportional to forces, as Galileo's writings suggested to Newton, we measure the acceleration of a small truck pulled along a railroad by various

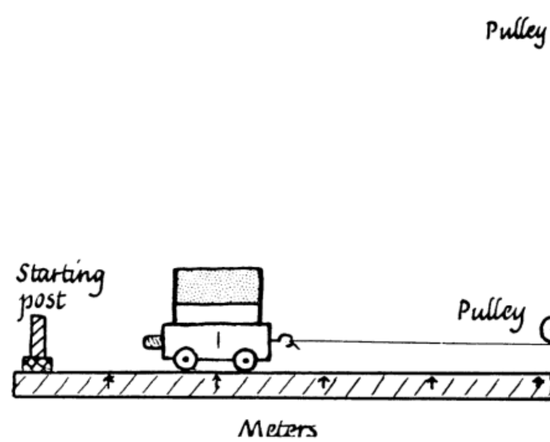



FIG. 7-10a. DEMONSTRATION EXPERIMENT illustrating the relations of force, mass, and acceleration. A small load W pulls a massive truck by means of a thread running over very good pulleys. The track is tilted slightly to compensate for friction. This is the apparatus used in the demonstrations II a,b,c. (For details of timing system, see Fig. 7-10b.)

forces. We tilt the track slightly to compensate for friction. That is not dishonest, if we publish our precaution. It is good to compensate for friction, or keep it small: the laws of motion do not fail when there is friction, but friction adds another force that must be measured separately if we want to know the resultant force. And it is the *resultant* force that appears in the simple laws.

Detailed arrangements of track and timing system depend on the equipment available. The track should be long, with steel rails as straight as possible and carefully supported. The truck should have the very best ball-bearing wheels. Electrical timing with a large clock as recorder is more convenient than schemes of ink spots or wavy traces. The clock can be started by an electric contact and stopped by an electric eye (= *photocell*) as in Fig. 7-10b. This may seem complex and mysterious at this stage. You will meet this machinery later—photocells, amplifiers, etc. All you need to know now can be got by watching the actual working of the railroad system and clock. You will see that the clock starts when the truck leaves the starting-post and stops when it reaches the electric eye. If you use the clock simply on this basis of direct observation you are doing nothing worse than when you use any clock—you assume its behavior is reasonable, but you keep an eye open for unwanted errors.

If you see this fundamental demonstration done with elaborate apparatus, just check it also with your own watch.

[6] How would you make sure the pinholes were properly lined up? An experimental physicist would probably use a flashlight. If he found it embarrassing to rely on the straightness of rays of light, he could use a taut thread, allowing for its sag, like a surveyor.