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—Edna St. Vincent Millay

Pure mathematics is, in its way,
the poetry of logical ideas.
—Albert Einstein

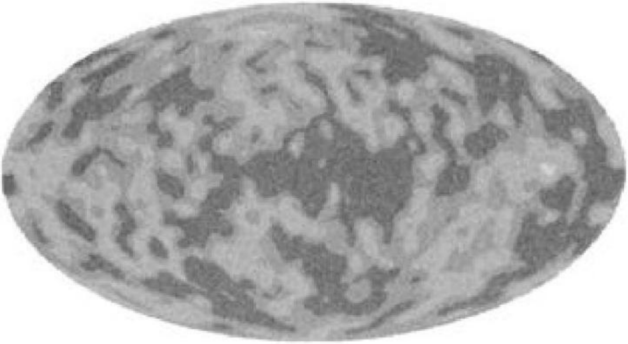
We have heard much about the poetry of mathematics, but very little of it has yet been sung … The most distinct and beautiful statement of any truth must take at last the mathematical form.
—Henry David Thoreau

PREFACE

On April 24, 1992, newspapers around the world reported an event that was hailed as “one of the major discoveries of the century”—what some would call “the missing link” and “the Holy Grail” of cosmology. The discovery was presented in the form of a picture that was in essence a snapshot of the universe at a dramatic moment in its evolution—the moment that space began. Before the time of the picture, there was only a conglomeration of elementary particles in a state of continual creation and annihilation. Then electrons and protons combined to form atoms of matter. For the first time there was space between the atoms, allowing light and other forms of radiation to travel freely. The “snapshot” depicts the pattern of rays that have reached us after traveling through space from that moment to the present. What was electrifying to scientists who had been studying those rays—the so-called cosmic microwave background radiation—was that there *was* a pattern in the picture. After decades of frustration at trying to detect even a ripple of variation in the apparently featureless sea of uniform background radiation, they were now successful in finding a possible link between the undifferentiated primeval “soup” predicted by the big bang theory of the creation of the universe and the later evolution into the highly differentiated stars and galaxies of the universe as we know it today. But reporters attempting to explain the precise nature of the picture were faced with at least one insurmountable obstacle: neither they nor their readers were prepared for the paradoxical nature of an image that depicts simultaneously a view outward in all directions from the earth and inward in all directions toward the big bang.

The big discovery of 1992 recalls in both obvious and more subtle ways the "discovery" of America just five hundred years earlier. If we look further back to the year 1000 A.D., Europeans generally pictured the earth as flat. It took great efforts of the imagination over the subsequent centuries to grapple with and understand the implications of a spherical earth, and to appreciate why people at the opposite side who were hanging upside down by their heels did not either fall off or suffer from perpetual headaches. The voyages of Columbus and those following in his wake gave a sense of reality and concreteness to the theoretical view of a round earth that had gradually been established before his time.

Now, as we approach the year 2000, there are few people left who believe in a flat earth, but most of the world's population continues to think in terms of a flat universe. Just as everyday experience led us to think of the earth as flat or planar rather than curved, so does our perception of the world around us lead us to view space as flat or "euclidean." It requires as great an effort of the imagination in the twentieth century to conceive of curved space as it did a thousand years ago to conceive of the earth as a gigantic ball somehow suspended or floating freely in an even more gigantic expanse of space. Nevertheless, the evidence is overwhelming that space is indeed curved, and only in that context can the cosmic microwave photograph of 1992 be fully understood.



The 1992 picture of variation in the cosmic background radiation (courtesy of George Smoot, NASA, and the COBE satellite).

What is the shape of the universe, and what do we mean by the curvature of space? One aim of this book is to make absolutely clear and understandable both the meaning of those questions and the answers to them. Little or no mathematical background is needed; the book takes the reader from the easily understandable mathematical ways of evaluating the world to those notions that are unfamiliar and further removed from everyday experience, while conveying the excitement and the power of the mathematical ideas that form the core of modern cosmology. The history and evolution of these ideas are often as fascinating as the ideas themselves, and are presented in an unfolding chronological narrative along with glimpses of the lives and personalities of some key players involved in the story. For those who wish to know more about the technical and mathematical underpinnings of the notions introduced, a section of notes at the back of the book provides further details as well as references to sources for further reading.

PRELUDE

Imagine sailing on a clear but windy day. The surface of the water is choppy and bright blue, reflecting and intensifying the color of the sky. Suddenly the weather changes—the wind ceases, the skies cloud over, and the surface of the ocean becomes calm and smooth. The water itself turns green and transparent, allowing a glimpse of a coral reef and a whole new world of colorful activity down below. If you dive below the surface to get a closer look, you'll find that your 20/20 vision in the air above produces only a blur underwater. But if someone provides you with a pair of goggles, then suddenly the world beneath the surface becomes just as clear and even more beautiful than your original view of the surface from above.

Imagine, similarly, going out to the middle of a desert on a clear and moonless night, far from city lights. Against a pure black background, the stars, the planets, the nebulae, the constellations, the Milky Way, stand out in a dizzying array. With the aid of a telescope, more and more exotic sights appear: majestic spiral galaxies, great spherical balls of light and color from past supernova explosions. Out of the "cosmic static" from the first radio telescopes, more and more refined astronomical instruments bring images of pulsars and quasars, as well as the ubiquitous cosmic microwave background radiation. These are all surface appearances, however, on the magnificent ocean of the cosmos. What lies below the surface and beyond one's field of view, what provides the underlying structure out of which all these phenomena evolved, cannot be seen without the necessary equipment: a pair of mind-goggles allowing the imagination to function outside its natural boundaries.

The goal of this book is to provide those mind-goggles, enabling the reader to move freely in the unfamiliar world of curved space-time. One cannot expect to plunge all at once into that world, but a bit of patience and persistence and the right tools will bring substantial rewards as new and unsuspected vistas emerge. Beyond that, the book is a celebration of the human imagination—the facility to make the kind of mental leaps without which the impact of the outer world on our senses would be mostly noise. Mathematical imagination and imagery, closely linked, provide the vision that allows us to see the hidden but exquisite structure below the surface.

CHAPTER 1
Measuring the Unmeasurable

THY SHADOW, EARTH, FROM POLE TO CENTRAL SEA,
NOW STEALS ALONG UPON THE MOON'S MEEK SHINE
IN EVEN MONOCHROME AND CURVING LINE
OF IMPERTURBABLE SERENITY.

—Thomas Hardy,
"At a Lunar Eclipse"

Over two thousand years ago, the philosopher-scientists of ancient Greece embarked on a project that was as daunting for those days as exploring the boundaries of the solar system would be today. It was to determine the size and shape of the entire earth. To the ancient Greeks, the earth was unimaginably big. Neither the Greeks nor any of the civilizations they came into contact with had roamed or sailed over more than a fraction of it. To go from the minuscule portions of the earth that could be directly measured to the immensities of unexplored and even undreamed-of distant lands required great feats of ingenuity. It also required the systematic development of an entirely new branch of learning that the Greeks would call *geometry*, meaning literally: "measuring the earth."

One of the best-known names from the early history of geometry is that of Pythagoras, whose lifetime spanned most of the sixth century B.C. But long before Pythagoras, the Egyptians had devised a simple method for constructing perpendicular lines, such as those framing the base of a pyramid. They used knots placed at equal intervals along a rope to divide the rope into lengths of 3, 4, and 5 spaces between knots. Placing pegs in the ground so that the rope formed a triangle, when stretched taut around the pegs, with side lengths of 3, 4, 5 respectively, they found that the angle between the sides of length 3 and 4 was a right angle, or 90 degrees. They also found that different side lengths would serve the same purpose when a certain condition was met. The key to getting a right angle was to have the square of the longest side equal to the sum of the squares of the other two sides, a relationship that we know as the "Pythagorean theorem." The Babylonians were also aware of this relationship. In fact, over a thousand years before Pythagoras, at the time of Hammurabi, "the Lawgiver," the Babylonians had developed mathematics to a much higher level than the Egyptians, including a more sophisticated system for representing numbers and some basic algebra, as well as geometry. Not only did they seem to be aware of the "Pythagorean theorem" but they also provided a long list of number triples, including such unlikely ones as (65, 72, 97) and (119, 120, 169), all representing the sides of a right triangle.

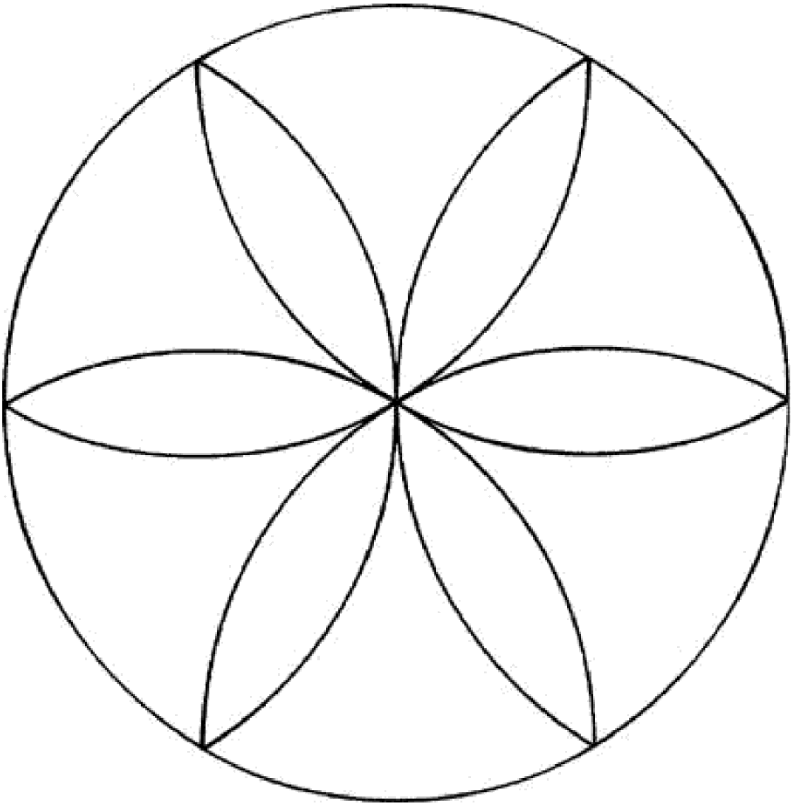
Why then was the theorem named after the latecomer Pythagoras? Despite the priority of the Egyptians and Babylonians, they gave no indication of having thought of the key mathematical notion of a proof. Pythagoras's name became attached to the theorem because he was reputedly the first person to provide such a proof. As it happens, there is no direct evidence that he did so. (It is not known if Pythagoras left anything in written form; if he did, none of his writings have survived to the present.) It seems most probable that the first proof of the "Pythagorean theorem" originated with his followers, the "Pythagoreans," sometime in the following century.

Euclid, who became the most famous of all Greek mathematicians, was born over two hundred years after Pythagoras. During the period between Pythagoras and Euclid, geometry developed on two parallel tracks. One consisted of the detailed study of particular shapes, such as triangles, rectangles, and figures bounded by circular arcs. The other was the development of the method of proof and the process of deductive reasoning, leading to new discoveries that would not have been found by direct observation. By the time Euclid arrived on the scene, a sizable body of geometric lore had accumulated.

The details of Euclid's life, even more than of Pythagoras's, remain shrouded in obscurity. Virtually all that can be stated with certainty is that he lived and worked in Alexandria during a period around 300 B.C. Unlike Pythagoras, however, he left writings that not only have survived to the present but have become the basis of much of modern science, as well as a model for all of mathematics.

The monumental work for which Euclid is best known is *The Elements*, a mathematical compendium in thirteen books, of which five are devoted to the geometry of two-dimensional figures, three are devoted to the geometry of three dimensions, and the remainder to other subjects.

Euclid's *Elements* made a deep impact on the psyche of the Western world. Originally viewed as both a tool and a model for research in mathematics and other sciences, *The Elements* gradually evolved into a basic component of a standard education—a piece of intellectual equipment that every young student was expected to wrestle with and internalize. The fascination of *The Elements* has at least four distinct components. First, there is the sense of certainty—that in a world full of irrational beliefs and shaky speculations, the statements found in *The Elements* were proven true beyond a shadow of a doubt. And although certain features in both the assumptions and the methods of reasoning used by Euclid have been questioned over the centuries, the astonishing fact is that after two thousand years, nobody has ever found an actual "mistake" in *The Elements*—that is to say, a statement that did not follow logically from the given assumptions. The second feature is the power of the method. Starting from a very few explicitly laid-out assumptions, Euclid produced a dazzling series of consequences. Third is the display of ingenuity employed in the proofs—not so different from the sort of ingenuity that adds to the appeal of a well-crafted detective story. Finally, the objects of the reasoning in the first books of *The Elements* are geometric shapes that have an aesthetic appeal of their own, quite apart from any formal reasoning that may be applied to them. Some combination of these features led Edna St. Vincent Millay to the sentiments expressed in her poem: "Euclid alone has looked on Beauty bare."



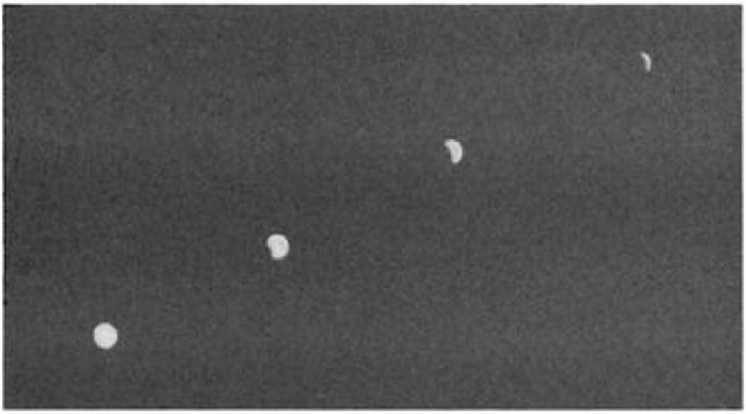
A geometrical figure.

Of all the shapes that were studied by mathematicians, one held a special fascination: the circle. Like euclidean geometry as a whole, this one shape—the circle—was destined to play a powerful role—for better and for worse—in all future attempts to describe the shape and the workings of the world and the universe.

How did the concept of a circle first enter human awareness? There are surprisingly few places in nature where one sees a true circle. The most striking example is undoubtedly the sun, a daily presence, even if too bright to view directly except when near the horizon or when filtered through a thin layer of clouds or fog. In some ways even more awe-inspiring is the full moon, taking shape gradually and transforming itself into a perfect circle once every twenty-eight days. Another, indirect example that star watchers became aware of is the course of the stars overhead each night, describing circular arcs across the sky, a pattern that is most noticeable for stars in the vicinity of the North Star. One of the occurrences of a circle on earth is the beautiful pattern of circular ripples caused by the first few raindrops on a still pool of water, or a pebble tossed into a quiet pond. For someone standing at the edge of the sea, or at the stern of a boat, the horizon itself takes the shape of an immense circle.

Perhaps it was the circular shape of the horizon that provided the first clue to the shape of the earth. The first concrete evidence of its shape in early times was not the result of direct observations of the earth itself, but rather came from watching the night skies. Although it is impossible to say when the first stargazers grasped the significance of their two key observations, both were noted by Aristotle, in the fourth century B.C.

The first observation had to do with lunar eclipses, the result of the sun, the earth, and the moon all lining up so that the earth temporarily blocks the sun's light from reaching the moon. The shadow of the earth gradually moves across the face of the moon, and it is clearly circular.



Four stages in a lunar eclipse, starting with the near-perfect circle of the full moon, and then successively revealing larger and larger circular bites out of the moon as the earth's shadow gradually moves across the face of the moon.

The second piece of evidence was more roundabout, but even more convincing. It required observing the skies not from a fixed point on earth, but from a number of places of differing latitudes. What became apparent was that as one travels to the south, the familiar constellations in the north gradually appear lower in the sky, while those in the south appear higher. Furthermore, new constellations, never seen at higher latitudes, appear near the horizon. The farther south one travels, the higher these new constellations appear in the sky, and the greater the number of new constellations that come into view. It eventually became apparent that such changes were exactly what would be expected if the earth were spherical. And so, over two thousand years ago, the idea of a flat earth simply did not fit the observed facts, and had to be discarded. The more challenging question was not the qualitative one of the shape of the earth, but determining its now known as Aswan. The third was an absurdly simple apparatus called a *gnomon*.

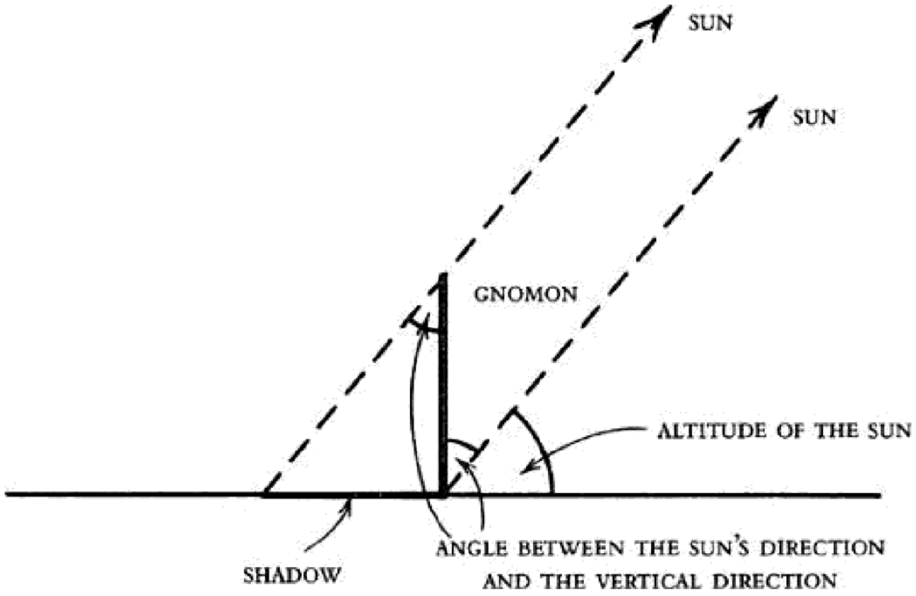
The gnomon had been in use for a very long time. It consisted of a vertical stick placed on a level piece of ground. The gnomon was a device that allowed one to follow the sun's shadow as the sun moves across the sky. Although the gnomon cannot be used to tell time in the manner of its more advanced cousin, the sundial, it does provide a surprising amount of useful information.

First, the gnomon gives the exact time once a day, at the moment that the sun is highest in the sky and the shadow of the gnomon is the shortest—at noon. In addition, it acts as a compass, since the shadow at noon points due north (at least for Europe and most of the Northern Hemisphere, including Alexandria. In any event, regardless of which hemisphere one is in, the sun's shadow at noon always lies on the north-south axis.)

The gnomon also serves as a primitive calendar, determining two key days of each year: the summer and winter solstices. If one places a mark where the shadow ends at noon on each day of the year, one finds that in winter, when the sun is low in the sky, the shadows are longer, while in the summer, with the sun high in the sky, the shadows are shorter. The shadow at noon goes through a yearlong cycle, from the shortest noon shadow in summer, gradually reaching its greatest length six months later, and then shortening again over the succeeding six months. The day on which the noon shadow is shortest, and the sun is highest, is called the *summer solstice*. The day six months later when the sun is lowest and the noon shadow the longest is known as the *winter solstice*. Counting the number of days from solstice to solstice also provided one of the earliest accurate measurements of the length of the year.

Finally, the gnomon could be used to determine the altitude of the sun—that is, the angular distance of the sun above the horizon at any given moment (at least on sunny days). All one had to do was measure the length of the shadow and the length of the stick. By drawing a right triangle to scale with those measurements, one can measure the angle opposite the shadow, and that angle will indicate how far off the sun's direction is from an overhead, vertical direction.

These uses of the gnomon were well known to Eratosthenes and his contemporaries. But it was the fortuitous geographical properties of Aswan that gave Eratosthenes his inspiration for determining the size of the earth. Aswan is almost due south of Alexandria. It also enjoys the special privilege of having the sun pass directly overhead at one moment of each year: at noon on the summer solstice. At that one moment each year, a gnomon in Aswan casts no shadow at all. (Aswan lies almost exactly on the Tropic of Cancer, the name given to the circle—about 23½ degrees above the equator—where the sun passes directly overhead at noon on the summer solstice.)



Gnomon and shadow.

By combining these facts with some simple but clever geometric reasoning, Eratosthenes was able to produce his remarkable pièce de résistance: the circumference of the earth. At noon on the summer solstice, he simply used his gnomon to determine the angle between the sun and the vertical direction in Alexandria. Since the sun at that moment is directly overhead at Aswan, he thereby knew the angle between the vertical directions at Alexandria and at Aswan. He found that angle to be $^1/_{50}$ of the circumference of a circle. That meant that the entire circumference of the earth is 50 times the distance between Alexandria and Aswan. Since the distance from Aswan to Alexandria is roughly 500 miles, by today's measurements, the earth must be approximately 25,000 miles the value of π was somewhere between $^{10}/_{71}$ and $3^{1}/_{7}$. That means that if the distance around the world, starting and ending at Alexandria, is 25,000 miles, then the distance straight through is somewhere between 7,955 and 7,960 miles —a remarkably small margin of error.

Thus, over two thousand years ago the size and shape of the earth were pretty well established. Unfortunately, with the crumbling of the ancient civilizations, a thousand years of accumulated wisdom was lost to the European continent. By good fortune, the decline of the West coincided roughly with the rise of Arabic civilization and culture, and much of ancient knowledge was translated and transferred there. Along with it came the desire to refine what was known. One example was the astonishing feat of al-Kashi of Samarkand, who in 1424 carried Archimedes' method of computing π to undreamed-of lengths, determining its value to sixteen decimal places. He did so not only for the sheer joy of calculating far beyond what anyone had calculated before but also for a very specific purpose: to determine the circumference of the universe to within the width of a horsehair. To get a sense of the degree of accuracy he achieved in computing the value of π, if the earth were a perfect sphere with a circumference of 25,000 miles, then al-Kashi's estimate for 71 would determine the diameter of the earth to within less than one ten-millionth of an inch. The only distantly comparable achievement up to that time was the value $^{355}/_{113}$ or $3^{16}/_{113}$ discovered in the fifth century by the Chinese mathematician Tsu Ch'ung-chih, which gives the correct value of π to six decimal places.

longer than today's "mile," so that this estimate of the earth's circumference loses a bit in translation. But that matters little. What is important is that in the early ninth century, the spherical shape of the earth was accepted as simple fact in the world of Islamic science. The curvature of the earth was known to account for the varying altitudes of the stars and the sun as one traveled north or south, and by measuring the change in the angle of the stars or the sun above the horizon over a precisely measured distance on the earth, ninth-century scholars were able to determine the size as well as the shape of the earth.

But most Europeans around the year 1000 were oblivious to all the discoveries in the Near and Far East, or, for that matter, to the accumulated knowledge of the ancient Greeks more than a millennium before; they were unable to see beyond their horizons either literally or figuratively, and for them the world was flat, the universe impenetrable.

CHAPTER II
Encompassing the Earth

> WHAT GOD SOEVER THIS DIVISION WROUGHT,
> AND EVERY PART TO DUE PROPORTION BROUGHT;
> FIRST, LEST THE EARTH UNEQUAL SHOULD APPEAR,
> HE TURNED IT ROUND, IN FIGURE OF A SPHERE.
>
> —Ovid, *Metamorphoses*, first decade A.D.
> Translation by George Sandys, 1626

One of the enduring myths of the Western world is that in order to gain support for his expeditions, Christopher Columbus had to first overcome a pervasive belief that the earth was flat rather than round and that by attempting to sail west to Asia he would risk sailing off the edge of the earth. The myth undoubtedly stems in part from a compression of the historical past, conflating the early Middle Ages, when a belief in a flat earth was indeed widespread in Europe, with the late Middle Ages—centuries later—by which time Europe had caught up with, and partially surpassed, the state of knowledge of ancient Greece and medieval Islam.

Ptolemy—astronomer, geographer, and mathematician—lived in Alexandria over a thousand years before Columbus, during the height of the Roman Empire. During the second century A.D., he worked to consolidate and extend the scientific

After the fall of Rome, mapmaking in Europe reverted to its earlier, more fanciful state, based on beliefs and hearsay, rather than on facts and science. It was not until the thirteenth century that Ptolemy's *Geography* once more became available, but at that time only in the original Greek, a language that was not widely known. Another two hundred years passed before it was translated into Latin; the first printed version dates from 1472. Columbus himself owned a copy printed in 1479.

The spherical shape of the earth is accepted as established fact in Ptolemy's *Geography*. Several subsequent books written between 1200 and 1500 continued the discussion of the shape of the earth; the title alone of one of the most notable of them provides clear evidence of how educated people in the fifteenth century viewed the world. It was called simply *The Sphere*. The book's author was known as Sacrobosco, a Latinized version of the name of an Englishman, John of Holywood, who wrote a number of books in the early part of the thirteenth century. Probably no other textbook in history has been as successful as Sacrobosco's—it remained in print and was still in use five hundred years after it was written. (If one were to count Euclid's *Elements* as a textbook it would undoubtedly own the record for longevity, but it was written in a quite different spirit and certainly was not intended as a text.)

What Sacrobosco did in his book was to adapt key passages from the *Almagest* of Ptolemy, add some newer material, and omit a number of technicalities, to arrive at a more accessible description of the workings of the universe, as best understood at the time. As the title indicated, the "sphere" was the key to everything. The earth was a sphere lying inside the great sphere of the fixed stars, while the sun, moon, and planets were attached to intermediate spheres. The evidence that the earth was round and not flat was taken straight from Ptolemy: