JIM BAGGOTT

QUANTUM REALITY

THE QUEST FOR THE REAL MEANING OF QUANTUM MECHANICS
—A GAME OF THEORIES

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ABOUT THE AUTHOR

Jim Baggott is an award-winning science writer. A former academic scientist, he now works as an independent business consultant but maintains a broad interest in science, philosophy, and history and continues to write on these subjects in his spare time. His previous books have been widely acclaimed and include the following:

***The Quantum Cookbook**: Mathematical Recipes for the Foundations of Quantum Mechanics*
***Quantum Space**: Loop Quantum Gravity and the Search for the Structure of Space, Time, and the Universe*
***Mass**: The Quest to Understand Matter from Greek Atoms to Quantum Fields*
***Origins**: The Scientific Story of Creation*
***Farewell to Reality**: How Fairy-tale Physics Betrays the Search for Scientific Truth*
***Higgs**: The Invention and Discovery of the 'God Particle'*
***The Quantum Story**: A History in 40 Moments*
***Atomic**: The First War of Physics and the Secret History of the Atom Bomb 1939–49*, short-listed for the Duke of Westminster Medal for Military Literature
A Beginner's Guide to Reality
***Beyond Measure**: Modern Physics, Philosophy, and the Meaning of Quantum Theory*
***Perfect Symmetry**: The Accidental Discovery of Buckminsterfullerene*
***The Meaning of Quantum Theory**: A Guide for Students of Chemistry and Physics*

PREAMBLE

I know why you're here.

You know that quantum mechanics is an extraordinarily successful scientific theory, on which much of our modern, tech-obsessed lifestyles depend, from smartphones to streaming to satellites. You also know that it is completely mad. Its discovery forced open the window on all those comfortable notions we had gathered about physical reality from our naïve interpretation of Isaac Newton's laws of motion, and unceremoniously shoved them out. Although quantum mechanics quite obviously *works,* it appears to leave us chasing ghosts and phantoms, particles that are waves and waves that are particles, cats that are at once both alive and dead, lots of seemingly spooky goings-on, and a desperate desire to lie down quietly in a darkened room.

But, hold on. If we're prepared to be a little more specific about what we mean when we talk about 'reality' and a little more circumspect about how we think a scientific theory might *represent* such a reality, then *all the mystery goes away*.

I'm not kidding. I have a bit of a reputation as the kind of guy you might find in the kitchen at parties; the kind who spoils all the fun, bursting the bubbles of excitable mystery and urban myth (what Americans sometimes call 'woo') with a cold scepticism and a calculating rationality. Spock, not Kirk (or McCoy). One commentator recently called me 'depressingly sane'.* This is a badge I'm happy to wear with pride. There are

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* This was theoretical physicist Sabine Hossenfelder, referencing my book *Farewell to Reality: How Fairy-tale Physics Betrays the Search for Scientific Truth,* in a tweet dated 11 March 2018.

many popular books you can buy about the weirdness and the 'woo' of quantum mechanics. This isn't one of them.

And in any case that's not why you're here.

But—let's be absolutely clear—a book that says, 'Honestly, there is no mystery' would not only be a bit dull and uninteresting (no matter how well it was written), it would also be completely untrue. For sure we can rid ourselves of all the mystery in quantum mechanics but only by abandoning any hope of deepening our understanding of nature. We must become content to use the quantum representation simply as a way to perform calculations and make predictions, and we must resist the temptation to ask: *But how does nature actually do that?* And there lies the rub: for what is the purpose of a scientific theory if not to aid our understanding of the physical world?

Let's be under no illusions. *The choice we face is a philosophical one.* There is absolutely nothing scientifically wrong with a depressingly sane interpretation of quantum mechanics in which there is no mystery. If we choose instead to pull on the loose thread we are inevitably obliged to take the quantum representation at face value, and interpret its concepts rather more literally. Surprise, surprise. The fabric unravels to give us all those things about the quantum world that we find utterly baffling, and we're right back where we started.

My purpose in this book is (hopefully) not to spoil your fun, but to try to explain what it is about quantum mechanics that forces us to confront this kind of choice, and why this is entirely philosophical in nature. Making different choices leads to different interpretations or even modifications of the quantum representation and its concepts, in what I call (with acknowledgements to George R. R. Martin) *the game of theories.*

Part I opens with a brief summary of everything you might need to know about quantum mechanics, which should hopefully set you up for what follows. I will then tell you about the rules of the game, based on a pragmatic but perfectly reasonable understanding of what we mean by 'reality', and the kinds of things we can hope to learn from a scientific representation of this. Part I concludes with Albert Einstein's great debate with Niels Bohr in the late 1920s and early 1930s, and the emergence of the anti-realist Copenhagen interpretation, which admirably sets the scene.

We will then go on in Part II to look at various attempts to play the game, from the legacy of Copenhagen, through relational quantum mechanics, to interpretations based on quantum 'information'. We will look at attempts to redefine quantum probability, by reformulating the axioms of quantum mechanics, introducing the notion of consistent histories, and quantum Bayesianism. We then turn our attention to realist interpretations based on the idea that quantum mechanics is a statistical theory. These include hidden variable theories of local (Bell's inequality), and 'crypto' non-local (Leggett's inequality) varieties.

Experimental evidence gathered over the past forty years or so comes down pretty firmly against local and crypto non-local hidden variables. So we turn to interpretations based on non-local hidden variables (such as so-called 'pilot wave' theories) or we try to fix problems associated with the 'collapse of the wavefunction' by introducing new physical mechanisms, including a possible role for human consciousness. We conclude with the notion that the wavefunction is real but doesn't collapse, which leads to many worlds and the multiverse.

If you will indulge me, through all of this I will make use of a no doubt overfanciful analogy or metaphor.[1] This is based on the notion that the game of theories involves navigating the 'Ship of Science' on the perilous 'Sea of Representation'. Yes, I've obviously read too many fantasy novels.

We sail the ship back and forth between two shores. These are the deceptively welcoming, soft, sandy beaches of Metaphysical Reality and the broken, rocky, and often inhospitable shores of Empirical Reality. The former are shaped by our abstract imaginings, free-wheeling creativity, personal values and prejudices, and a variety of sometimes pretty mundane things we're obliged to accept without proof in order to do any kind of science at all. These become translated into one or more *metaphysical preconceptions*, which summarize how we think or even come to believe reality should be. These are beliefs that, by their nature, are not supported by empirical evidence. So, if you prefer you could think of these preconceptions as intuitions or even articles of faith, echoing one of my favourite Einstein quotes: 'I have no better expression than the term "religious" for this trust in the rational character of reality and in its being accessible, to some extent, to human reason.'[2]

Within the sea I have charted two grave dangers. The rock shoal of Scylla lies close to the shores of Empirical Reality. It is a rather empty instrumentalism, perfectly empirically adequate but devoid of any real physical insight and understanding. Charybdis lies close to the beaches of Metaphysical Reality. It is a whirlpool of wild, unconstrained metaphysical nonsense. The challenge to theorists is to discover safe passage across the Sea of Representation. In *Quantum Reality* I want to explain why this has proven so darn difficult, and why I have a very bad feeling about it.

So, welcome. You're here because you want some answers. Please take a seat and make yourself comfortable, and I'll go and put the kettle on.

PROLOGUE

Why Didn't Somebody Tell Me About All This Before?

My first encounter with quantum mechanics occurred in my very first term as an undergraduate, studying for a bachelor's degree in chemistry in a rather damp and gloomy Manchester, England, in the autumn of 1975.

Looking back, it's no real surprise that all the students in my class (me included) were utterly baffled by what we were taught. Until that moment, we had all been blissfully unaware that there was anything more to be learned about the physical world beyond the smooth continuity and merciless certainties of Newton's clockwork mechanics.

Our understanding of atoms was limited to the 'planetary model' associated with the names of physicists Niels Bohr and Ernest Rutherford. If we had thought about it at all (and I can tell you that we really hadn't), then we would have supposed that the classical theories we use to describe planets orbiting the Sun could simply be extended to describe little balls of electrically charged matter orbiting the central nucleus of an atom. Yes, the forces are different, but surely the results would be much the same.

But now we were told that the physics of atoms and molecules is governed by a very different set of laws, with which even chemists must come to terms. Nothing had prepared us for this. In our first lecture we chomped our way through Max Planck's discovery of the quantum, Einstein's 'light-quantum' hypothesis, Bohr's quantum theory of the atom, Louis de Broglie's wave-particle duality,* Erwin Schrödinger's wave mechanics, and Werner Heisenberg's uncertainty principle.

I thought my head was going to explode.

Mechanics is that part of physics concerned with the how and why of stuff that *moves,* governed by one or more mathematical *equations of motion*. In hindsight, our problems were compounded by the fact that the evolution of our understanding of classical mechanics had stopped with the school textbook version of Newton. We were not being trained to be physicists, and so missing from our education was the elaborate reformulations of classical mechanics, first by Joseph-Louis Lagrange in the eighteenth century, and then by William Rowan Hamilton in the nineteenth. These reformulations weren't simply about recasting Newton's laws in terms of different quantities (such as energy, instead of Newton's mechanical force). Hamilton in particular greatly elaborated and expanded the classical structure and the result, called *Hamiltonian mechanics,* extended the number of situations to which the theory could be applied.

We were therefore confronted not only with this extraordinary thing called the quantum *wavefunction,* but also with the challenge of writing down something called the 'Hamiltonian' for a specific physical system or situation, such as the orbit of an electron in an atom or the vibrations of a chemical bond holding

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* More than forty years later, I can still hear my lecturer pointing out as an aside that de Broglie is pronounced 'de Broy'.

two atoms together, without really understanding where either of these things had come from.*

But, make no mistake, I was completely hooked. I filled my notebooks with equations that looked... well, they looked *beautiful*. I still didn't really understand what any of it meant, but I learned how to use quantum mechanics as best I could and set aside any concerns. I went on to complete a doctorate at Oxford University and a couple of years of postdoctoral research at Oxford and at Stanford University in California, before returning to England to take up a lectureship in chemistry at the University of Reading. Although I was never blessed with any great ability in mathematics, I learned a great deal more about quantum mechanics from Ian Mills, professor of chemical spectroscopy in my department, and I take some pride in a couple of research papers I published on the quantum theory of high-energy molecular vibrations.

Then, in 1987, whilst working for a couple of months as a guest researcher at the University of Wisconsin-Madison, I happened upon an article that sent me into a tailspin. This was written by N. David Mermin.[1] It told of something called the Einstein–Podolsky–Rosen 'thought experiment', which dates back to 1935, and some laboratory experiments to probe the nature of quantum reality that had been conducted by Alain Aspect and his colleagues in 1982.

I felt embarrassed. I had come to this really rather late. *Why didn't somebody tell me about all this before?*

I had allowed my (modest) ability in the *use* of quantum mechanics to fool me into thinking that I had actually understood

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* I've written a technical book, suitable for readers with a background in physical science and some capability in mathematics, called *The Quantum Cookbook: Mathematical Recipes for the Foundations of Quantum Mechanics*. This was published by Oxford University Press in 2020 and I consider it a 'companion' to this volume. It is the book that I would have found really helpful when I was 18.

it. Mermin's article demonstrated that I really didn't, and marked the beginning of a 30-year personal journey. I'm now the proud owner of several shelves overflowing with books on quantum mechanics, science history, and philosophy, and a laptop filled with downloaded articles. I've written a few books of my own, the first published in 1992.

I can happily attest to the fact that, like charismatic physicist Richard Feynman, I still don't understand quantum mechanics.[2] But I think I now understand *why*.

PART I

THE RULES OF THE GAME

1

THE COMPLETE GUIDE TO QUANTUM MECHANICS (ABRIDGED)

Everything You've Ever Wanted to Know, and a Few Things You Didn't

Here's what I've learned over the past forty years or so.

Nature is lumpy, not smooth and continuous

We now know that all matter is composed of atoms. And each atom is in turn made up of light, negatively charged electrons 'orbiting' a nucleus consisting of heavy, positively charged protons (two up quarks and a down quark), and electrically neutral neutrons (one up quark and two down quarks).[*] Atoms are discrete. We can say that they are 'localized'. Atoms are 'here' or 'there'. In itself this is not particularly revelatory.

But despite what a few ancient Greek philosophers had argued, two and a half thousand years before, towards the end of the nineteenth century atoms were really rather controversial. After

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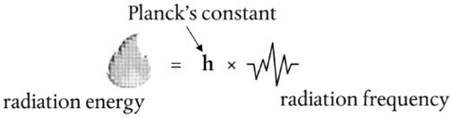
[*] I've put 'orbiting' in inverted commas because the electron doesn't orbit the nucleus in the same way that the Earth orbits the Sun. In fact, it does something a lot more interesting, as we'll soon see.

all, why believe in the existence of atoms when you can never hope to see them or gain any kind of evidence for them?

In fact, it was a determination to refute the existence of atoms that led Max Planck to study the properties and behaviour of so-called 'black-body' radiation trapped inside cylindrical vessels made from platinum and porcelain.* When such a vessel is heated, its interior glows like a furnace. As the temperature rises, the light radiation released inside glows red, orange-yellow, bright yellow, and ultimately brilliant white. Planck was interested in finding a theory to describe the variations in the pattern and intensities of different frequencies (or wavelengths, or colours) of the radiation as the temperature is raised.

What Planck found in an 'act of desperation' in 1900 turned him into a committed atomist, but it took a few more years for the real significance of his discovery to sink in. Planck had concluded that the radiation inside the cavity is absorbed and emitted by its walls as though it is composed of discrete bits which he called *quanta*. This is summarized in an equation now known as the Planck–Einstein relation:



This might not seem very profound. But think about it. Radiation frequency is smoothly and continuously variable—there are no sudden jumps or breaks in the rainbow spectrum of colours: instead they blend seamlessly from one colour to another. If

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* 'Black-body' doesn't refer in any way to the colour of the walls of the cavity, but rather to the way they absorb and emit the radiation trapped inside. In theory, a 'black' body absorbs and emits radiation 'perfectly', meaning that the radiation doesn't depend on what the walls are made of.

energy = radiation frequency, then this would imply that energy too must be smooth and continuous. But this is not what Planck had found. For any given frequency Planck's constant (represented by the letter *h*) represents the smallest amount of energy that can be absorbed or emitted by an object. Energy isn't taken up or given out smoothly and continuously by the object, but rather in discrete bits determined by *h*. Planck's constant is the telltale sign of all things quantum.

Planck initially attributed this behaviour to the atomic nature of the material that formed the walls of the vessels. But it was Einstein who really launched the quantum revolution when in 1905 he rather outrageously suggested that the *radiation itself* is 'quantized' in localized, discrete bits or lumps of energy. This is Einstein's 'light-quantum' hypothesis, and it is the reason why we refer to the above relation today using both Planck and Einstein's names. He was right, of course. We now know these lumps of light-energy as *photons*.

So it's not only matter that comes in lumps, but also radiation. Put more and more energy into an electron inside an atom and it will 'orbit' the nucleus at greater and greater average distances until it is ripped out of the atom completely. But you can't increase this energy smoothly and continuously. The electron will absorb energy only at very discrete intervals organized in an *atomic spectrum* (see Figure 1).

These intervals form a ladder with rungs in a distinctive pattern. It was Niels Bohr in 1913 who figured out that this pattern is characterized by one or more *quantum numbers* and, unlike a real ladder, the quantum rungs get closer and closer together the higher in energy you go. Pump just the right amount of energy into an electron in an atom, sufficient to climb from one rung to the next, and the electron's orbit appears to change *discontinuously*, in a 'quantum jump'.

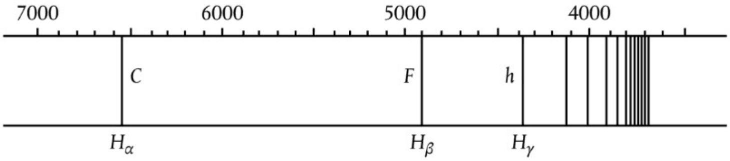

Figure 1 This picture shows a series of lines in the atomic spectrum of hydrogen, which consists of a single proton orbited by a single electron. Energy increases from left to right, and the spectrum shows that energy is not absorbed or emitted continuously, but only in discrete amounts. This spectrum appears in a 1910 textbook *Lærebog i Physik*, by Christian Christiansen, who taught Niels Bohr at the University of Copenhagen. Wavelengths are recorded in ångstroms (tenths of a nanometre or billionth of a metre) along the top, with the characteristic spectral lines H_α (656.3 nanometres—red), H_β (486.1 nanometres—blue) and H_γ (434.0 nanometres—violet) clearly marked.

So far as we know, there is nothing in reality that isn't quantized, possibly including space and time.

Waves are particles and particles are waves

I have to admit that French physicist Louis de Broglie is a bit of a hero of mine. He seems to have had little impact on science after winning the Nobel Prize in 1929, but his contribution six years earlier was more than enough to leave a lasting mark on human history.

Einstein's light-quantum hypothesis was greeted with considerable scepticism at the time. When he was recommended for membership in the prestigious Prussian Academy of Sciences in 1913, its leading members—Planck among them—acknowledged his remarkable contributions to physics, which by this time included his special theory of relativity (the general theory would follow a few years later). In accepting his nomination, they were prepared to forgive his lapses of judgement: 'That he may sometimes have missed the target in his speculations, as, for example, in his hypothesis of light-quanta, cannot be really held too much against him, for it is not possible to introduce really new ideas even in the most exact sciences without sometimes taking a risk.'[1]

But in his short paper, Einstein had suggested that it might be possible to gain evidence for the quantum nature of light by studying the *photoelectric effect*. Shine light on metal surfaces of a certain frequency and intensity and electrons will get kicked out. Now, the energy of a classical wave is related to its amplitude—the height of its peaks and depths of its troughs—think of the difference between gently rolling surf and a tsunami. This energy is reflected in the *intensity* of the wave or, if you prefer, its *brightness*. If, as everybody believed, light is described purely in terms of waves, then increasing the light intensity increases the energy and should therefore smoothly increase the number and energies of the electrons ejected from the surface.

But this is not what was observed in early experiments. The Planck–Einstein relation suggests that it is the light frequency—not the intensity—that is all important. Light of the wrong frequency, no matter how intense, just won't cut it. Only light-quanta (photons) with sufficient energy will knock the electrons from the surface. Increasing the intensity of the light simply increases the number (but not the energies) of the ejected electrons.

At the time this kind of behaviour was very counterintuitive, but it was nevertheless shown to be correct in further experiments performed about ten years later, and led to the award of the Nobel Prize in Physics to Einstein in 1921.

This was a great achievement, but it also posed a big problem. There was an already well-established body of evidence in favour of a wave theory of light. Push light of a single colour through a narrow aperture or slit, cut with dimensions of the order of the wavelength of the light, and it will squeeze through, bend around at the edges and spread out beyond. It 'diffracts'. A photographic plate exposed a short distance away will reveal a diffuse band, rather than a narrow line with the same dimensions as the slit.[*]

Cut two slits side by side and we get *interference*—evidenced by alternating bright and dark bands called interference fringes. As the waves from both slits spread out and run into each other, where wave peak meets wave peak we get a bigger peak (which we call constructive interference), and where peak meets trough we get cancellation (destructive interference)—see Figure 2a. Constructive interference gives rise to the bright fringes. Destructive interference gives rise to the dark fringes. This kind of behaviour is not limited to light—such wave interference is fairly easily demonstrated using water waves, Figure 2b.

But waves are inherently *delocalized*: they are here *and* there. Einstein's light-quantum hypothesis didn't negate all the evidence for the delocalized wave-like properties of light. What he was suggesting is that a complete description somehow needs to take account of its localized particle-like properties, too. He had some ideas about how this might be done, and we'll come to consider these later on in this book.

Okay, so light exhibits some peculiar behaviours, but matter must surely be different. It's fairly straightforward to show that material particles such as electrons behave pretty much how we would expect. For example, we observe distinct tracks in a device called a cloud chamber—see Figure 3a.[†] This picture shows a

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[*] However, look closely and you'll see that the edges of this band show a distinct *diffraction pattern* of alternating light and dark 'fringes'.

[†] The cloud chamber was invented by Charles Wilson. It works like this: an energetic, electrically charged particle passes through a chamber filled with

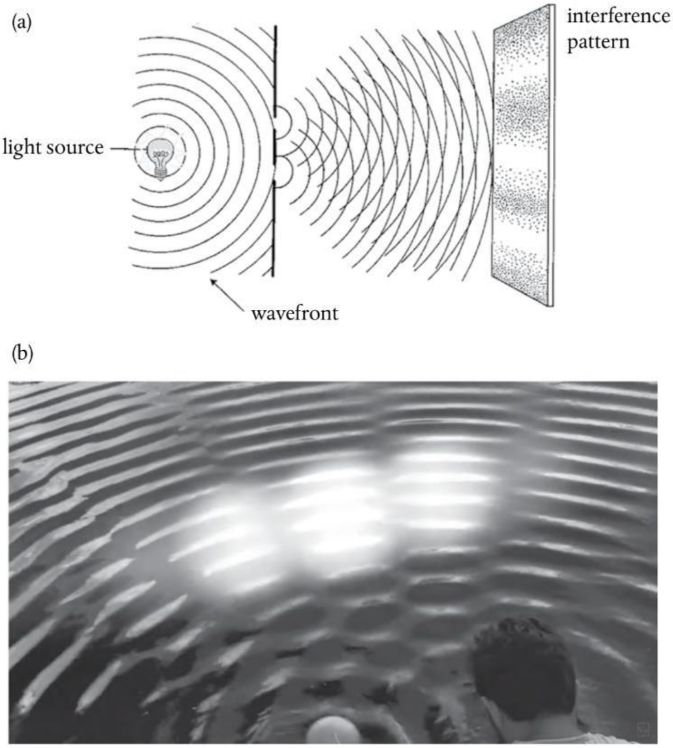

Figure 2 (a) When passed through two narrow, closely spaced slits, light of a single wavelength produces a pattern of alternating light and dark fringes. These can be readily explained in terms of a wave theory of light in which overlapping waves interfere constructively (giving rise to a bright fringe) and destructively (dark fringe). (b) Such interference is not restricted to light, and can be fairly easily demonstrated with water waves.[2]

bright track left by a positively charged alpha particle (the nucleus of a helium atom, consisting of two protons and two neutrons), and a series of fainter tracks left by negatively charged electrons, their curved motions caused by the application of a magnetic field.

vapour. As it passes, it dislodges electrons from atoms in the vapour, leaving charged ions in its wake. Water droplets condense around the ions, revealing the particle trajectory.

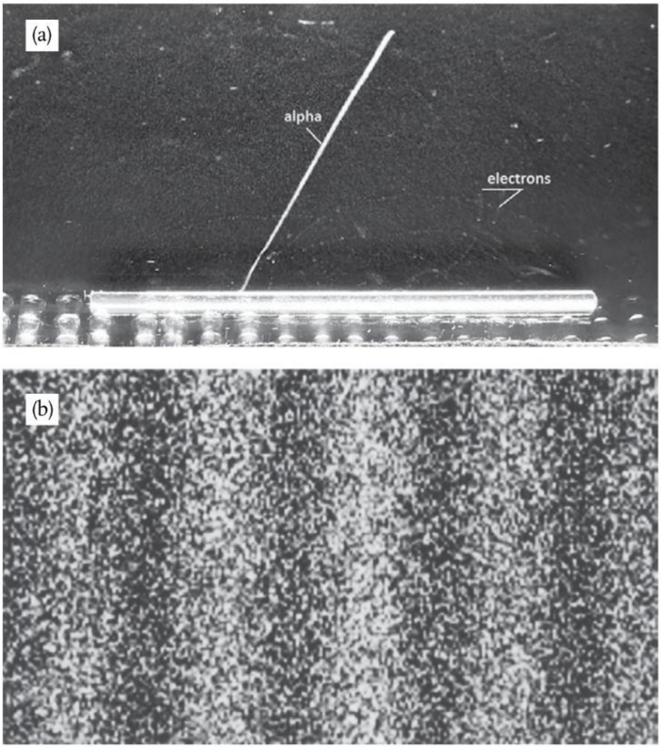

Figure 3 (a) Tracks caused by the motions of an alpha particle and electrons ejected from a rod of radioactive thorium inside a cloud chamber. (b) A two-slit interference pattern generated using electrons.

The simplest explanation for such tracks is that they trace the paths or *trajectories* of individual particles as they pass through the chamber.

And here we come to de Broglie's historically profound insight. Why force a distinction? If light waves can also be particles (photons, though this name hadn't yet been coined), *could particles like electrons also be waves*? The idea seems completely absurd and, indeed, some physicists dismissed it as 'la Comédie Française'. We're so used to thinking of elementary particles like electrons as small, localized bits of charged matter that to imagine them any other way requires some considerable mental effort.

Readers familiar with televisions in the days before plasma and LCD screens may recall that these consisted of one or more electron or 'cathode ray' guns, each of which would produce a beam of electrons. The beams were then accelerated and modulated to produce broadcast images on a phosphorescent screen.

So, imagine we pass a narrow beam of electrons through a plate in which we've cut two small, closely spaced holes or slits.

Our instinct might be to imagine that in a two-slit experiment, the electrons in the beam will follow paths through either one slit or the other, like machine-gun bullets, producing two bright lines on the screen marking where the electrons have passed through. We would expect each line to be brightest in the centre, showing where most of the electrons have passed straight through the corresponding slit unimpeded, becoming a little more diffuse as we move away, signalling electrons that have caught the edges of the slit and scattered on their way through. But these experiments have been done, and this is not what we see. Instead of two bright lines characteristic of particles following straight paths through the slits, we get a two-slit interference pattern—Figure 3b.

Electrons can also be waves.

De Broglie's idea was just that—an idea. He was able to develop a direct mathematical relationship between a wave-like quantity—wavelength—and a particle-like quantity—linear momentum*—such that

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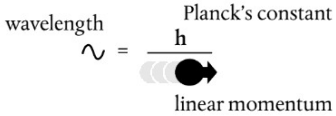
* In classical mechanics, linear momentum derives from the uniform motion of an object travelling in a straight line, calculated as the object's mass × velocity. However, in quantum mechanics, the calculation of linear momentum is rather different, as we'll see very soon.



But this was not a fully fledged wave theory of matter. That challenge fell to Erwin Schrödinger, whose formulation—first published early in 1926 and called wave mechanics—is still taught to science students today.

Everything we think we know about a quantum system is supposed to be summarized in its wavefunction

Schrödinger's theory is really the classical theory of waves in which we make use of the de Broglie relation to substitute wavelength for linear momentum. This requires a bit of mathematical sleight of hand and some assumptions that prove to be unjustified. Although Schrödinger published a much more obscure derivation, this is what it really boils down to. The result is *Schrödinger's wave equation.*

It's helpful to stop and think about this for a minute. The classical wave equation features a *wavefunction,* which you can think of as describing a familiar sine wave, oscillating smoothly and continuously between peak and trough. The wave equation then describes the motion of this wave in space and time. Into this, we've now injected Planck's constant and linear momentum, a very particle-like property. If we adopt the classical expression for momentum as mass × velocity, you can see that this is now a wave equation that features something that has a mass, and this gets incorporated into the solutions of the equation—the wavefunctions.

How can a wave have mass? This is just one mind-bending consequence of wave–particle duality. And we're just getting started.

What's quite fascinating about all this is that, right from the very beginning, physicists were scratching their heads about Schrödinger's wavefunction. It's pretty obvious how the wavefunction should be interpreted in classical wave theory but, aside from now featuring particle-like properties such as mass and momentum, in Schrödinger's wave mechanics the wavefunction had taken on an altogether different significance.

In classical mechanics, there are no real issues with the way we interpret the concepts represented in the theory. We think we know what mass is. We know what velocity and acceleration are. These are things we *observe* directly—by simple observation we can tell the difference between something moving slowly and something moving fast. When we put our foot down and go from nought to sixty in some incredibly short time, or when we loop the loop on a rollercoaster, we *feel* the acceleration. We can calculate linear momentum and we know what this means. These things, called physical 'observables', sit right on the surface of the classical equations of motion. We don't have to dig any deeper for some kind of hidden meaning for them. It's obvious what they are and how they should be interpreted.

But now look at what Schrödinger's wave mechanics asks us to do. You want to know the linear momentum of an electron moving freely through a vacuum? Then you need to solve the wave equation and identify the relevant wavefunction, determine the rate of change of this wavefunction in space, and multiply the result by minus the square root of minus 1 times Planck's constant divided by 2π.* This procedure returns the linear momentum

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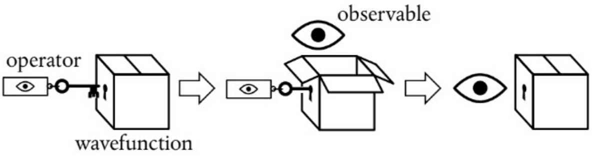
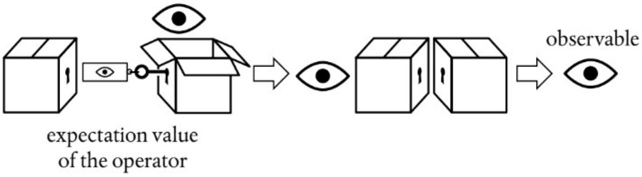
* The square root of minus 1 is an 'imaginary number', usually written as *i*. This might seem obscure, but it crops up all the time in mathematics and physics. All you need to remember is that $i^2 = -1$.

multiplied by the wavefunction, from which we can then deduce the momentum.

In Schrödinger's wave mechanics (and, more generally, in quantum mechanics), we calculate observables such as momentum and energy by performing specific mathematical *operations* on the relevant wavefunction. Such manipulations are then summarized collectively as *operators* for the observables. The operators are mathematical recipes, which we can think of as 'keys' which unlock the wavefunction (depicted below as a box), releasing the observable before closing again. The logic is as follows:



The description given in the paragraph above summarizes the mathematical operator (the key) for linear momentum in quantum mechanics. There's one further small step. I won't give the details here, but it is quite straightforward to deduce something called the *expectation value* of the operator, which is a kind of average value. It has the helpful property that



When two identical boxes face off against each other as 'mirror images', as shown above, and if everything is being done properly, then these combine together to produce the result 1. This leaves us with just the observable, so the expectation value provides a useful recipe for calculating the values of observables such as momentum and energy.

Whoa. You don't need to be a rocket scientist to realize that something has fundamentally changed. It's as though nature has chosen to hide its secrets in the quantum wavefunction, hence the locked box pictogram. To discover the value of an observable, we need to open the box with the right key (the operator). Opening the box with one kind of key gives us one kind of observable, such as momentum. A different observable will require a different key.

We never had to do anything like this in classical mechanics. The observables were always right there, in front of us, staring us in the face.

No, seriously, electrons really do behave like waves

At this point I want to bring you back to the electron interference pattern shown in Figure 3b. We might shrug our shoulders at this, and acknowledge the wave nature of electrons without thinking too deeply about what this might mean. But let's push the experiment a stage further. Let's wind down the intensity of the electron beam so that, on average, only *one* electron passes through the slits at a time. What then?

What we see is at first quite comforting. Each electron passing through the slits registers as a single bright dot on the phosphorescent screen, telling us that 'an electron struck here'. This is perfectly consistent with our prejudices about electrons as particles, as it seems they pass—one by one—through one or other of the slits and hit the screen in a seemingly random pattern—see Figure 4a.

But wait. The pattern isn't random. As more and more electrons pass through the slits we cross a threshold. We begin to see individual dots group together, overlap, and merge. Eventually we get a two-slit interference pattern of alternating bright and dark fringes, Figure 4e.

We can quickly discover that if we close one or the other slit or try to discover which slit each individual electron passes through then we will lose the interference pattern. We just get behaviour characteristic of particles following straight-line paths. If we try to look to see how we get wave behaviour, we get particle

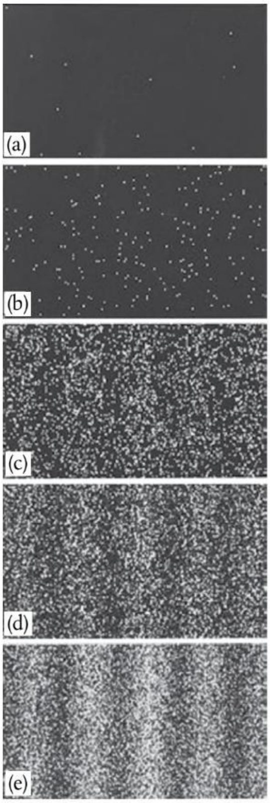


Figure 4 We can observe electrons as they pass, one at a time, through a two-slit apparatus by recording where they strike a piece of photographic film. Photographs (a) to (e) show the resulting images when, respectively, 10, 100, 3,000, 20,000, and 70,000 electrons have been detected.

behaviour. If we don't look to see how we get wave behaviour, we get wave behaviour. Left to itself, it seems that the behaviour of each electron must somehow depend on the existence of the slit through which it *does not pass,* which is decidedly odd.

Alternatively, we conclude that the wave nature of the electron is an *intrinsic* behaviour. *Each individual electron behaves as a wave,* described by a wavefunction, passing through both slits simultaneously and interfering with itself before striking the screen.

So, how are we supposed to know precisely *where* the next electron will appear?

The wavefunction gives us only probabilities: in quantum mechanics we can only know what might happen, not what will happen

A wave alternates between positive amplitude, largest at a peak, and negative amplitude, largest at a trough. We calculate the intensity of the wave as the *square* of its amplitude, which is always a positive number. So, in two-slit interference described purely in terms of waves we imagine a resulting wave which, when squared, produces a pattern which alternates between regions of high intensity (bright fringes) and zero intensity (dark fringes), as shown in Figure 5a.

But, by its very nature, this pattern of intensity is spread across the screen. It is *distributed* through space, or delocalized. And yet we know that in the experiment with electrons, as illustrated in Figure 4, we see electrons detected one at a time, as single bright spots, in only *one* location on the screen. Each electron hitting the screen is localized.

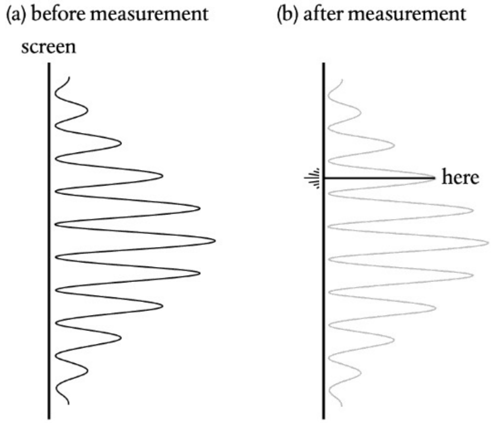


Figure 5 (a) Before measurement, the square of the electron wavefunction predicts a distribution of probabilities for where the electron might be found, spread across the screen. (b) After measurement, the electron is recorded to be found in one, and only one, location on the screen.

How does this work?

Schrödinger had wanted to interpret the wavefunction literally, as the theoretical representation of a 'matter wave'. He argued that atoms are simply the diffraction patterns of electron waves captured and wrapped around atomic nuclei. But to make sense of one-electron interference we must reach for an alternative interpretation suggested later in 1926 by Max Born.

Born reasoned that in quantum mechanics the square of the wavefunction is a measure not of the intensity of the electron wave, but of the *probability* of 'finding' its associated electron.* The alternating peaks and troughs of the electron wave translate into a pattern of quantum probabilities—in this location (which will become a bright fringe) there's a higher probability of

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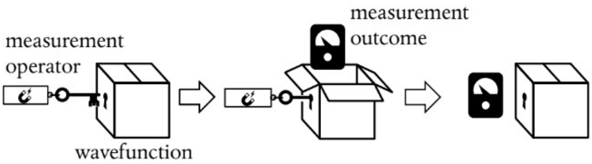
* To be clear, because the wavefunction might contain i, the square root of -1, we multiply it by its *complex conjugate,* in which i is replaced by $-i$ (since $-i \times i = -i^2 = +1$), so we always get a positive result. This is called *modulus square* of the wavefunction. It is actually the two locked boxes facing off against each other.

finding the next electron, and in this other location (which will become a dark fringe) there's a very low or zero probability of finding the next electron.

Just think about what's happening here. Before an electron strikes the screen, it has a probability of being found 'here', 'there', and 'most anywhere' where the square of the wavefunction is bigger than zero.

Does this mean that an individual electron can be in more than one place at a time? No, not really. It is true to say that it has a probability of being found in more than one place at a time and there is definitely a sense in which we think of the electron wavefunction as delocalized or distributed. But if by 'individual electron' we're referring to an electron as a particle, then there is a sense in which this *doesn't exist* as such until the wavefunction interacts with the screen, at which point it appears 'here', in only one place, as shown in Figure 5b.

That this might be a bit of a problem was recognized in the late 1920s/early 1930s by John von Neumann. If 'measurement' is just another kind of quantum process or transition, then von Neumann argued that this suggests a need for a 'measurement operator', such that



The measurement outcome is then just the expectation value of the measurement operator.

Just like the distributed interference pattern shown in Figure 5a, the wavefunction in question may consist of different measurement possibilities, such as the pointer of the gauge above pointing to the left or to the right. Von Neumann realized that there is nothing in the mathematical structure of quantum mechanics that explains how we get from many *possible* outcomes to just one *actual* outcome. So, to ensure that the structure is mathematically robust and consistent, he had no choice but to *postulate* a discontinuous transition or jump which gets us from the possible to the actual. This postulate is generally known today as the 'collapse of the wavefunction'. It is absolutely central to the ongoing debate about how quantum theory is to be interpreted.

Quantum probability is not like classical probability

One more thing. That there's a 50% probability that a tossed coin will land 'heads' simply means that it has two sides and we have no way of knowing (or easily predicting) which way up it will land. This is a classical probability born of ignorance. We can be confident that the coin continues to have two sides—heads and tails—as it spins through the air, but we're ignorant of the exact details of its motion so we can't predict with certainty which side will land face up.

Quantum probability is thought to be very different. When we toss a quantum coin* we might actually be quite knowledgeable about most of the details of its motion, but we can't assume that 'heads' and 'tails' *exists* before the coin has landed, and we look.

Einstein deplored this seeming element of pure chance in quantum mechanics. He famously declared that 'God does not play dice'.[3]

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* We'll see how this can be done in practice in later chapters.

For a specific physical system or situation, there is no such thing as the 'right' wavefunction

Physics is a so-called 'hard' or 'exact' science. I take this to mean that its principal theoretical descriptions are based on rigorous mathematics, not on words or phrases that can often be ambiguous and misleading. But mathematics is still a language, and although we might marvel at its incredible fertility and 'unreasonable effectiveness',[4] if not applied with sufficient care it is still all too capable of ambiguity and misdirection.

Centuries of very highly successful, mathematically based physics have led us to the belief that this is all about getting *the right answer*. Nature behaves a certain way. We do *this*, and *that* happens. Every time. If the mathematics doesn't predict *that* with certainty every time we do *this*, then we're inclined to accept that the mathematical description isn't adequate, and we need a better theory.

In quantum mechanics, we're confronted with a few things that might seem counterintuitive. But this is still a mathematically based theory. Sure, we've swopped the old classical observables such as momentum and energy for mathematical operators which we use to unlock their quantum equivalents from the box we call the wavefunction. But—to take one example—the frequencies (and hence the energies) of the lines in an atomic spectrum are incredibly precise—just look back at Figure 1. If quantum mechanics is to predict what these should be, then surely this must mean discovering the precise expression for the wavefunction of the electron involved?

And it is here that we trip over another of quantum mechanics' dirty little secrets. There is really no such thing as the 'right' wavefunction. All we need is a function that is a valid solution of the wave equation. Isn't this enough to define the 'right' one? No, not really. Whilst there are some mathematical rules we need to philosophers have happily challenged this authority, and we shouldn't assume that the version taught today will still be taught in another 90 years' time.

Eager readers will also note that I've deliberately held back some of the more infamous examples of quantum weirdness—such as Schrödinger's cat and the Einstein–Podolsky–Rosen experiment. Please be patient: we will come to these in Chapter 4. I first want to give you some context in which to think about them.

To summarize, we've seen that experimental discoveries in the first decades of the twentieth century led to the realization that physical reality is inherently lumpy. In the classical mechanics of everyday life, we can safely ignore Planck's constant and the lumpiness it implies, and assume everything is smooth and continuous. But at the level of molecules, atoms, and subatomic particles, from which everything in the visible universe is constructed, Planck's constant comes into its own and we can no longer ignore the duality of waves and particles.

De Broglie opened Pandora's Box in 1923. Schrödinger gave us his wave equation and his wavefunctions a few years later. The all-too-familiar observables of classical mechanics became locked away inside the quantum wavefunction, requiring mathematical operators to liberate them from their prison. Born said that the wavefunctions are utterly inscrutable; they tell us only about quantum probabilities. Heisenberg (and Bohr) explained that the heart of quantum mechanics beats uncertainly. Nature suffers a peculiar arrhythmia.

And then the debates began. What is quantum mechanics telling us about the nature of physical reality? And just what is this thing called reality, anyway?