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Jean Bricmont
Physics Department
UCLouvain
Louvain-la-Neuve
Belgium

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What Are the Issues Raised by Quantum Mechanics?

Although this book belongs to the "popular physics" category, its main purpose is cultural rather than scientific. We shall try to explain to the lay reader the basic principles of quantum theory, and emphasize their paradoxical nature, but our main goal is to unravel the incredible amount of confusion, pseudo-science and bad philosophy that accompanies most popular discussions of quantum mechanics.

But this will also plunge us into the deepest questions about our understanding of the world and of our place in it.

First of all, what is quantum mechanics? It is the theory of the elementary constituents of matter, such as atoms or electrons, and of radiation, that emerged in 1900 and was developed in the late 1920s. This theory has led to the most spectacularly well-confirmed predictions ever made in science. Some experimental results agree with the theoretical predictions up to one part in a billion. The theory underpins all modern electronics and telecommunications. It explains the stability of atoms and of stars, and lies at the foundation of the whole field of particle physics, as well as of solid state physics, chemistry, and thus, in principle, of biology. It is truly our most fundamental theory of nature. Yet, to quote the famous American physicist Richard Feynman, winner of the 1965 Nobel Prize in Physics, "nobody understands quantum mechanics" [79].[1]

While stunningly successful in its predictions and its practical applications, quantum mechanics has enjoyed a parallel career as alleged grounds for a wide

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[1] In this book, we shall not give too many or too long quotes. We refer those who want more precisions to my more detailed but more technical book *Making sense of Quantum Mechanics*, [36]. However, this book will be self-contained.

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range of speculations. It has been claimed that quantum mechanics proves the existence of God, free will, and the afterlife, or that it justifies belief in the direct influence of mind on matter and telepathy. There is a sort of "therapy" called quantum healing. Quantum mechanics has been linked to Jungian psychoanalysis, to vitalism, to all sorts of New Age beliefs, to Eastern mysticism and to dialectics (Hegelian or Marxist), among other systems of thought (see Chap. 11 for references).

Although most physicists dismiss these ideas as unscientific, there is no shortage of famous physicists, starting with Niels Bohr and Werner Heisenberg,[2] as well as many of their followers, who have claimed that quantum mechanics signals the end of "objective reality" or that, after the advent of quantum mechanics, physics no longer deals with reality but only with "our knowledge of it". We shall refer below to those views as those of the "Copenhagen" interpretation. This school of thought is named so because Bohr lived and worked in Copenhagen. There exists also a rather widespread impression that, thanks to quantum mechanics, a cat can be both alive and dead at the same time.

A number of physicists maintain that quantum mechanics implies the existence of multiple universes that proliferate endlessly, in which copies of ourselves live 'parallel' lives, each unaware of the others.

Another claim which is often made is that quantum mechanics shows that the deterministic world-view of classical physics is no longer tenable.[3]

To whet the reader's appetite, we shall start by quoting what some famous physicists have said about what quantum mechanics means, in particular concerning the disappearance of "objective reality". Of course, the quotes here may look strange, but we will explain later what motivates them.

Werner Heisenberg, one of the founding fathers of quantum theory wrote that:

> […] the idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees exist, independently of whether or not we observe them […] is impossible […]
>
> Werner Heisenberg [100, p. 129]

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[2] We refer to the Glossary for a brief biographical note on the scientists mentioned in this book. Niels Bohr was Danish and received the 1922 Nobel Prize in Physics, for his work on the structure of atoms, and Heisenberg was German and received the 1932 Nobel Prize in Physics, for his work on quantum mechanics.

[3] The notion of determinism will be explained in Chap. 3.

He added: “the natural laws formulated mathematically in quantum theory no longer deal with the elementary particles themselves but with our knowledge of them.” [101, p. 15]

Concerning Niels Bohr, the founder of the “Copenhagen interpretation”, Aage Petersen, who was his assistant for many years, characterized his views as follows:

> When asked whether the algorithm of quantum mechanics could be considered as somehow mirroring an underlying quantum world, Bohr would answer: “There is no quantum world. There is only an abstract physical description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature.”
>
> Aage Petersen [150, p. 12]

The German physicist Pascual Jordan, who was a very important contributor in the early days of quantum mechanics insisted that, if one measures the position of an electron: “the electron is forced to a decision. We compel it *to assume a definite position*; previously, it was, in general, neither here nor there; it had not yet made its decision for a definite position […].”[4] He made a similar statement concerning the measurement of velocity.

The American physicist John Archibald Wheeler, who studied with Bohr and who made important contributions both to nuclear physics and to cosmology, is famous for saying [200, p. 192]: “No elementary phenomenon is a phenomenon until it is a registered (observed) phenomenon.” He also wrote: “The past is not really the past until it has been registered. Or put another way, the past has no meaning or existence unless it exists as a record in the present.” [47, pp. 67–68].[5]

Eugene Wigner, co-recipient of the 1963 Nobel Prize in physics for his contributions to quantum and nuclear physics stressed “the essential role played by the consciousness of the observer” [201, p. 251], because of quantum mechanics.[6]

The American physicist and Cornell university professor David Mermin, well known for his work in statistical and condensed matter physics and who also worked a lot on foundations of quantum mechanics, wrote in 1981: “We now know that the moon is demonstrably not there when nobody looks.” [124, p. 397].

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[4] Reference [109], quoted and translated by M. Jammer [108, p. 161].

[5] We will come back to the ideas of Jordan and of Wheeler in Chap. 4.

[6] To be fair to Wigner, one must add that his ideas on the role of consciousness in quantum mechanics changed over time (see Esfeld [72]).

But not everybody agreed with the Copenhagen interpretation. Its most famous critics, before World War II, were Albert Einstein and Erwin Schrödinger.

In a letter to Schrödinger, Einstein referred to Bohr as the "Talmudic philosopher" for whom "reality is a frightening creature of the naive mind" [66]. Einstein also referred to Bohr as [68] "the mystic, who forbids, as being unscientific, an enquiry about something that exists independently of whether or not it is observed […]".

Schrödinger complained that "Bohr's […] approach to atomic problems […] is really remarkable. He is completely convinced that any understanding in the usual sense of the word is impossible." [171]. He also wrote: "If I were not thoroughly convinced that the man [Bohr] is honest […] I would call it intellectually wicked." […] [177]

Schrödinger did not even try to hide his feelings when he wrote to his friend Max Born, who was asserting "time and again that the Copenhagen interpretation is practically universally accepted": "Have you no anxiety about the verdict of history? Are you so convinced that the human race will succumb before long to your folly?" [178].

After World War II, the critique of the mainstream view was taken up for the main part by the American physicist David Bohm and the Irish physicist John Bell. The latter was once interviewed by people who recalled that: "We first asked Bell over the telephone whether he himself felt he had demonstrated that 'reality doesn't exist'. He responded by warning us that he is an impatient, irascible sort who tolerates no nonsense." [11, p. 86].

Can anybody seriously ask a physicist whether he has proven that "reality doesn't exist"?

We shall come back, in Chap. 10, to the historical disputes among physicists concerning quantum mechanics, but all this shows that there is indeed something very bizarre about quantum theory. The intention of this book is to separate the wheat from the chaff. We mean to explain, in the simplest possible terms, what is so bizarre about quantum mechanics, while also trying to show that its mysteries can nevertheless be understood in rational terms.

There are three main conceptual issues associated with quantum mechanics, to which we shall refer below as being *the three fundamental questions*.

1. *The role of the "observer"*. Since Copernicus, modern science has de-centered human beings from its explanations of reality, first by realizing that the Earth is not at the center of the Universe and then by showing that humans are not the object of a special act of creation, but rather the result of a lengthy contingent evolution. Quantum mechanics seems to have put humans back

at the center of the picture: it is sometimes claimed that it abolishes the distinction between subject and object or that it gives an active role to human consciousness within the theory. But if the human observer has a role in shaping reality, one must ask how reality was shaped before humans existed. If humans got there in the first place through evolution, how did that work? Biology is based on chemistry, whose mechanisms are explained ultimately through quantum mechanics. But what role could the human subject have had during this whole process, before its appearance as *Homo sapiens*?

2. *The issue of determinism.* Determinism means that future events are determined by preceding ones. So, if a system is deterministic and if its present situation is given, all its future states are fixed (see Chap. 3 for a more detailed discussion).
   However, as we shall see, quantum mechanical predictions are essentially statistical. This means that, if the present situation of a quantum mechanical system is given, quantum mechanics only assigns various probabilities to what the future state of that system may be. Does that imply that quantum mechanics signifies the end of a deterministic world-view? Does it explain or justify "free will"?
3. *The issue of locality.* One of our most basic experiences of the world is that when we act on it, we act on it *locally*. For example, I can act on something by touching it. I can communicate with someone else by speaking; but this means that a sound wave propagates from place to place between us. Even if I use radio, TV or the Internet to communicate, all these means rely on waves propagating at a finite speed from where I am to the recipient of my message. That is what one calls locality: every action from one place to another results from a propagation of something (waves for example) at a finite speed between those places.
   There is nothing in our experience of the world that suggests that one might act *instantaneously at a distance.*
   However, in quantum mechanics the non-existence of instantaneous actions at a distance is not so obvious. So, our third issue will be whether quantum mechanics does imply the existence of instantaneous actions at a distance. If yes, does that justify beliefs such as telepathy? And does that conflict with the theory of relativity's notion that "nothing travels faster than light"?

A first goal of the book will be to explain why quantum mechanics has raised such issues and to give the traditional answers to those questions. Roughly speaking, these answers are that quantum mechanics has given a fundamental role to measurements or observations within physics and has refuted determinism. On nonlocality, the traditional answers are ambiguous and often confused.

On the other hand, the answers that we will try to defend in this book are, in a nutshell:

1. *The role of the "observer"*. There is no need whatsoever to give a special role to the observer or even to observations in order to account for the quantum phenomena.
2. *The issue of determinism*. There is a way to account for the quantum phenomena in a deterministic theory, although a rather special one.
   Those answers to [1] and [2] are based on the works of Louis de Broglie, a French Nobel Prize in physics, David Bohm and John Bell.
3. *The issue of locality*. Certain facts discovered thanks to quantum mechanics do imply that there exist in Nature instantaneous actions at a distance. This discovery follows from an argument partly due to Albert Einstein, Boris Podolsky and Nathan Rosen and partly to John Bell. This does *not* justify unscientific beliefs, such as telepathy, but it does create a tension with the theory of relativity.

As we shall explain below, the main problem with the usual formulation of quantum mechanics is that it is perfectly capable of predicting, with spectacular precision, the statistical results of experiments (nobody denies that), but is not saying anything definite about what is happening in the physical world outside the laboratories. Physicists do have pictures of what is going on in the world, but those pictures are not part of the theory, which speaks only of what happens when quantum objects are being 'measured'. And, sometimes, these pictures are contradicted by logical but relatively unknown consequences of the quantum theory itself.

Since the views defended here are not considered orthodox by most physicists, there is a serious ethical problem in defending a heterodox view of science in a popular book. Why not first convince the scientific community before exposing one's own views to the general public? There are three answers to that objection: one is that I shall carefully distinguish between what is generally accepted and what is not.[7]

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[7] Chapters 2 and 4 are completely standard. Chapter 5 is based on standard but not very widely known results, and therefore the conclusions drawn there are not universally accepted. This is even more true of Chap. 7, which is based on easy-to-prove theorems and experimental facts, but where the conclusions are even less generally accepted than those of Chap. 5. Chapter 8 is the one chapter of this book that is highly controversial (but which is also central from our point of view). In Chap. 9 we criticize a popular but by no means generally accepted theory. Finally, Chaps 3, 6, 10 and 11 concern philosophy and history of science, subjects on which there is never a consensus.

The second answer is that there are many popular books explaining views different from mine and I shall refer to several of them, so that the reader can decide which view is the most plausible (see "Further Reading" at the end of the book). Finally, it is not really true that a "scientific consensus" exists on the issues discussed in this book. There used to be one, and it is still the basis of most textbooks on quantum mechanics. But right from the very beginning of the theory, there were famous dissenters, notably Einstein, but also de Broglie and Schrödinger. Later, David Bohm and John Bell were also critical of the orthodoxy, even though their voice was barely audible. But now, any conference on "foundations" of quantum mechanics will see a variety of positions and interpretations confronting each other, and none of them can claim to be either the orthodox view as presented in textbooks or a new orthodoxy.

It should also be emphasized that, contrary to popular books praising, say, alternative medicines, there is nothing "anti-scientific" in this book: we are not denying any application or experimental prediction of quantum mechanics. We are only concerned with what quantum mechanics means, not with its empirical correctness.

There is a cast of characters that will appear repeatedly in this book: Einstein, Bohr, Heisenberg, Schrödinger, de Broglie, Bohm, Bell, Feynman, Wheeler, Wigner, and many less important figures, arguing among themselves about the issues raised here.

By showing that science does not necessary produce a consensus over every major subject, in particular the one treated here, we hope to give a more positive image of science as an open endeavor rather than as a producer of dogmas. The uncertainties are challenges rather than weaknesses of the endless work in search of scientific knowledge. There is nothing anti-scientific in this view of science.

This book is not written especially for physicists, but if aspiring physics students read it, they are likely to be told during their studies that the issues raised here are irrelevant or "purely philosophical" or even "metaphysical". These claims are also found in the writings of physicists defending the mainstream views. Two arguments are often given to justify these claims:

1. The quantum theory works perfectly well, in all known circumstances; it is not contradicted by any experiment and leads to many technical applications.
2. The goal of physics is solely to predict results of experiments performed in laboratories and to produce technical applications.

The first point is correct, but it is precisely *because* it works so well that trying to understand why it works makes sense. Obviously if quantum mechanics worked half of the time, so to speak, there would be no reason to try to understand it in depth. Many models in physics are known to be applicable within certain limits and, once we know that, there are no further questions to be raised about those models. But quantum mechanics works on all known scales and is not contradicted by any experiment whatsoever.[8] Isn't it therefore worthwhile to ask *why* it works so well?

For the second point, there are several answers. The goal of science has always been, at least in part, to understand the world. Otherwise, why would anybody worry about the origin of the Universe or about distant galaxies? Certainly such studies, by themselves, have no technological applications. And of course, celestial mechanics, which gave rise to modern science, had no applications at all in its early development. Moreover, the theory of evolution had no application when it was introduced, even though it greatly changed our understanding of the world.

In fact, for most people, what is interesting in science is what it tells us about our vision of the world and of ourselves in it.

The idea that the only goal of physics is to predict results of experiments performed in laboratories inverts the means and the goal. Experiments are needed to test our theories in order to avoid falling into idle speculation or "metaphysics", but our theories are about the world, not about the experiments themselves.

Of course, it may be that it is simply impossible to understand the quantum world and that we have to content ourselves with predicting results of experiments. One could say that our experiments amount to "asking questions" about Nature; we do get answers and they can be predicted, at least statistically, but nothing more can be said; in particular, one cannot understand what is going on inside the experimental apparatuses.

Why not? After all, who are we but somewhat evolved creatures? Why should we expect to be able to understand the world as it is? Isn't the fact that quantum mechanics looks ununderstandable simply a consequence of the limitations of our minds? That may be the case, but one needs some argument to reach this pessimistic conclusion, rather than simply settling for the assertion.

Besides, there is a serious issue of coherence raised by the notion that the only goal of physics is to predict results of experiments performed in laboratories. If indeed that was all there is to physics, why do experiments in the first place?

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[8] Putting aside the problem of having a quantum theory of gravitation, which is a deep unsolved problem, but that cannot be regarded as a refutation of quantum mechanics.

The need to finance costly experiments is "sold" to politicians and the public by saying that we are discovering the fundamental laws of Nature. But if, in quantum mechanics, we give up the idea of understanding the world and restrict physics to be "exclusively about piddling laboratory operations", as John Bell puts it [12, p. 34], then how can we claim that we are trying to find the fundamental laws of Nature? What would the funders say if they read the statements by physicists who claim that their goal is merely to predict results of experiments performed in laboratories, and nothing else? Wouldn't they at least be puzzled and ask for some clarification? Isn't it therefore simply a matter of intellectual honesty to ask ourselves how we would clarify those statements? Physicists may have answers to those questions, but it may be worthwhile to see what they are and to discuss them.

Although nothing in this book will be very technical and we refer to [36] (and references therein) for more details, we shall put some extra material in the footnotes and appendices, either for the sake of precision or to provide the reader with more references.

We should also warn the reader that there will be nothing "fancy" in this book: no Big Bang, no Black Holes, no String Theory, no Quantum Gravity.... It is our contention that many of these fancy topics, about which several popular books have already been written, are difficult to grasp if the basic conceptual problems of quantum mechanics (the subject of this book) are not clarified first. Furthermore, we claim that, in order to achieve that clarification, it is sufficient to study the simplest physical situations.

We shall not quite follow the preacher's maxim: "First, tell them what you are going to tell them, then tell them, then tell them what you have told them", but we shall not refrain from repeating ourselves. This may be bad style, and we apologize to the reader who may be annoyed by repetitions, but we believe that it is easy enough to overlook a crucial point if it is only made once.

To keep a difficult subject as clear and simple as possible, we shall mainly rely on elementary drawings, illustrating both experiments and theory. The only mathematical concept that we use is the one of function, but only in very simple cases. The drawback of this approach is that it obliges us to ask the reader to take for granted some mathematical results, that will be stated verbally, without formulas.

Moreover, the book can be read in different ways. We put an asterisk on the title of sections that can be skipped at first reading and give a detailed summary at the end of each chapter for those who find that chapter either too difficult or too easy in order to allow them to continue with the rest of the book. Some readers might find it useful to start by reading the summary before reading the details of the chapter. Finally, all the main theses of the book will be summarized in Chap. 12 (the reader may want to jump to it to see where we are headed).

The intention of this book is not to give final answers to the conceptual problems of quantum mechanics, but rather to open the reader's mind to the possibility that answers can be given beyond what is taught in standard quantum mechanics courses or in most popular books on the subject. The student I once was, who could not understand what he was told about what quantum mechanics *meant*, would have been delighted to read such a book.

Before getting started, and although our goal is *not* to explain quantum mechanics as a physical theory but only to discuss its conceptual aspects, it may be useful to explain briefly where quantum mechanics came from (but reading this section is not necessary to understand the rest of the book).

1.1 Historical Background*

1.1.1 Pre-quantum Physics

Painting in broad strokes, we shall distinguish four periods in the history of physics, before quantum mechanics. First, the Newtonian revolution in the seventeenth and eighteenth centuries gave us laws that govern the motion of planets, projectiles, satellites etc. It all relied on the law of universal gravitation saying that bodies attract each other through a force proportional to the product of their masses and decreasing with the square of their distance. This, plus the idea that the acceleration of a body is proportional to the force exerted on it by other bodies allowed Newton and his followers to derive the trajectories of the planets in the solar system, that had been previously stated by Copernicus, Kepler and others.[9]

This was one of the major conceptual revolutions in the history of mankind: while previously various disciplines could record empirical regularities, it was only Newton (and other scientists at that time) who could use mathematical formulas (that Newton largely developed himself)[10] to compute and predict how objets will behave in hitherto unobserved circumstances (like launching a new projectile).

In the nineteenth century, there were two new developments. First, the discovery of new forces that were not of a gravitational nature: electricity and

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[9] See Appendix 7. A for more details.

[10] This was the theory of derivatives and integrals, also developed independently by Leibnitz.

magnetism. After several stages, the laws governing those forces were unified by the Scottish physicist James Clerk Maxwell into a theory called electromagnetism. The latter postulates the existence of waves, also called fields, that are created by charged particles but that also guide their motion. Light is an example of an electromagnetic wave, but there are many others, like X-rays, or radio and TV waves. Roughly speaking, the emitter of a radio or TV station transforms words and pictures into electromagnetic waves and the latter produce in your radio or TV a motion of charged particles, electrons, that then generates sound and light.

One could think of those waves as water waves on which a little boat (the charged particle) floats, except that (and that is a big difference!) here there is no water: those waves are supposed to propagate in a vacuum. This, as well as the nature of the gravitational force which also acts without any medium, is quite mysterious and we shall come back to that in Chap. 6.

The third period was the development of statistical methods. The industrial revolution was advanced by the steam engine and thermodynamics was describing how those engines work.[11] During the second half of the nineteenth century, thanks to the combined work of the Austrian physicist Ludwig Boltzmann, the American physicist William Gibbs and of Maxwell and Einstein, one managed to explain the laws of thermodynamics by applying statistical reasonings to the motions of myriads of molecules or atoms, whose very existence was disputed at that time.[12]

The last stage, which was quite revolutionary, was marked by the two theories of relativity, the so-called special one, developed in 1905, due mainly to the works of the Dutch physicist Hendrik Lorentz, the French mathematician Henri Poincaré, and Albert Einstein, and the general one developed around 1915, due mainly to Einstein and the German mathematician David Hilbert.

The special theory of relativity basically modified Newton's laws of motion in order to make them compatible with the newly discovered laws of electromagnetism. Indeed, when electromagnetism was developed, it was realized that this theory was not compatible with some aspects of the classical laws of Newton and that was a major problem. On the other hand, the general theory of relativity replaced Newton's theory of gravitation. We shall briefly discuss the meaning of the special theory of relativity in Sect. 7.7.

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[11] The most famous law of thermodynamics is the one saying that entropy increases, but we shall not discuss it.

[12] The existence of atoms was definitively established by Einstein in 1905 when he derived the laws of motion of objects that are very small but visible with a microscope, bombarded by those invisible atoms (this motion is known as Brownian motion).

1.1.2 Quantum Physics

Quantum physics emerged from troubles within the classical world view sketched in the previous subsection. Those came from different sources. One of them was the so-called specific heats of solids, which is the way the temperature of a body changes when it absorbs a certain amount of heat. There was a well defined classical prediction for those quantities that turned out to be completely wrong at low temperatures. Similarly, there was a type of electromagnetic waves[13] whose behavior was radically at variance with what was classically expected.

However, these worries did not look serious enough to cause a major revolution.

That second problem was "solved" in 1900 by the German physicist Max Planck, who decided, in a completely ad hoc way, to treat those waves, whose energies were previously thought to take a continuum of values, as if they were made of integer multiples of a fixed amount of energy, called "quanta" of energy, and related to the frequency of the wave; in that way, he was able to deduce the observed behavior of those waves. That was a major progress, but with no understanding of why it worked. Planck received the Nobel Prize in Physics in 1918 for that discovery.

A next step was taken by Einstein in 1905, with his explanation of the photoelectric effect, namely the fact that light can kick electrons out of atoms only if it has a sufficiently high frequency (classically, one would expect that this kicking out phenomena would depend on the intensity of the light and not on its frequency). This was explained by Einstein by postulating that light is made of some sort of particles, quanta of light, called photons, whose energy is proportional to their frequency, so that a high frequency would mean a high energy of the photons, hence an ability to kick out electrons. Einstein was awarded the Nobel Prize in Physics in 1921 for "his discovery of the law of the photoelectric effect".[14]

In 1907, Einstein applied Planck's method of quanta of energies to account (more or less) for the specific heats of solids. His method was refined in 1912

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[13] Called the black body radiation, namely the radiation inside a closed cavity whose walls are opaque to radiation.

[14] One might wonder why Einstein did not get the Nobel Prize for his two theories of relativity. This is because they were still considered somewhat philosophical, even though, by 1921, they had several experimental confirmations. In particular the criticism of relativity by the philosopher Henri Bergson played a role in the attitude of the Nobel committee, even though Bergson's objections were based on a complete misunderstanding of Einstein's theory, see http://nautil.us/issue/35/boundaries/this-philosopher-helped-ensure-there-was-no-nobel-for-relativity and [38].

by the Dutch physicist Peter Debye, with results quite in agreement with observations.

In 1913 came Bohr's model of the atom, which is still taught in elementary physics and chemistry classes. This model accounted for the fact that the radiation emitted by atoms was again taking a discrete set of values rather than a continuous one (a discrete set is, for example, any finite set or the set of integers $1, 2, 3\ldots$ or their inverses $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$). His model described the atom as a miniature solar system, with the nucleus in place of the sun and the electrons circling around it like the planets. But they circled on a well-defined set of orbits, having different energies. The discrete values of the emitted energies came from electrons jumping from one orbit to another and emitting the energy difference between those orbits.

This was another major success in accounting for the observed phenomena, but still without any theoretical understanding of why it worked.

Things were so puzzling that in 1911 Einstein, seeing an insane asylum in Prague, said to the physicist and philosopher Philip Frank: "These are the madmen that do not occupy themselves with the quantum theory."[15]

What is described here is called the old quantum theory, or the pre-quantum theory. The breakthrough came during the years 1924–1927. First Louis de Broglie suggested that, in the same way that waves such as light might be associated with particles, the photons (as was suggested by Einstein), matter particles such as electrons might be associated with waves, but he did not have a full-fledged theory about those waves (we shall discuss this idea in depth in Chap. 8).

Then, independently of de Broglie and of each other, first Werner Heisenberg, and slightly later Erwin Schrödinger, developed the modern quantum theory. Heisenberg found a way to compute in a concrete situation the discrete values taken by the energy of a system. This was then generalized by him together with other German physicists, Max Born and Pascual Jordan. Important work in that direction was also due to the Swiss physicist Wolfgang Pauli and the British physicist Paul Dirac.[16]

Schrödinger on the other hand associated to physical systems a mathematical concept, the wave function (discussed in Chap. 4) and wrote down equations telling how this object changes over time. He could also show that his method and the one of Born, Heisenberg and Jordan led to the same results.[17]

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[15] See [83, p. 98].

[16] Their method was basically algebraic and relied on the use of matrices, with which most physicists were not very familiar at that time.

[17] Schrödinger's method was more analytic and was based on differential equations. It is basically his method that is taught and used nowadays.

For a while, there was a great puzzlement about the meaning of the newly introduced concepts. But things were wrapped up, so to speak, at the Solvay Conference, held in Brussels in October 1927, where all the great physicists of the time met and the so-called "Copenhagen interpretation" was generally accepted.[18]

The transistor, on which modern electronics is based, was discovered thanks to quantum mechanics, which allows sometimes electrons to jump over a barrier that they could not go through classically. Lately quantum mechanics has found applications in cryptography, as well as in teleportation of information and quantum computing; we shall briefly discuss these applications in Sect. 7.6.

Quantum mechanics also allowed chemists to understand how atoms are bound together in molecules and it lays at the basis of atomic and nuclear theory.

After World War II, quantum mechanics was further extended to a quantum theory of electromagnetic waves and is at the basis of all the physics of elementary particles. It has also found numerous applications in astrophysics and cosmology.

1.2 Outline of the Book

Let us first say that this outline can be skipped over, as it is mainly intended to be a useful reference for the reader.

The first mystery raised by quantum mechanics is that, as physicists often say, quantum objects can be in two places at once or be in a "superposition" of states with different and mutually incompatible properties. There are good experimental and theoretical reasons why they employ this language and we shall explain them in Chap. 2.

This idea of superposition is at the basis of the notion that 'observations' play a central role in the physical theory, since, when an observation is made on a system which is in a superposed state, the system is supposed to 'jump' or to 'collapse' into one of those states (we never see objects having mutually incompatible properties). In other words, since we never observe directly a superposed state, observation is supposed to destroy these superpositions.

This will do justice to the way physicists often speak about quantum phenomena. The phenomena are strange and all the talk about observations

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[18] We shall discuss it throughout the book, but especially in Chaps. 4, 5 and 10.

affecting reality is not purely arbitrary, nor based on pure prejudice, although, as we shall see later, it is not inevitable either.

Of course, an obvious question is: who is "observing"? A purely physical device in the laboratory that records the result of an experiment or a human subject? This question is often quite central in popular discussions of quantum mechanics.

In Chap. 3, we will make a small "philosophical" detour in order to define what determinism means, what it implies concerning "free will" and how probabilities are used in physics.

Next we shall explain, in Chap. 4, the language used by physicists to predict what happens with these "superposed states".

We shall then try to understand, in Chap. 5, what that language means. We shall see that some natural ways to understand it unfortunately run into very serious difficulties.

Indeed, it is in relation to the phenomenon of superposition that Schrödinger introduced his famous *Gedankenexperiment*[19] of a cat who, if one follows the prescriptions of quantum mechanics to their logical conclusions, would be in a "superposed" state of being "both alive and dead". Of course, Schrödinger regarded this as a *reductio ad absurdum* of the quantum mechanical formalism.

In Chap. 6, we will make another "philosophical" detour in order to clarify certain notions such as "realism" and "observations", that are often encountered in discussions of quantum mechanics. We will argue that there is no way to solve the problems of quantum mechanics by modifying our philosophy, for example by giving up "realism".

In Chap. 7, we shall turn to the second mystery: the fact that quantum mechanics does imply that there exists a certain subtle form of action at a distance in the world. We shall also discuss to what extent this is compatible with Einstein's theory of relativity.

In Chap. 8 we shall explain, without mathematics, how to formulate quantum mechanics without referring to "observers". This is a theory due to the French physicist Louis de Broglie and the American physicist David Bohm. In two words, that theory says that 'matter moves' or, more precisely, that quantum particles do have positions at all times and therefore also have trajectories and velocities.

Contrary to what is often alleged, this is not contradicted by any known quantum facts or arguments.

The theory was introduced at approximately the same time as ordinary quantum mechanics and its "Copenhagen" interpretation, between 1924 and

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[19] This means a thought experiment, viz. an experiment that illustrates the theory and is not really performed in laboratory, but using a German word in English always makes things look more serious.

1927, by Louis de Broglie, but it was rejected at the time by a large majority of physicists, and ignored even by critics of the Copenhagen school, like Einstein and Schrödinger. The theory was then abandoned by its founder, only to be rediscovered and completed by David Bohm in 1952, then further developed and advertised by John Bell.

In this theory, there is no special role given to the observer or to the "measuring device". All the usual quantum predictions are recovered in the de Broglie–Bohm theory, and the nonlocality of the world is also to some extent explained by that theory.

The de Broglie–Bohm theory is far from being the only theory proposed as an alternative to ordinary quantum mechanics. A rather popular alternative is the idea of "many-worlds", which says that the Universe constantly splits into zillions of copies of itself, and, in each of these worlds, a copy of each of us lives an independent life, while being unaware of what happens in the other worlds.

While the "many-worlds" idea may have a certain science fiction kind of attraction in the minds of some people (hence, its popularity), we shall show in Chap. 9 that it is not really consistent.

It is natural to ask why the views advocated here, those of de Broglie, Bohm and Bell are often ignored. To explain that, one has to re-examine the history of quantum mechanics, which is what we shall do in Chap. 10. We claim that the ideas of Einstein, Schrödinger, de Broglie, Bohm and Bell were not really understood and, for that reason, were never really refuted. In their famous debate, Bohr did not really reply adequately to Einstein; Schrödinger's cat paradox was ignored; and the theories of de Broglie and Bohm were rejected without being examined. Finally, Bell's result on nonlocality was almost universally misunderstood.

Because of the philosophical problems to which it is linked, quantum mechanics has had a much greater cultural impact than most scientific theories. It has certainly inspired postmodernist philosophy as well as cultural studies, various pseudo-sciences and even contemporary art. In Chap. 11, we will make a brief tour of the ways in which quantum mechanics has been mixed with pseudo-science, mysticism, religions, philosophy, politics, ideology and social sciences and discuss to what extent this impact is due to misunderstandings and misrepresentations of quantum mechanics.

Finally, we will summarize our main theses of this book (Chap. 12) and provide a glossary of the main concepts encountered in the book, as well as some biographical details about the scientists encountered here. We will also give to the reader a bibliography of "further readings", including books written from a perspective different from ours.

2

The First Mystery: Interference

In *Alice Through the Looking-Glass*, the Queen says to Alice that she used to believe six impossible things before breakfast. In this book, we'll ask you to believe only two "impossible things" (before or after breakfast, as you wish): the *superposition of states* or the idea that particles can be in two different mutually incompatible states at once and *nonlocality*, which means that, in some circumstances, one can act at a distance, arbitrarily far, and instantaneously.

But, unlike what the Queen was speaking about, these "impossible things" are well-established facts. The first "impossible thing" will be explained in this chapter and in Chap. 4. We shall see in Chap. 8 that this "impossibility" is not only possible, but is not really that surprising. The second "impossible thing" will be explained in Chap. 7 and will be partly clarified in Chap. 8, but will nevertheless remain baffling.

We invite the reader to put herself or himself mentally in the shoes of a Sherlock Holmes or a Hercule Poirot and pay attention to what is really proven in the experiments below as opposed to what is often asserted in loose talk about them, and that we shall also explain.

As in every good detective novel, the reader has to be patient before learning about the denouement of the plot, which will come only in Chap. 8. Until then, we shall not try to demystify quantum mechanics too much, but rather explain the language physicists use and why they use it.

Indeed, before being demystified, the reader has to understand what is strange in quantum mechanics and, in some sense, "mystifying".

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2.1 The Double-Slit Experiment

To discuss the double-slit experiment, we shall follow the presentation by Feynman [78], and consider the behavior of three types of objects, bullets, waves and electrons, in a situation where they move towards a wall with two slits or holes in it, and where there is a second wall, behind the first one, on which the arrival of those objects is recorded.

Let us start with bullets. This is illustrated in Fig. 2.1: the little box on the left of each part (a, b, c) of the figure emits the bullets. They are sent one by one towards a wall with two slits in it, each of which can be open or not. If they miss the slits, they are absorbed by the first wall. If they pass one of the slits, they arrive somewhere on a second wall, behind the first one and that arrival is recorded. There is no aiming of the bullets towards one of the slits.

Since the bullets cannot always be sent with exactly the same initial position or the same initial velocity, there is some "random" distribution of the bullets on the second wall. In Fig. 2.1 one sees what happens when a set of bullets are sent when only the upper slit is open (part a), when only the lower slit is open (part b), and when both slits are open (part c). Each blue dot on the second wall represents the detection of one bullet, and the blue curves indicate the density of impacts of the bullets.

In part (c) of Fig. 2.1, when both slits are open, the density of impacts of bullets is simply the sum of the densities when each slit is open (see part (a) and (b) of Fig. 2.1).

There is no particular mystery here.

Consider now a wave, say a water wave, sent through the slits. Then, we get interference effects, shown in Figs. 2.2, 2.3 and 2.4. To understand that, think of throwing two pebbles in a pond, at some distance from each other. One

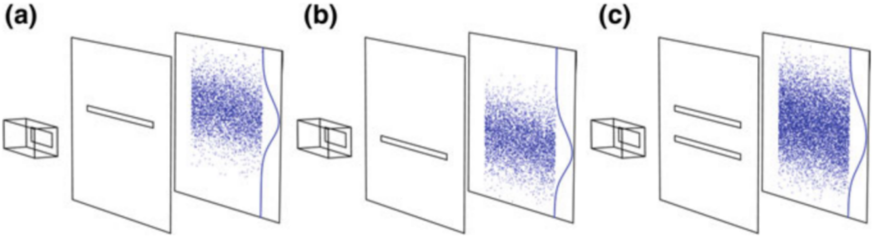

Fig. 2.1 The double-slit experiment with bullets. The little box on the left of each part **a**–**c** of the figure emits the bullets. Part **a** shows what happens when only the upper slit is open, part **b** when only the lower slit is open, and part **c** when both slits are open. Each *blue dot* on the second wall represents the detection of one bullet, and the *blue curves* indicate the density of impacts of the bullets (A. Gondran cc by-sa 4.0)

Fig. 2.2 The double-slit experiment with waves when only the upper slit is open. The intensity of the wave is shown in *white* (more intense) and *blue* (less intense). The *blue curve* on the right of the figure indicates the intensity of the wave on the second wall (A. Gondran cc by-sa 4.0)

Fig. 2.3 The double-slit experiment with waves when only the lower slit is open. The intensity of the wave is shown in *white* (more intense) and *blue* (less intense). The *blue curve* on the right of the figure indicates the intensity of the wave on the second wall (A. Gondran cc by-sa 4.0)

can see that the waves generated by the pebbles interfere: at some places the interference is constructive and the waves add to each other, at other places it is destructive and the waves subtract each other.

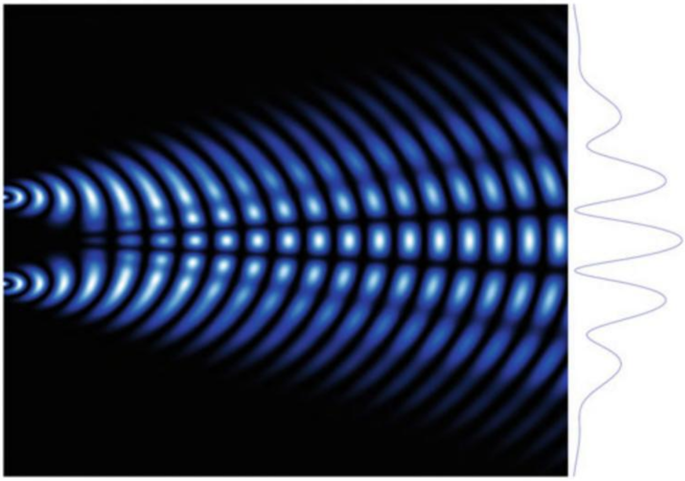

Fig. 2.4 The double-slit experiment with waves when both slits are open. The intensity of the wave is shown in *white* (more intense) and *blue* (less intense). The *blue curve* on the right of the figure indicates the intensity of the wave on the second wall. This exhibits an interference effect (A. Gondran cc by-sa 4.0)

In the two slits experiment with waves, if only one slit is open, we get the results of Figs. 2.2 and 2.3, which are not very different from what one gets with bullets, see parts (a) and (b) of Fig. 2.1.

But, when both slits are open, the wave goes through both slits and each slit acts as a source of a wave propagating towards the second wall (like the two waves produced by the two pebbles). The intensity of the wave indicated by the blue curve on the right of Fig. 2.4 is *not* the sum of the intensities of the waves indicated by the blue curves on the right of Figs. 2.2 and 2.3. Note that at some places the intensity of the wave on the second wall is less than what it is when only one slit is open, but that is exactly what one would expect for waves, because of interference.

In order to make the comparison with what happens with bullets easier, we have included in Fig. 2.5 a three dimensional picture with all three situations arising with waves (only the upper slit open, only the lower slit is open, and both slits open).

So far, so good; there is nothing surprising here and these two behaviors are called "classical": one for particles (bullets), one for waves.

The surprises come when one does the experiment with electrons. Electrons are little particles with a negative electric charge, and they surround the nucleus in atoms. When moving freely, they carry electricity.

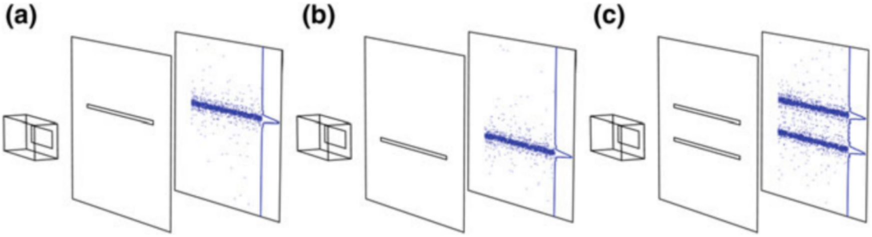

Fig. 2.8 The double-slit experiment with electrons, similar to Fig. 2.6, but where one puts the second wall sufficiently close to the first one (the distance between the walls is less by a factor of ten than in Fig. 2.6, although we have rescaled the figure so that this distance looks the same as in Fig. 2.6). We see that the interference pattern observed in part **c** of Fig. 2.6 essentially disappears (A. Gondran cc by-sa 4.0)

This is sometimes expressed by saying that, if we *look* or if we *know* through which slit the particle went, then it behaves like a particle, but if we do not know through which slit it went, it behaves like a wave.

And this leads to the famous expression: *the wave-particle duality*. Electrons are supposed to have a dual nature: sometimes they are particles, sometimes waves. Moreover, which "nature" they have seems to depend on whether we "look" at them or not!

In the age of Twitter, Sean Carroll, a theoretical physicist at the California Institute of Technology, who is also a cosmologist and author of several popular books, considers that the best answer to "how to summarize quantum mechanics in five words?" comes from the physicist and science writer Aatish Bhatia whom Carroll quotes:

> Quantum mechanics in 5 words. Don't look: wave. Look: particle.
>
> Sean Carroll [40, p. 35]

This is the first time that we refer to *our knowledge* of something as having apparently an impact on a physical situation. What this really means is of course one of the main subjects of this book!

To summarize, the double-slit experiment with electrons leads to an apparent *dead end*: indeed, what does each particle do?

1. Does it go through one slit? But then, why does the opening of the other slit affect the density of particles detected at a given point on the second wall? How come that this density is lower at some places when both slits are open than when only one slit is open?

2. Does it go through both slits? No, because one always detects the particle in one piece at a given place (there is no half-electron), and, by placing the second wall right next to the first one, one can determine through which slit the particle went, see Fig. 2.8.

These phenomena are sometimes described by saying that the particle goes through both slits when they are both open and through one slit otherwise. But what does it mean for a particle to go through two slits whose separation is far greater than the size of that particle? And how does the electron, while moving towards the wall with the slits, "know" whether one or both slits will be open, so as to know whether it should behave as a wave or as a particle?

This double-slit experiment is an example of what Niels Bohr called "complementarity": we can either check through which slit the particle went, when both slits are open, and then the particle behaves as a particle, or we can ignore which slit the particle went through, and then the particle behaves as a wave. But we cannot combine both pictures into a single coherent whole.

Note that "complementary" is used here in a non-habitual fashion: the word usually means that two pictures, say of a person viewed from the front and from the back, may "complement" each other in the sense that they yield a more precise image of that person. But one must stress that, for Bohr, the wave description and the particle one are "complementary" in the sense that they *exclude* each other.

In any case, these "ways of speaking" do not cast much light on what is really going on.

That the double-slit experiment is mysterious is acknowledged by most physicists. For example, in a standard textbook of quantum mechanics, written by two famous Soviet physicists, Lev Landau and Evgeny Lifshitz, one reads:

> It is clear that [the results of the double-slit experiment] can in no way be reconciled with the idea that electrons move in paths. [...] In quantum mechanics there is no such concept as the path of a particle.
>
> Lev Landau and Evgeny Lifshitz [114, p. 2]

And, after describing the double-slit phenomenon, Richard Feynman wrote:

> Nobody knows any machinery. Nobody can give you a deeper explanation of this phenomenon than I have given; that is, a description of it.
>
> Richard Feynman, [79, p. 145]

Coming back to the three fundamental questions raised in Chap. 1, what does the double-slit experiment suggest?

1. It suggests some sort of "reality created by the observer", since knowing through which slit the particle goes (by putting a detector behind one of the slits) seems to affect its behavior. But nothing can be concluded so far, as to what sort of "observer" we are talking about. Does it have to be a human subject or merely some purely physical interaction with the detector behind the slit in the first wall?
2. As for determinism, it seems that one cannot predict or control where the particle will be detected on the second wall. But that in itself may not be terribly surprising, since it is also true for the bullets, unless one is able to control very precisely their initial position and initial velocity. But one could expect that, the smaller the particle, the more difficult it is to control those values and that our incapability to predict where the electrons will be detected is thus not that strange and, by itself, does not prove indeterminism.
3. There seems also to be something nonlocal going on, since opening one slit affects the behavior of the particle going through the other slit (when only that slit is open), even if both slits are quite distant from each other. But there is no proof that the effect is instantaneous. Moreover, one can check that the size of the effect (the interference pattern) depends on how far apart the slits are and where the second wall is placed. For example, if the slits are far apart, or if the second wall is close to the first one, the interference effects will be small (see Fig. 2.8). So, we cannot conclude from this that any genuine nonlocality, namely any instantaneous action at a distance, exists.

2.2 Delayed Choices

John Wheeler invented a clever experiment, called the "delayed-choice" experiment, that makes the mystery of interferences even more troubling.

One can modify the double-slit experiment as follows (see Figs. 2.9 and 2.10): insert lenses behind the slits that will focus the incoming particles toward two counters C_1 and C_2 that may detect them. If one detects the particle on one of those counters, one will be tempted to conclude that the particle went through the upper slit if counter C_1 detects it, and that it went through the lower slit if counter C_2 detects it.[1]

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[1] As we will see in Chap. 4, quantum mechanics does not assign paths to particles so that saying that the particle went through one slit or the other is not really allowed by the usual formalism. Besides, we will see in Sect. 8.2 that, in a theory that does assigns paths to particles, one can determine through which slit the particle went, but the result is not the one stated here.

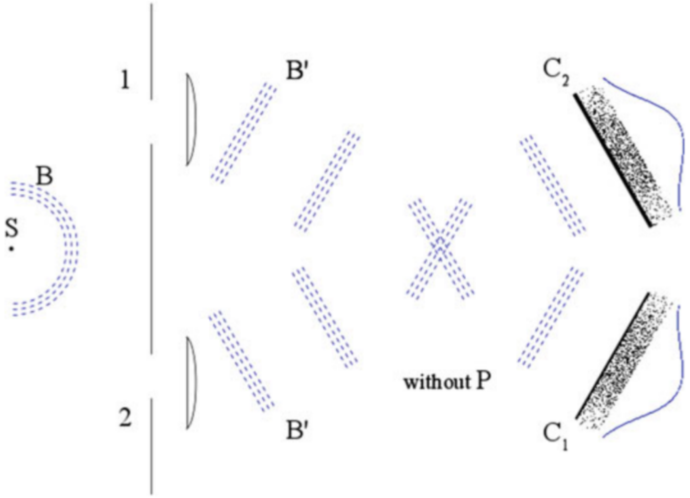

Fig. 2.9 The delayed double-slit experiment with electrons when both slits are open. The source indicated S sends a beam of particles denoted B towards both slits. One inserts lenses, behind the slits, that will focus the incoming beams of particles, denoted B', toward two counters C_1 and C_2 that may detect them. The resulting density of detections of particles on each counter is similar to what one obtains when only one slit is open, namely there are no interference effects. If one detects the particle on one of those counters, one will be tempted to conclude that the particle went through the upper slit if counter C_1 detects it, and that it went through the lower slit if counter C_2 detects it (A. Gondran cc by-sa 4.0)

But one can also insert a detection plate in the region where what appears to be the particles trajectories cross each other, as in Fig. 2.10 (the plate is denoted by P in that figure). Then, one will see an interference pattern as in part (c) of Fig. 2.6, and according to the standard way of speaking, one will say that the particle went through both slits.

But one can choose to insert the detection plate *after* the passage of the particles through the slits. So, it looks like we can decide whether the particle went through both slits or through only one of them by inserting or not the detection plate after the particle had supposedly decided to go through one slit or both!

This is the basis of the claim by Wheeler, that "the past is not really the past until it has been registered" [47, p. 68].

Moreover, Wheeler invented an ingenious scheme where such "experiments" would not take place in the laboratory, but on a cosmic scale: light sent by distant quasars can pass on either side of a galaxy [198]. The experiment here concerns photons instead of electrons, since light is composed of the former, but the phenomena are similar in both cases. The two sides of the galaxy are

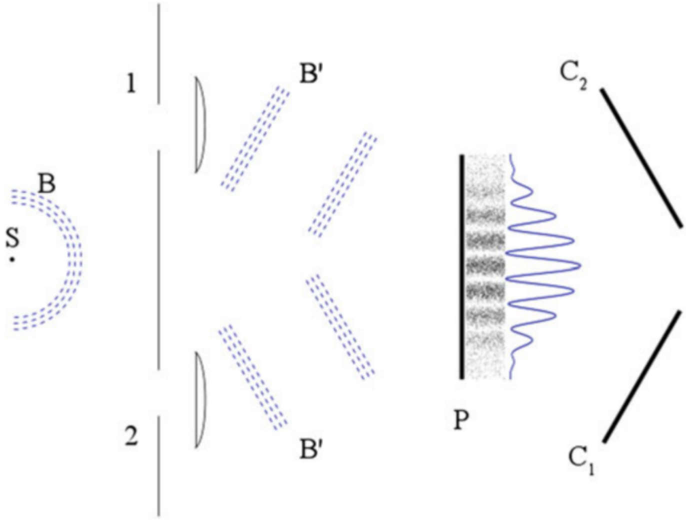

Fig. 2.10 The delayed double-slit experiment with electrons when both slits are open. As in Fig. 2.9, the source indicated S sends a beam of particles denoted B towards both slits. One inserts lenses, behind the slits, that will focus the incoming beams of particles, denoted B', toward two counters C_1 and C_2 that may detect them. But, contrary to what happens in Fig. 2.9, here one also inserts a detection plate (denoted by P in the figure) in the region where what appears to be the particles trajectories cross each other. Then, one sees an interference pattern as in part c of Fig. 2.6, and, according to the standard way of speaking, one will say that the particle went through both slits. Since one can choose to insert the detection plate P *after* the passage of the particle through the slits, it looks like we can decide whether the particle went through both slits or through only one of them (as in Fig. 2.9) by inserting or not the detection plate after the particle had supposedly decided to go through one slit or both (A. Gondran cc by-sa 4.0)

like the two slits here. Then, when the photon reaches the Earth, one can choose to either put some equivalent of the detection plate or not to put it: if we do not put it, we can detect on which side of the galaxy the light went, and, if we do put it, we can "observe" that it went on both sides at once.

If we accept Wheeler's reasoning, this implies that we could decide *now*, by choosing which kind of experiment to perform on the light coming from distant quasars, what happened billions of years ago! In other words, the choices we are making now do not only "create reality", but they also "create" the past. If this were true, it would give us, humans, a more fantastic role in Nature than what most of science fiction can imagine.

3

"Philosophical" Intermezzo I: What Is Determinism?

3.1 Definitions

Since we used the word "determinism" in the previous chapters, it might be good to pause for a moment and to define what this word means and what it implies.

3.1.1 Determinism and Randomness

To define determinism in a physical theory, one can simply say that, if the state of a physical system is given at some time, then there are laws specifying what this state will be at all later times.

When we say that "there are laws", we do not assume that we know those laws but simply that certain systems follow certain laws. After all, we should have no doubt that the planets were moving according to the laws of gravitation before we discovered those laws or even before humans existed.

Let us first define the *state* of a physical system. If one considers the sun and one planet (forgetting about everything else in the Universe), and if the precise positions and velocities of both bodies are given, then their positions and velocities are determined for all future times by Newton's laws of motion, and they can even be expressed in a rather explicit formula.[1] If one considers the sun and all the planets (again, forgetting about everything else) and one specifies the precise positions and velocities of all these bodies at a given time,

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[1] The reason why one need to specify both positions *and* velocities and not positions alone is due to a property of Newton's laws, that we will not go into.

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then their positions and velocities are again determined for all future times by those laws of motion, but there does not exist an explicit formula expressing what those positions and velocities will be.

This extends in principle to the whole Universe, as long as one considers only gravitational forces and one remains within the framework of classical physics.

This also extends to the tossing of a coin, the throwing of a dice or a roulette ball on the wheel: if one specifies exactly the way those objects are thrown (their position, velocity, the way they rotate etc.), then again their future is determined.

In these examples, the state of the systems under consideration is defined by the exact positions and velocities of all the bodies of the system at a given time. Once such a state is specified, the laws of motion determine the state of the system at all later times.

The important word here is "exact": everyone knows that, if the coin, the dice or the roulette ball are thrown a little bit differently, the result may be heads instead of tails, a 6 instead of a 2, or the ball landing on the red instead of on the black.

The essential point that defines a deterministic dynamics is that it associates to each exact state of the physical system at a given "initial" time a *unique* state for all future times. We will call *initial conditions* the exact state of the system at that initial time.

In physics, one also considers non-deterministic dynamics. This is defined as a dynamics where each initial state does *not determine* a unique later state, but several possible states.

It is a dynamics defined by assigning to each state a series of other states with certain probabilities.

The simplest examples are again given by coin tossing and dice throwing. Suppose that we do not know (as is the case in practice) the initial conditions in a particular tossing or throwing. Then, we assign certain probabilities to the outcomes.

In the case of a coin, assuming that there are no tricks or bias, the probability of heads or tails is $\frac{1}{2}$, irrespective of what happened previously. For a dice, each face has probability $\frac{1}{6}$, again irrespective of what happened previously.

But one could imagine a biased coin, where heads would fall with probability $1/4$ and tails with probability $3/4$. Or a biased dice where one face would appear with probability $1/2$ and each of the five other faces with probability $1/10$. Obviously, the sum of the probabilities of the different possible outcomes must always be equal to 1.

Almost all of the applied sciences use probabilistic dynamics, of course more fancy ones than the examples given here, but based on the same idea.

The outcomes of such non-deterministic processes are often called "random". To define (intuitively) a random sequence of results, consider the repeated tossing of a non biased coin, with results noted H (heads) and T (tails). This sort of experiment is typical of what one calls random and thus can be used to explain that notion.

What does one expect when one tosses many times such a coin? First of all, one expects both H and T to appear about half of the time.[2] But one expects also each successive pair of the form HH, HT, TH, TT to appear about a quarter of the time. Each of the eight series composed of three successive results HHH, HHT, HTH, etc. should appear one-eight of the time. More generally, in a random sequence, every finite series of symbols, made of H's and T's, should appear with a frequency that depends only on its length (the longer the length, the smaller the frequency)[3] so that two finite series of the same length appear with the same frequencies.[4]

This notion of random sequence may become more intuitive if one considers examples of sequences that are not random. For example, the sequence HTHTHTHT…is just the repetition of the pair HT, which therefore occurs more than a quarter of the time, and is not random. Another example is given by the sequence HHTTHTHHHTTHTHHHTTHTHHHTTHTHHHTTHTH…, which is obtained by the repetition of the series HHTTHTH, and is also not random (the series HHTTHTH is of length 7 and thus should occur, in a random sequence, only a fraction $\frac{1}{2^7} = \frac{1}{128}$ of the time).

We will call *apparently random* (for reasons that will be clear below) a sequence satisfying the above definition.

Suppose we are given a sequence that is apparently random according to the above definition. We can raise a fundamental question. Is that sequence *truly random*, or, to use a synonym, *intrinsically random*? What these expressions mean is that no conceivable deterministic explanation of the appearance of this random sequence could be given. This notion has to be contrasted with the previous definition (apparently random) that referred to a statistical property of the sequence (two finite series of the same length appear with the same

---

[2] This expectation will be justified trough the law of large numbers in Sect. 3.4.1.

[3] To define the frequency with which some series of symbols occurs in a sequence of results, one counts the number of those series of symbols in that sequence and one then divides that number by the total number of results.

[4] Thus, every finite series of results of length n will occur a fraction of the time equal to $\frac{1}{2^n}$. This of course makes sense only for an infinite random sequence. Since, in practice, every sequence has a finite length, the definition given here has to be considered as an idealization.

frequencies), while the notion of truly or intrinsically random refers to the impossibility for the sequence to be produced by a deterministic mechanism.

We already saw that the results of coin tossing are not *truly* random. This gives a simple example of a sequence of results that appears random, but that is in reality deterministic: if we give a more detailed description of the system, namely the exact initial conditions of the state of the coin for each tossing, then the results are determined by those initial conditions.

Conversely, any deterministic system can look indeterministic if we do not describe it in sufficient detail. The coin tossing and the dice throwing described above provide examples of that situation. If we give the initial conditions in detail, then those processes are deterministic, but if we do not, then they may *appear to be* random.

One might ask a further question: can one find a criterion that would allow us to determine, whenever the behavior of a system is *apparently random*, whether it is *truly random*? The answer is "no": although we shall not prove it, the examples of the coin tossing and the dice throwing illustrate why this is so: one cannot think of anything more apparently random than the throwing of a coin or a dice. In fact these examples epitomize the notion of randomness. But if even those system are in fact deterministic, once one gives a more complete description of their states, how can one hope to find a criterion that would prove that a system is truly random and does not simply look random because of our incomplete description of their states?

This does not mean that there cannot be any truly random systems in Nature, but that, in order to prove that fact, one has to give other arguments than simply observe that they appear random.

3.1.2 Determinism and Predictability

This brings us to another important distinction, the one between *determinism* and *predictability*. In the examples of the coin tossing or the dice throwing, we said that, if we knew the initial conditions with enough precision, we could in principle calculate on which face they would fall. But here and in many other phenomena, we cannot, in practice, know the initial conditions with enough precision to be able to predict the face on which they will fall.

It may also happen that, although the initial conditions can be known with great precision, the calculation of the evolution of the system between its initial state and its final one may be too complicated to be carried out.

Yet another possibility is that *there are deterministic laws* governing a given physical situation, but that we do not know them.