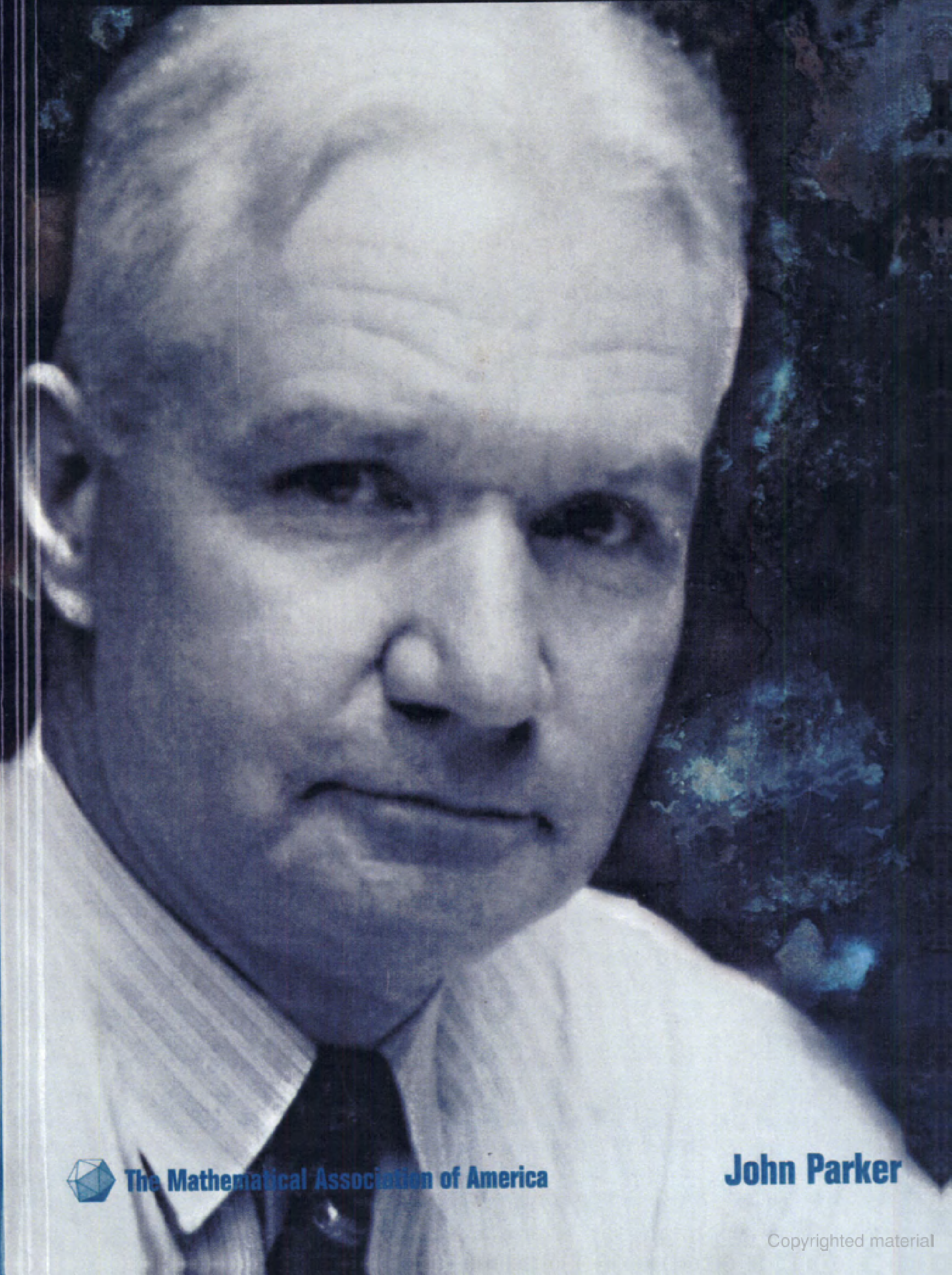
R.L. MOORE

Mathematician & Teacher



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John Parker

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Much, however, is owed to one of the most stalwart figures and prime movers of the Project and a tireless contributor to its aims and objects, the late Dr. Ben Fitzpatrick. Harry Lucas recruited Ben Fitzpatrick to the Moore Project in 1996, and he became Lucas's main liaison contact with the mathematics community as foundation work began, and thereafter in all aspects of the project. He was especially successful in instigating and collating a substantial archive of Oral History for The Legacy of R.L. Moore Collection now housed at the Archives of American Mathematics at the Center for American History, The University of Texas at Austin.

Other tape-recordings and transcripts from this archive include those conducted by Douglas Forbes, for his 1971 dissertation: *The Texas System: R.L. Moore Original Edition.* Tape-recorded contributions consulted by the author include those by R.D. Anderson, Steve Armentrout, Joanne Baker, Mrs. B.J. Ball, Lida Barrett, Mary Bing, Ed Burgess, Howard Cook, J.L. Cornette, Jerome Dancis, James Dorroh, W. Eaton, J.W. Green, M.E. Hamstrom, F. Burton Jones, I.W. Lewis, Lee

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1

Roots and Influences (1882–1897)

Early life and family connections, a generation away from two US presidents and the first president of the Confederacy. Educated privately by a Scot who had served Maximilian in Mexico.

Moore at age ten

The rough and tumble of Texas frontierism in the 1890s may seem a curious and perhaps unexpected point in time to discover the inspirations of a man who was to take his place at the cutting edge of mathematical research in the United States over the ensuing decades. In fact, it offered a setting in which the very genesis of the do-it-yourself principles of what became universally known as the Moore Method emerged and went on to influence his entire career. The defining moment occurred when, at the age of fourteen, Robert Lee Moore decided he would apply for entry to The University of Texas at Austin, founded in the year of his birth, to pursue his fascination with mathematics. To achieve that goal, he had to have calculus but it was not taught at the Waldemar Malcolmson Academy, a private school run by a colorful, enigmatic Scot who had otherwise provided him with a very decent education, and most importantly of all instilled within him a yearning for higher education.

Malcolmson did, however, possess an old calculus book that he lent to his student for private study, although he offered no other tuition. Moore quickly lost patience with the imprecise language of the book and wrote to The University of Texas to ask for a copy of the calculus in use there. He received it by return post and began teaching himself. He recalled in later life he would read a statement of a theorem, while covering the portion of the page that gave the proof. If he failed to prove the theorem after a reasonable length of time, he would uncover the first line of the proof, read that, and then try to prove the theorem without further assistance. If this failed, he would uncover the next line and so on until a satisfactory resolution was achieved. It was not a pleasant experience for him to uncover even a part of the proof and, in those instances that he uncovered much of the proof, he felt he had failed.[1] This was, in essence, the beginning of R.L. Moore's journey of self-discovery that would in turn lead to the creation of a teaching style in which he encouraged his own students to find their own way through the mathematical maze, with a minimal reliance on textbooks, and in many situations, none at all. This précis is, of course, an over-simplification of what eventually transpired and, indeed, the development of the Moore Method, running parallel to Moore's own career, covers far broader concerns than calculus. We will

[1] J. Eyles, The Importance of R.L. Moore's Calculus Class, dissertation, 1998, p. 9; from the Center for American History, UT Austin.

delve deeply into the many influences brought to bear on the way in which he taught his students, resulting from his upbringing in those heady days in Texas as well as other subliminal aspects that were surprisingly unknown to himself until much later in his life.

Many of them long pre-dated the moment that Robert Lee Moore joined this world,[2] at a time when America was still in the healing process from the wounds and consequences of the Civil War. With him came an ancestry reflecting that conflict in a quite remarkable way and, when combined with his own future career, we are provided with a classic American tale. It is one that cuts a swathe across defining events in a century of history both in mathematics and the social and political changes that saw the emergence of one nation under the flag, all men supposedly equal under the law, though not necessarily at peace with one another, nor issues of the Civil War fully resolved. Yet, the probability is that Moore himself did not fully appreciate the extent or importance of his own historical connections until quite late in a lifetime immersed in mathematical study and teaching. In fact, he reached adulthood before discovering that he did not come from dyed-in-the-wool Southern stock and it did not especially please him.[3]

This discovery was made during the compilation of a family tree for which over a period of several years he painstakingly researched through distant generations of the American-born families and their European connections and at the end of what, by any measure, was a masterly example of genealogical detective work, he produced a chart of lineage that would embrace two American presidents, the president of the Confederacy and three European royal houses.[4] Moore traced his ancestry through 300 years of American history to one John Moore who was born in England and settled in Sudbury, Mass. where he bought land and built a house in 1642. He married Elizabeth Whale, later moved to Hartford, Connecticut, and died on 6 January 1673. It was from Hartford 185 years later that the family tree, so steeped in New England traditions, developed a southern branch that was to plunge them into the midst of the Civil War and set sibling against sibling.

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[2] 'No, he never really joined the world' wrote Joe B. Frantz, in his book *The Forty-Acre Follies*, an unofficial history of The University of Texas at Austin where he was a professor of history for 40 years.

[3] Joe B. Frantz, page 113.

[4] See Appendix One: The Moore Genealogy.


R.L. Moore's parents, Charles and Louisa Anne

Moore's paternal grandparents, John Stephens Moore and Caroline Abigail Cowles Moore, were married in 1823 and had six children, five boys and one girl, the first born at Farmington, CT and the others at Glastonbury, CT between 1825 and 1846. John Moore was a physician, practicing in Vermont and Connecticut until the mid-1850s when he informed his family that they were all going on an adventure to the mountainous regions of North Carolina where he wished to study natural drugs and Indian remedies. When the family eventually returned to Connecticut, the eldest son Henry remained in North Carolina and eventually moved to Kentucky. His letters home encouraged R.L. Moore's father, Charles J. Moore, to join him there.

Charles rode straight into the developing bitterness over the abolition issue that reached its point of no return when the Supreme Court's ruling in the Dred Scott case made slavery legal in all territories. Charles arrived in Kentucky in 1858 — the year that Senator William H. Seward of New York referred to differences between North and South as an 'irrepressible conflict'.[5] Such was the feeling in the South by then that

[5] William Henry Seward, On the Irrepressible Conflict, delivered at Rochester, New York, October 25, 1858.

Charles had to hide his northern origins and accent, otherwise he would have found neither work nor lodgings. He all but discarded his northern connections as he settled into his new surroundings and became in every respect except birth, a Southerner. When Jefferson Davis and Robert E. Lee ordered the attack on Fort Sumter, Henry and Charles Moore volunteered for service in the Confederacy. Hartford, Connecticut County History would record that the brothers 'gave valiant service to the South'.[6] In the north, meanwhile, two of their brothers enlisted for the Union cause while Henry and Charles in the south went into the Orphan Brigade, 2nd Kentucky Regiment, which came into being on 28 October 1861 at Bowling Green, Kentucky and fought as a unit for the first time at Shiloh. During 1862 the brigade found itself in Vicksburg. Charles Moore was lucky to survive. After just six weeks, the brigade had lost more than 1,300 men, many more from malaria and fever than from battle wounds, with just 542 left standing. The unit was pulled out only to return a few months later, replenished with new recruits in an attempt to save the Confederate garrison from a six-week siege by the Union army. They failed and instead moved to take up a rearguard position with the Army of Tennessee, fought in the Battle of Chattanooga, were in action continuously during 100 days of the Atlanta campaign and suffered 1,860 men killed or wounded. At the end of it, less than fifty men of the original brigade survived without wounds. Both the Moore brothers did indeed survive and with the south desolate and destroyed, joined the rest in attempting to pick up the pieces of their lives. There was no question of them returning North and, in any event, their roots were planted firmly into Southern soil.

The two brothers soon came into contact with another Moore family from Virginia. The upshot of this contact was that the two brothers became engaged to two sisters of that family. Henry married Mary E. Moore and on 10 May 1866 Robert Lee Moore's parents, Charles and Louisa Anne were wed, thus forging the important link on his mother's side of Moore's ancestral chart, one generation away from former US President Zachary Taylor, and to Jefferson Davis who was Taylor's son-in-law. Across the Southern states, the Reconstruction years following the war were marked by a great movement of population, black and white, as they attempted to escape desolation and poverty in a ruined economy. Charles Moore surveyed his options and decided to migrate

[6] County Records, City Hall, Hartford, CT.

Robert Lee as a toddler

west from Kentucky with their, then, four children to Dallas, Texas in 1877 to set up a hardware, grain and grocery store just off the town square.[7] It turned out to be a good choice, even if the formative years of the Moore children would be set against the backdrop of what was initially a wild and wondrous place. Dallas was still very much a frontier town, having expanded from John Neely Bryan's trading post for westward bound wagon trains to classic boomtown in barely 25 years. The Confederacy had used it as an administrative center, as did the incoming Union troops and it retained its importance after the war as commercial enterprises blossomed. The addition of a railhead and the influx of farm tool manufacture and the buffalo trade set the population expansion into fast mode. This and the new order of social life after the war also brought its own challenges. With the cotton industry almost in ruins, Dallas saw an influx of families from other less prosperous Southern towns, hoping to rebuild their fortunes. They were to be joined by the first Texas slaves to be freed on 19 June 1865 and others followed, making the trek from distant parts to join the bustle of Dallas

[7] On 22 November 1963, shots rang out over the site of the Moores' long-demolished store, as President John F Kennedy was assassinated. The store would have stood on what was by then the famous grassy knoll.

Moore's Feed Store, Dallas, Texas

growth. It is also worth noting that the Ku Klux Klan made its first recorded appearance there as early as 1868.

The first passenger train, the Houston and Texas Central, steamed into Dallas amid great excitement on 16 July 1872 and by the end of that year, the population had doubled. New businesses and buildings began to appear daily. Hotels were being built, water and gas utilities arrived, telegraph lines were installed as the town established itself as a major dispersal center for raw materials, to be shipped to the South and East. Prosperity also attracted outlaws, and some famous names flickered across the Dallas scene. Belle Starr, who had consorted with the James Gang and the Younger Brothers, was already notorious when she arrived in Dallas, first as a dance hall singer and later running a livery stable from which she sold stolen horses and harbored outlaws. Wyatt Earp's pal John 'Doc' Holliday practiced as a dentist there for three years before he shot a man and 'was invited to leave town'.[8] The out-

[8] Timeline, Dallas Historical Society (`www.dallashistory.org`).

law Sam Bass also operated from the Dallas area, and robbed four trains in two months during the spring of 1878 before he was shot dead by Texas Rangers in an ambush near Round Rock. There was refinement, too. Patrons at the Dallas Opera House were able to see James O'Neill in *The Count of Monte Cristo,* while in 1887 customers paid $15 a head to witness Edwin Booth perform *Hamlet*. It was into this cauldron of activity that Charles Jonathan Moore brought his wife and four children, Henry, Jennings, Eleanor and Arthur in 1877, to which were added Robert Lee in 1882,[9] and Caroline Louisa in 1887. Only fragmentary accounts of their lifestyle are available, but it is clear from family correspondence that Charles had, by no means, relaxed the traditional Southern values he had embraced so wholeheartedly before and during the Civil War. His northern roots remained hidden and his children were adults before they learned that they had relatives in Connecticut. In fact, the first recorded visit by a member of the Southern branch of the Moore family to those in the North was in 1907 when Robert Lee stopped off at the home of Charles's younger brother Arthur in Glastonbury while he was at Princeton. There are indications that Charles Moore may have run his home as a not-so-benevolent dictator. Given the surroundings in which he was raising his children, the era and his personal adoption of the Southern code, this was not an unnatural or even unexpected aspect of family life and on Robert Lee's part there appeared to be no lingering animosity in later life. He kept up regular correspondence with his father after he had left home for university and the letters, returned to him after the old family home was sold, displayed nothing but respect and a strong bond of affection. Charles had, after all, ensured that all of his children were given as good a start as he could afford and in Dallas where education had never been a great priority, that meant private schooling. Public school funding was not adequately supported by taxation in Dallas until the early 1900s and a number of private establishments sprung up to fill the void.

Robert Lee, along with two of his brothers, was fortunate indeed in the school chosen for them by their parents. They placed the boys in the hands of a dour Scot named Waldemar Malcolmson who ran an establishment with his wife close to the Moore homestead on Live Oak Street. He called himself a professor, though the grounds for such a title

[9] 'A name which fit, except that he lacked that Southern immortal's naturally gentle and generous nature,' Joe B. Frantz, *The Forty-Acre Follies*, page 113.

were somewhat unclear, as was his past. It hardly mattered given the alternatives but, as it turned out, Malcolmson was to be a significant guiding influence on the young Robert Lee and once again, there were interesting historical connections that would add additional weight to the boy's education; not for him, the attentions of a young and inexperienced school teacher, as was often the case in these establishments.

Malcolmson was a well-educated man of the world who spoke fluent French and Spanish. He had traveled to Mexico in 1861 as an officer in a tri-nation army formed under French leadership by Napoleon III to bring to heel the government of Benito Juárez, which owed a huge foreign debt to a consortium of European nations, mainly France, Great Britain and Spain. Napoleon III sent Archduke Maximilian of Austria as the new puppet 'king' of Mexico along with a large army to ensure his safe installation on the Mexican throne. Juárez was driven northward but counter-attacked and Napoleon's forces, crippled by disease that claimed many thousands of lives, evacuated Mexico in March 1867. Maximilian refused to abdicate, and his small remaining force was surrounded, starved, and finally capitulated on May 15, 1867. In spite of petitions from European leaders, and the Pope, Maximilian was executed on a hill outside Querétaro the following month. Malcolmson, who is believed at one stage to have been on the staff of Maximilian, escaped and with many others made a hasty retreat north into Texas, eventually arriving in Dallas with his wife in the early 1870s.

They founded a school under the name of the Dallas Lyceum, although it changed names several times. Robert Lee joined the school register when he reached his eighth birthday. Malcolmson ran a good school, and became noted for the quality of instruction and the variety of subjects. They included arithmetic, algebra, trigonometry, geography, history and penmanship. Seniors were invited at extra cost to select courses in higher mathematics, German, Spanish, French and Latin. Anne Atkins, who attended his school in the 1880s, left this account of Malcolmson:

'He was a rare type and a real scholar, with an odd approach to many things and unorthodox to a degree. He believed that Bacon wrote Shakespeare and was indifferent as to truths in the world around us. But when it came to mathematics, French or Latin, he ranked with the best and prepared another young playmate so well that he entered The University of Texas with the highest grades and became one of the most distinguished members of the State Bar, reading French and Latin with the greatest ease to the end of his life. The professor had an odd way of impressing facts upon us and one thing I have never forgotten: the names of the presidents of the United States up to Cleveland's first administration. This was through learning the final initial of each. From the beginning he gave me, I have gone to a wide development of the French language and literature.'[10]

Other surviving papers from the era in collections maintained by the Dallas Historical Society include an intriguing statement of intent to his clients by Professor Malcolmson that provides a basis for speculating that it might well have sown the seed for the unique style of teaching eventually adopted by Robert Lee Moore: 'The method of instruction adopted is concise and lucid, not burdening the minds of young scholars with useless matter, but omitting nothing that is necessary to a thorough, practical course of instruction. The Lyceum aims to be second to no institution of learning in Dallas and a student will be able to receive instruction in all the branches taught at schools of the highest grade without leaving the building.' And so, Robert Lee Moore began his studies with Malcolmson and remained largely under his tuition until it was time to move on to university. It was a good grounding, and one in which the teacher encouraged his pupils into home study in areas in which they showed an aptitude. In Moore's case, his fascination with mathematics was evident before he entered his teens and a collection of grade reports among his papers relating to his time at Malcolmson's school reflect an unbroken line of As for mathematics. On that basis, Malcolmson had already singled out Moore as a potential candidate for The University of Texas at Austin, which had opened its doors in 1883, and the teacher was anxious from his own standpoint to have students passing through his establishment to successful higher education.

He had already made contact with a number of the teachers and departmental heads at Austin, including George Bruce Halsted, a brilliant, if eccentric, mathematician who accepted a professorship at Austin in 1884. We will meet Halsted later, looming large in our subject's early career, although he was already in the foreground of Moore's immediate future as he dedicated himself to securing a place at Austin. Encouraged by Malcolmson, he began earnest preparations to gain entry to the university. He wrote Dr. Halsted and obtained the calculus and other books for mathematical study in preparation for his anticipated

[10] Collections and other documents, Dallas Historical Society.

arrival at Austin and, as an additional brainteaser, he learned shorthand, which he used to make copious notes in the margins of books.

Knowing full well he would also need Latin for a university place, he wrote in his newly commenced diary, written on pages of an Ivory Soap order pad[11] doubtless picked up from his father's store, that he intended to begin studying Latin at the very latest in January 1898. He was considered sufficiently proficient by the summer of that year and duly applied for enrollment to The University of Texas while still only fifteen years old. With his application went a letter of recommendation from Waldemar Malcolmson dated 19 July 1898: '…the hardest [working] student I have ever had, nothing pleases him but continuous study, he can read Spanish at sight having studied with me for seven years. In mathematics, he has used Professor Halsted's books as well as others on the same subject and has gone through his Calculus. He takes special pleasure in the study of mathematics in many cases solving, without aid, the most difficult propositions in an original manner…. [T]his boy Moore is without exception the student who has shown the greatest application to his studies. It in fact amounts to a passion with him.'[12]

The University of Texas was as old as Robert Lee Moore, building work having begun in 1881 after political machinations had delayed funding. The Texas Constitution of 1866 directed the legislature to put the university in operation at an early date. In 1871 the legislature established the Texas Agriculture and Mechanical College, but the university was postponed. The Texas Constitution of 1876 specified that the legislature, as soon as practicable, was to establish, organize, and provide for the maintenance and support of a 'university of the first class' to be located by vote of the people and styled The University of Texas, for promotion of the study of literature and the arts and sciences. An agricultural and mechanical branch was mandated. The same article of the constitution made A&M a branch of the university and also ordered the legislature to establish and maintain a college or branch university for instruction of black youth, though no tax was to be levied and no money

[11] Commencement date, June 1897, partly illegible and some notes in shorthand and Latin; R.L. Moore Papers in the Archives of American Mathematics at the Center for American History, The University of Texas at Austin (referred to as the R.L. Moore Papers in the AAM).

[12] Waldemar Malcolmson to Dr. Garrison (The University of Texas), 19 July 1898, from the R.L. Moore Papers in the AAM.

appropriated out of the general revenue for such a school or for buildings of The University of Texas.

This prohibition prevented establishment of a branch of the university for African-Americans, although Austin was selected for its site in 1882. The campus of the main university originally consisted of the forty-acre tract on College Hill set aside when Austin became the state capital, and it opened in 1883 with a strong staff, predominantly weighted in teachers from the South, who included three former colonels in the Confederate army and one professor who had been chief of staff to General Stonewall Jackson. The arrival of Professor Halsted, a Princeton man, the following year rather put the cat among the pigeons. While evidently in sympathy with Southern ideals, his eccentricity did not do him any favors among those stern ex-colonels. By the time R.L. Moore made his application, the university was well established and only the previous year had taken steps to raise entry standards and to strengthen its catalog. Beneficial from Moore's point of view was the fact that the School of Mathematics had established a new instructorship to enable the teaching force to divide the freshman class into six sections instead of two, with two instructors of mathematics fully committed. The university catalog pointed out that smaller sections would enable students to get more individual instruction from his professor; enthusiasm and interest would be aroused and 'a desire created to pursue the subject in its higher branches'.[13] In R.L. Moore's case that is exactly what happened. He took his entry examination in May 1898 and on 11 July, recorded in his diary 'received news today from the University; examinations papers have been pronounced satisfactory'.[14] He enrolled in UT to begin an association that would last more than seventy years, entering a school with almost four hundred students, and he never once wavered in his intention to become a mathematician. He arrived in Austin on 6 September, staying initially at the Driskill Hotel until he found lodgings. He had been given an advice sheet from the university that informed him:

'Some students do their own cooking and housework, and are thus enabled to live at an expense not exceeding $5 a month. They serve as waiters in boarding houses, or do other work in private families, which relieves them of expense of board. Regular board, with furnished room,

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[13] *The University Record* (The University of Texas, Austin, Texas, 1898) Vol. 1, No. 1, December 1898, pp. 32–33.

[14] Ivory Soap pad diary.

Young Master Moore

can be obtained near the University, at prices varying from $12.50 to $20 a month. A large number of students pay the former price. In University Hall board, furnished room, lights and fuel may be obtained for $15 a month. Two large student clubs have, during the present session, further reduced the price of board and lodging… [to] an average expense of $11.25 monthly for each member.' Traylor adds: 'Many of the students support themselves by doing work in private families, milking cows, making fires, cooking, tending the horse; others waited on the tables in boarding houses or attended to the rooms; others taught, acted as clerks, stenographers, typists, accountants, or surveyors.'[15]

[15] Traylor, pp. 13–14.

So, only the prices changed down the decades. As to the academic standards that confronted him in 1898, Austin had put together a strong catalogue and had brought together a broad based and experienced staff. As already noted, admission standards had already been raised as demand for university education continued to rise. For a BA course, for example, candidates were to take entrance examinations in English, History and Mathematics and in Latin or Greek. They would be tested on their knowledge of the elements of English Grammar and English Composition, and 'especially upon their ability to write paragraphs of idiomatic English properly spelled and punctuated.... subjects for such paragraphs will be taken from the books named below, whose subject matter the candidates must be familiar with: [Joseph Addison's fictitious] Sir Roger de Coverly Papers in *The Spectator,*[16] Dryden's *Palamon and Arcite*, Cooper's *The Last of the Mohicans*, and Burke's *Speech on Conciliation with America*.'[17]

In Latin, candidates were to become proficient in grammar, 'with special stress upon inflections and the syntax of the simple sentence; the translation of elementary English prose into Latin; in *Viri Romae*, any books of Caesar, the four lives of Nepos that bear upon Roman history (Hamilcar, Hannibal, Cato, Atticus), any four orations of Cicero and the first book of Virgil's *Aeneid* with the scansion of the dactylic hexameter. Greek would be examined on inflections and syntax; in any three books of Xenophon's *Anabasis*; in the translation of easy Greek at sight; in the translation of elementary English prose into Greek. Knowledge of accent [was] required.'[18]

The courses offered at that time in the School of Pure Mathematics included: 1] Spherics, Solid Geometry, Algebra, Plane and Spherical Trigonometry, with Applications to Surveying and Navigation. 2] Conic Sections, Analytical Geometry. 3] Calculus for Physics and Engineering. 4] Differential and Integral Calculus. 5] Integral Calculus, Differential

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[16] Addison's contributions to *The Spectator* are said to have perfected the essay as a literary form. His prose style was the model for pure and elegant English until the end of the 18th century; his comments on manners and morals were widely influential in forming the middle-class ideal of a dispassionate, tolerant, Christian world citizen. His fictitious Sir Roger De Coverly Papers, according to William Makepeace Thackeray, give a full 'expression of the life of the time; of the manners, of the movement, the dress, the pleasures, the laughter, and the ridicules of society..

[17] *The University Record,* Vol. 1, No. 2, April 1899, pp. 102–103.

[18] Ibid.

Calculus, and Differential Equations, for Physics, Engineering, and Economics. 6] History of Elementary Mathematics. 7] Advanced Integral Calculus: Definite Integrals, Differential Equations, Functions of a Complex Variable. 8] Modern Geometry, Metric Geometry, Recent Geometry. 9] Geometry of Position. 10] Theory of Equations, Theory of Functions. 11] History of Mathematics. 12] Noneuclidean Geometry. 13] Hypergeometric Functions. 14] Algebra of Logic.[19]

On his makeshift Ivory Soap pad diary, Moore kept up a running commentary of his life at the University. The entries, once again, pointed to a young man intensely interested in study, the day's classroom activity, work done, comments of teachers and so on, while reflecting little social activity apart from church and long walks. Many of the entries were timed late at night, Moore having spent the evening studying. His first two sets of grades provided him with As in his mathematics and Latin courses, a B in physiology and hygiene, a C in English and Ds and Es in the remainder. He also took an examination in Spanish with the freshmen and passed, thus to get a credit for Freshman Spanish. Later, he also obtained a credit for Freshman German. But it was mathematics that had gripped his imagination and Halsted soon recognized this fact. Moore records a conversation he had with his teacher in which he was admonished for talking too fast. The diary relates a conversation entry for 12 January 1899 when Halsted was talking of recommending two of his students for a teaching fellowship at the University that year. Halsted had clearly penciled him in as a future candidate for that role:

'Mr. Moore, if you are going to make mathematics as a profession, you should pay more attention to the pedagogical and learn to talk slow.'

'How old are you, Mr. Moore?' [asked Halsted.] 'Sixteen,' I replied.

'It is a great advantage to be young,' Halsted continued. 'You are gifted in mathematics.... I do not think there is anyone here more so.... But there are other things to be considered ... and you being young have time to see to them. Demand in mathematics is always greater than the supply.'

Moore added a further note in his diary of a later conversation in which Halsted told him, 'If you go on, your career is assured.' In February, Halsted confirmed he intended at some point in the future to put Moore's name forward when he promised, 'If everything turns out

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19 *The University Record*, Vol. 1, No. 2, April 1899, pp. 105–109, 111.

all right in the mathematical departments, I don't know but what I will recommend you as a student assistant.'

This additional praise came after his student's encouraging start to a course on noneuclidean geometry, which began in the New Year of 1899. Thus, Halsted was already guiding the young Moore along parallel lines to his own interests as is indicated in Moore's diary entries for May, 1899 when he makes reference to studying Bolyai. The work was a translation of Bolyai's original by none other than Halsted himself, published under the title of *The Science Absolute of Space Independent of the Truth or Fallacy or Euclid's Axiom XII.* Bolyai is mentioned increasingly in Moore's notes and it is possible, even at that early stage of Moore's career, that Halsted had developed a somewhat romantic notion that here, in this new University of Texas, from the unlikely place of Dallas, he had discovered a new János Bolyai. The ingredients were not dissimilar and Halsted knew the story well. Born in Transylvania, Bolyai had mastered the calculus at the age of thirteen under the guidance of his father, Farkas. He became an accomplished linguist and between 1820 and 1823 he prepared a treatise on a complete system of noneuclidean geometry, but before the work was published, he discovered that Carl Gauss had anticipated much of his work. Although Gauss had never published his work in this area, this was a severe blow to Bolyai. However, when Bolyai's work was published in 1832 as an Appendix to an essay by his father, Gauss wrote to a friend saying, 'I regard this young geometer Bolyai as a genius of the first order.'[20]

To Bolyai's father he wrote, 'To praise it would amount to praising myself. For the entire content of the work … coincides almost exactly with my own meditations which have occupied my mind for the past thirty or thirty-five years.'[21]

That Halsted had Bolyai in his mind as he began to take a deeper interest in Moore, noting his characteristics, intensity and ability, was to some extent borne out by a letter Halsted wrote to him a few years later with glowing largesse: 'I have always held that your work is the most profound, unexpected and epoch-making ever contributed to mathemat-

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[20] *The University Record*, Vol. 1, No. 2, April 1899, pp. 105–109, 111.

[21] Information on Bolyai can be found at `www.gap.dcs.st-and.ac.uk/~history/mathematicians/bolyai.html`.

Moore and unknown person in cap and gown

ics by an American.... Like John Bolyai, you had from the very first *one* who appreciated you.'[22]

That kind of support for Moore's ability had been growing in the mind of his teacher from the early days, and his praise was carefully administered at the beginning so as not to spoil his protégé. Moore seems to have understood.

Having been invited to Halsted's house where a reception was being given for a member of the staff, Moore indicated in his diary that he had no wish to involve himself in this social activity: 'I tried to excuse myself on ground of examinations ... telling him something like "I will try to come".... But I won't go, very sorry that I can't well do it for I would hate to offend such a true friend as I think Dr. Halsted is to me and whom I like so much and have so much respect for.'[23]

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22 Halsted to Moore, 12 December 1916, R.L. Moore Papers in the AAM.
23 R.L. Moore's entry in his Ivory Soap pad diary, 8 June 1899, R.L. Moore Papers in the AAM.

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Of Richest Promise (1897–1902)

Professor G.B. Halsted had introduced his protégé to the work of Bolyai, Lobachevsky and Hilbert, and Moore, at 18, makes his name producing a 'delightfully elegant and simple proof' demonstrating the redundancy of one of Hilbert's geometry axioms, which draws the attention of E.H. Moore.

1898 Christmas holidays R.L. with his sister, Caroline Louisa Moore

Robert Lee Moore's confidence in and respect for George Bruce Halsted never wavered and late in life, after years of being able to reflect and consider, he insisted there was no other person he would rather have studied under in his early Texas days. Some mathematicians, and a number of authors of papers on Moore, have found this curious, given the personalities he encountered in the early stages of his mathematical adventure, notably his doctoral mentor Professor E.H. Moore. Despite accumulating many accolades for his work, there was always a question mark against Halsted among the mathematical community, one that became more pronounced as the years passed, and his eccentricities began to form a greater part of conversation than his prodigious output.

It was to Moore's personal dismay that he would find himself the unfortunate catalyst to growing animosity towards Halsted at The University of Texas, which escalated into open warfare with the hierarchy and finally a parting of the ways. The bitterness that Halsted harbored for the rest of his life was regularly reflected in his letters to Moore, not surprisingly in that it marked the beginning of the decline of his own career. The endnote eventually came from Halsted himself in a mordant comment he wrote for a Princeton biographical questionnaire: 'I am working as an electrician as there is nothing in cultivating vacant lots.'[1] As we will see, Moore would be blameless in this affair and fortunately it did not erupt until he had acquired three years of solid tuition and guidance from Halsted, followed by many more years of regular exchanges by correspondence until, as Halsted admitted, the student eventually became the teacher. Moore was therefore able to garner all that Halsted could offer him and benefit from an impressive mathematical legacy that was as star-studded as Moore's personal genealogy chart. Nor would he ever forget that it was Halsted who, as they turned into the 1900s, provided linkage to the past, the present and the future of the American mathematical research community in which Moore was to play such a major part. In doing so, he became a pivotal figure in the expansion of this professional family tree that expanded far and wide across the United States of America to embrace some of the most gifted mathematicians of the twentieth century.

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[1] L.E. Dickson, Biography, Dr. George Bruce Halsted (1853–1922), *Amer. Math. Monthly* 1 (1894), 337–340; and Halsted entry by H. S. Tropp in the *Dictionary of Scientific Biography*, New York, 1970.

Moore would undoubtedly have figured among them anyhow, but Halsted's own background and experiences are important because they reflect developments that brought outstanding additions to Moore's mathematical pedigree. Halsted had grown up with the finest. The son of Oliver Spenser and Adela Meeker Halsted, of Newark, New Jersey, he was the seventh child in a family of ten children. His brother, father, an uncle, a grandfather, a great uncle and a great-grandfather were all Princeton graduates and the Halsted family had given funds for an early astronomical observatory. Halsted was awarded his B.A. there in 1875 and his M.A. three years later. In 1876, however, he became the first doctoral student of Professor James Joseph Sylvester on his appointment to the chair of mathematics at The Johns Hopkins University. Recognized as one of the founding fathers of the American school of mathematical research, he and Halsted struck up an immediate rapport. Unfortunately it did not last.

Halsted had virtually begged to be taken on by Sylvester[2] who had a history that was, and still is, well known across the boundaries of international mathematics. After schooling at the Royal Institution in Liverpool and St. John's College, Cambridge, he was refused graduation because, at the time, it was necessary to swear a religious oath to the Church of England. Being Jewish, he could not take that oath so could not graduate and had difficulty in finding work. At the age of 27, he won the chair at Thomas Jefferson's nonsectarian University of Virginia at Charlottesville, although his appointment lasted only three months. Halsted loved to tell one of the many stories that add spice to the mathematical history of America:

'As Sylvester would not sign the 39 articles of the Established Church, he was not allowed to take his degree, nor to stand for a fellowship to which his rank in the tripos entitled him. Sylvester always felt this religious disbarment bitterly. His denunciation of the narrowness, bigotry, and intense selfishness exhibited in these creed tests was a wonderful piece of oratory in his celebrated address at Johns Hopkins. No one who saw will ever forget the emotion and astonishment exhibited by James Russell Lowell while listening to this unexpected climax.

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2 Karen Hunger Parshall, "America's First School of Mathematical Research: James Joseph Sylvester at The Johns Hopkins University 1876–1883," *Arch. Hist. Exact Sci.* 38 (1988), 153–196. See also Parshall, *James Joseph Sylvester Life and Work in Letters*, Oxford University Press, 1998, and bibliography.

James Joseph Sylvester

Thus barred from Cambridge, he accepted a call to America from the University of Virginia. In Sylvester's class was a pair of brothers, excruciatingly pompous. When Sylvester pointed out one day the blunders made in a recitation by the younger of the pair, this individual felt his honor and family pride aggrieved, and sent word to Professor Sylvester that he must apologize or be chastised. Sylvester bought a sword cane, which he was carrying when waylaid by the brothers, the younger armed with a heavy bludgeon.

'The younger brother stepped up in front of Professor Sylvester and demanded an instant and humble apology. Almost immediately he struck at Sylvester, knocking off his hat, and then delivered with his heavy bludgeon a crushing blow directly upon Sylvester's bare head. Sylvester drew his sword cane and lunged straight at him, striking him just over the heart. With a despairing howl, the student fell back into his brother's arms screaming out, "I am killed! He has killed me!" Sylvester was urged away from the spot by [an onlooker], and without waiting to collect his books, left for New York, and took ship back to England. Meantime a surgeon was summoned to the student, who was lividly pale, bathed in cold sweat, in complete collapse, seemingly dying, whispering his last prayers. The surgeon tore open his vest, cut open his shirt and at once declared him not in the least injured. The fine point of the sword cane had struck a rib fair, and caught against it, not penetrating. When assured that the wound was not much more than a mosquito-bite, the dying man arose, adjusted his shirt, buttoned his vest, and walked off, though still trembling from the nervous shock.'[3]

Back in London, Sylvester worked for a time as an actuary for an insurance company and tutored part-time in mathematics. One of his private pupils was Florence Nightingale. In 1846 he became a law student at the Inner Temple, and in 1850 was admitted to the bar. It was while working as a lawyer that Sylvester met Arthur Cayley and began the long partnership in which they created the theory of algebraic forms. From 1855 to 1870 Sylvester was professor of mathematics at the Royal Military Academy, Woolwich. From that position, he built up an international reputation. His style has been described as flamboyant and his output powerfully imaginative. He wrote in haste and was continually coining new technical terms, few of which actually survived. Recognition eventually superceded religious bigotry and he became the second president of the London Mathematical Society and the first recipient of the Gold Medal that the Society awarded to honor its first president, Augustus DeMorgan.

His retirement from the military academy at the early age of fifty-five came at an opportune time. The organization of The Johns Hopkins University in Baltimore was already under way and Sylvester was widely tipped for the chair of mathematics. There were detractors, including many of America's own mathematicians. At Harvard, mathematician Benjamin Peirce, then receiving attention for his own innovative work on linear associative algebras, stepped in with support. He had known Sylvester since his first visit to America and wrote to university president Daniel Gilman:

'If you inquire about him you will hear his genius universally recognized but his power of teaching will probably be said to be quite deficient... Among your pupils sooner or later, there must be one who has a genius for geometry. He will be Sylvester's special pupil the one pupil who will derive from the master knowledge and enthusiasm and that one pupil will give more reputation to your institution than ten thousand who will complain of the obscurity of Sylvester, and for whom you will provide another class of teachers.'[4] The upshot was, of course, that

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[3] Quoting Halsted, writing in *Amer. Math. Monthly* (1894), 294–298, quote from 296.

[4] From the Gilman Papers, Special Collections Division, Milton S. Eisenhower Library, The Johns Hopkins University, as quoted in Karen Hunger Parshall and David E. Rowe, *The Emergence of the American Mathematical Research Community 1876–1900: J.J. Sylvester, Felix Klein and E.H. Moore*, American Mathematical Society, 1994, pp. 73–74.

Sylvester was appointed and as Parshall and Rowe tell us, his arrival on American shores, 'did more for American mathematics than resuscitate Peirce's work. It signaled the beginnings, however modest, of the entry of the United States into the international mathematical arena, a development which would take place within the context of neither the college nor the federal government nor the general scientific society but within that of the emergent American university.'[5] As for the one genius of geometry that Peirce spoke of, was this to be George Bruce Halsted? Or perhaps we should rephrase that and say it could have been him, but for some peculiar twists of fate. Halsted was in no doubt where he wanted to be in 1876 as Johns Hopkins was being set up. He had heard that the university was to offer a limited number of graduate fellowships and in what was to become a familiar trait, he drew up letters plastered with unashamed egotism about his background and sent them to the powers-that-be in Baltimore, including one to president Daniel Gilman. In it he spoke of 'such overmastering anxiety to be a partaker in your rich feast of learning that I cannot wait a single day, but would this very instant lay before you my humble petition'.[6]

Regardless of the over-gilded CV, Halsted, then 23, was chosen along with Thomas Craig, a graduate of civil engineering from Lafayette College, Pennsylvania, to become the first two candidates to receive the fellowship award. The Johns Hopkins mathematics department opened for business with seven undergraduates and eight graduates and in that fall of 1876 Halsted took on the role of research assistant to Sylvester who began the semester in a manner that one of his students described as 'fiery and passionate'.

By the second academic year, however, Halsted was showing a clear divergence from his professor's continuing theme of invariant theory and veered increasingly towards geometrical research, notably non-euclidean geometry that was to take up of much of his time in the years ahead. He began his investigations in 1877 and published a bibliography in two parts in 1878 and 1879, entitled *Bibliography of Hyperspace and Non-Euclidean Geometry*. This, he reported in his fellowship application for that year, had been a substantial task, completed on his own account without aid, discovering what had been written in virtually

[5] Parshall and Rowe, p. 49.

[6] Letter, Halsted to D.C. Gilman, 10 April, 1876, Gilman Papers, quoted in Parshall and Rowe, p. 78.

G.B. Halsted

every language. It was, he said, a major contribution towards ending the 'woeful ignorance' that existed in America on the subject. Halsted's tendency towards outspokenness also came to the fore in 1878 when he was preparing a lecture on Modern Logic. He had planned to refer to the contributions of Charles Peirce,[7] son of the mathematician Benjamin Peirce, but then declined to do so, claiming that Sylvester thought the work to be pretentious. Sylvester made it known to Peirce that this was a gross misrepresentation of his views and Peirce who, according to his own biographer Joseph Brent, had his 'own characteristic attraction to controversy', resumed cordial relations with both. Even so, Halsted left Johns Hopkins in the fall to return to Princeton where he completed his dissertation on *Basis for a Dual Logic* and was awarded his PhD at Johns Hopkins *in absentia*. Back at Princeton, Halsted became an instructor in the mathematics department and it appeared he had his mind set on advancing his career with that institution.

In 1950, Katherine S. Eisenhart, wife of Luther Pfahler Eisenhart, Dean of the Princeton Graduate School, compiled a selection of notes on earlier Princeton faculty members. In her references to Halsted, she mentions a 'curious little booklet' he published in 1882, entitled *Some*

[7] Peirce, of course, went on to become the principal founder of America's pragmatic school of philosophy.

Testimonials and Credentials. The pages contained a summary of his career, along with letters from a number of eminent mathematicians and scientists carefully chosen to bolster the Halsted image. Among them was one from Henry Burchard Fine who had been a student of Halsted at Princeton, and was by then a teacher in mathematics. He wrote that Halsted had steered him into a mathematical career.[8] If the intended purpose was to gain a bid for a professorship, it did not succeed. Mrs. Eisenhart identified the reason why Princeton shunned his self-promotional efforts: 'He was a man of ability in his chosen field, but certain eccentricities prevented him from attaining the success either as a teacher or writer which his powers seemed, in his youth, to promise.'[9] The following year therefore found Halsted attending an interview for the chair at The University of Texas, which had opened its doors just twelve months earlier. His credentials were impressive. The appointment was confirmed and he took his place among the ex-colonels of the Confederacy in 1884 and remained until 1902. He re-organized the mathematical curricula, popularized a series of faculty lectures and devised various methods to try to put The University of Texas on the map. He also pursued a fairly broad base of mathematical teaching and research with some vigor, particularly in the area of noneuclidean geometry. He would state fiercely that after two thousand years geometry was almost as Euclid had left it, circa 300 BC. He maintained that Cartesian coordinates, algebracized geometric notions and projective geometry were adaptations or refinements of euclidean geometry.

As we have seen from the early recollections of R.L. Moore, he began to focus particularly on the work of the Hungarian Bolyai in his outlining of a 'hyperbolic' geometry in 1833. Whereas Euclid assumed that in the two-dimensional plane, given a point not on a given line, one and only one line can be drawn parallel to the given line, this new hyperbolic geometry asserted that infinitely many lines could be drawn parallel to the given line. A surface known as the pseudo-sphere is a realization of this hyperbolic geometry. Halsted translated Bolyai's supplement into English in 1890 and Lobachevsky's papers the following

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[8] Fine later developed into a distinguished administrator at Princeton and into a gifted expositor. Princeton named a building Fine Hall, as a home for the mathematics department.

[9] Katherine S. Eisenhart, pp. 31–33 in a typescript prepared in 1950, deposited in the Princeton University Library.

year. Professor Robert Greenwood, who arrived later at the mathematics department at Austin, was able to study his work and formed a precise view: 'In a very real sense Halsted introduced and facilitated the study of noneuclidean geometry in [the United States], and also to some extent in other English speaking areas. Halsted visited eastern Europe and became acquainted with several of Lobachevsky's followers in Kazan.... In Austin, Halsted authored some geometry textbooks (in English), and one of these was translated into French. As U.T. Austin grew, two lines of mathematical curricula were developed. Thomas Ulvan Taylor was added to the staff in 1888 as an assistant professor of mathematics. Taylor taught the courses which later came to be designated as "applied" mathematics and Halsted taught those courses which later became "pure" mathematics.'[10]

Halsted also began the tradition of attracting quality students and achieved success with a number of students prior to the arrival of Robert Lee Moore. Some were to loom large and imposing in the young man's future. They included Milton Brockett Porter from Sherman, Texas, who was a student at The University of Texas from 1889–1892. He went on to Harvard where he received his M.A. in 1895 and his PhD in 1897. He returned to Texas as an instructor in pure mathematics in 1897 and accepted a similar post at Yale in 1898, subsequently being promoted to an Assistant Professorship, which he held until returning to The University of Texas (to replace Halsted) as Head of Pure Mathematics in 1903. A gifted mathematician of high ideals and character, he remained at The University of Texas for the next forty-two years, and then as Emeritus Professor from 1945–1960. Another was Harry Yandell Benedict who came to Austin in 1889, graduating in 1892. During 1891–1892 he was a Fellow in Pure Mathematics, and in 1892–1893 a teacher in Pure and Applied Mathematics at the University. He moved to the University of Virginia and during 1893–1895, he was an assistant in astronomy at that university. He resigned that post to study at Harvard, graduating from there with a PhD 'in mathematics, especially astronomy',[11] in 1898, having held two scholarships while at Harvard. Benedict, affectionately known to all as

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[10] R.E. Greenwood, *History of the Various Departments of Mathematics at the University of Texas at Austin: 1883–1983,* p. 7. Unpublished manuscript, R.E. Greenwood Papers at the Archives of American Mathematics.

[11] *The University Record,* Vol. 1, No. 2, April 1899, p. 223, as quoted in Traylor, p. 27.

Dean Benny, spent one year at Vanderbilt University, took entire charge of the department of mathematics at that institution and was subsequently elected president of The University of Texas, 1927–1937.

Leonard Eugene Dickson was another student who credited early contact with Halsted for the successful start to his career, which he pursued in an outstanding manner. Dickson was from Cleburne, Texas, where his father was a successful merchant. He graduated in 1893 and was awarded a teaching fellowship at The University of Texas in pure mathematics. He completed the course for his Master of Arts degree under Halsted's supervision. He then applied for doctoral fellowships at Harvard and Chicago and received an offer from both, electing to join Professor Eliakim Hastings Moore who, like Sylvester, was in the process of founding a mathematics department of substance at the University of Chicago. Dickson received his PhD in 1896 for a dissertation entitled *The Analytic Representation of Substitutions on a Power of a Prime Number of Letters with a Discussion of the Linear Group.*[12] Dickson went to Europe for a year, studying with Sophus Lie at Leipzig and with Jordan, Appel, Picard, Hermite and Painlevé at Paris. He returned to become an assistant professor of mathematics at the University of California during 1897–1899, resigning that post to become an associate professor of mathematics at The University of Texas where he remained for only a year before being recruited by Professor E.H. Moore at the University of Chicago, there to remain pursuing an outstanding career until 1939.[13] The staffing line-up in pure mathematics at The University of Texas as Robert Lee Moore began his own work in earnest was therefore impressive: Professor George Bruce Halsted, Associate Professor L.E. Dickson, instructors H.Y. Benedict and T.M. Putnam, a promising young mathematician from the University of California who had studied with Dickson. R.L. Moore was exposed to all of these brilliant minds at various times and as can be seen from his early diary notes, dating from the end of 1898, Halsted was leading the teenager whom he described as a 'young man of marvelous genius, of richest promise'[14] down the path of his own specialist interests. In the summer of 1900 Halsted, while blowing his

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[12] His work there was to be a precursor to the arrival in Chicago of Halsted's current (and last) protégé, R.L.Moore, which will be discussed in Chapter Four.

[13] A.A. Albert, Leonard Eugene Dickson (1874–1954), *Bull. Amer. Math. Soc.* 61 (1955), 331–346.

[14] George Bruce Halsted, *Science Magazine,* Friday, Oct. 24, 1902, p. 645, as quoted in Traylor, p. 36.

own trumpet for achievements at The University of Texas, singled out Moore, still only eighteen, as one of the stars of the future in that field.

He went public with this view in the June summary[15] of the Pure Mathematics Department at Austin, which he described as a 'bloom period'. He noted that for sixteen years, his department had pursued a decidedly geometric character, believing that this was the 'most remunerative as it is the most charming part of all mathematics'. There were courses in Modern Synthetic Geometry, Recent Geometry of the Triangle and Circle (the Lemoine-Brocard Geometry), Geometry of Position (Projective Geometry), Noneuclidean Geometry, and these were further strengthened by one in Group Theory by Dickson. Halsted then revealed that Dickson had accepted the position of an Assistant Professorship at the University of Chicago, pointing out that the head of department, Professor E.H. Moore, had finally included in his program 'the magic name non-Euclidean geometry'. In announcing Dickson's departure, Halsted added in the same summary that R.L. Moore was one of two students that year showing 'that the splendid quality of Texas youth is of undiminished vigor'. Indeed, the academic year of 1900–1901 was to bring significant results for Moore, and instant fame within the mathematical world. Before going on towards completion of this mathematical backdrop to R.L. Moore's own progression, it would be remiss not to mention the other side of the Halsted coin, that is the folklore surrounding him during his time at Austin. His eccentricities, wrote T.U. Taylor, were widely discussed: 'Of all the rare and odd professors that have been on the faculty of The University of Texas, I think George Bruce Halsted will rank as number one … for about sixteen years his sayings and doings in the classroom and in public lectures were the talk of the campus and the town.'[16] Halsted married Miss Maggie Swearingen in June 1886. They had three sons, Arthur, Halcyon, and Harbeck. The last two sons received M.D. degrees; the eldest (born in 1887) became an electrical engineer. Professor Halsted built a house for his family on the northeast corner of Twenty-fourth and Guadalupe Streets in Austin, which became the object of humorous comment. It was erected to his own design high above the ground with

[15] *The University Record,* Vol. 1, No. 3, June 1900, pp. 165–166, as quoted in Traylor, pp. 30–31.

[16] Among various recollections of Halsted contained in T.U. Taylor's book, *Fifty Years on Forty Acres*, published by Alec Book Company, Austin, 1938.

brick columns tall enough for a man to walk under without bumping his head. One theory popular at the time in the university chambers of gossip was that Halsted feared that a break in the dam on the Colorado River would flood Austin and wash away his home, whereas with his design, the water would merely flow underneath the building.

Another theory was that he built his house high so that he could stable his Shetland ponies, pigs and chickens under his home and would not need a barn or other shelter for them. Halsted and his Shetland ponies became a familiar sight around Austin. He had constructed a cart made from a single-plank mounted on two axles and connected by shafts to the animals. Taylor recalled: '[He] drove over town at a rattling pace, irrespective of traffic laws or right or left side of the street. One day in the faculty meeting, one of the professors happened to refer to the police court and ended with the remark that he had never been before a police court. Immediately, Dr. Halsted interjected: "I have the advantage over you. I have been there several times."'

Students, of course, had a great deal of fun amongst themselves. The 1895 edition of their annual yearbook, *Cactus*, included a mention that Professor Halsted won the Roman Chariot Race for that year! Some were able to record another incident that occurred after he'd made a trip to Mexico and found a plant whose juice, he decided, would revive youth in old age. 'He brought a boiled-down syrup from it to class and made each of his students, who weren't exactly suffering from the onset of senility, drink some of the potion. Unfortunately, we have no statistics on how the students held up sixty years later,' wrote Joe Frantz.

Similarly, his appearance at national meetings of mathematical and scientific bodies, at which he was in regular attendance, often raised a few eyebrows, not to mention some telling looks down noses. He was also well known for speeches outside the mathematical sphere, which he apparently delivered as a performance, like a hammy actor in a touring repertory company reciting Shakespeare not very well. An example survives. He gave the address on the eve of George Washington's birthday in 1899 to the Charles Broadway Rouss Camp of Sons of Confederate Veterans. It took the form of a collective deification of George Washington, Robert E. Lee and the Texas Ranger Sul Ross. The prose was purple and the applause loud, as he spoke in these terms:

'What words can fitly paint the mighty, the peerless paladins of the South. But for purity of contour, perfection of form, grace of power, elegance of mode, adorable, heart-winning Southern dignity, the world inevitably singles out Robert E. Lee…'

He had the speech printed as a pamphlet by Ben C. Jones and Co, Austin and presented a copy to Robert Lee Moore.[17] Notwithstanding these idiosyncrasies, Halsted worked hard to draw attention to the work being done at Austin and to promote scientific and mathematical discussion of all kinds, and especially about his own research. At a time when rail links made it difficult and expensive for potential lecturers and speakers of note to attend, he participated in the affairs of the Texas Academy of Science to encourage regular contact and discussion between educators and scientists. He was its president in 1894 and his retiring address, *Original Research and Creative Authorship, the Essence of University Teaching,* was reprinted in *Science*, the journal of the American Association for the Advancement of Science. He was also the mathematics section chairman and vice-president of the AAAS.[18] The mixture of peculiarities with the underlying importance and prodigious nature of his study, research and involvement in the mathematics and scientific communities at large combined to establish Halsted as a character who, sadly, was to be reviled as much as he was revered. Whether Halsted was a good teacher was also a matter of debate. Some said he was not, because of his tendency to talk about everything under the sun in class, other than the topic at hand, such as his travels and experiences and opinions. He was also impatient, insulting or chastising toward those he considered laggards, which, among seniors, occasionally led to heated exchanges and on one famous occasion, fisticuffs. If judged on results and publications, however, Halsted had an excellent record although again, it may be said that he paid particular attention to those who might develop into star performers. His colleague, T.U. Taylor, was in no doubt: 'The one outstanding thing that must be said to [his] credit was that he inspired men to study and research and in this respect he made a genuine contribution to American scholarship in mathematics.'

R.L. Moore certainly fell into that category and at his youthful age was quite clearly in awe of his teacher. His diary entries largely concerned his work, apart from attempting to talk a certain Miss Joynes into

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[17] G.B. Halsted, *Washington, The Ideal of the South; Resurgent in Lee and Ross: Address Before the Charles Broadway Rouss Group of Sons of Confederate Veterans,* February 21, 1899, R.L. Moore Papers in the AAM.

[18] His retiring vice-presidential address in 1902, The Message of Noneuclidean Geometry was published in the *Proceedings of AAAS* 53 (1903–1904), 349–371.

going to church with him or fighting with his roommate over his persistence in washing his nose in the bowl and opening the blinds when Moore was still in his night gown. Moore had to slap him, and grazed his head.[19] The time of 11:15 pm appeared regularly on his jottings, indicating that he used it as a break point when he finished his studies, and took time out to make a few notes in his diary before retiring. He had piled on the courses and the credits to such an extent that he gained his B.S. and M.A. degrees simultaneously in 1901, having been given an option on a B.S. or B. Lit. and choosing the former. He had already taken classes as a student assistant and for the academic year of 1901–1902 was granted a teaching fellowship, much to the amusement and admiration of some of his colleagues. As one of them recalled in the students' magazine, *The Alcalde*, in 1917:

'It is reasonable to assume that as a boy he attended some sort of school. It is alleged that he never studied algebra or geometry but was born with knowledge of them. The writer of this article has no recollection of learning the abc's and he does not believe that R.L. Moore can remember when he did not know geometry. However that may be…during the year of 1901–02 … he taught … a course in analytical geometry administered from Puckle's *Conic Sections*.… Though Moore was at the time only some nineteen years old, noticeably younger than the average students in his class, he handled the subject after the manner of a veteran and his subjects (pupils) even more so. Never, from the first week did the class doubt his knowledge of Conic Sections or his intention to run his class. He often criticized the text, a thing at that time new to most of us. One morning when class assembled, there appeared on the board the following: "Quiz For Math 2: 1] Find the equation of the boundary of Moore's intellect. 2] Find the points of intersection of this curve and Puckle's *Conic Sections*." It was during this year that he distinguished himself by proving the redundancy of Hilbert's set of geometric assumptions. This was the real beginning of his work.'[20]

Moore was obviously enjoying himself, although he seldom showed great emotion in the entries in that same Ivory Soap pad diary that he began on the first day he arrived on the Austin campus. He simply noted that he 'like[d] the business [of teaching] *very* well'.[21] In another entry

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19 Ivory Soap pad diary, 8 June 1899, R.L. Moore Papers in the AAM.
20 *The Alcalde* 5 (March 1917), 427.
21 Ivory Soap pad diary, 6 December 1901, emphasis in original, R.L. Moore Papers in the AAM.

for the same day, he mentioned what was to develop into the most important discovery of his short career. 'My, what a fine thing that Hilbert's geometry is. How fine, how pure, and free from ideas of space. Now at last it seems it has been rendered possible for one to write out in this year, 1901, a geometry in which every proposition is arrived at from certain explicit assumptions with the use of no other assumptions whatever except "the laws of pure logic." How pure!! How unexpected. No, I don't believe I would have thought last session this year I would be studying so fine a thing as this "geometry" of Hilbert.' Given an inherent talent for mathematics such as R.L. Moore early evinced, the direction in which it would take him was clearly determined by the kinds of mathematical problems on which his mentors and colleagues were concentrating, as well as his own tastes. R.L. Moore had arrived at the point of eureka, although he hadn't quite realized it. Raymond L.Wilder, future president of the American Mathematical Society and one of Moore's own early doctoral students, would record in reflections of that era:[22]

'Geometry was finally being put on a satisfactory axiomatic basis and Halsted's greatest interest at the time was in geometry, particularly [then] in Hilbert's recently published *Grundlagen der Geometrie* (1899). The outstanding characteristic of this work was its attempt to found the geometry of the plane and three-space on a rigorous axiomatic basis. Not only did Halsted apparently acquaint R.L. Moore with Hilbert's work, but Moore was induced to check one of the axioms (Axiom II 4) for independence.'

In fact, Halsted had been in touch with the renowned German mathematician, David Hilbert, who, in 1895, was appointed to the chair of mathematics at the University of Göttingen, where he continued to teach for the remainder of his career. After the publication of the *Grundlagen der Geometrie* (which became *The Foundations of Geometry*, in English in 1902), much of the book's popularity was due to a list of 23 research problems he set out at the International Mathematical Congress in Paris in 1900. His famous speech, 'The Problems of Mathematics', was translated by Halsted who described it as brilliant. It was an in-depth appraisal of mathematics of that period and offered a preview of the mathematical problems that he believed

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[22] R.L. Wilder, The mathematical work of R.L. Moore: Its background, nature, and influence, *Arch. Hist. Exact Sci.* 26 (1982), 73–97, quote from p. 74.

would arise in the twentieth century. Halsted, meanwhile, was hurriedly preparing a Supplementary Report for *Science* on Hilbert's set of axioms. Hilbert had called one group of his assumptions the Axioms of Arrangements but Halsted renamed them the Betweenness Assumptions.

Of these assumptions, the fourth one stated:

Any four points, *A, B, C, D*, of a straight line can always be so arranged that *B* lies between *A* and *C* and also between *A* and *D*, and furthermore *C* lies between *A* and *D* and also between *B* and *D*.

In his translation of Hilbert's statements, to arrive at the above, Halsted had come to an interpretation of 'angeordnet'. He discussed this with R. L. Moore, posing the question as to whether that fourth assumption might be demonstrated from the other assumptions. Based on Halsted's translation, we have:

If *A, B, C* are points of a straight (line), and *B* lies between *A* and *C*, then *B* also lies between *C* and *A*. If *A* and *C* are points of a straight, then there is always at least one point *B*, which lies between *A* and *C*, and at least one point *D*, such that *C* lies between *A* and *D*. Of any three points situated on a straight there is always one and only one which lies between the other two.

DEFINITION. The system of two points *A* and *B*, which lie upon a straight *a*, we call a *sect*, and designate it with *AB* or *BA*. The points between *A* and *B* are said to be points of the sect AB or also situated *within* the sect *AB*; all remaining points of the straight are said to be situated *without* the sect *AB*. The points *A*, *B* are called *endpoints* of the sect *AB*.

Let *A, B, C* be three points not co-straight and *a* a straight in the plane *ABC* striking none of the points *A*, *B*, *C*: if then the straight *a* goes through a point within the sect *AB*, it must always go either through a point of the sect *BC* or through a point of the sect *AC*.

At the time, Halsted was preparing to write a geometry textbook, using Hilbert's axioms as a basis for it and had written to Hilbert, asking whether he 'recognized any desirability for change'. Hilbert's answer to Halsted was received 14 April 1902 and read, in part, 'Instead of II 4 (the fourth assumption), I believe it suffices simply to say: If *B* lies between *A* and *C* and *C* between *A* and *D*, then lies also *B* between *A* and *D*; and then to prove my old II 4 as theorem.'

Halsted suggested that Moore might attempt to fill in the proof. His student went away to work on the task and before the day was out he was able to demonstrate Hilbert's new axiom, eliminating II 4 and

David Hilbert

reducing The Betweenness Assumptions from five to four. Moore would recount this discovery many times in the years ahead. He was so excited that he dashed over to the campus late in the evening. Looking up toward the old main building, he saw the light shining from Halsted's window and hurried to his room. Halsted wrote down Moore's oral arguments and sent the paper to the *American Mathematical Monthly*. He also wrote Professor E.H. Moore, at the University of Chicago, to draw attention to his namesake's achievement, knowing full well that E.H. Moore had also been working on Hilbert's conclusions. In fact, Professor Moore had already established the redundancy himself and had written a paper on it, published in the *Transactions of the American Mathematical Society*[23] the previous January. However, the young Moore's proof was decidedly shorter and more elegant. In fact, said the professor in a note to Halsted, it was 'delightfully simple' and he took the trouble to write to R.L. Moore to explain exactly what had happened to stave off any disappointment:

'Apparently he [Dr. Halsted] has not called your attention to the fact that the redundancy was pointed out by me and proved in my paper, which I am sending under separate cover, on the projective axioms of

[23] On the Projective Axioms of Geometry, *Trans. Amer. Math. Soc.* 3 (1902), 142–168.

geometry, published in the January number of the *Transactions*. In accordance with correspondence with him [Halsted], it was in connection with this paper of mine that he wrote to Hilbert and received Hilbert's response which led to your work on the subject. You will see that it was my desire to survey the whole system of projective axioms, and to exhibit a new system, and, in that connection to show that Hilbert's axioms I 4 and II 4 were in his system redundant, and moreover, to furnish a satisfactory account of the roles of the axioms I 3, 4, 5 which had been held by Schur[24] to be redundant. As to the axiom II 4, you will see that, by considerations of the other linear axioms alone, and so in particular without the use of II 5, or of my axiom 4, I prove on page 151 that the axiom II 4 is a result of the statement 21 which statement is the statement of your theorem I. Thus to complete the proof of the redundancy of II 4, in Hilbert's system, I should today make use of your proof of theorem I. The proof that I give, in that it involves my triangle transversal axiom 4, is necessarily much longer. I have supposed that you might be interested in understanding how your paper impresses me, and remain with considerable interest in the progress of your mathematical career.'[25]

Halsted was not impressed by E.H. Moore's letter. He would later write to R.L. Moore:[26] 'Neither Hilbert nor anyone else acknowledged that E.H. Moore's obscure and bungling adumbration proved anything before I published your beautiful proof.'

Despite the touch of rancor in Halsted's part, Professor Moore had obviously made a note of the young Moore's ability, which was Halsted's intention in the first place, and it augured well for the future. At The University of Texas, however, Dr. Halsted did not come out of these exchanges as well as his student, and already an aura of disquiet had descended over Halsted's tenure there. There had been many disagreements between him and the Board of Regents, especially over his outspoken views, which, as a senior university figure, might be taken as the official University of Texas line. The animosity had been building between them for some time and now it was to affect R.L. Moore. Halsted had proposed him for an instructorship vacancy in the universi-

[24] See Schur's statement in Chapter Four.

[25] E.H. Moore published a copy of the letter to R.L. Moore in The Betweenness Assumptions, *Amer. Math. Monthly* 9 (1902), 142–158.

[26] Halsted to Moore, March 19, 1904; R.L. Moore Papers in the AAM.

ty teaching staff, which was to occur for the 1902–1903 year. Moore had recorded this in his diary, with an uncommon show of excitement in May 1902. The euphoria was to be short-lived.

The following month, Halsted was informed that his candidate, R.L. Moore, had been rejected in favor of Mary E. Decherd who had graduated from the university with a B.S. in 1892 and had since been teaching at Austin High School. The daughter of an Austin family whose four children all received degrees from The University of Texas, she had also completed some graduate work for her Masters at the University of Chicago. There were, as Joe Frantz pointed out, Decherds all over the Austin area. They were people of some prominence with local civic connections, although 'none compared with R.L. Moore for raw promise'. Halsted was furious at this news, and promptly fired off an article published in *Science*[27] in which he pointedly criticized the university for its decision. He pulled no punches and the piece resounded through the halls of The University of Texas hierarchy:

'If the keenest, brightest, most gifted of the young people reject the scientific career, then fellowships serve only a dull, stale, tired clique of incompetents. Even after the possession of the rare and precious gift of scientific genius has been clearly, competitively proven ... the exquisite bud in its tender incipiency may be cruelly frosted.... This young man of marvelous genius, of richest promise, I recommended for continuance in the department he adorned. He was displaced in favor of a local schoolmarm. Then I raised the money necessary to pay him, only five hundred dollars and offered it to the President here. He would not accept it.... The bane of the state university is that its regents are the appointees of a politician. If he were even limited by the rule that half of them must be academic graduates, there would be some safety against the prostitution of a university, the broadest of human institutions, to politics and sectionalism, the meanest provincialism.'

Through no fault of his own, R. L. Moore found himself at the center of a huge row and given Halsted's notoriety, onlookers throughout the mathematics community watched with interest. Halsted confirmed by letter to Moore on 8 September that he had raised the $500 himself to fund his employment: 'He rejected it. He would not have you even if you cost him nothing. Of course, I made a fuss about it. He sent a letter

[27] Published in The Carnegie Institution, *Science* 16 (24 October 1902), 642–652, quote from p. 645.

after me, saying that after the present session, "my services would not be required."... So you see there is no hope for you here.'[28]

Halsted's departure suddenly became more imminent. He published another article, mentioning R.L. Moore, in the December issue of the prestigious *Education Review*, which was available in the third week of November for those awaiting the next installment of this rather public brawl. In a paper entitled 'The Teaching of Geometry', Halsted was highly critical of published aids to teaching which, he said, were in a 'hopeless muddle'. America awaited a *Geometry for Beginners* written by someone familiar with the new, penetrating and critical research in the principles of geometry. Meanwhile 'every conscientious teacher must better from his own studies the book he is still forced to use. He has no right to dump into a helpless scholar what he knows to be trash'.

University administrators acted swiftly, and in a later letter to Moore, Halsted wrote: 'My professorship has been declared vacant.... My salary is stopped and the blow has affected me considerably. It will, I fear, hurt my chances, which were good, from getting a place at another university.'[29] In another note, his consuming anger was evident: 'I like all my predecessors, am left after all the honors I have conferred on the University, to starve insofar as [they] can accomplish it. If I succeed in getting a place, I will want you at once and the new president will not be the malicious fool to refuse to take you, even "gratis".'[30]

The Board of Regents issued a statement in the most pointed terms, confirming that with immediate effect, Halsted was no longer connected with The University of Texas. And so Halsted's departure was confirmed and R.L. Moore was caught in the turbulence.

28 Halsted to Moore, 8 September 1902, R.L. Moore Papers in the AAM.

29 Halsted to Moore, 19 January 1903, R.L. Moore Papers in the AAM.

30 Halsted to Moore, 8 September 1902, R.L. Moore Papers in the AAM.

3

On to Chicago (1903)

Arrival at the University of Chicago, then a hotbed of mathematical development and progress under E.H. Moore; scene setting and influences including such luminaries as John Dewey, Bolza, Maschke, Klein and Weierstrass.

'Bobby' Moore

That Robert Lee Moore did not receive the Austin instructorship temporarily sent his youthful career into the sidings. Neither did The University of Texas cater for doctoral studies at that time and the plan that he should continue his studies with Halsted, with the clear hope of following L. E. Dickson to Chicago, could not be fulfilled. Halsted, meanwhile, was still desperately trying to find a position that would enable him to take Moore with him but that possibility very quickly diminished to the point of despair, as the professor made clear in another letter to Moore about their joint predicament:

'As yet I have not secured any place for next year. Of course, I would rather have you with me than anyone else in the world but it is very doubtful if I will have any choice of assistant the first year.... I feel sure you can get good places henceforth. If you can afford it, why not go to Germany and take a PhD under Hilbert? Write to me often.... Yours Always, Sincerely, G.B. Halsted'[1]

Moore, however, could not afford to go to Germany and in a couple of his diary entries, he mentioned 'having to go to papa' again. In early November 1902, further exchanges between himself and Halsted came to naught, with the latter now suggesting that he should write directly to Professor E.H. Moore and Dr. Dickson about the possibility of taking his studies towards a doctorate at Chicago. '[Y]our future is assured,' Halsted told him, 'I wish I could say as much for my own. The strain of my position here [is] now terrible.'[2]

R.L. Moore's situation was no more encouraging either and although he wrote to Professor Moore in Chicago, there was no fellowship available. In the fall of 1902, he accepted a teaching post at Marshall High School, in the north east of Texas, 260 miles from Austin but closer to his Dallas home. He hoped it would be a stopgap appointment, teaching students almost as old as himself and certainly a good deal worldlier after the cocooned life of study he had led thus far. The gloom was apparent in his first diary entry in the Ivory Soap book, on 9 December 1902: 'Tonight, I was walking along and passed by one of the Yates boys (George, I believe). After I had passed on a certain distance, someone (I think it was he) called out: "Look at that bull-headed school teacher. Hasn't he got a cap, that bull-headed teacher?" Am I going to ask him if he was that person who called out if I see him on the street

[1] Halsted to Moore 22 October 1902; R.L. Moore Papers in the AAM.

[2] Halsted to Moore, 13 November 1902; R.L. Moore Papers in the AAM.

by himself some time and is a fight going to follow? Well, that perhaps will be "seen" in the future.'

This was to be his first and last diary entry during his time at Marshall. It was not a happy place for him, as was evident from the continuing correspondence with Halsted, whose own dilemma stumbled from bad to worse. At least the exchanges between them provoked a fairly regular flow of mathematical challenges and discussion, which was a good deal more than the young teacher could expect at his new place of employment. Although some have suggested the months spent there would allow him to 'grow' as a teacher, it was unlikely he gained much from the experience and he certainly did not enjoy it. He stood in front of a classroom of long rows with girls on one side of the room and boys on the other. Some of the tough young farm lads delighted in playing him up. One of his former pupils, Inez Hughes, recalled that Moore turned up at school one day with his shoes tied with cord laces. After lunch he discovered a pair of shoelaces on his desk with verse attached:

An eagle flew from North to South
With Bobby Moore in his mouth,
But when he saw he had a fool,
He dropped him in the Marshall School.[3]

Moore never learned the identity of the author of that piece of doggerel and after some early attempts by students to disrupt lessons, he chose the course of all-inclusive discipline, keeping the entire class after school if any one of them had misbehaved. He was considered short-tempered by his students, which some saw merely as a challenge to taunt him further. After a while, things settled down and, as a rule, the students came to like him. 'We found we could get on his good side by asking a lot of questions, so we did.'[4]

Meanwhile, Moore's correspondence with Dr. Halsted during this unhappy period took an interesting turn. It occurred, coincidentally, as a debate had opened up between Halsted and John Dewey who, like Halsted, had gained his doctorate from Johns Hopkins University, his in philosophy in 1884. Both had long ago attended the lectures on the logic of Charles Peirce that resulted in the rift between Halsted and Sylvester. In response to Halsted's paper published in the *Education Review* in December 1902 in which he chastized teachers for dumping 'into a

[3] Quoted by Traylor, in *Creative Teaching*, p. 44.
[4] Ibid.

helpless scholar what he knows to be trash', Dewey published a follow-up paper. It was entitled 'The Psychological and the Logical in Teaching Geometry' and it discussed, among other things, the difference between truth and the whole truth. It was a wide-ranging treatise in which he proclaimed that while, for example, Halsted's favorite starter for beginners, the definition of a straight line, was a matter for mathematics alone, there was a need for making clear that the content of a book or lesson for a given grade of pupils was not just a matter for mathematics. It was a psychological matter as well.[5]

The professor found himself compelled to offer some guidance on matters psychological, logical and spiritual over a period of several exchanges on the issue of using unproven statements as an initial premise for reasoning. In the first, which appeared to be the result of Moore questioning religious assumptions, he stated: 'We *must* postulate God and we must study and follow Jesus, and strive to get in our minds the Holy Spirit. Your tendency, I fear, is too critical. Do not go on in any doubting. *Will* to believe. Go to Church but only to select out the good and what you can approve. Cultivate love for all men, even those you find it very hard to like. Begin to do definitely helpful acts every day towards someone.'[6] The theme of this correspondence continued for some weeks, during which time Halsted had finally found a new position, as a professor of mathematics at St. John's College, Annapolis, Maryland, from where he wrote to R.L. Moore:

'[Y]ou should read William James *The Will to Believe* and also his later Gifford Lectures. I supposed you had intellectually passed the point where you supposed there was any tenable meaning in the phrase you use "simply in search of the truth". You might be in search of what postulates created or accepted by you would make the best universe for life but to suppose there is an objective truth independent of our will is absurd. As Tennyson says, "For this is truth to me, and that to thee."

'Every successful and happy community has been and is held together by a postulated God. I know you can postulate a Godless universe but such "truth" will make you isolated and unhappy and is in no sense truer than the truth of your happy neighbor, the [C]hristian who sees that after he has postulated God then Christ is divine, as containing and revealing

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[5] Albert C. Lewis: *Reform and Traditions in Mathematics: The Example of R.L. Moore*, 15 April 1999.

[6] Halsted to Moore, 5 February 1903; R.L. Moore Papers in the AAM.

the proper nature and will of God. You are in a very primitive state of the so-called *scientific* mind, which does not see that your creative postulating belief makes that which you can call true. "Will to believe" is "will to *make true*." Accept the plan of creating your universe first expounded by Jesus. You are a good example of the consequences of creating a non-Christian universe. That you do not feel inclined to bow your head when the others do, even if only out of sympathy, is to me a sort of self-deception that begins alienation from fellow men and so begins dreadful unhappiness.

'Changes come to the mass of mankind gradually and are minute variations of their created universe. The wayward are *eliminated*. By all means go to church and bow your head. Your illustration from geometry turns against you. The latest understanding of the parallel-postulate is that it is a *definition*, and whoever believes it is right. All we need to see is that another space could be postulated (Bolyai's or Riemann's) just as I see that a [G]odless universe can be postulated. There is no "our space" except the space we make and in the making of it, we make it by definition Euclidean or non-Euclidean. I could very well tell you "will to believe" Euclid's parallel-postulate. If it made such a difference for you to believe God, I would urge you *will* to believe it.'[7]

The discussion by correspondence continued and again although it is impossible to be specific about the questions, the answers in Halsted's response[8] provide the clues as to the thoughts and requirements of Moore's search for 'the truth':

'The universe is not *entirely* created by our *individual* self since we are imitative animals and accept the language used about us. Yet your supposedly cogent illustration is badly chosen. You ask "Suppose one should will to put his hand in a fire and also at the same time to believe he felt no pain. Could he create such a state of affairs?" I answer, Yes, certainly he could. In fact, that very state of affairs has often been created.... Of course, if society, ancestors and contemporaries agree to believe certain things, then certain others logically follow. They agree to think the essences of things together into a personal God. Though you have the power to believe otherwise, and to create a scheme antagonistic, yet as one of the things which follow from social life is dependence on fellows, then also follows unhappiness from the making of sympa-

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[7] Halsted to Moore, 26 February 1903; R.L. Moore Papers in the AAM.

[8] Halsted to Moore, 31 March 1903; R.L. Moore Papers in the AAM.

thy more difficult, the self-isolation of an antagonistic scheme. We believe that society eliminates the wayward. I postulate that you at least partially understand me. That seems a universal assumption on which to base social life.

'The creation of a Godless universe has usually had as a concomitant unhappiness. This sentence I may safely assume to convey to you a meaning. I believe, and so do you, that if an individual creates in a certain way he thereby creates, as a concomitant, unhappiness.... Since we both believe this, we may put it among the assumptions. It gives meaning to my sentence about "Every successful and happy community has been held together by a postulated God." The fact undeniable that "many and important things true for one man are not true for another", has only relevancy insofar as it enables us to conceive that there might be a man who could believe the opposite of this statement. It remains for you, understanding it, to make it part of your universe or to try to make the opposite belief part of your equipment for action. I cannot coerce your creation. I can only suggest alternatives and say which I believe leads to happiness. Remember we have in the language the words perversity and pervert.'

Moore was clearly at some sort of crossroads spiritually and mathematically but for the time being this was set aside. There was, included in that last note, also a reference to the possibility that Moore might soon hear good news from Chicago. In fact, Halsted knew it was highly likely. He had already received a letter from Oskar Bolza, one of E.H. Moore's partners in the mathematics mission being undertaken at the University of Chicago. Bolza revealed that R.L. Moore had applied for a fellowship at the university and Bolza was now seeking a reference regarding his mathematical abilities and attainments. Overjoyed at the news, Halsted dashed off a note to his protégé: 'I have just written a tremendously strong letter to Professor Bolza in your favor...and I think you may surely count on receiving the appointment.'[9]

That prophecy proved correct and Moore, in a further uncommon display of moderated emotion in his Ivory Soap diary: 'Gee whiz! I have become a Fellow in Mathematics in the University of Chicago.' When confirmation arrived, he wrote to his father and of course to Halsted, who replied: 'I greatly rejoice.'[10]

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[9] Halsted to Moore, 16 March 1903; R.L. Moore Papers in the AAM.
[10] Halsted to Moore, 14 April 1903; R.L. Moore Papers in the AAM.

He and Moore both, and they had good reason. The mathematics department of the University of Chicago had, in its short history, attracted international attention and acclaim as a place of significant and exciting developments in an otherwise bland and largely under-performing arena of American scientific and mathematical research. Chicago was well on the road to establishing itself as one of the outstanding universities in the United States offering true graduate education in mathematics. There were only two others at that time, Johns Hopkins, whose development we have already examined, and Clark University, founded two years before Chicago. These three shone out like beacons at a time when, compared with their counterparts in Europe, mathematics courses taught at the majority of American universities were, according to one view, 'paltry, the spirit of research almost nonexistent and the quality of the faculty vastly inferior'.[11]

Chicago's first president, William Rainey Harper, set about generating great innovations in every aspect of higher education and the process was still in its youth when R.L. Moore was finally able to extricate himself from Marshall High School to arrive in Chicago in September 1903. He was in for a culture shock. This city was the modern, northern equivalent of the Wild West, and more. It was bigger, much bigger, brasher and far more violent than any place in Texas. Chicago had experienced an incredible rate of growth, expanding in seventy years from a small trading post on the swampy river mouth near the southwestern tip of Lake Michigan to a city of over one million by 1900, and rising. Its people were drawn largely from the northeastern states and across Europe. The strategic inland water location provided the powerhouse that drove its climb towards becoming one of the most renowned and wealthiest commercial centers in the world. The boomtown rose with the Civil War. Just as Dallas was a major supply center for the Confederacy, Chicago had been a major logistical base for the Union, although downstate in southern Illinois there had been sympathy toward the Confederate cause. The city had survived the Great Fire of 1871, when four square miles of property, including the business district, was burned to the ground, killing 250 people and rendering 90,000 homeless. The cost of the damage was $200,000,000 and the rebuilding program saw the birth of steel-framed skyscrapers, with the completion

[11] Quoting David E. Zitarelli, in Towering Figures in American Mathematics 1890–1950, *Amer. Math. Monthly* 108 (2001), 606.

of the Home Insurance Building in 1885, followed, during the next decade, by twenty-one buildings ranging from twelve to sixteen stories.

That was fine, but great festering slums were being created just as fast, with the population swelling during the second great wave of European immigration. Now, Russian Jews, Italians, Poles, Serbs, Croatians, Bohemians and others from Eastern Europe joined the earlier immigrants predominantly from Scandinavia, Germany, Britain and Ireland. The 1900 census showed that more than three-quarters of Chicago's population was made up of the foreign-born and their children, and it now surpassed Philadelphia as America's second most populous city.

By 1903 when R.L. Moore arrived, it was a principal center of not just the economy of the United States, but of social insurgency, immigration, housing and education. It also boasted the most elegant brothels in the country, which entertained royalty from abroad and millionaires from the newly sprawling suburbs. Scholars were attracted by this new seat of learning, while writers and authors arrived for inspiration from its brawling streets. 'I have struck a city,' wrote Rudyard Kipling. 'A real city, and they call it Chicago. The other places do not count.... Having seen it, I urgently desire never to see it again. It is inhabited by savages.'[12]

In the middle of this sprawling, steaming morass of human flotsam and jetsam sitting uncomfortably alongside boiling commercialism arose the University of Chicago, founded in 1891 by the American Baptist Education Society and Standard Oil magnate John D. Rockefeller. He later described the University as 'the best investment I ever made' although its president, Dr. William Rainey Harper, was its true creator. He envisaged a university that went well beyond the liberal arts colleges of the 1890s, where classical education ruled. He wanted important additions such as the graduate research pioneered in Europe, managed by a faculty of dedicated teachers and researchers who were to follow specific guidelines that precluded stocking the students' minds with knowledge of what had already been accomplished. He demanded new horizons, new research and new lines of investigations. The mission, he insisted, could not be allowed to fail and in forcing into existence a hotbed of new thinking, Harper provided unparalleled opportunities for the linchpins in a new generation of American

[12] *American Notes,* 1889.

mathematicians and scholars. Frederick Rudolph, professor of history at Williams College, has written: 'No episode was more important in shaping the outlook and expectations of American higher education during those years than the founding of the University of Chicago, one of those events in American history that brought into focus the spirit of an age.'[13]

The whole focus of the Chicago enterprise emerged through the foresight of Dr. Harper who was not only a great barnstorming manager of university affairs but an exceptional role model for all who gathered around him. Among them were some brilliant personalities to add to the chart of R.L. Moore's mathematical ancestry and, once again, a detour from the main thrust of our story is necessary to place them in the chronology of his exposure to a rich seam of research and development in progress immediately prior to, and after, his arrival at Chicago.

Harper himself was not a mathematician. He had pursued Hebraic studies at Muskingum College, New Concord, Ohio, from which he graduated in 1870 at the age of fifteen. He received his PhD from Yale at the age of nineteen for studies in the Indo-Iranian and Semitic languages and moved into academy teaching before being awarded a professorship in Hebrew at the Baptist Union Theological Seminary in Chicago. He authored a number of textbooks[14] and aids for the teaching of Hebrew. He returned to Yale in 1886 as a professor in Semitic languages and in 1889 was simultaneously appointed Woolsey professor of biblical literature. Even as the ground plans for the University of Chicago were approved, Harper was among those short-listed to become president. He held back until the last, and was finally persuaded by Rockefeller himself by being given virtual carte blanche to organize a liberal and comprehensive outlook for the new university where students would be given the freedom to do research. He announced his intention to divide the traditional collegiate program into two parts, devoting the first two years to general education and the last two years to higher study. He would keep the university open all year round and other innovations included the introduction of correspondence courses.

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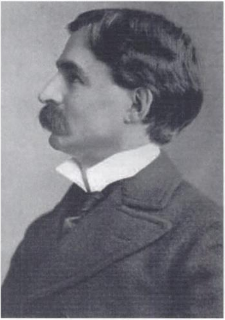
[13] Frederick Rudolph, *The American College and University: A History*, 1962.

[14] Among his more important books are *Religion and the Higher Life* (1904); *A Critical and Exegetical Commentary on Amos and Hosea* (1905); *The Prophetic Element in the Old Testament* (1905) and *The Trend in Higher Education* (1905). Harper remained at Chicago as president and head of the department of Semitic languages until his death in 1906.

More important were the people he drew in, carefully selecting those who would head up the teams to attract strong faculty and students alike in a research-orientated environment. They would include such luminaries as John Dewey whose influence on the American education field, for good or ill, was to reverberate through the twentieth century. He left Michigan in 1894 to become professor of philosophy and chairman of the Department of Philosophy, Psychology, and Pedagogy at the University of Chicago. Dewey's achievements there had brought him national fame, which came to fruition at the time of R.L. Moore's arrival.

The emphasis on new thinking and the pedagogical development became a priority in Harper's plan, and none more so than the Department of Mathematics which had been highlighted as a crucial area in Chicago's objectives, which in turn had great bearing on young careers, like that of R.L. Moore. Harper wanted to appoint a man to bring European thinking and personalities into the new institution and there were a number of established American or European mathematicians he could have selected to head the new department. In fact, he had taken advice and a short list of six well-known names had been drawn up for him. But, as with many of his decisions, Harper confounded those around him by choosing a virtual unknown who was not even on the list. He settled on Eliakim Hastings Moore, a relatively untried assistant professor with no great publishing history, then residing not far away at Northwestern University in Evanston, Illinois.

Moore, son of a Methodist minister from Athens, Ohio, was barely 28 years old when Harper first approached him and had published only four papers, apart from his doctoral dissertation, which had caused hardly a ripple of interest. Harper had spotted something others had missed, the possibility of genius inside the excitable persona of E.H. Moore. It was undoubtedly a gamble, but it paid off handsomely. Harper had completed a thorough research job on Moore's background, which was solid enough. It included experience among some of Germany's finest mathematicians, although it was not what might be termed a spectacular CV. He also saw that the assistant professor was undoubtedly hemmed in by his present surroundings at Northwestern, whose work was not especially adventurous, although a good deal better than some. E. H. Moore had studied at Yale and worked one summer with Ormond Stone, director of the Cincinnati Observatory, whose mathematical output was substantial. His own mentor was Hubert Anson Newton, pro-



Eliakim Hastings Moore

fessor of mathematics at Yale who inspired and guided Moore into research. Newton was professor of mathematics at Yale from 1855 until his death in 1896, although in the latter stages of his career he turned towards astronomy.

He was not noted for his own research work, but in recognizing the limited facilities for the training of future mathematicians in the United States, encouraged his most talented mathematics students to undertake research by self-discovery and especially to take account of the work being done abroad. In fact, he was so impressed by E.H. Moore that after he had earned his PhD at Yale in 1885, Newton personally financed a year's study in Germany. There, armed with an introduction from Newton, Moore met and worked with great mathematicians of the day, including Schwartz, Klein and Weierstrass. He returned in confident mood, took work first as an instructor in the Academy at Northwestern University, then as a teacher at Yale University for the next two years before returning to Northwestern in 1889 as assistant professor. He was still there when Harper began to cast his net for his own staff and it must surely have been Hubert Newton, an old friend and former colleague at Yale, who tipped him off about the genius of E.H. Moore.

Harper made the approach, but Moore did not rush to accept. He played a canny game. There were apparently several exchanges, especially over salary, before Moore finally accepted the offer of a professorship and the appointment as acting head of mathematics at the University of Chicago, starting in the fall of 1892. Four years later, recognizing the remarkable job Moore was accomplishing, Harper made him permanent head, a position he held until 1931. From the outset, Harper and Moore were in complete agreement over the need to attract universally brilliant scholars to the University, and especially recruit faculty members from Germany. The first important addition was Oskar Bolza who in 1892 was engaged in a rather bitter dispute at the fiscally challenged Clark University.

Nine of the eleven members of the permanentfaculty had put in their resignations en masse, and although they eventually withdrew them the situation remained volatile. Clark had been cutting back heavily on equipment and research, in which the university had boasted a leadership role in America. Financially, however, the chickens were coming home to roost. Harper immediately targeted a number of the unhappy members of Clark faculty and in what became known as Harper's Raid[15] secured the services of no less than fifteen of them, including instructors and fellows. Fed up with the lack of resources at Clark, Bolza himself had decided to return to Germany.

E.H. Moore had other ideas. Bolza was the one man he especially wanted from Clark because he had the experience and the connections that Moore himself had sampled in Germany. Bolza had studied mathematics under Christoffel and Reye at Strasbourg, under Hermann Schwarz at Göttingen and Karl Weierstrass at Berlin. He was present for the lecture course on the calculus of variations given by Weierstrass in 1879, which was a defining moment in Bolza's own search for mathematical direction. Finally, and most importantly from E.H. Moore's standpoint, he achieved his doctorate from the University of Göttingen in 1885 supervised by Felix Klein, best known for his work in non-euclidean geometry, the connections between geometry and group the-

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[15] The Clark crisis, outlined in *American Higher Education: A Documentary History*, Wilson Smith and Richard Hofstadter (eds.), University of Chicago Press, 1961, Vol. 2, pp. 759–761.

ory, and for results in function theory. Plagued by ill-health and depression, Klein had by then curtailed his own research work but had set out to inspire others with the foundation at Göttingen of what became a model for mathematical research centers the world over. It was an experience that E.H. Moore, who had witnessed it himself first hand in 1886, wanted to tap into.

Nor was that the end of the story as far as the Klein connection was concerned. Bolza's closest friend and another of Klein's students was Heinrich Maschke who had migrated to the United States in 1888. Having acquired Bolza, E.H. Moore now approached Maschke, who worked for the Western Electrical Instrument Company, Newark, New Jersey. He was glad of the opportunity to join his old friend Bolza and the two went to Moore's department as professors of mathematics. So was created one of the most influential mathematical research teams in the United States whose work as a forceful and inspirational triumvirate was to become the guiding spirit for so many brilliant young mathematicians over the next decade and beyond, including R.L. Moore. As these arrangements were being completed, the Chicagoans were presented with a heaven-sent opportunity to make a very firm stamp of their authority on the American mathematical scene when the Windy City was chosen as the venue for the 1893 World Fair.

To coincide with that event, E.H. Moore's team began work on organizing the Chicago Mathematical Congress, a weeklong extravaganza of lectures and papers from America's own mathematical hierarchy, including one from Dr. G.B. Halsted,[16] as well as from abroad. On the first day, a paper by David Hilbert entitled 'Invariantentheorie' was the star attraction, and the Congress concluded with a lecture by Felix Klein.[17] Never had such an influential collection of views been assembled under one roof in the United States and it was in the vein of what might be termed calm assertiveness that the mathematics department of the University of Chicago under Moore, Bolza and Maschke grew in stature and performance. It was a team totally dedicated to mathematical excellence and Archibald's description hit the right note:

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[16] Halsted's Congress lecture was entitled 'Some Salient Points in the History of Non-Euclidean and Hyper-Spaces.'

[17] The title was *Concerning the Development of the Theory of Groups during the Last Twenty Years*. Klein then went straight on to deliver the Evanston Colloquium Lectures on Mathematics, from 28 August to 9 September 1893.

'These three men supplemented one another remarkably. Moore was a fiery enthusiast, brilliant, and keenly interested in the popular mathematical research movements of his day. Bolza, a product of the meticulous German school of analysis led by Weierstrass, was an able and widely read research scholar. Maschke was more deliberate than the other two, sagacious, brilliant in research, and a most delightful lecturer on geometry.'[18]

Into this hotbed of exploration and discovery stepped R.L. Moore, arriving in the fall of 1903 when the triumvirate was at its peak. It was also a measure of the Texas Moore's ability — and Halsted's very strong recommendation of it — that he was offered a fellowship at such an important time in the Chicago experiment. It was, after all, not the norm to take a young man teaching high school in some out-of-the-way Texas town for such a position, but more importantly at this juncture was the fact that all eyes were on the Chicagoans. They were being watched by the whole mathematics community, a scrutiny that became rather more focused when Professor Moore became president of the American Mathematical Society in 1901 and even more so when he received a good deal of publicity during his term of office for suggesting the use of what he described as 'laboratory' teaching techniques at Chicago. To put it bluntly, his department had gained a reputation but now had to prove the worth of his pedagogical experiments. Moore chose his 'stars' carefully. Only the best would do and indeed, as will be seen, he experienced difficulty in teaching those who were slower on the uptake. Of his early PhD students L.E. Dickson was outstanding. As already noted, Dickson arrived from Texas having gained his B.S. and M.S. under G.B. Halsted and was described by E.H. Moore as 'the most thoroughly prepared student I have ever had'. He joined Moore's department in 1900 for what was the beginning of a long and fruitful career, remaining at Chicago until his retirement in 1941.

In 1901 his book, *Linear groups with an exposition of the Galois field theory,* was published as an expanded version of his 1896 doctoral thesis. It is described by Karen H. Parshall as 'a unified, complete, and general theory of the classical linear groups — not merely over the prime field GF(p) as Jordan had done, but over the general finite field GF(pn), and he did this against the backdrop of a well-developed theo-

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[18] *Semicentennial History of the American Mathematical Society, 1888–1938*, New York, 1938.

L.E. Dickson

ry of these underlying fields.... [H]is book represented the first systematic treatment of finite fields in the mathematical literature'.[19]

Dickson went on to publish 17 books and many articles and papers. He was awarded numerous honors and degrees from rival universities, became president of the American Mathematical Society (1917–1918) and supervised more than 60 PhD students. It has been said that mathematicians at this level might consider it fortunate indeed to have one such thoroughly brilliant student in even a decade but E.H. Moore brought on three others who were to arrive in Chicago at his behest at a particularly relevant time. The first was Oswald Veblen, widely considered to be E.H. Moore's first true prodigy, followed quickly by R.L. Moore and George David Birkhoff. Before examining their arrival and friendship in more detail, it is timely to inject a recollection of E.H. Moore's style and attitudes as he drew in graduate students of such high caliber and who themselves were to carry forward this great flair and influence on the creativity of the American mathematical school. For

[19] K.H. Parshall, A study in group theory: Leonard Eugene Dickson's *Linear groups*, *Math. Intelligencer* 13 (1991), 7–11.

this we can accurately rely on the personal descriptions of L.E. Dickson himself, in a biographical account he jointly authored with another outstanding Chicago student of the earliest era, Gilbert Bliss, who began his doctoral studies working on the calculus of variations. He gained his PhD in 1900, producing a dissertation, *The Geodesic Lines on the Anchor Ring,* which was supervised by Bolza. Bliss joined Dickson on the Chicago faculty in 1908, following the untimely death of Maschke in that year and, like his comrade, remained until retirement. Their joint summary of experiences with E.H. Moore may be seen to hold the seeds of inspiration for the style of instruction eventually developed by R.L. Moore. In the biographical memoir, Dickson and Bliss identified E.H. Moore's success as an educator as being due to his profound interest in mathematics and his faculty for inspiring his colleagues, and especially the strongest graduate students, with some of his own enthusiasm. He pursued educational experiments fearlessly and in every one he undertook, as at every other stage of his leadership in his department, he had one permanent characteristic: 'He believed in the exercise of individuality in classroom instruction, and he gave his colleagues unlimited freedom in the development of their classroom methods. He expected and insisted on success, and he was always sympathetically interested in a new proposal or procedure, but so far as is known to us he never prescribed a textbook.... Two of the characteristic qualities of his research were accuracy and generality. He was a master of mathematical logic, and his originality in making one or more theories appear as special instances of a new and more general one was remarkable.'[20]

His classes were of indeterminate length because he would bring before his students topics that he found of absorbing interest and he, and they, would investigate those ideas together until the topic was exhausted. Formal class schedules, time of day nor mealtime had any bearing on his pursuit of that topic. His style was not to everyone's taste. Dickson and Bliss recalled that students not so far advanced found difficulty in keeping up with him. At times, because of his total absorption in the subject at hand, he would be unaware offense had been given by something he had said, and would be extremely gentle in his expressions of regret when it was called to his attention that someone's feelings had been hurt by his impatience. Weak students often shunned his

[20] *Biographical Memoir of Eliakim Hastings Moore (1862–1932)*, by G.A. Bliss and L.E. Dickson, National Academy Biographical Memoirs, Vol. XVII, 1936, p. 89.

classes because of the demand placed on them. Strong students and those of quickest mind were attracted to him. He became a teacher who taught the teachers of mathematics. His teaching skills were the strongest when dealing with graduate students, although with students not so far advanced he was less accessible. His own mind was so quick and his concentration so thorough that it was difficult for him to await the comprehension and the slower development of understanding among those who were less experienced. However, he did edit an arithmetic book for use in elementary schools in 1897, and in 1903–1904 and following years, he influenced radically the methods of undergraduate instruction in mathematics at the University of Chicago. He gave courses in beginning calculus himself, casting aside textbooks, and concentrating instead on the fundamentals of the topic and their graphical interpretation.

He also put forward a proposal for laboratory courses, meeting two hours each day, which E.H. Moore had outlined in his presidential retirement address before the American Mathematical Society, the principal theme of which was a call for reforms across the American educational system in the teaching of science and mathematics:[21]

'This program of reform calls for the development of a thoroughgoing laboratory system of instruction in mathematics and physics, a principal purpose being as far as possible to develop on the part of every student the true spirit of research and an appreciation, practical as well as theoretic, of the fundamental methods of science.... To provide for the needs of laboratory instruction, there should be regularly assigned to the subject two periods, counting as one period in the curriculum.... This pedagogic principle of concentration is undoubtedly sound. One must, however, learn how to apply it wisely.'

The laboratory methods apparently intrigued R.L. Moore and he mentioned it to G.B. Halsted in a letter to him on 2 November 1903. Halsted was similarly intrigued, though sceptical: 'What have you found out about them? Do you find anything admirable or desirable in it? Or is it merely a fad?'[22] R.L. Moore reserved judgment and appears to have made no further reference to it in correspondence with Halsted, perhaps because it was apparent from his tone that Halsted believed it

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[21] Retirement address, *On the Foundations of Mathematics*, delivered before the AMS, at its ninth annual meeting on 29 December 1902; *Bull. Amer. Math. Soc.* 9 (1902–3), 402–424.
[22] Halsted to Moore, 15 November 1903; R.L. Moore Papers in the AAM.

to be a ridiculous notion. There were certainly pros and cons that created debate among those at the hub and, as with many such new plans, the amount of work required of the instructor was considerable. As Dickson and Bliss pointed out in their memoir, the two-hour period that was the feature of the experiment had to be abandoned because of the very practical difficulty in finding hours on schedules which would not interfere with the offerings of other departments. In the end, it proved an unworkable burden upon the curriculum schedules. Perhaps the most notable edict was also included in E.H. Moore's presidential retirement address:

'The teacher should lead up to an important theorem gradually in such a way that the precise meaning of the statement in question, and further, the practical truth of the theorem is fully appreciated and the importance of the theorem is understood and indeed the desire for the formal proof of the proposition is awakened before the formal proof itself is developed. Indeed, in most cases, much of the proof should be secured by the research work of the students themselves.'

E.H. Moore's grand plan, with its inherent flaws such as the pressures it brought to bear on the curriculum, eventually led to the demise of the laboratory experiment. Perhaps the overriding aspect that came out of these discussions in the Chicago Mathematics Club was E.H. Moore's insistence on great care being taken in using precise language. This reinforced a trait already evident in R.L. Moore's earliest work, and even in his letters to Halsted and his own notebooks, where he precedes even the most mundane statements with his 'Suppose…' Those who came to his classes in years hence quickly discovered that precision and clarity in written and spoken statements were vital ingredients of any Moore Method experience.

4

A Veritable Hothouse (1903–1905)

Under the general guidance of E. H. Moore, Bolza and Maschke, R. L. Moore takes his place at Chicago with nervous enthusiasm. Befriended by Veblen and in the company of Dickson, Birkhoff (G. D.), Bliss and Co., E. H. Moore's finest group of young mathematicians begin their journey to the cutting edge of American mathematics. Letters and papers provide R. L. Moore's running commentary.

Mister Robert Lee Moore

An aura of excitement, competitiveness and newness developed at Chicago that was in many ways unique. There was so much going on that the sheer drama of the moment — for such it surely was — combined effectively with the inspirational qualities of instructors to lead them into territory previously uncharted in American methods of tuition and in the subject matter upon which they focused. E.H. Moore's own courses might, in retrospect, be looked upon as elitist in that they ought to have carried a health warning for the faint-hearted or slower students. It might also be added that an additional ingredient that gave rise to the (R.L.) Moore Method which is ultimately at the heart of these chapters, must surely have come to him in his days at Chicago (1903–1905) when he was exposed to that dynamic scenario created by E.H. Moore, Bolza and Maschke, each guiding 'students along lines of investigation consonant with his own evolving research interests'.[1]

The latter probably had a greater influence than the system in injecting into the psyches of the aspirants the qualities and ambitions that produced a fair number of mathematical stars who went on to substantial endeavor as E.H. Moore and his team skillfully identified areas of modern work, often aligned to European studies, and pushed forward the boundaries of research in the United States. In doing so, Professor Moore set up a program at Chicago that by 1900 put it ahead of the half dozen or so rival universities with graduate programs in mathematics and thus attracted a continuing flow of talent into his own program. By 1915, the University of Chicago had produced 63 mathematics doctoral students, well ahead of any other American establishment.[2]

As history now shows, the sparkling collection of students who gathered at Chicago in those early years included a number who were brilliant enough to take their place at the cutting edge of America's mathematical development in the first half of the twentieth century, which saw crucial advances in group theory, algebra, differential geometry, topology and logic. But, of course, that was Moore's prime objective in his aborted plan for the 'laboratory' periods for which he directed that students and instructors should combine their efforts in the acquisition of

[1] Parshall and Rowe, *The Emergence of the American Mathematical Research Community*, p. 393.
[2] Analysis and studies by Dell Dumbaugh Fenster and Karen Hunger Parshall in *The History of Modern Mathematics*; Boston, Academic Press, Inc., 1994, pp. 179–227.

Heinrich Maschke

skills and knowledge. He made that quite clear in his 1902 AMS presidential address, "On the Foundations of Mathematics":

'Some hold that absolutely individual instruction is the ideal and a laboratory method has sometimes been used for the purpose of attaining this ideal.... The instructor utilizes all the experience and insight of the whole body of students. He arranges it so that the students consider that they are studying the subject itself, and not the words, either printed or oral, of any authority on the subject ... I am convinced that the laboratory method ... is the best method of instruction for students in general and for students expecting to specialize in pure mathematics.'[3]

Consequently, students and instructors at Chicago were inspirational to each other, a fact no better demonstrated than in the work of five, in particular, who went on to contribute significantly to the American mathematical world for the ensuing four decades: Leonard Dickson, Gilbert Bliss, Oswald Veblen, Robert Lee Moore, and G.D. Birkhoff. Of particular interest for this account was the relationship that developed between R.L. Moore and the brilliant young doctoral student Oswald Veblen, later to achieve great fame at Princeton. The two struck up an immediate friendship, both on a personal level and in a very close and fervent relationship in their early work. The central interest in this

[3] Retiring presidential address to the AMS Ninth Annual Meeting, *Bull. Amer. Math. Soc.* 9 (1902–3), 402–424, quote from pp. 419–420.

coming together of two young minds was R.L. Moore's astute approach to geometry and axiomatics that emerged under Halsted, and which had brought him to the attention of E.H. Moore in the first place.

Veblen, likewise, was engaged in similar research and both came into Professor Moore's sphere at a time when he himself was pursuing the foundations of geometry and analysis. It will be recalled from early references in Chapter Two that Hilbert had published his *Grundlagen der Geometrie* in which he pursued the work of the German mathematician Moritz Pasch and the Italians, Giuseppe Peano and Giuseppe Veronese, and others. Hilbert stated that his aim was to discover a complete set of independent axioms and 'to deduce from these the most important geometrical theorems in such a manner as to bring out as clearly as possible the significance of the different groups of axioms and the scope of the conclusions to be derived from the individual axioms'.[4] Hilbert proposed twenty such axioms and he analyzed their significance in *Grundlagen der Geometrie*, thus putting geometry in a formal axiomatic setting. His work became a major talking point, highlighting the axiomatic approach to mathematics, which he followed up with a famous presentation at the Paris congress in 1900. He opened with one of his most quoted statements:

'Who of us would not be glad to lift the veil behind which the future lies hidden; to cast a glance at the next advances of our science and at the secrets of its development during future centuries? What particular goals will there be toward which the leading mathematical spirits of coming generations will strive? What new methods and new facts in the wide and rich field of mathematical thought will the new centuries disclose?'[5]

Hilbert then posed 23 problems and challenged mathematicians the world over to resolve fundamental issues. He went on to stress the undeniable importance of definite problems for the progress of mathematical science in general: 'We hear within ourselves the constant cry: There is the problem, seek the solution. You can find it through pure thought.'[6]

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[4] D. Hilbert, *The Foundations of Geometry*, Open Court Publishing, La Salle, Illinois, 1965, p. 1.

[5] Hilbert's famous speech *The Problems of Mathematics* was delivered to the Second International Congress of Mathematicians in 1900 and published in *Bull.Amer. Math. Soc.* 8 (1902), 437–479.

[6] Hilbert references: M. Toepell, On the origins of David Hilbert's Grundlagen der Geometrie, *Arch. Hist. of Exact Sci.*, 35 (4) (1986), 329–344. H. Weyl, David Hilbert and his mathematical work, *Bull. Amer. Math. Soc.* 50 (1944), 612–654. H. Weyl, David Hilbert. 1862–1943, *Obituary Notices of Fellows of the Royal Society of London* 4 (1944), 547–553.

With these words ringing in the ears of like-minded teachers, and their students, urged on by reviews of Hilbert's book and papers by such personalities as Poincaré, Halsted and Hedrick, the whole topic was ready meat for seminar discussion and Oswald Veblen was drawn into it as soon as he arrived at Chicago in the fall of 1901. Veblen's background included schooling in Iowa, a B.A. from the University of Iowa in 1898 and then working there for a year as a laboratory assistant. He spent a year at Harvard University before leaving to undertake graduate research at Chicago and arrived at the very moment when Professor Moore was introducing a seminar on the foundations of geometry and analysis in which his principal reference for his lectures was Hilbert's *Grundlagen der Geometrie.*

The course was especially topical in view of a paper published that year contesting Hilbert's work and which, for E.H. Moore, introduced an up-to-the-minute level of discussion. The paper, by Friedrich Schur,[7] claimed redundancies in Hilbert's two sets of axioms. As he discussed the contested elements in his class, Moore there and then discovered that while redundancies existed, they related to one axiom of connection and one of order. As already noted, Moore published the new mathematical result ahead of R.L. Moore's discovery of the same redundancy that led to the Chicago professor praising Halsted's protégé for a 'delightfully simple proof'.

Veblen was so inspired by his exposure to his professor's method and subject matter that he chose to pursue the issue in his studies.[8] His abilities and sustained interest drew the close attention of E.H. Moore, Bolza and Maschke and they engaged the 21-year-old's receptive brain in an intensive program of instruction that laid the basis for the important work he was later to achieve in the fields of foundations of geometry, projective geometry, differential invariants and spinors.[9] Such was the tenor of Veblen's work that he progressed rapidly towards the decision to focus his doctoral dissertation on euclidean geometry, entitled *A System for Axioms for Geometry.* E.H. Moore was so impressed by his student's achievement in this direction that he made a point of mention-

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[7] Friedrich Schur, Über die Grundlagen der Geometrie, *Math. Ann.* 55 (1901), 265–292.

[8] Parshall and Rowe, *The Emergence of the American Mathematical Research Community*, p. 383.

[9] R.C. Archibald, *A Semicentennial History of the American Mathematical Society 1888–1938*, New York, 1938.

Oskar Bolza

ing it at the beginning of his presidential address to the American Mathematical Society in 1902:

'In his dissertation on Euclidean geometry, Mr. Veblen, following the example of Pasch and Peano, takes as undefined symbols "point" and "between" or "point" and "segment." In terms of these two symbols alone he expresses a set of independent fundamental postulates of Euclidean geometry, in the first place developing the projective geometry, and then as to congruence relating himself to the point of view of Klein in his *Erlangen Programm* [sic], whereby the group of movements of Euclidean geometry enters as a certain subgroup of the group of collineations of projective geometry.'

In Veblen's system then there were only two primitive notions, point and order (the points *a*, *b*, *c* occur in the order *abc*).[10] When Veblen completed his thesis in 1903 he became an associate in mathematics at the University of Chicago and immediately became active in advising other young mathematicians, and the following year the friendship established with R.L. Moore on his arrival into this maelstrom of activity was apparent from the version of the dissertation revised for publi-

[10] *Oswald Veblen (1880–1960), A Biographical Memoir*, Saunders Mac Lane, National Academy of Sciences, Washington, DC, 1936; passim.

cation. It not only mentioned R.L. Moore's work at Texas but carried a footnote: 'I wish to express deep gratitude to Professor E.H. Moore who has advised me constantly and valuably in the preparation of this paper and also to Messrs N.J. Lennes and R.L. Moore who have critically read parts of the manuscript.'[11] It was a proud moment for the young Moore, then poised to receive his own fellowship, to be mentioned in such a way.

The rapport between R.L. Moore and Oswald Veblen began with this common interest in the foundations of axiomatic geometry, in which they were immersed in the ongoing theme of investigations, and developed into a lively mathematical friendship that was to continue over many years. This relationship may be seen as one of the defining elements of their respective careers. They were clearly fascinated, excited and undoubtedly filled with a youthful zest that turned into passion. It would form the central plank of the work of both men for the first decade or more of the twentieth century, by which time they had themselves moved to new pastures, though they remained constantly in touch. Quite apart from the thrills attained from their chosen lines of research, the enthusiasm with which encouragement was heaped upon them in this hothouse environment of study was an experience that identifiably underscored their future progress, both in their work and development as researchers and teachers.[12] A further contributor to this arena of research during R.L. Moore's early days at Chicago was

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[11] As quoted in Traylor, p. 59.

[12] Veblen remained at Chicago until 1905 when he moved to Princeton University to begin a long and illustrious career which kept him at the university until 1932. He established Princeton as one of the leading centers in the world for topology research, his first of many works on the subject being published just before he left Chicago. His continuing interest in the foundations of geometry led to his work on the axiom systems of projective geometry and together with John Wesley Young he published *Projective Geometry* (1910–18) which they introduced with the words: 'Since it is more natural to derive the geometrical disciplines associated with the names of Euclid, Descartes, Lobachevsky, etc. from projective geometry than to derive projective geometry from one of them, it is natural to take the foundations of projective geometry as the foundations of all geometry.' Veblen's 1922 work, *Analysis Situs* was seen as the first systematic coverage of the basic ideas of topology and contributed much to the development of modern topology. Soon after Einstein's theory of general relativity appeared Veblen turned his attention to differential geometry. This work led to important applications in relativity theory, and much of his work also found application in atomic physics. He followed E.H. Moore as an active member of the American Mathematical Society, serving as president in 1923–1924 and was honored by other mathematical societies around the world.

Gilbert Bliss, who brought to the table the added dimension of having become one of the first American mathematicians to benefit from further study in Europe. He entered the University of Chicago in 1893, receiving his B.S. four years later, and then began graduate studies in mathematical astronomy. In 1898, inspired by a lecture from Bolza, he began his doctoral studies working on the calculus of variations, and after receiving his doctorate in 1900, spent the following two years at the University of Minnesota before leaving for Göttingen. There, he was exposed to the work of several of the most famous European mathematicians of that era including Hilbert and Klein. He was thus well equipped with up-to-date knowledge when he returned to the United States and the University of Chicago in 1903 at the time of R.L. Moore's arrival. Bliss remained for a year before moving to the University of Missouri, but he, Veblen and R.L. Moore were all to meet up again at Princeton.[13] The fifth influential member of the early 1900's group at Chicago was G.D. Birkhoff, first as an undergraduate before moving to Harvard, returning to enter the Chicago graduate program in 1905 for his doctorate. By then, he had completed graduate work at Harvard and jointly authored with H.S. Vandiver a paper which contained a number-theoretic result that turned out to be pivotal in Wedderburn's proof of his eponymous theorem. The coincidence of that, apart from the fact that Birkhoff[14] also became one of the most

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[13] Bliss returned to Chicago on the death of Maschke and remained until his retirement. Throughout his professional life, he took a major role in mathematics in the United States, and especially in the American Mathematical Society: Colloquium Lecturer, 1909, Vice President 1911 and President 1921. He received many awards for his work and most famously worked on ballistics during World War I, designing new firing tables for artillery. In 1918 he joined Veblen in the Range Firing Section at the Aberdeen Proving Ground, a military weapons testing site established in 1917 in Harford County, Maryland. There he very effectively applied methods from the calculus of variations to solve problems relating to correcting missile trajectories for the effects of wind, changes in air density, rotation of the Earth and other perturbations.

[14] A leading American mathematician of the early twentieth century, who formulated the ergodic theorem, which transformed the Maxwell-Boltzmann ergodic hypothesis of the kinetic theory of gases (to which exceptions are known) into a rigorous principle through use of the Lebesgue measure theory. Birkhoff taught at the University of Wisconsin, Madison (1907–1909); Princeton University (1909–1912); and Harvard University (1912–1944). He was a stimulating lecturer and director of research. In the mid-twentieth century, many of the leading American mathematicians had either written their doctoral dissertations under his direction or had done postdoctoral research with him. Birkhoff served as president of the American Mathematical Society (1924–1926), as dean of the

highly respected mathematicians of the first half of the century, was that his co-author, Vandiver, was to become a life-long (if uneasy) associate of R.L. Moore after they both landed in Austin, Texas.

Evidence that R.L. Moore, as the youngest in the Chicago group, received guidance and stimulation from all those around him can be gleaned from the ongoing entries in his Ivory Soap pad diary. Generally, the diary (where it is possible to decipher the oft-illegible scrawl of a young man in a hurry or, late at night, very tired) reads like the naïve jottings of a very unworldly student who has missed his youth while his face was buried in his work. There is virtually no mention of any social activity, sports, girlfriends or other interests that might otherwise amuse and engage someone of his age.

The diary concerns itself mainly with reflections and comments about his work, intentions and notes of anything faintly praiseworthy of himself, or advice as to how he might proceed. His entries include mentions of Leonard Dickson remarking, for example, that those were 'pretty theorems this morning, Mr. Moore, those were all right'.[15] G.A. Bliss, meanwhile, gave him advice on presentation in preparation for talks to the university's Mathematics Club, formed in the 1890s by E.H. Moore and which was recognized as another area of group inspiration. He records advice from Bliss: 'You know, most people don't understand a paper, anyway, but if you present things in a certain way you will get a reputation of being a person who can't present things well and you are too good a man to have that said of you. We want to let people see what kind of material we have here. Don't you see?'[16]

There were numerous diary entries noting encouragement from E.H. Moore himself. He was seemingly keen to see Moore establishing himself in mathematics on a broader front than university study, and suggested quite early on that he should join the American Mathematical Society as soon as he could afford it: 'I think you are in mathematics for good, are you not?'[17]

R.L. Moore replied: 'I hope so.'

Harvard faculty of arts and sciences (1935–1939), and as president of the American Association for the Advancement of Science (1936–1937). His works include *Relativity and Modern Physics* (1923), *Dynamical Systems* (1928), *Aesthetic Measure* (1933), and *Basic Geometry* (1941; with Ralph Beatley).

[15] Ivory Soap pad diary, 31 March 1904; R.L. Moore Papers in the AAM.

[16] Ibid., 6 March 1904.

[17] Ibid., March 1904.

He also noted Professor Moore's numerous references to him taking up summer teaching posts at the university. This, along with the professor's teaching effort and style, clearly made a lasting impression, not least in the way that the younger Moore was able to observe his precision, clarity, and logical correctness in his presentations. In fact, the manner in which he delivered his presentations was thought by some to show that E.H. Moore had a speech impediment. But as Dickson and Bliss pointed out in their memoir, 'This arose from his speaking habit which was designed to allow accurate description of what he wished to state. It would seem at times that he would begin to say a word, testing it almost to see if it would convey what he desired and, determining that it would not, switch in mid-breath to another word.'

This precision was applied not merely to mathematical statements, but all aspects of everyday conversation, so that those who conversed with him were forever on their mettle to state exactly what they meant. R.L. Moore himself recalled[18] that one day when late for an appointment, he began by explaining that his alarm clock did not go off at the appointed time. Then, remembering his professor's requirement for accurate statement, corrected himself and added that it may have gone off but he had not heard it. Stumbling over that excuse, he corrected himself again by saying that of course it was possible that the alarm had gone off and that he had heard it and switched it off only then to have fallen asleep again. At this point, R.L. Moore was relieved of the need for further explanation. It was another lesson learned in the need for precise and clear statements, in both mathematical presentations and general conversation. Equally, the manner in which E.H. Moore approached his teaching of mathematical concepts had distinct influence on R.L. Moore. According to Traylor,[19] much of the content in R.L. Moore's course "Introduction to the Foundations of Analysis" which he offered at The University of Texas years later was seen by him first with E.H. Moore or Oswald Veblen.

Veblen, in his dissertation, gave a system of axioms for geometry in which the so-called Heine-Borel property appeared as Axiom XI: If there exists an infinitude of points, there exists a certain pair of points A, C such that if σ (σ denotes a set or class of elements, any one of which is denoted alone or with an index or subscript) is any set of seg-

18 In conversation with Traylor, *Creative Teaching*, p. 49.

19 Traylor, *Creative Teaching*, pp. 45–63.

ments of the line *AC*, having the property that each point which is *A*, *C* or a point of the segment *AC* is a point of a segment σ, then there is a finite subset $\sigma_1, \sigma_2, \ldots, \sigma_n$ with the same property. He called Axiom XI his Continuity Axiom, noting that Schoenflies had called that property the Heine-Borel theorem. Borel is given credit for first stating the theorem in 1895, as a theorem of analysis, but Heine in 1871 used that notion in the proof of a theorem of uniform continuity. Veblen credited fellow student N.J. Lennes with the idea of the equivalence of his Axiom XI with the Dedekind cut axiom while R.L. Moore also made further contributions.

Veblen stated as axioms: [Axiom II:] If points *A, B, C* are in the order *ABC*, they are in the order *CBA*. [Axiom III:] If points *A, B, C* are in the order *ABC*, they are not in the order *BCA*. [Axiom IV:] If points *A, B, C* are in the order *ABC*, then *A* is distinct from *C*. He then refers to R. L. Moore as having suggested that 'If *A* is a point, *B* is a point, *C* is a point would be a more rigorous terminology for the hypotheses of II, III, IV, inasmuch as we do not wish to imply that any two of the points are distinct'.

R.L. Moore was therefore closely in touch with Veblen's work as he honed his dissertation to perfection and conversely, it was Veblen, in his capacity as R.L. Moore's supervisor in his doctoral studies — under the overall direction of E.H. Moore — who suggested topics for investigation toward his own thesis. The choice eventually became *Sets of metrical hypotheses for geometry* in which he gave a set of assumptions concerning point, order, and congruence that, together with other order assumptions, a continuity assumption, and a weak parallel assumption, were sufficient for the establishment of ordinary euclidean geometry.

Veblen meanwhile published a paper in 1905 entitled 'Theory of plane curves in non-metrical analysis situs' in which he argued a proof of the fundamental theorem proposed by Jordan: A simple closed curve lying wholly in a plane decomposes the plane into an inside and an outside region. The setting for this theorem was taken by Veblen to be a space satisfying axioms I–VIII, XI of his dissertation, stating explicitly that nothing is assumed 'about analytic geometry, the parallel axiom, congruence relations, nor the existence of points outside a plane'. Again, discussion over these issues had taken place between himself and R.L. Moore and in fact there was input from another unexpected source, G.B. Halsted.

Throughout his time at Chicago, and almost two decades afterward, R.L. Moore had maintained a long and detailed correspondence with his original mentor, G.B. Halsted as he moved through the last and somewhat unchallenging posts of his illustrious, if bumpy, career. In 1903 he took up a position at St. John's College, Annapolis, Maryland, where he stayed for just a few months before moving to Kenyon College, Gambier, Ohio (1903–1906). He remained prodigious in his output, writing papers and articles, notably translating Poincaré's 1902 review of Hilbert into English. He also pursued his interest in mathematical education and often took up his pen to criticize the careless way that mathematics was presented in the textbooks. In 1902, he also began work on his book, *Rational Geometry,* eventually published in July 1904.

As will be seen from extracts from the correspondence below,[20] R.L. Moore made a significant contribution to this work and, as an extraordinary sequence of letters demonstrates, the pupil eventually began to match, and then surpass, the teacher in his modern mathematical concepts. In all the correspondence, the formality of past contact was maintained, every one beginning 'Dear Mr. Moore', and usually ending 'Yours always, G.B. Halsted'. The selection of edited extracts published below is, of course, a one-sided affair since the letters received by Halsted from Moore were not available. However, it is not difficult to imagine the full exchanges, or indeed to discover the hope held by Halsted that at some point in the future, he and his pupil might be reunited professionally.

November 15, 1903: 'Your letter of November 13 has just arrived. You certainly had a hard night of it to keep you up to 2:15. Be careful. You cannot do the best mathematical thinking that way.'

November 25, 1903: 'Your letter with the proofs and your notes came duly, and I find you have lost none of your keenness and subtlety… I especially credit you for your beautiful proofs, of which I have made an appendix, but I wish also to mention my indebtedness to you in my very brief preface where I shall mention no one else but Hilbert. Do you care to suggest the phraseology?... Please give a little meditation to the "Construction problem" and let me hear from you, as I do not see my way clear how to change it and introduce the distinctions…'

January 16, 1904: 'I have been so severely sick that I had two physicians and the case was diagnosed as tuberculosis and I was ordered to

[20] Halsted to Moore, various dates, R.L. Moore Papers in the AAM.

give up work and leave for a milder climate at once; but I would not, and told the doctor I was going to get well, and so I am now free from fever and much better, but I will hardly undertake now to change the chapter on Constructions to fit the very searching discussion which you wrote me. Now that I am back at work, I hope to be able to send you some new proof sheets.'

January 24, 1904: 'I have just received your letter of Jan. 19 and I think your five postulates for Constructions therein contained are the best yet. But as the insertion of them now in the book would necessitate the entire re-making of every printed page thereafter … I doubt if I will attempt any sweeping changes in the text. I send you by this mail so much of the text as I now have, and in its present form. You will notice great changes and will perhaps see other changes which should still be made. Among so many changes surely some will have escaped me. If you will kindly indicate these, I may still have time to insert them.… My health has much improved and I am inclined to think the physician was hasty in his diagnosis.'

March 19, 1904: 'I have just received your note of March 16. The proof sheets you sent came safely. What I say in the Appendix is: "The above proofs are due to my pupil R.L. Moore, to whom I have been exceptionally indebted throughout the making of the whole of this book." If you would prefer some other form of acknowledgement kindly let me know. I prefer that yours should be the only name, because I owe so much more to your criticism than to all others. Neither Hilbert nor anyone else acknowledged that E.H. Moore's obscure and bungling adumbration proved anything before I published your beautiful proof. [Postscript:] What suggestions have you as to 'The Value of Non-Euclidean Geometry to the Teacher' [a paper Halsted had prepared for delivery in April]?'

April 13, 1904: 'Your note written April 7 but posted April 11th has just reached me. There was a tremendous meeting at Columbus and the paper on 'The Value of Non-Euclidean Geometry to the Teacher' was enthusiastically received. I am glad that you have been awarded the fellowship for 1904–5. I have myself to teach Elementary Mechanics … I wish you would give me the benefit of your investigations in the choice of a suitable book … and I hope you will hit upon something satisfactory. I send you now for the first time the whole of the [proofs for] *Rational Geometry*. To the footnote … I have added that I have been exceptionally [double underlined] indebted to you throughout the making of all the book. No other name but yours is to be mentioned as in any way helping me.'

During the summer months there was considerable correspondence between them in regard to Moore's proof of $AB = BA$ for a line segment joining A and B, and by the fall, Halsted was reporting his meetings with some old friends.

October 22, 1904: 'I was very glad to hear from you again ... I was with Poincaré almost constantly at St. Louis and am now just finishing translating his great address there. I have the original MS now before me. No copy exists. When my translation is published of course you will get a copy. I will send you the first. Poincaré now appreciates your proof. He has seen it in my book, of which he thinks very highly.'

December 12, 1904: 'I was exceedingly interested in your speaking on the Non-Euclidean and using my Bolyai. Have you notes written out that you could send me?.... The *Rational Geometry* is to have the unexampled honor of translation into French. Meantime ... M. Dehn[21] has made an onslaught upon it from which I hope to get in good time for the French translation. I send you herewith my letter to him, as I wish to ask you also what you think on all these points. Why is the angle-sum-excess in a circle-arc-triangle not proportional to the area? Explain all this to me. Please return the letter when you answer the questions. I enclose postage. Please be thinking how we should modify the book in the French translation. You know I have grown to rely implicitly on your judgment.'

December 15, 1904: 'Please tell Dr. Dickson that I give you full authority to change, add to or take from my article.[22]... What you say about Hilbert is true, but your axioms are better than his and amount to cutting out a part of his assumptions. As to Dehn, I saw no reason to attach importance to the treatment of equivalence for tetrahedra, and have entirely omitted it.... Perhaps a few words might be inserted in the book to make this standpoint plain. Please let me know if there is anything radically untenable in these positions.'

The correspondence between pupil and professor diminished somewhat as Moore began the most important era of his mathematical life,

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[21] Max Dehn, one of Hilbert's former students who eventually emigrated to the United States in the 1930s. It is also worth noting that Dehn features in a recent work by Benjamin Yandell, *The Honor's Class: Hilbert's Problems and Their Solvers*, 2002.

[22] For publication in the *Transactions of the American Mathematical Society* of which Professor Dickson was now editor.

working towards his doctorate. The remaining letters from Halsted contain a brief commentary of the build up to that event.

May 23, 1905: 'The article, my *Hilbert's Foundations of Logic and Arithmetic* is already in print and will appear in the forthcoming number of the *Monist*... I wish I could have you with me next year, but I have not been able to make arrangements. I have been asked to be Director of a new University for Mexico. If it goes through[23] I want you as one of the 3 Professors of Mathematics.... Will you undertake to write a *Rational Algebra* with me? If we go to Mexico it will be issued in English and Spanish and be the official textbook. If not, it will supercede all American algebras for use where the new rigor is growing into appreciation. I will of course adopt it here. *Rational Algebra: A textbook for colleges based on Hilbert's Axiomatic Founding of the Number System and Utilizing the New Graphic Methods. By Dr. G.B. Halsted and Dr. R.L. Moore*'[24]

July 18, 1905: 'My Dear Dr. Moore, Congratulations on your Professorship.'

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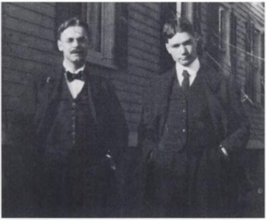
23 It did not.

24 The joint project was never activated.

5

Uneasy Progress (1905–1908)

R.L. Moore is credited in Veblen's dissertation for assistance given and Veblen supervised Moore's PhD, secured in the fall of 1905. The work set the scene for much of Moore's early research which was, like Veblen's, devoted to the axiomatic foundations of geometry. His dissertation was also closely related to Veblen's, whose axioms I and III–X it utilized. His first appointment, however, did not call upon these talents...



E. H. Moore and R.L. Moore in Chicago

Robert Lee Moore was still only 22 years old when he received his PhD from The University of Chicago and there has always been some confusion as to just who acted as his principal adviser during his two years of doctoral studies. In 1938, the issue first raised itself publicly. In that year, Raymond C. Archibald named Veblen[1] as R.L. Moore's doctoral adviser in his *Semicentennial History of the American Mathematical Society*. Eyebrows were raised at the time because, by then, E.H. Moore had been dead six years and both Veblen and R.L. Moore had risen to the status of being among the top five most influential mathematicians in the United States. Moore himself was president of the American Mathematical Society in that important semicentennial year, a role that Veblen had already fulfilled. Moore was also known across the land as the first American to be honored with the American Mathematical Society's sponsorship as Visiting Lecturer,[2] which had taken him to universities throughout the country.

Both were universally respected and Veblen, especially, had achieved international recognition in that decade as a founding professor of the Institute for Advanced Study established at Princeton. He was largely responsible for the selection of the Institute's early mathematics faculty that contained such names as Einstein, Alexander, Morse, von Neumann and Weyl and 'his effect on mathematics, transcending the Princeton Community and the country as a whole, [would] be felt for decades to come.... his assistance was decisive for the careers of dozens of men'.[3]

It was with some interest, therefore, that readers of Archibald's book would discover that Veblen had more than a friendly interest in the launching of R.L. Moore's mathematical career. As already noted, Veblen himself was a student of E.H. Moore at the time of R.L. Moore's arrival at Chicago, and after gaining his own PhD he stayed on as an associate in mathematics. Reverting back to that time, a collection of mathematical papers discovered in the R.L. Moore files[4] (and read by the present author) clearly indicate that there was input from E.H.

[1] In *Semicentennial History of the American Mathematical Society, 1888–1938*, New York, 1938, p. 209.

[2] In 1931–1932.

[3] Deane Montgomery, in Oswald Veblen, a biographical memoir, *Bull. Amer. Math. Soc.* 69 (1963), 26–36. Montgomery is quoting a statement by the faculty and trustees of the Institute for Advanced Study.

[4] R.L. Moore Papers in the AAM.

Moore during his doctoral studies. In the same collection, there is a single sheet headed *Vita,* written in R.L. Moore's own hand. It was the draft for inclusion in his published version of his dissertation and certainly provided room for ambiguity. Indeed, R.L. Moore set the scene for confusion when he wrote: 'In the University of Chicago, I was a Fellow in 1903–1904 and 1904–1905 and an assistant in mathematics in the summer of 1904. Let me here express my thanks to my instructors Professors Bolza, Dickson, Laves, Maschke, Moulton, Moore, and Drs. Bliss and Lunn and in particular to Dr. O. Veblen who has given me so much and continued assistance and many suggestions in conjunction with the preparation of this thesis.' There was a further sentence that had been crossed out of the draft, but perfectly legible, which said: 'In fact, part of the work was done in conjunction with his [Veblen's] collaboration.'

Twenty-three years later, he went further and allowed Veblen unequivocal credit. On 12 March 1938 when Archibald was finalizing the manuscript for his *Semicentennial History,* he wrote to Moore at Texas: 'My dear Moore ... One interesting item has just come to me which I wish to have checked by yourself. The statement was made that although you really got your doctor's degree at Chicago, it was done under Veblen and not under E.H. Moore. Do you agree with this? Just a postcard reply is all that is necessary.' R.L. Moore allowed the Archibald reference to stand. In his dissertation entitled *Sets of Metrical Hypotheses for Geometry,* Moore gave axioms for euclidean geometry using as primitive notions *point, order* and *congruence.* As R.L. Wilder points out,[5] much of Moore's early work was, like Veblen's, devoted to the axiomatic foundations of geometry. His dissertation was also closely related to Veblen's, whose axioms I and III–X it used. Alternative sets of axioms were considered, some of them being systems in which ordinary ruler and compass constructions are possible, as well as a set for Bolyai-Lobachevskian geometry. In showing that every circle is a Jordan curve, he had to use a definition given by Veblen in 1905[6] in terms of order and continuity conditions.

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[5] In The Mathematical Work of R.L. Moore: Its Background, Nature and Influence, *Arch. Hist. Exact Sci.,* 26, (1982) 73–97. R.L. Wilder, a president of both the AMS and MAA, was Moore's fourth doctoral student.

[6] O. Veblen, Theory on plane curves in non-metrical analysis situs, *Trans. Amer. Math. Soc.* 6 (1905), 83–98.

The Veblen influence continued for some time, elements of it evident in Moore's work for many years afterward. Their close working relationship under the auspices of E.H. Moore, Bolza and Maschke ended in the summer of 1905, however, when Veblen went to Princeton to launch what was to become an outstanding career solely from that base while R.L. Moore himself took up relatively low-grade work in terms of both pay and stature, at the University of Tennessee.

The appointment in the fall of 1905 began what turned out to be a particularly unhappy tenure. Departing from the excitement and friendship of Chicago appears to have had an adverse psychological effect on a young man who had had little practice in the art of general communication with life outside of the university campus. Even his letters home, evident from those retained in his papers which he wrote his father[7], were generally concerned with his work, often in complex detail. Whether his father understood was another matter.

There is scant evidence during his time at Chicago of any extracurricular activities or interest, other than walking, and only a fleeting reference to the opposite sex in his diary. Even that was with the critical eye of a man whose time in the north had not tempered his Southern-based opinions one bit. He noted with obvious disdain that during his last days at Chicago, he had joined a party that went riding in a sleigh and a female student actually sat with the driver! He wrote that a greater freedom existed among Northern people, especially in regard to 'relations between the sexes (in public! etc etc)' than in the south.[8] It is also to the last remaining pages of that diary that we may turn to discover the apparent concerns and unhappiness that had descended upon him after the intensity of Chicago where he had crammed a very great deal of study into his two years. Indeed, it might be said that he had hardly lifted his head since joining Dr. Halsted at The University of Texas and now, suddenly, it all seemed to be catching up with him. At the time, he was apparently working hard on lists of axioms for the positive integers and their arithmetic, on which he seemed to have become obsessed at the expense of getting his dissertation into publishable form.

That task had been urged upon him as a matter of urgency by both E.H. Moore and Veblen, but he had yet to complete it. A bout of malaria

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[7] The letters were returned to him after his father's death and form part of the R.L. Moore Papers in the AAM.

[8] The Ivory Soap pad diary, 19 May 1904, pp. 104–106.

Oswald Veblen

picked up after being bitten by a mosquito while out camping with his brother during the summer holidays of 1905 merely added to his malaise and he had even let drift his correspondence with Halsted about which he had been meticulous in the past. On February 6, 1906, Halsted wrote: 'I have not heard from you in a very long time. Please send me your present address.' Moore responded immediately and recounted his illness, to which Halsted replied on February 13, 1906: 'I was horrified to learn that you had been bitten by a Texas mosquito. It gives me great pleasure to send you … a complimentary copy of my Poincaré, *Science and Hypothesis.* A new edition of my *Rational Geometry* is called for. I think I will make it rather a simplification attractive to schools, assuming more theorems (the harder ones). I wish you would now look over the book and suggest some better and simpler substitutes all the way through.'

Moore, still affected by the malaria, was not in the mood for such additional labor, however, as his diary entries for February and March 1906 quite clearly indicate:

'Shucks! Is there much use in such diary keeping? This is Sunday. Am I losing both the desire and capacity for continued close concentrated work? Or if not both desire and capacity, at least capacity. Is my mental laziness a result of that malaria and this jaundice or not? In

Chicago, part of the time I don't know but that I had a tendency to loaf, but if so hasn't that tendency *increased*? If so, its intensity approaching a maximum and will it begin to decrease after a while and will I work as hard and as long at a time as ever? At one time, before I went to The University of Texas and for a while after I went there, didn't I have a ... conscious scramble against quitting studying for the day till sundown? Didn't I have that sort of scuffle for at least part of the time during every year there? If so, is that laziness, this disinclination to get right down to hard, long, continued concentration...— a sort of reaction against that sundown business of my earlier days? If so, will things finally adjust themselves and will I before long ... work pretty hard and regularly? I don't know but that it might be hard or impossible to describe exactly my condition but I'm not sure but that I might not suggest [that] procrastination has become one of my traits. For example, sometime last month, I got back my thesis [manuscript] and a letter from Professor Moore who seemed desirous that I should prepare part of my work as a separate paper to be published in *Transactions*, possibly in April and other parts ... to appear possibly in October.

'Now I don't know but that with a certain amount of [strenuous effort] and concentration that the first paper might have been prepared in a week but here it is, March 25 [1906], and I haven't got it ready for him yet. It's true I have, part of the time, been undecided as to how to treat a certain part of the subject,... and I have been vacillating between different methods of treatment. *But* I decide that a certain method seems good and almost no sooner do I feel that it does indeed seem good than I am *liable* to feel ... a strong inclination to postpone for a while the actual working out and writing out of the detail of it.

'Then another day I begin to consider the method and find an objection to it. Think of another method, which has some advantage, but in turn postpone detailed consideration and writing of it.... Procrastination! If I set a certain time limit, then don't know but that I may get pretty busy for a while and work pretty late ... and get something done by about that time. For example Lennes seemed to want me to...'

There, the diary ended and not another word was written in the Ivory Soap pad that had been his companion since he entered The University of Texas in 1898. His mention of Lennes, incidentally, demonstrated a further element of the close cooperation that E.H. Moore has inspired among his students. Although eight years older than R.L. Moore, Lennes received his M.S. in 1903 and then spent a couple of years

R.L. MOORE
Mathematician & Teacher

John Parker

Some say that the only possible effect of the Moore Method is to produce research mathematicians—but I don't agree. The Moore method is, I am convinced, the right way to teach anything and everything—it produces students who can understand and use what they have learned. It does, to be sure, instill the research attitude in the student—the attitude of questioning everything and wanting to learn answers actively—but that's a good thing in every human endeavor, not only in mathematical research.

— **Paul R. Halmos**,
Santa Clara University

The story of the mathematician R.L. Moore and his students is worth presenting to a wide audience not only because of their scientific contributions but especially because their creativeness in mathematics went hand in glove with their inspiration of creativeness in the classroom. The author has drawn on a large array of published and unpublished sources and does not overlook the criticisms and controversies that have been associated with Moore and the Moore Method. This personal history thus becomes a study in the art of teaching and in the discovery and nurture of talent.

— **Albert C. Lewis**,
C. S. Peirce Edition Project

I first studied under Moore in 1941. I found him to be an inspirational kind of teacher, and a man totally dedicated to his students, more so than any other teacher I've known.

— **Richard D. Anderson**,
Boyd Professor Emeritus, Louisiana State University,
Past President, MAA and Past Vice-President, AMS

R.L. Moore: Mathematician & Teacher presents a full and frank biography of a mathematician recognized as one of the principal figures in the 20th Century progression of the American school of point set topology. He was equally well known as creator of the Moore Method (no textbooks, no lectures, no conferring) in which there is a current and growing revival of interest and modified application under inquiry-based learning projects in both the United States and the United Kingdom.

Parker draws on oral history, with first-person recollections from many leading figures in the American mathematics community of the last half-century. The story embraces some of the most famous and influential mathematical names in America and Europe from the late 1900s in what is undoubtedly a lively account of this controversial figure, once described as Mr. Chips with Attitude. He was the first American to become a Visiting Lecturer for the American Mathematical Society, was a member of the National Academy of Sciences, published 68 papers and a book that is still referred to seventy years later and that has been the subject of literally hundreds of papers by other mathematicians around the globe.

Three of Moore's students followed him as president of the American Mathematical Society, and three others became vice-presidents. Five served as president of the Mathematical Association of America, and three became members of the National Academy of Sciences.

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