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Rethinking Knowledge

The Heuristic View

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Chapter 1

Introduction

Abstract By invading many areas traditionally covered by philosophy, modern science has made philosophy increasingly irrelevant, and has presented it with the challenge to legitimate itself. This challenge has not been successfully met, so much so that several scientists, and even some philosophers, have concluded that philosophy is dead and has dissolved into the sciences. The question then arises whether philosophy can still be fruitful, and what kind of philosophy can be such. This book attempts to give an answer to this question, reviving the ancient view that philosophy is the acquiring of knowledge. This involves rethinking knowledge and methods to acquire knowledge, including mathematical knowledge, which raises special problems. In addition to describing the aim of the book, the introduction briefly describes the parts and the individual chapters of the book, and outlines some conventions adopted in the book.

1.1 Philosophy and the Birth of Modern Science

“One ought either to philosophize or say goodbye to life and depart hence, because,” as compared with philosophy, “all other things seem to be a lot of nonsense and foolishness” (Aristotle, *Protrepticus* Düring, 110).

Is this praise of philosophy even conceivable today? The question is justified, because in the seventeenth century philosophy suffered a trauma from which it has not recovered yet, the birth of modern science. The latter has invaded many areas traditionally covered by philosophy. As a result, the role of philosophy has become problematic, and philosophy has come to need legitimation in the face of science.

Thus Gadamer states that, “since the seventeenth century,” what “we today call philosophy is found to be in a changed situation. It has come to need legitimation in the face of science in a way that had never been true before” (Gadamer 1998, 6).

In fact, since the seventeenth century, a great deal of philosophy has been an attempt to provide an answer to the trauma caused by the birth of modern science. There have been both radical answers and moderate answers.

1.2 Radical Answers

A radical answer is that, with the birth of modern science, philosophy has nothing left to speak of, because all questions about which one can say anything sensible belong to the sciences.

Thus, Wittgenstein states that “the correct method in philosophy would really be the following: to say nothing except what can be said, i.e. propositions of natural science – i.e. something that has nothing to do with philosophy” (Wittgenstein 2002, 6.53). For, “philosophy is not one of the natural sciences” (ibid., 4.111).

This answer, however, is unsatisfactory, because it assumes that the whole field of knowledge is exhausted by the present sciences. Such assumption is unjustified because, as research proceeds, new questions arise which are essentially beyond the present sciences, and may even give rise to new sciences.

Another radical answer, but opposite to the previous one, is that the sciences do not give knowledge, only philosophy provides access to knowledge.

Thus, Heidegger states that “science does not think” (Heidegger 1968, 8). It “is the disavowal of all knowledge of truth” (Heidegger 1994, 5). Therefore, “no one who knows will envy scientists – the most miserable slaves of modern times” (ibid., 6). Science aims at exactness and security but, with its “insistence on what is demonstrable,” it blocks “the way to what-is” (Heidegger 1972, 72). Only philosophy opens the way to what-is, because “philosophy is the knowledge of the essence of things” (Heidegger 1994, 29).

This answer, however, is unsatisfactory, because the assumption that only philosophy opens the way to what-is has no foundation. There is no special source of knowledge which is available to philosophy but not to science.

1.3 Moderate Answers

A moderate answer is that we must admit that philosophy is not yet a science. While no one would doubt the objective character of mathematics and natural sciences, the same cannot be said of philosophy. A revolution in philosophy is necessary if philosophy is to acquire the character of a rigorous science.

Thus, Husserl states that philosophy is still “incapable of giving itself the form of actual science” (Husserl 2002, 250). It “does not merely have a doctrinal system at its disposal that is incomplete and imperfect in one respect or another, but has none whatsoever. Anything and everything is controversial here, every position-taking is a matter of individual conviction” (ibid., 251). But, “the highest interests of human culture demand the elaboration of a rigorously scientific philosophy” (ibid., 253). So, “if a philosophical revolution is to prove itself in our time, it must always be animated by the intention to found philosophy anew in the sense of rigorous science” (ibid.).

This answer, however, is unsatisfactory, because philosophy has no specific field of its own, therefore all attempts to give philosophy the character of a rigorous science have been unsuccessful. Husserl wants to develop a philosophy that is a “universal science of the world, universal, definitive knowledge, the universe of truths in themselves about the world, the world in itself” (Husserl 1970, 335). He aims at “the discovery of the necessary concrete manner of being of absolute subjectivity” in a “life of constant ‘world-constitution’” (ibid., 340). He also aims at “the new discovery, correlative to this, of the ‘existing world’,” which “results in a new meaning for what, in the earlier stages, was called world” (ibid.). His attempt, however, fails because, starting from absolute subjectivity, Husserl arrives at the discovery, not of the existing world, but only of a world as a correlative of subjectivity.

Another moderate answer, but opposite to the previous one, is that we must abandon the idea that science is the paradigmatic human activity, and hence philosophy should try to become a science. Science has no privileged position with respect to philosophy, both science and philosophy must be evaluated in terms of their capacity to achieve the aims we would like to achieve through them.

Thus, Rorty criticizes the logical empiricists who maintained that “science was the paradigmatic human activity,” and “what little there was to say about other areas of culture amounted to a wistful hope that some of them (e.g., philosophy) might themselves become more ‘scientific’” (Rorty 1991–2007, I, 46). Science has no privileged position with respect to philosophy. Both science and philosophy must be evaluated in terms of their capacity “to be reliable guides to getting what we want” (Rorty 1999, 33).

This answer, however, is unsatisfactory, because Rorty states that we philosophers “are not here to provide principles of foundations or deep theoretical diagnoses, or a synoptic vision” (ibid., 19). When we are asked what we “take contemporary philosophy’s ‘mission’ or ‘task’ to be,” the best we “can do is to stammer that we philosophy professors are people who have a certain familiarity with a certain intellectual tradition,” and “can offer some advice about what will happen when you try to combine or to separate certain ideas, on the basis of our knowledge of the results of past experiments” (ibid., 19–20). But “we are not the people to come to if you want confirmation that the things you love with all your heart are central to the structure of the universe” (ibid., 20). That is, we philosophers are not the people to come to if you want to have an answer to the questions you are most interested in. This makes philosophy into a marginal activity with little utility.

1.4 Death of Philosophy?

The inadequacy of these answers to the trauma caused by the birth of modern science, makes one doubt that trying to legitimate philosophy is a feasible enterprise. This doubt is strengthened by the fact that, in the past century, some philosophers have affirmed that philosophy is dead and has dissolved into the sciences.

Thus, Quine states that philosophy, and specifically epistemology, or theory of knowledge, has dissolved into psychology, because “the stimulation of his sensory receptors is all the evidence anybody has had to go on, ultimately, in arriving at his picture of the world. Why not just see how this construction really proceeds? Why not settle for psychology?” (Quine 1969, 75). The “old epistemology aspired to contain, in a sense, natural science; it would construct it from sense data. Epistemology in its new setting, conversely, is contained in natural science, as a chapter of psychology” (ibid., 83). Epistemology “simply falls into place as a chapter of psychology and hence of natural science” (ibid., 82).

Heidegger states that “the sciences are now taking over as their own task what philosophy in the course of its history tried to present in part, and even there only inadequately, that is, the ontologies of the various regions of beings (nature, history, law, art)” (Heidegger 1972, 58). The “development of philosophy into the independent sciences” is “the legitimate completion of philosophy. Philosophy is ending in the present age. It has found its place in the scientific attitude of socially active humanity” (ibid.). The “end of philosophy means the completion” of philosophy, where “however, completion does not mean perfection as a consequence of which philosophy would have to have attained the highest perfection at its end” (ibid., 56). On the contrary, completion means “the end of philosophy in the sense of its complete dissolution into the sciences” (Heidegger 1998, 259, footnote).

But, if philosophy is dead and has dissolved into the sciences, then it is impossible to try to legitimate it in the face of science.

1.5 Criticisms by Scientists

That philosophy is dead and has dissolved into the sciences is also the opinion of many scientists.

Thus Hawking states that questions such as “What is the nature of reality? Where did all this come from? Did the universe need a creator?” are traditionally “questions for philosophy, but philosophy is dead. Philosophy has not kept up with modern developments in science, particularly physics. Scientists have become the bearers of the torch of discovery in our quest for knowledge” (Hawking and Mlodinow 2010, 5).

Dyson states that, “compared with the giants of the past,” the present philosophers “are a sorry bunch of dwarfs,” which compels us to ask: “When and why did philosophy lose its bite? How did it become a toothless relic of past glories?” (Dyson 2012).

Krauss states that “science progresses and philosophy doesn’t,” and “the worst part of philosophy is the philosophy of science; the only people” who “read work by philosophers of science are other philosophers of science. It has no impact on physics what so ever,” so “it’s really hard to understand what justifies it” (Andersen 2012).

Therefore, Pinker asserts that “philosophy today gets no respect. Many scientists use the term as a synonym for effete speculation” (Pinker 2002, 11). What a difference

between this spiteful attitude toward philosophy, and the appreciative attitude of Galileo, who stated that he had “studied for a greater number of years in philosophy than months in pure mathematics” (Galilei 1968, X, 353).

1.6 Why Still Philosophy?

Since some philosophers and many scientists have affirmed that philosophy is dead and has dissolved into the sciences, we must ask: Why still philosophy? Can philosophy still be fruitful, and what kind of philosophy can be such? In particular, what kind of philosophy can be legitimized in the face of science? Asking these questions is nothing really new, because philosophy has always called into question everything, including itself. But, with the birth of modern science, such questions have become more pressing, as well as more difficult and embarrassing.

Some philosophers, however, scorn these questions. Thus, Popper states that a philosopher “should try to solve philosophical problems, rather than talk about philosophy” (Popper 1974, 68).

Rorty states that questions about “the nature of philosophical problems” are “likely to prove unprofitable” (Rorty 1992, 374).

Williams states that “philosophy is not at its most interesting when it is talking about itself” (Williams 2006, 169).

But this amounts to taking for granted that philosophy can still be fruitful. This is unjustified because, as we have seen, it contrasts with the opinion of some philosophers and many scientists. This makes it necessary to call philosophy into question.

1.7 Aim of the Book

This book aims to give an answer to the question whether philosophy can still be fruitful, and what kind of philosophy can be such. Briefly, its answer is that philosophy can still be fruitful only if it aims at knowledge and methods to acquire knowledge, because knowledge plays a central role in human life. To a large extent, we are what we know, we reflect reality, and reality is for us what we have access to and we know. Generally our aspirations, desires and hopes essentially depend on what we know.

The view that philosophy aims at knowledge and methods to acquire knowledge may be called the heuristic view, because ‘heuristic’ is said of methods which guide to acquire knowledge. Developing the heuristic view of philosophy requires a rethinking of logic and a rethinking of knowledge. A rethinking of logic has been carried out in Cellucci 2013a. A rethinking of knowledge is the aim of this book. There are, however, some minor overlappings with Cellucci 2013a, which are motivated by the desire to make the book self-contained.

The questions about knowledge discussed in this book do not exhaust all questions about knowledge. Every investigation is a potentially infinite task, and this book is no exception. However, the questions discussed in this book are essential for the development of a fruitful philosophy.

1.8 Organization of the Book

In order to highlight the organization of the book, the text is divided into five parts, after the present Introduction which occurs as Chapter 1.

Part I examines the nature of philosophy. Chapter 2 presents the heuristic view, according to which philosophy aims at knowledge and methods to acquire knowledge. Chapter 3 discusses the foundationalist view, according to which philosophy aims to justify already acquired knowledge. Chapter 4 argues that the main motivation for the foundationalist view, namely, to save knowledge from sceptical doubt, is unfounded, because absolute scepticism is not logically irrefutable. Chapter 5 argues against the view that philosophy is a humanistic discipline, opposed to the sciences.

Part II examines the nature of knowledge. Chapter 6 explains the central role that knowledge plays in human life. Chapter 7 examines several views about the relation of knowledge to reality. Chapter 8 discusses the view that the aim of science is truth, and considers several concepts of truth. Chapter 9 maintains that the aim of science is plausibility, it distinguishes plausibility from truth, probability, and warranted assertibility, and discusses the relation of science to common sense. Chapter 10 considers the relations of knowledge to certainty, objectivity, intuition, and deduction.

Part III examines the methods to acquire knowledge. Chapter 11 maintains that it is unjustified to say that there is no method to acquire knowledge. Chapter 12 considers various methods to acquire knowledge. Chapter 13 discusses various models of science, and to what extent they are capable of accounting for models in science. Chapter 14 maintains that knowledge is problem solving by the analytic method. Chapter 15 maintains that perceptual knowledge is also problem solving by the analytic method. Chapter 16 discusses the relation of knowledge to error. Chapter 17 considers the relation of knowledge to mind.

Part IV examines the nature of mathematical knowledge. Chapter 18 maintains that mathematics is problem solving by the analytic method. Chapter 19 discusses the nature of mathematical objects, mathematical definitions, and mathematical diagrams. Chapter 20 argues that mathematics is not theorem proving. Chapter 21 examines various notions of demonstration. Chapter 22 considers the question of mathematical explanation of mathematical facts, and the question of mathematical explanation of empirical facts. Chapter 23 discusses the nature of mathematical beauty, and its role in mathematical discovery. Chapter 24 considers the relation of mathematics to the world.

Part V ends and completes the book. Chapter 25 examines the connection between knowledge and the purpose and meaning of human life. Chapter 26 summarizes some of the main theses of the book.

1.9 Conventions

Constant use of ‘he or she’ may be clumsy, while constant use of ‘she’ may give rise to misunderstandings. Therefore, I use the generic ‘he’, while stipulating here that I mean it to refer to persons of both genders.

When I quote Greek expressions, I use the so-called scientific transliteration from the Greek to the Latin alphabet.

When I quote from ancient Greek philosophers, and even from some modern philosophers, translations are mine unless stated otherwise. This is motivated by the fact that every translation is an interpretation, and the interpretations given in this book are often different from those on which current translations are based. Moreover, current translations of different works of the same author by different translators may be inconsistent with each other, so quoting from them would lead to misunderstandings.

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Part I
The Nature of Philosophy

Chapter 2

The Heuristic View

Abstract As a response to the increasing irrelevance of philosophy, this chapter lists the characteristics philosophy should have in order to be fruitful. This results in a view of philosophy that may be called the heuristic view, according to which philosophy aims at knowledge and methods to acquire knowledge. In listing the characteristics philosophy should have according to the heuristic view, the chapter systematically compares them with those of classical analytic philosophy. This is motivated by the fact that, in the past century, classical analytic philosophy has been the prevailing philosophical tradition. The characteristics of philosophy listed in the chapter are not intended to suggest that philosophies without such characteristics are bad, but only that they cannot be expected to be legitimized in the face of science.

2.1 The Characteristics of Philosophy

This part of the book examines the nature of philosophy. In the Introduction, the question has been raised as to whether philosophy can still be fruitful and what kind of philosophy can be such. This chapter answers the question, listing the characteristics philosophy should have in order to be fruitful and legitimized in the face of science. This results in the formulation of a view of philosophy that, as already stated in the Introduction, may be called the heuristic view of philosophy.

In listing the characteristics philosophy should have according to the heuristic view, this chapter systematically compares them with those of classical analytic philosophy. This is motivated by the fact that, in the past century, classical analytic philosophy has been the prevailing philosophical tradition.

Here, ‘classical analytic philosophy’ refers to the philosophical tradition started by Russell and Moore, which developed through Wittgenstein and logical positivism, knowing a moment of particular fortune with the Oxford ordinary language school of philosophy, and had several followers in the second half of the twentieth century. Dummett includes Frege in this tradition, saying that Frege “was the true father of analytical philosophy” (Dummett 2007, 27). This, however, is controversial. For example, Carl states that “Frege is not an analytic philosopher” (Carl 1994, 24).

In this chapter, special reference is made to Wittgenstein, since he is widely recognized as the single most significant figure in classical analytic philosophy.

The reference to classical analytic philosophy, rather than to contemporary analytic philosophy, is motivated by the fact that, as Glock points out, while classical analytic philosophy is “a historical tradition held together by ties of influence on the one hand, family resemblances on the other,” this “tradition is currently losing its distinct identity” (Glock 2008, 231).

The characteristics listed below are not intended to suggest that philosophies without such characteristics are bad, but only that they cannot be expected to be legitimized in the face of science.

2.2 Philosophy and the World

According to the heuristic view, philosophy is an inquiry which primarily aims at acquiring knowledge about the world, including ourselves, since we are part of the world. Like mathematics and science, philosophy is a means by which we make the world understandable to ourselves.

This contrasts with classical analytic philosophy. Thus, Wittgenstein states that “philosophy gives no pictures of reality” (Wittgenstein 1979, 106). It does not give any elucidation of reality, and in fact “one of the greatest impediments for philosophy is the expectation of new, deep, unheard of, elucidations” (Wittgenstein 2005, 309).

Dummett states that “philosophy is concerned with reality, but not to discover new facts about it: it seeks to improve our understanding of what we already know” (Dummett 2010, 10).

But, if philosophy is not concerned to discover new facts about reality, then it becomes self-referential. Thus, Moore states: “I do not think that the world or the sciences would ever have suggested to me any philosophical problem. What has suggested philosophical problems to me is things which other philosophers have said about the world or the sciences” (Moore 1942, 14). In fact, most classical analytic philosophy is self-referential. When it occasionally gets involved with issues outside its own limited domain, its only aim is to cleanse its terrain of alien regrettable intrusions.

Dummett admits that “the layman or non-professional expects philosophers to answer deep questions of great import for an understanding of the world” (Dummett 1991, 1). And “the layman is quite right: if philosophy does not aim at answering such questions, it is worth nothing. Yet he finds most writing by philosophers of the analytical school disconcertingly remote from these concerns,” and this complaint “is understandable” (ibid.). According to Dummett, however, the complaint is “unjustified,” because “philosophy can take us no further than enabling us to command a clear view of the concepts by means of which we think about the world” (ibid.). Philosophy “concerns our view of reality” only “by seeking to clarify the concepts in terms of which we conceive of it” (ibid., 11).

On the contrary, as the layman expects, philosophy should actually aim at answering deep questions of great import for an understanding of the world. To say

that philosophy concerns our view of reality only by seeking to clarify the concepts in terms of which we conceive of it, does not relieve philosophy from the obligation to give pictures of reality. For, in order to enable us to command a clear view of the concepts in terms of which we conceive of reality, philosophy ought to put those concepts in relation with reality. But if philosophy gives no pictures of reality, how can it do that?

2.3 Philosophy and Globality

According to the heuristic view, philosophy is not limited to sectorial questions, it gives a global view. So, there cannot be a philosophy of mathematics alone, or of physics alone, or of biology alone, etc. For, the question of the nature of mathematics, or physics, or biology, etc., cannot be adequately approached locally, that is, in isolation from all other knowledge, but only globally, as part of a general approach to knowledge.

This contrasts with classical analytic philosophy. Thus, Carnap states that philosophers must limit themselves to sectorial questions because, “if we allot to the individual in philosophical work as in the special sciences only a partial task,” then “stone will be carefully added to stone and a safe building will be erected at which each following generation can continue to work” (Carnap 2003, xvii).

But if philosophers confine themselves to sectorial questions, then they have no overall plan. This leads them to focus on smaller and smaller questions, thus confirming the motto: Some people know more and more about less and less, until they know everything about nothing, and these are the philosophers.

Classical analytic philosophy adopts the Socratic method of questions and answers, but it retains only the outward form of that method, not the substance, namely, the serious search for answers to general questions. There is no evidence that a minute work on sectorial questions may lead to what is essential. With such a minute work, one risks being merely “a maker of words, incapable of doing any” worthwhile “work” (Plato, *Epistulae*, VII 328 c 5–6). Therefore, philosophy must not be limited to sectorial questions, it must give a global view. As Plato says, “anyone who can have a global view is a philosopher, and anyone who can’t isn’t” (Plato, *Respublica*, VII 537 c 7).

Carnap’s argument for confining oneself to sectorial questions implies a view of philosophy which assimilates philosophy to what Kuhn calls ‘normal science’, whose aim “is not major substantive novelties” (Kuhn 1996, 35). Normal science accumulates details within a settled paradigm and theory, without questioning or challenging the underlying assumptions of the theory. Only within normal science the individual adds stone to stone, working on a small piece of a broader project. But, as confining science to normal science would lead to a narrow view of science, so confining philosophy to sectorial questions would lead to a narrow view of philosophy.

2.4 Philosophy and Essential Problems

According to the heuristic view, philosophy must not deal with peripheral questions, but with great essential problems in the sense of science. Aiming at acquiring knowledge about the world, including ourselves, philosophy seeks the big picture: to understand the world and our place in it. To this purpose, philosophy necessarily must concern itself with great essential problems.

This contrasts with classical analytic philosophy. Thus, Wittgenstein states that in philosophy “there are no great essential problems in the sense of science” (Wittgenstein 2005, 301). Philosophy “is a tool which is useful only against the philosophers” (Wittgenstein 1932–1933, 11).

But, if in philosophy there are no great essential problems in the sense of science, if philosophy is a tool which is useful only against the philosophers, why should we continue to practice it? How could we avoid concluding that philosophy is only a crossroads of many routes leading nowhere?

Russell himself, though being one of the fathers of classical analytic philosophy, declares: “The new philosophy seems to me to have abandoned, without necessity, that grave and important task which philosophy throughout the ages has hitherto pursued,” namely, “to understand the world” (Russell 1995b, 170). It “cares only about language, and not about the world” (Russell 1960, 15). According to the new philosophy, “the desire to understand the world” is “an outdated folly” (Russell 1995b, 162). Wittgenstein makes philosophy, “at best, a slight help to lexicographers, and at worst, an idle tea-table amusement” (ibid., 161). But, “if this is all that philosophy has to offer, I cannot think that it is a worthy subject of study” (ibid., 170).

In fact, the assumption that in philosophy there are no great essential problems in the sense of science, has produced a new kind of scholasticism, characterized by an argumentative style made of dreary distinctions concerning minute, inconsequential questions, incapable of making significant contributions to an understanding of the world, including ourselves. It is no wonder then that, as Fodor states, today “nobody reads philosophy,” in particular “it’s mainly the laity that seems to have lost interest. And it’s mostly Anglophone analytic philosophy that it has lost interest in” (Fodor 2004, 17).

2.5 Philosophy and Knowledge

According to the heuristic view, since philosophy is an inquiry which primarily aims at acquiring knowledge about the world, questions about knowledge are central to philosophy.

This contrasts with classical analytic philosophy. Thus, Searle states that since, in the seventeenth century, “the possibility of certain, objective, universal knowledge seemed problematic” and “the very existence of knowledge was in question,” Descartes “took epistemology as the central element of the entire philosophical

enterprise” (Searle 2008, 4). As a result, “we had three and a half centuries in which epistemology was at the centre of philosophy” (ibid., 5). But today, “because of the sheer growth of certain, objective, and universal knowledge, the possibility of knowledge is no longer a central question in philosophy,” so questions about knowledge no longer “lie at the heart of the philosophical enterprise” (ibid.).

But this is unjustified. If philosophy is an inquiry which primarily aims at acquiring knowledge about the world, then necessarily epistemological questions have a central place in it. The purpose of epistemology is not to inquire into the possibility of certain, objective, universal knowledge, because there is no such knowledge. By Gödel’s second incompleteness theorem, even mathematical knowledge cannot be proved to be certain by absolutely reliable means. The purpose of epistemology, or theory of knowledge, is rather to inquire into methods to acquire knowledge, fallible knowledge and yet knowledge. As Kitcher says: “What might we want from a theory of knowledge? There’s an obvious answer. A theory of knowledge should enable us to get more of it” (Kitcher 2011, 508). This transfers epistemology from the context of justification to the context of discovery, which includes the context of justification since, in the process of discovery, hypotheses are accepted only when they are shown to be plausible, namely such that the arguments for them are stronger than the arguments against them, on the basis of the existing knowledge.

Dummett replaces epistemology with the theory of meaning as the centre of philosophy. He states that Descartes made epistemology “the foundation of philosophy because he had conceived the task of philosophy as being that of introducing rigour into science” (Dummett 1973, 676). But Descartes was wrong, because “the fundamental part of philosophy which underlies all others” is “the theory of meaning” (ibid., 669). Epistemology deals with questions of justification, but “until we have first achieved a satisfactory analysis of the meanings of the relevant expressions, we cannot so much as raise questions of justification,” since “we remain unclear about what we are attempting to justify” (ibid., 667). For this reason, Frege made “the theory of meaning” the “foundation of all philosophy, and not epistemology, as Descartes misled us into believing” (ibid.). Thus Frege “effected a revolution in philosophy,” and so we can “date a whole epoch in philosophy as beginning with the work of Frege” (ibid., 669).

But the theory of meaning cannot be the foundation of philosophy, since the main philosophical questions are not questions about the use of language, but questions about the world. Moreover, Frege did not replace epistemology with the theory of meaning as the centre of philosophy. His main purpose was to give a secure foundation for mathematics, which was a question of justification within Descartes’ epistemological tradition (see Cellucci 1995). As Descartes conceived the task of philosophy as being that of introducing rigour into science, Frege conceived the task of philosophy as being that of introducing rigour into mathematics. Indeed, in order to achieve a secure foundation for mathematics, Frege asked that “the fundamental propositions of arithmetic should be proved, if in any way possible, with the utmost rigour” (Frege 1960, 4).

2.6 Philosophy and the Armchair

According to the heuristic view, philosophy is not a product of thought alone, hence it is not an armchair subject, it essentially needs inputs from experience.

This contrasts with classical analytic philosophy. Thus, Dummett states that philosophy “is a discipline that makes no observations, conducts no experiments, and needs no input from experience: an armchair subject, requiring only thought” (Dummett 2010, 4). Philosophy is like “another armchair discipline: mathematics. Mathematics likewise needs no input from experience: it is the product of thought alone” (ibid.).

Nagel states that “philosophy is different from science” because, “unlike science it doesn’t rely on experiments or observation, but only on thought,” since philosophy “is done just by asking questions, arguing, trying out ideas and thinking of possible arguments against them” (Nagel 1987, 4).

But this is based on the assumption that philosophy is not an inquiry which primarily aims at acquiring knowledge about the world, and hence is not concerned to discover new facts about reality. As argued above, this assumption is unjustified. Since philosophy is an inquiry which primarily aims at acquiring knowledge about the world, it necessarily needs inputs from experience, and hence cannot rely only on thought. By thought alone, we can at most reformulate what we already know in other terms.

In particular, saying that philosophy is like another armchair discipline, namely mathematics, is unjustified because mathematics is not an armchair subject, it essentially involves interactions with the world beyond the armchair. As Atiyah points out, “almost all mathematics originally arose from external reality” (Atiyah 2005, 226). In fact, several mathematical problems have an extra-mathematical origin, and the solutions of mathematical problems are only plausible, so they are evaluated in terms of their compatibility with the existing knowledge, and hence with experience.

2.7 Philosophy and the Sciences

According to the heuristic view, philosophy is continuous with the sciences, in the sense that it aims at a kind of knowledge which differs from scientific knowledge in no essential respect, and is not restricted to any area. Thus, the objectives of philosophy are not essentially different from those of the sciences, and philosophy is an activity which is not essentially different from the sciences. The only difference between philosophy and the sciences, is that philosophy deals with questions which are beyond the present sciences. The latter are what we already know, philosophy is about what we do not yet know.

This contrasts with classical analytic philosophy. Thus Wittgenstein states that, in philosophy, “we are not doing natural science” (Wittgenstein 1958, II, xii.230).

Indeed, “natural science” is “something that has nothing to do with philosophy” (Wittgenstein 2002, 6.53). The “word ‘philosophy’ must mean something whose place is above or below the natural sciences, not beside them” (ibid., 4.111).

Dummett states that “philosophy stands in complete contrast with sciences: its methods wholly diverge from those of science, and its objective differs to an equal extent,” moreover “the results of philosophy differ fundamentally in character from those of the sciences” (Dummett 2010, 7).

But if philosophy stands in complete contrast with sciences, if its objectives, methods, and results differ fundamentally in character from those of the sciences, how could philosophy possibly contribute to our knowledge of reality?

Dummett also states that saying that philosophy is continuous with the sciences is a form of scientism, where “scientism is the disposition to regard the natural sciences as the only true channel of knowledge” (ibid., 35). Scientism implies that “the idea that philosophy has a subject matter or a method of its own must be discarded: if it is to contribute to knowledge at all, it must be continuous with the natural sciences,” and its task reduces “to that of adding ornamentation to the theories of the scientists” (ibid.).

But saying that philosophy is continuous with the sciences does not amount to regarding the natural sciences as the only true channel of knowledge, or to reducing the task of philosophy to that of adding ornamentation to the theories of the scientists. Scientism is the disposition to regard, not the natural sciences, but the present natural sciences, as the only true channel of knowledge. There are areas of experience which the present natural sciences are incapable to deal with. Dealing with them requires new ideas, not devised by any of the present sciences, and it is the task of philosophy to devise them. It is in this sense that the present sciences are what we already know, and philosophy is about what we do not yet know.

Dummett’s view that philosophy stands in complete contrast with sciences is shared, for opposite reasons, by several scientists, who believe that philosophy is, at best, an ornamentation or a parasitic commentary on the achievements of the sciences, and that the sciences are philosophy-free. But, as Dennett says, “there is no such thing as philosophy-free science; there is only science whose philosophical baggage is taken on board without examination” (Dennett 1996, 21).

2.8 Philosophy and the Results of the Sciences

According to the heuristic view, philosophy makes use of the results of the sciences, and this is essential to its progress. Indeed, since philosophy is continuous with the sciences, like the sciences, in order to obtain new knowledge, philosophy must start from the existing knowledge.

This contrasts with classical analytic philosophy. Thus, Wittgenstein states that “one might also give the name ‘philosophy’ to what is possible before all new discoveries” (Wittgenstein 1958, I, § 126). Therefore, philosophy is independent of any scientific discovery, in particular “no mathematical discovery can advance it” (ibid., I, § 124).

But if, in dealing with philosophical problems, no use is made of the results of the sciences, then philosophy ends up with repeating old idioms, neglecting that often they are based on obsolete views of the world. This is not only acknowledged but even theorized by Wittgenstein, who states that “no new words have to be used in philosophy – the old, ordinary words of language suffice” (Wittgenstein 2005, 309).

Contrary to Wittgenstein’s claims, in order to deal with new philosophical problems, philosophy must be able to use whatever is known, starting from the results of the sciences, introducing new idioms adequate to the questions dealt with. Old idioms are often based on obsolete scientific theories, or simply on prejudices.

2.9 Philosophy and Method

According to the heuristic view, the method of philosophy is the same as that of the sciences. Indeed, since philosophy is continuous with the sciences, its method cannot be essentially different.

This contrasts with classical analytic philosophy. Thus, Wittgenstein states that “philosophers constantly see the method of science before their eyes, and are irresistibly tempted to ask and answer questions in the way science does,” but this “leads the philosopher into complete darkness” (Wittgenstein 1969a, 18). Actually, the method of philosophy is completely different from that of science, because “the task of philosophy” is “to clarify the use of our language, the existing language. Its aim is to remove particular misunderstandings” (Wittgenstein 1974, 115). Therefore, the method of philosophy is the analysis of language, which removes “misunderstandings concerning the use of words” by “substituting one form of expression for another; this may be called an ‘analysis’ of our forms of expression, for the process is sometimes like one of taking a thing apart” (Wittgenstein 1958, I, § 90).

Dummett states that the aim of “philosophy is the analysis of the structure of thought,” and “the only proper method for analyzing thought consists in the analysis of language” (Dummett 1978, 458). For, “there can be no account of what thought is, independently of its means of expression,” namely language, and “the structure of the sentence reflects the structure of thought” (Dummett 1991, 3). Therefore, “the philosophy of language is the foundation of all other philosophy” (Dummett 1978, 442).

But the method of philosophy cannot be the analysis of language, because philosophy is an inquiry which primarily aims at acquiring knowledge about the world, and the analysis of language is inadequate to that purpose. For, questions about the world are not questions of words but questions of things. As Kant states, “in matters over which one has quarreled over a long period of time, especially in philosophy, there has never been at the basis a quarrel of words but always a true quarrel over things” (Kant 2007, 179).

Moreover, the assumption that the only proper method for analyzing thought consists in the analysis of language contrasts with studies on non-linguistic thought. They are important not only with regard to animal thought, whose existence was already acknowledged by Chrysippus, who stated that a hunting “dog makes use of

the fifth undemonstrable syllogism when he come to a crossroads” (Sextus Empiricus, *Pyrrhoniae Hypotyposes*, I, 69). They are also important with regard to human thought, for example because several scientists and mathematicians affirm that their most creative work does not involve language.

Thus, Einstein states that “the words or the language, as they are written or spoken, do not seem to play any role in my mechanism of thought. The psychical entities which seem to serve as elements in thought” are, “in my case, of visual and some of muscular type” (Einstein 1954, 142–143).

Hadamard states: “Words are totally absent from my mind when I really think,” and “do not reappear in my consciousness before I have accomplished or given up the research” (Hadamard 1954, 75).

Identifying the method of philosophy with the analysis of language is a result of the assumption of classical analytic philosophy, that questions about knowledge no longer lie at the heart of the philosophical enterprise. With respect to this assumption, Hawking states that, while in the eighteenth century philosophers “considered the whole of human knowledge, including science, to be their field,” in the twentieth century they have “reduced the scope of their inquiries so much that Wittgenstein, the most famous philosopher of this century, said, ‘The sole remaining task for philosophy is the analysis of language’. What a comedown from the great tradition of philosophy from Aristotle to Kant!” (Hawking 1988, 185). Similarly, Rota states that, in the twentieth century, “the classical problems of philosophy have become forbidden topics,” and this has led to “the shrinking of philosophical activity to an impoverished *problématique*, mainly dealing with language” (Rota 1997, 98).

Rather than with the analysis of language, the method of philosophy must be identified with that of the sciences. The latter is the analytic method, which is both a method of discovery and justification. The analytic method is the method according to which, to solve a problem, one looks for some hypothesis that is a sufficient condition for solving the problem, namely, such that a solution to the problem can be deduced from it. The hypothesis is obtained from the problem, and possibly other data already available, by some non-deductive rule – such as induction, analogy, metaphor, and so on – and must be plausible, namely such that the arguments for it are stronger than the arguments against it, on the basis of the existing knowledge. But the hypothesis is in its turn a problem that must be solved, and is solved in the same way. That is, one looks for another hypothesis that is a sufficient condition for solving the problem posed by the previous hypothesis, it is obtained from the latter, and possibly other data already available, by some non-deductive rule, and must be plausible. And so on, *ad infinitum*. (For more on the analytic method, see Chap. 12, where it is also argued that Plato first formulated the analytic method as the method of philosophy).

2.10 Philosophy and the Aim to Acquire Knowledge

According to the heuristic view, since philosophy is continuous with the sciences, aiming at acquiring knowledge is part of the deepest nature of philosophy.

This contrasts with classical analytic philosophy. Thus, Wittgenstein states that philosophy “arises neither from an interest in the facts of nature, nor from a need to grasp causal connections;” on the contrary, it is “essential to our investigation that we do not seek to learn anything new by it” but only “to understand something that is already in plain view” (Wittgenstein 1958, I, § 89). In philosophy, “it is not that a new building has to be erected, or that a new bridge has to be built, but that the geography, as it now is, has to be judged” (Wittgenstein 1978, V, § 52). Philosophy has no impact on the growth of knowledge, it “leaves everything as it is” (Wittgenstein 1958, I, § 124). It “simply puts everything before us, and neither explains nor deduces anything” (ibid., I, § 126). It “only states what everyone admits” (ibid., I, § 599).

Dummett states that “philosophy does not advance knowledge: it clarifies what we already know” (Dummett 2010, 21). It “does not seek to observe more, but to clarify our vision of what we see” (ibid., 10).

But a philosophy thus meant has little reason to exist. Therefore, Dummett states that today “it is by no means obvious that universities, and thus ultimately the state, should support philosophy” but for the historical precedent that “the history of Western universities goes back 900 years” and “philosophy has always been one of the subjects taught and studied in them” (ibid., 2). When the first Western universities came into being, philosophy “was not sharply differentiated from what we call ‘natural science’” (ibid.). It was then easy to find a justification for philosophy. But, in the twentieth century, “the distinction between philosophy and the natural sciences came to be generally admitted” (ibid., 3). Therefore, finding a justification for philosophy became difficult. Indeed, “if universities had been an invention of the second half of the twentieth century, would anyone have thought to include philosophy among the subjects that they taught and studied? It seems very doubtful” (ibid., 2). It “would be easy to conclude that this is an anachronism” (ibid.).

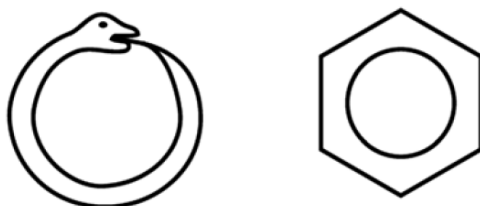
This is the conclusion to which the rejection of the view that philosophy primarily aims at acquiring knowledge leads. While, in the philosophical tradition, philosophy was inspired by the hope that it would contribute to the advancement of knowledge, classical analytic philosophy conceives of philosophy in such a way that it becomes incapable of making any contribution to our knowledge of the world. Then, as Dummett says, it would be easy to conclude that it is an anachronism.

2.11 Philosophy and the Aim to Obtain Rules of Discovery

According to the heuristic view, philosophy aims not only at acquiring knowledge, but also at obtaining rules of discovery, since nothing guarantees that new knowledge can be acquired by the existing rules of discovery.

This contrasts with classical analytic philosophy. Thus, Hempel states that philosophy cannot aim at obtaining rules of discovery, because hypotheses cannot be obtained by “any process of systematic inference” (Hempel 1966, 15). They “are not derived from observed facts, but invented in order to account for them” (ibid., 15). Their discovery “requires inventive ingenuity; it calls for imaginative, insight-

ful guessing” (ibid., 17). That is, it calls for intuition. To support this view, Hempel mentions the case of Kekulé who one evening, while sitting by a fire, sank into half-sleep and had a dream. He saw atoms fluttering before his eyes, long chains often combined in a denser fashion, all in motion, twisting and turning like snakes, until one of the snakes seized its own tail. This suggested to Kekulé the hypothesis that benzene forms a closed ring of six carbon atoms.



According to Hempel, Kekulé’s case shows that, to solve a problem, scientists essentially depend on intuition, only thanks to which they may arrive at “the discovery of important, fruitful theories in empirical science” (ibid.). But, contrary to Hempel’s claim, Kekulé’s case does not show that. Kekulé himself does not say he had an intuition, but only a dream. Moreover, at Kekulé’s time it was well known that the behaviour of a molecule depended on its structure, and the structures already tried for benzene were inadequate. Therefore, Kekulé was well aware of the need to find a new structure, and had already considered various possibilities on that regard. Seeing that a snake biting its tail formed a stable structure suggested to him, by an analogical inference, that a structure for benzene could be a closed ring. Furthermore, Kekulé formulated the hypothesis that the structure of benzene is a closed ring only after comparing the hypothesis with the existing data. Therefore, he formed his hypothesis through a rational process. There is also the question whether Kekulé’s report of the event is reliable. It has been argued that Kekulé made up the whole episode. This has been subject of some controversy (see [Rocke 2011](#), Chap. 10).

2.12 Philosophy and the Birth of New Sciences

According to the heuristic view, since philosophy deals with questions which are beyond the present sciences, it must try unexplored routes. By so doing, when successful, philosophy may even give birth to new sciences. This is one of its greatest values.

This contrasts with classical analytic philosophy. Thus, Dummett states that “no practicing philosopher would explain the value of the subject merely as a matrix out of which new disciplines could develop” (Dummett 2010, 4). Philosophy is “what is left when the disciplines to which it gave birth have left the parental home” (ibid.). So, “it was not until the nineteenth century that it made sense to ask for an example of a philosophical problem, as opposed to a problem of some other kind.” (ibid., 8). In particular, as opposed to a scientific problem.

But trying unexplored routes, possibly giving birth to new sciences, is one of the greatest values of philosophy. So, in the seventeenth century, philosophy gave birth to modern science, which originated from Galileo's philosophical revolution: the decision to renounce Aristotle's aim to penetrate the essence of natural substances, dealing only with some of their phenomenal properties mathematical in kind, such as location, motion, shape, or size (see Chap. 8).

Since the seventeenth century, philosophy has given birth to several other sciences. For example, in the twentieth century, Turing's philosophical analysis of the computational behaviour of human beings gave birth to computer science. The philosophical character of Turing's analysis is apparent from "the enthusiastic philosophical reception of Turing's approach," which "stands in stark contrast to the very limited attention given to it in print in the following decade" (Mosconi 2014, 38).

There is no reason to think that philosophy will not give birth to new sciences also in the future. For example, knowledge has an important role in evolution. Even simple organisms such as the prokaryotes cannot survive if they do not acquire knowledge about the environment. But the current theories of evolution disregard the role of knowledge in evolution, they do not take into account knowledge processes. Therefore, there is need for a new science which completes the current theories of evolution with a theory of knowledge. As another example, although it is evident that the mind is intimately linked to brain processes, and that the study of such processes will give us much insight into operations of the mind, it should also be understood that the mind involves processes that are external to the mind (see Chap. 17). The present psychology and cognitive science do not take into account such processes. Therefore, there is need for a new science which considers not only the processes internal to the mind but also processes external to it.

These are just two examples of possible new sciences, but other examples could be devised. There is much space for philosophy, because the things which we do not yet know, even on basic questions, are numerous, and philosophy is about what we do not yet know. In this connection, it is worth recalling Seneca's prediction: "Veniet tempus quo posterī nostri tam aperta nos nescisse mirentur [A time will come when our posterity will marvel that we were ignorant of such obvious things]" (Seneca, *Naturales Quaestiones* 7.25). Seneca's prediction should act as a warning also for us.

Of course, trying unexplored routes, philosophy moves on a magmatic ground, so it can offer no theories but only viewpoints. The proper place for theories are the new sciences to which philosophy may possibly give birth. But this does not make philosophy less continuous with the sciences. Obtaining knowledge is a profoundly unitary enterprise.

2.13 Philosophy and the History of Philosophy

According to the heuristic view, philosophy makes use of the experience of the philosophers of the past. Without it, philosophers would keep reinventing the wheel, or hunting down trails that are known to be dead ends. Therefore, the history of philosophy is relevant to philosophers.

This contrasts with classical analytic philosophy. Thus, Wittgenstein states: “What has history to do with me? Mine is the first and only world! I want to report how I found the world. What others in the world have told me about the world is a very small and incidental part of my experience of the world” (Wittgenstein 1979, 82, 2.9.16). The “reason why I give no sources is that it is a matter of indifference to me whether the thoughts that I have had have been anticipated by someone else” (Wittgenstein 2002, 4). A philosopher is like “a king” who has been “brought up in the belief that the world began with him” (Wittgenstein 1969b, § 92).

In fact, in formulating problems and dealing with them, several classical analytic philosophers behave like that king, not only with respect to the philosophical tradition, but also with respect to their own history. They tend to consider only the problems and the solutions their generation proposes, ignoring those of the previous generations. By so doing, they think to behave like those scientists who, while knowing only the most recent literature in their field, give contributions to their discipline. But it is not so. Those scientists deal with questions posed by the world, though very limited ones, and, being very limited, in some cases knowledge of the most recent literature alone may be enough to deal with them. Conversely, philosophers who only know the most recent literature do not deal with question posed by the world, but only with puzzles posed by their colleagues, which are generally irrelevant to an inquiry into the world.

Of course, the history of philosophy is not a substitute for philosophy itself. In this regard, it is worth recalling Kant’s complaint that “there are scholars for whom the history of philosophy” is “itself their philosophy” (Kant 2002, 53). According to them, “nothing can be said that has not already been said before” (ibid.). But they are wrong, because the world poses ever newer problems and challenges. Such scholars “must wait until those who endeavor to draw from the wellspring of reason itself have finished their business, and then it will be their turn to bring news of these events to the world” (ibid.).

Nevertheless, being aware of the philosophy of the past is important, on the one hand, to acquire awareness of the implications of certain philosophical assumptions, and, on the other hand, to avoid retracing routes already covered. As Santayana says, “those who cannot remember the past are condemned to repeat it” (Santayana 1948, 248).

2.14 Philosophy and Intuition

According to the heuristic view, philosophy makes no use of intuition, because the method of philosophy is the same as that of the sciences, and intuition plays no role in the method of the sciences.

This contrasts with classical analytic philosophy. Thus, Wittgenstein says that philosophy requires intuition because philosophy is about what lies in front of everyone’s eyes, and the philosopher may grasp it because “God grant the philosopher insight into what lies in front of everyone’s eyes” (Wittgenstein 1998, 72). Even knowledge of logical relations requires intuition, because “I can’t come to this insight through a logical inference, I must see it” (Wittgenstein 1975, 336).

Philosophy requires intuition like mathematics, in which “no investigation of concepts, only insight into the number-calculus can tell us that $3 + 2 = 5$ ” (Wittgenstein 1974, 347). Even “in order to follow the rule ‘Add 1’ correctly a new insight, intuition, is needed at every step” (Wittgenstein 1969a, 141).

But, being subjective and arbitrary, intuition is unreliable. Bealer states that “denying that intuitions are evidence leads to epistemic self-defeat; it is impossible to have a coherent epistemology without admitting intuitions as evidence” (Bealer 1996, 32, footnote 26). But this is unjustified. It is possible to have a coherent epistemology without admitting intuitions as evidence, if one assumes that the method of philosophy is the same as that of the sciences, namely, the analytic method, and, as it will be argued in Chap. 12, intuition plays no role in the analytic method.

2.15 Philosophy and Emotion

According to the heuristic view, philosophy is concerned with emotions, not only because emotions are important to the quality and meaning of our life, but also because they are a help to reason and play an important role in knowledge, including mathematical and scientific knowledge.

This contrasts with classical analytic philosophy. Thus, Russell states that, although “emotions are what makes life interesting, and what makes us feel it important,” when “we are trying to understand the world, they appear rather as a hindrance. They generate irrational opinions” which “cause us to view the universe in the mirror of our moods” (Russell 1995a, 175–176).

But it is not so, emotions are a help to reason, because reason is the capacity to choose appropriate means to a given end (see Cellucci 2013a, Chap. 14). And emotions help us to choose appropriate means to the end of survival, because only individuals who perceive a favorable occasion as a positive emotion, and danger as a negative one, can survive.

Generally, emotions play an important role in knowledge. They play an important role in the choice of problems and hypotheses to solve them. We are able to seek solutions only for a small part of the problems that arise. Emotions may help us to choose which problems to consider and which to disregard. Only if we feel strongly involved in a problem we may have the drive to do the hard work that finding a solution may require. Besides, in solving a problem we may be faced with so many alternative hypotheses, that it would be unfeasible to consider all of them. Emotions may help us to choose which hypothesis to consider and which to disregard.

Even Carnap admits that, also in “the most rational of sciences, namely physics and mathematics,” the “basic orientation and the direction of interests are not the result of deliberation, but are determined by emotions” (Carnap 2003, xvii). (For more on the role of emotions in knowledge, see Cellucci 2013a, Chap. 15).

2.16 Philosophy and the Solvability of Problems

According to the heuristic view, philosophy cannot demand and expect conclusive solutions to the questions belonging within it. Solutions to philosophical problems are always temporary and are bound to be replaced sooner or later by others.

This contrasts with classical analytic philosophy. Thus, Wittgenstein states that “philosophical problems must be solvable really completely, in contrast to all others” (Wittgenstein 2005, 310).

But conclusive solutions to philosophical problems are impossible. Philosophical problems, being problems about the world, are like scientific problems, and the latter cannot be solved conclusively. A philosopher is like a scientist who, as Poincaré says, “is always searching and is never satisfied” (Poincaré 1910, VII). Indeed, every solution is based on hypotheses which are only plausible, so it is always provisional and bound to be replaced with another one as new data emerge. Even Wittgenstein acknowledges that “in this work more than any other it is rewarding to keep on looking at questions, which one considered solved, from another quarter, as if they were unsolved” (Wittgenstein 1979, 30, 13.11.14). Therefore, it is arbitrary to insist that philosophical treatments of problems are valuable only if they provide conclusive solutions.

Dennett states that, confronted with the alternative of either (A) solving a major philosophical problem “so conclusively that there is nothing left to say,” or (B) writing “a book of such tantalizing perplexity and controversy that it stays on the required reading list for centuries to come,” some philosophers “admit that they would have to go for option (B)” (Dennett 2013, 429). Conversely, scientists “tend to opt for (A) without any hesitation,” and “shake their heads in wonder (or disgust?) when they learn that this is a hard choice for many philosophers” (ibid.). But, for once, philosophers are right and scientists are wrong. For, not only no major philosophical problem, but also no major scientific problem, can be solved so conclusively that there is nothing left to say.

2.17 Philosophy and Progress

Although solutions to philosophical problems are always temporary and are bound to be replaced sooner or later by others, they advance knowledge. There is progress everywhere, even in philosophy.

This contrasts with classical analytic philosophy. Thus, Wittgenstein states that the solution of philosophical problems produces no progress, because in philosophy we are simply destroying “houses of cards and we are clearing up the ground of language on which they stand” (Wittgenstein 1958, I, § 118). People complain that “philosophy really doesn’t make any progress, that the same philosophical problems that occupied the Greeks keep occupying us. But those who say that don’t understand” that it must be so because “our language has remained constant and

keeps seducing us into asking the same questions” (Wittgenstein 2005, 312). As long as language will remain constant, human beings “will continue to bump up against the same mysterious difficulties, and stare at something that no explanation seems able to remove” (ibid.).

But, if the solution of philosophical problems produces no progress, then philosophy is a futile activity which gets us nowhere. Even Dummett admits that, “if philosophy makes no progress, it is not worth wasting any time on” (Dummett 2010, 148).

As to language, even supposing that our language has remained constant since the Greeks and keeps seducing us into asking the same questions, this does not mean that the solution of philosophical problems produces no progress. The world changes all the time, there are no two instants in which it is exactly the same, so the world poses ever newer questions and challenges. Once again, this shows that it is arbitrary to restrict philosophy to the method of the analysis of language.

2.18 Philosophy and Professionalization

According to the heuristic view, philosophy cannot be a professional activity, because it has no special field to investigate, or special techniques of its own to use.

This contrasts with classical analytic philosophy. Thus, Carnap states that, “while the attitude of the traditional philosopher is more like that of a poet,” the analytic philosophers “have taken the strict and responsible orientation of the scientific investigator as their guideline for philosophical work” (Carnap 2003, xvi). The “individual no longer undertakes to erect in one bold stroke an entire system of philosophy. Rather, each works at his special place within the one unified science” (ibid.).

As Rescher points out, this has led to “the turning of philosophy from global general, large-scale issues to more narrowly focused investigations of matters of microscopically fine-grained detail” (Rescher 1993, 731). As a result, today “philosophical investigations make increasingly extensive use of the formal machinery of semantics, modal logic, compilation theory, learning theory, and so forth. Ever heavier theoretical armaments are brought to bear on ever smaller problem-targets” (ibid.).

Austin even claims that, in order to become a professional philosopher, “first we may use the dictionary – quite a concise one will do, but the use must be thorough” (Austin 1970, 186). There are two ways of using it, “one is to read the book through, listing all the words that seem relevant,” the other one “is to start with a wishful selection of obviously relevant terms, and to consult the dictionary under each: it will be found that, in the explanations of the various meanings of each, a surprising number of other terms occur” (ibid., 186–187). We will then “look up each of these, bringing in more for our bag from the ‘definitions’ given in each case” (ibid., 187). If we will continue like that, after a while “it will generally be found that the family circle begins to close, until ultimately it is complete” (ibid.). So we will “arrive at the meanings of large numbers of expressions” and we “shall comprehend clearly

much that, before, we only made use of ad hoc. Definition, I would add, explanatory definition, should stand high among our aims” (ibid., 189).

But so one does not get at the nature of things, only at the opinion of the authors of the dictionary. Admittedly, the Socratic endeavour was the pursuit of definition. For, Socrates thought that, until one knows the essence of a thing, one cannot answer any other questions about it, and a definition states what a thing is in itself, its essence. But, by stating the essence of things, Socratic definitions are of things, not of words. For example, Socrates asks the question: “What do you think knowledge is?” (Plato, *Theaetetus*, 146 c 3). By this question, he does not want to know what the word ‘knowledge’ means, but rather “to know what knowledge is in itself” (ibid., 146 e 9–10).

Actually, a professional philosophy is impossible. A philosopher cannot be a professional in the same sense as a mathematician, or a physicist, or a biologist, because philosophy has no special field of its own. As sciences claim special fields, philosophy’s field of inquiry changes. It remains the unexplored ground, but what ground is unexplored changes with time. On the other hand, a philosopher cannot be a professional in the same sense as a doctor, or a lawyer, or an engineer, because philosophy has no special techniques of its own. Although the method of philosophy is the same as that of the sciences, namely the analytic method, the latter is only a general framework, and its application requires experience specific to the field. But a philosopher moves on an unexplored ground, on which there is still very limited experience.

Therefore, a philosopher is, and always will be, a great amateur. And yet, just because a philosopher moves on an unexplored ground, on which there is still very limited experience, philosophy is always exposed to the risk of failure but is also capable of surprising developments. Just like those thanks to which, trying unexplored routes, through hazardous though sometimes fortunate moves, philosophy has given birth to new sciences.

2.19 The Heuristic View vs. Philosophy as Criticism of Principles

The heuristic view of philosophy, outlined above, should not be confused with Russell’s view of philosophy as criticism of principles.

Admittedly, Russell states that philosophy “aims primarily at knowledge” (Russell 1997, 154). It aims at a knowledge that “does not differ essentially from scientific knowledge,” because “there is no special source of wisdom which is open to philosophy but not to science, and the results obtained by philosophy are not radically different from those obtained from science” (ibid., 149). The only difference between philosophy and science is that “science is what we know, and philosophy is what we don’t know,” but “philosophical speculation as to what we do not yet know has shown itself a valuable preliminary to exact scientific knowledge” (Russell 1950, 24).

So far so good. But then Russell states that philosophy aims at the kind of knowledge “which results from a critical examination of the grounds of our convictions, prejudices, and beliefs” (Russell 1997, 154). It “examines critically the principles employed in science” (ibid., 149). It wants “to see whether they are mutually consistent and whether the inferences employed are such as seem valid to a careful scrutiny” (Russell 1995a, 239).

This view seems unrealistic. Scientists do not have to wait for philosophers to examine critically the principles employed in their sciences, in order to see whether they are mutually consistent, or whether the inferences employed are valid. This is an integral part of their work, and they are much more competent to the task than philosophers, who do not have the necessary qualifications. Moreover, a philosophy in accordance with Russell’s view could not be expected to give birth to new sciences.

Conversely, a philosophy in accordance with the heuristic view would permit to give an affirmative answer to the question raised at the beginning of this chapter – whether philosophy can still be fruitful, and what kind of philosophy can be such. A philosophy with such characteristics would be fruitful because it aims at knowledge and methods to acquire knowledge, it pursues this aim by trying unexplored routes, and may even give birth to new sciences.

A remark about the use of the term ‘sciences’. Since, according to the heuristic view, philosophy is an inquiry which primarily aims at acquiring knowledge about the world, including ourselves, throughout this book ‘sciences’ means ‘natural and human sciences’, unless stated otherwise.

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thing first arises,” like “the foundation of a house” (Aristotle, *Metaphysica*, Δ 1, 1013 a 4–5). In particular, a principle is like the foundation of a house which is unshakable, being based on intuition, since it is “intuition that apprehends the principles” (Aristotle, *Analytica Posteriora*, B 19, 100 b 12). And “intuition” is “always true” (ibid., B 19, 100 b 7–8).

From antiquity to the present day, the foundationalist view has had a large following.

For example, Russell states that “what we firmly believe, if it is true, is called knowledge, provided it is either intuitive or inferred” from “intuitive knowledge from which it follows logically” (Russell 1997, 139). A body of knowledge must be arranged “in deductive chains, in which a certain number of initial propositions form a logical guarantee for all the rest. These initial propositions are premisses for the body of knowledge in question” (Russell 1993, 214). They “must be knowledge which is independent of inference” (Russell 1992b, 157). Moreover, they must be knowledge which is absolutely certain, namely knowledge “of whose truth there can be no doubt” (ibid., 178). In order to be such, they must be “intuitive knowledge,” namely “immediate knowledge of truths,” and hence “self-evident truths” (Russell 1997, 109). Such truths must be “self-evident in a sense which ensures infallibility” (ibid., 135). All other knowledge “consists of everything that we can deduce from self-evident truths by the use of self-evident principles of deduction,” so “all our knowledge of truths depends upon our intuitive knowledge” (ibid., 109).

From the first assumption of the foundationalist view – that there is immediately justified knowledge, and all other knowledge is deduced from it – it is clear that, according to this view, knowledge is based on the axiomatic method.

The second assumption of the foundationalist view – that immediately justified knowledge is absolutely certain, being based on intuition – derives from the fact that the main motivation for this view is to save knowledge from sceptical doubt.

Thus, Russell states that he “was troubled by scepticism and unwillingly forced to the conclusion that most of what passes for knowledge is open to reasonable doubt” (Russell 1971, III, 220). The question which worried him was: “Is there any knowledge in the world which is so certain that no reasonable man could doubt it?” (Russell 1997, 7). This kind of knowledge was essential to stop scepticism, by which “the whole attempt to get behind to something more solid, and worthy to be called knowledge, is futile” (Russell 1992b, 159).

3.2 The First Assumption of the Foundationalist View

Is the foundationalist view adequate? In order to answer this question, let us examine the first assumption of the foundationalist view, that there is immediately justified knowledge, and all other knowledge is deduced from it.

This assumption is motivated by the infinite regress argument, according to which, since human capacities are finite, we cannot go through an infinite series of

premisses. So a series of premisses in a science cannot be infinite, it must have a beginning, and hence there must be immediately justified knowledge.

Thus, Russell states that, “since human capacity is finite, what is known of a science cannot contain more than a finite number of definitions and propositions,” so “every series of definitions and propositions must have a beginning, and therefore there must be undefined terms and unproved propositions. The undefined terms are understood by means of acquaintance. The unproved propositions must be known by self-evidence” (Russell 1992b, 158).

The infinite regress argument, however, is inadequate. Although, by the finiteness of human capacities, we cannot go through an infinite series of premisses, this does not mean that the series of the premisses cannot be infinite, but only that, at each stage, we can only go through a finite initial segment of the series. And yet, as in the analytic method, we can go through longer and longer finite initial segments.

Of course, if the series of the premisses is infinite, there will be no immediately justified premisses, so no knowledge will be definitive. But this does not mean that there will be no knowledge. There would be no knowledge only if the premisses, or hypotheses, occurring in the infinite series were arbitrary. But they need not be arbitrary. As in the analytic method, they must be plausible, namely the arguments for them must be stronger than the arguments against them, on the basis of the existing knowledge. If the hypotheses are plausible, then there will be knowledge, albeit provisional knowledge always in need of further consideration, since new data may always emerge.

That the infinite regress argument is inadequate means that the first assumption of the foundationalist view is unjustified. As in the analytic method, the process of problem solving may consist in formulating hypotheses which are plausible, where each hypothesis is in its turn a problem which must be solved, and is solved by formulating another hypothesis which is plausible, and so on, *ad infinitum*.

3.3 The Second Assumption of the Foundationalist View

Let us now examine the second assumption of the foundationalist view, that immediately justified knowledge is absolutely certain, being based on intuition. The foundationalist view tries to motivate this assumption in several ways. One of the most significant ones is Russell’s.

According to Russell, immediately justified knowledge consists, on the one hand, of “the general truths of logic” and, on the other hand, of “the particular facts of sense” (Russell 1993, 78). The latter are “truths of perception,” namely truths which are “immediately derived from sensation” (Russell 1997, 113). Both the general truths of logic and the truths of perception are intuitive truths, and hence are absolutely certain. Indeed, “the more we reflect upon these, the more we realize exactly what they are, and exactly what a doubt concerning them really means, the more luminously certain do they become” (Russell 1993, 78). Admittedly, “verbal

doubt concerning even these is possible,” but “real doubt, in these two cases, would, I think, be pathological” (ibid.).

In view of the illusions of sense, Russell’s claim that real doubt about the truths of perception would be pathological may seem odd. But Russell argues that “there are no such things as ‘illusions of sense’. Objects of sense, even when they occur in dreams, are the most indubitably real objects known to us” (ibid., 92–93). What is illusory in the illusions of sense “is only the inferences to which they give rise; in themselves, they are every bit as real as the objects of waking life” (ibid., 93).

This view on the truths of perception is not peculiar to Russell, but goes back to the Greeks. Thus Aristotle states that “perceptions are always true, while most of the imaginings turn out to be false” (Aristotle, *De Anima*, Γ 3, 428 a 11). Epicurus states that “our sensations” are “the standards of truth,” since “there is nothing which can refute sensations or convict them of error” (Diogenes Laertius, *Vitae Philosophorum*, X, 31). Even objects of sense “occurring to madmen and to people in dreams are true” (ibid., X, 32). Conversely, “falsehood and error always lie in that which is added by our judgment” (ibid., X, 50).

Kant reasserts this view: “The senses do not err; yet not because they always judge correctly, but because they do not judge at all,” indeed, “error, and thus also illusion as leading to the latter, are to be found only in judgments” (Kant 1998, A293/B350). Thus, to the human being “the full moon, which he sees ascending near the horizon through a hazy air, seems to be further away, and also larger, than when it is high in the heavens,” but this “is an error of the understanding, not of the senses” (Kant 2007, 258). The “illusion is not ascribed to the senses, but to the understanding, whose lot alone it is to render an objective judgment from the appearance” (Kant 2002, 86).

3.4 Frege’s and Russell’s Foundational Programs for Mathematics

Russell’s assumption, that immediately justified knowledge includes the general truths of logic, is the basis of Russell’s logicist program: To show that “all pure mathematics deals exclusively with concepts definable in terms of a very small number of fundamental logical concepts,” and that “all its propositions are deducible from a very small number of fundamental logical principles” (Russell 2010, xliii). If this could be shown, it would follow that all mathematical knowledge is absolutely certain, because the fundamental logical principles are intuitive truths, and hence are absolutely certain.

Russell’s logicist program is an extension of Frege’s logicist program, according to which all arithmetical truths are deducible from a very small number of fundamental logical principles, so arithmetic is merely “a further development of logic” (Frege 1984, 145). Logical principles flow from intellectual intuition, which is “the logical source of knowledge” (Frege 1979, 267). On the other hand, geometrical

truths are not deducible from logical principles, indeed “our knowledge of them flows from a source very different from the logical source, a source which might be called spatial intuition” (Frege 1980, 37). Namely, Kant’s pure sensible intuition. Russell’s logicist program is an extension of Frege’s logicist program since it assumes that, not only arithmetical truths, but also geometrical truths, are deducible from a very small number of fundamental logical principles.

Frege’s logicist program, however, fails because, by Gödel’s first incompleteness theorem, there are arithmetical truths which cannot be deduced from logical principles. *A fortiori*, Russell’s logicist program fails.

3.5 Russell’s Alternative Foundational Program for Mathematics

Russell, however, makes another attempt to show that that all mathematical knowledge is absolutely certain. He argues that the axioms of mathematics are absolutely certain, because their logical consequences are true.

Indeed, Russell states that there is a “close analogy between the methods of pure mathematics and the methods of the sciences of observation” (Russell 1973, 272). In mathematics, “we tend to believe the premisses because we can see that their consequences are true, instead of believing the consequences because we know the premisses to be true. But the inferring of premisses from consequences is the essence of induction” (*ibid.*, 273–274). Thus, “the reason for accepting an axiom” is “always largely inductive” (Whitehead and Russell 1925–1927, I, 59). Hence, “the method in investigating the principles of mathematics is really an inductive method, and is substantially the same as the method of discovering general laws in any other science” (Russell 1973, 274).

This attempt also, however, fails because from a false axiom one can deduce true consequences. In order to assert that an axiom is true, one ought to be able to show that all of its logical consequences are true, but this is generally unfeasible. (For more on this, see Chap. 21).

Russell himself acknowledges that, by considering the logical consequences of an axiom, “infallibility is never attainable, and therefore some element of doubt should always attach to every axiom and to all its consequences” (Whitehead and Russell 1925–1927, I, 59). Thus he admits that his attempt is inadequate with respect to the aim of showing that all mathematical knowledge is absolutely certain.

3.6 Hilbert's Foundational Program for Mathematics

In addition to Russell's and Frege's attempts, other attempts have been made to show that mathematical knowledge is absolutely certain, because it is based on intuition. A very influential attempt has been Hilbert's foundational program.

According to Hilbert, the hard core of mathematics consists of knowledge about "concrete objects that are intuitively present as immediate experience prior to all thought" (Hilbert 1967b, 464). Such knowledge is absolutely certain, because it is based on concrete intuition.

However, mathematics as a whole also includes knowledge about abstract objects, obtained by abstract operations. Such knowledge is problematic, because "abstract operation with general concept-scopes and contents has proved to be inadequate and uncertain" (Hilbert 1996c, 1121).

Therefore, Hilbert states his consistency program: To demonstrate by absolutely reliable means – namely, means based on concrete, sensible intuition – that mathematics as a whole is consistent. For, the "extension by the addition" of abstract objects to concrete objects "is legitimate only if no contradiction is thereby brought about in the old, narrower domain" (Hilbert 1967b, 471). If this could be demonstrated, it would follow that all mathematical knowledge is absolutely certain.

Hilbert's consistency program, however, fails because, by Gödel's second incompleteness theorem, it is impossible to demonstrate by absolutely reliable means that mathematics as a whole is consistent. (For details on Hilbert's consistency program, see, for example, Cellucci 2007, 38–62).

3.7 Russell's Foundational Program for Empirical Knowledge

Russell's assumption that immediately justified knowledge also includes the truths of perception, is the basis of his foundational program for empirical knowledge: To show that all empirical knowledge can be "inferred in some sense" from the truths of perception, "though not necessarily in a strict logical sense" (Russell 1993, 75). If this could be shown, it would follow that, since the truths of perception are intuitive truths and hence are absolutely certain, all empirical knowledge is absolutely certain.

But Russell's foundational program for empirical knowledge fails, because it is based on the assumption that "the 'evidence of the senses' is proverbially the least open to question" (ibid., 74). Then, in order to see that the truths of perceptions are actually truths, we would have no means more reliable than the evidence of the senses itself. Indeed, Russell states: "If the reality is not what appears, have we any means of knowing whether there is any reality at all? And if so, have we any means of finding out what it is like?" (ibid., 16). Such questions "are bewildering," and philosophy cannot answer to them, although it can at least ask them, and "show the

Thus Rorty states that, if we “drop the notion of epistemology as a quest” for “truth, we are in a position to ask whether there still remains something for epistemology to be. I want to urge that there does not” (Rorty 1980, 210).

Williams states that “a non-foundational picture will be too trivial to be thought of as an epistemological theory. It is reasonable to take this to mean that epistemology as a serious theoretical enterprise is dead after all” (Williams 1999, 198).

But this is unjustified. For reasons that will be explained in Chap. 8, epistemology cannot be a quest for truth, but this does not mean that there remains nothing for epistemology to be. As it will be argued in Chap. 9, it can be a quest for plausibility. The purpose of epistemology is not to inquire into the possibility of certain, objective, universal knowledge, because there is no such knowledge, it is rather to inquire into methods to acquire knowledge, fallible knowledge and yet knowledge.

Moreover, a non-foundational picture need not be too trivial to be thought of as an epistemological theory. A non-foundational picture such as the heuristic view, described in Chap. 2, is by no means too trivial to be thought of as an epistemological theory. Indeed, it puts questions about knowledge at the centre of philosophy, reviving the view that philosophy is an enquiry which primarily aims at knowledge and methods to acquire knowledge.

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4.3 Aristotle's Argument Against the Two Schools

Aristotle agrees with the two schools that in none of the cases (I)–(III) there is knowledge.

In particular, Aristotle agrees that, when the members of the first school claim that in case (I) there is no knowledge, “they are right, because it is impossible to survey infinitely many items” (Aristotle, *Analytica Posteriora*, A 3, 72 b 10–11). Moreover, if the series of the premisses “does not terminate and there is always something prior to what one looks for, then there will be demonstrations of everything” (ibid., A 22, 84 a 1–2).

Aristotle also agrees that, when the members of the first school claim that in case (II) there is no knowledge, they are right, because mere assumptions need not be true, while, in order to have knowledge, the premisses “must be true, since it is impossible to have knowledge of what is not the case” (ibid., A 2, 71 b 25–26).

Aristotle also agrees that, when the members of the second school claim that in case (III) there is no knowledge, they are right, because “it is impossible for the same thing at the same time to be both prior and posterior to something” (ibid., A 2, 72 b 27–28). Moreover, demonstrations which proceed in a circle or reciprocally “say nothing more than that this is the case if this is the case, and it is easy to prove everything in this way” (ibid., A 3, 72 b 34–35).

But, while agreeing with the two schools that in none of the cases (I)–(III) there is knowledge, Aristotle claims that this does not mean that there is no knowledge. Indeed, none of (I)–(III) is the case, therefore the sceptical doubts raised by the two schools are unjustified.

For, according to Aristotle, “we have knowledge of things through demonstrations,” where a demonstration is a deduction which proceeds “from premisses that are true, and primitive, and immediate” (ibid., A 2, 71 b 17–21). And from premisses that are absolutely certain, being based on intuition, since “it is intuition, and not discursive thinking, that apprehends the primitive things,” it “is intuition that apprehends the unchanging and first terms in the order of demonstrations” (Aristotle, *Ethica Nicomachea*, Z 11, 1143 a 36–1143 b 3).

That the premisses are true means that, with respect to them, “falsity does not exist, nor error, but only ignorance” (Aristotle, *Metaphysica*, Θ 10, 1052 a 1–2). That the premisses are primitive means that they are principles, because “I call the same thing primitive and principle” (Aristotle, *Analytica Posteriora*, A 2, 72 a 6–7). That the premisses are immediate means that no other proposition is prior to them, because a premiss is “immediate if there is no other proposition prior to it,” so “a principle of a demonstration is an immediate proposition” (ibid., A 2, 72 a 7–8). That the premisses are absolutely certain, being based on intuition, means that “the intuition” on which the premisses are based “concerns things about which there is no falsity” (Aristotle, *De Anima*, Γ 6, 430 a 26–27).

But, if we have knowledge of things through demonstrations, and a demonstration is a deduction which proceeds from premisses that are true, primitive,

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