



Spinoza's Epistemology through a Geometrical Lens

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ABBREVIATIONS

Works by Descartes

- AT *Oeuvres de Descartes*. 11 vols. Edited by C. Adam and P. Tannery. Paris: J. Vrin, 1996.
- CSM *The Philosophical Writings of Descartes*. Vols. 1–2. Translated and edited by John Cottingham, Robert Stoothoff, and Dugald Murdoch. Cambridge: Cambridge University Press, 1984–85.
- CSMK *The Philosophical Writings of Descartes*. Vol. 3. Translated and edited by John Cottingham, Robert Stoothoff, Dugald Murdoch, and Anthony Kenny. Cambridge: Cambridge University Press, 1991.
- DM *Discourse on Method, Optics, Geometry, and Meteorology*. Revised Edition. Translated by Paul J. Olscamp. Indianapolis, IN: Hackett Publishing Company, 2001.

Works by Spinoza

- CM *Cogitata Metaphysica (Metaphysical Thoughts)*. Cited by part and chapter.
- Curley *The Collected Works of Spinoza*. 2 vols. Edited and Translated by Edwin Curley. Princeton, NJ: Princeton University Press, 1985, 2016. Cited by volume and page.
- DPP *Renati des Cartes Principiorum Philosophiae Pars I & Pars II (Descartes' Principles of Philosophy)*

E	<i>Ethics</i>
Ep.	Letters
G	<i>Spinoza Opera</i> . 4 vols. Edited by Carl Gebhardt. Heidelberg: Carl Winter, 1925. Cited by volume and page.
KV	<i>Korte Verhandeling van God de Mensch en deszelfs Welstand</i> (<i>Short Treatise on God, Man, and His Well-Being</i>). Cited by part, chapter, and paragraph number.
TIE	<i>Tractatus de Intellectus Emendatione</i> (<i>Treatise on the Emendation of the Intellect</i>). Cited by paragraph number.
TTP	<i>Tractatus Theologico-Politicus</i> (<i>Theological-Political Treatise</i>). Cited by chapter and paragraph number.

Translations of Spinoza are from Curley unless otherwise indicated. In citing Spinoza's *Ethics* (and the DPP), I use the following abbreviations: a = axiom, app = appendix, c = corollary, d = definition, defaff = definitions of the affects, dem = demonstration, lem = lemma, post = postulate, p = proposition, pref = preface, s = scholium. Thus, E2p40s2 stands for *Ethics*, Part 2, Proposition 40, Scholium 2.

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- Fig. 5.1 The figure depicts the reconvergence of light rays from an object (V, X, Y) via refraction in the eye's cornea and lens (L), and the observation (by P) of the image thereby formed at the back of the eye (R, S, T) from within an enclosed chamber (Z). (*Discourse on Method for Rightly Directing One's Reason and Searching for Truth in the Sciences, Together with the Optics, Meteorology, and Geometry, Which Are Essays in This Method.* Leiden: Maire, 1637) 127
- Fig. 5.2 The figure depicts Spinoza's contention that parallel rays entering a circular lens from any point are refracted so as to come together in a single point. (*Opera Posthuma.* J. Ricuwerstsz, 1677, p. 532) 128



Introduction

Spinoza's philosophy has at its heart a hierarchical trio of kinds of knowledge (*cognitio*): imagination (*imaginatio*), reason (*ratio*), and intuitive knowledge (*scientia intuitiva*).¹ First and humblest, imagination consists

¹ *Cognitio* presents a difficult choice for the translator. While “knowledge” is the more common translation (and the one used by Curley), a number of commentators opt for “cognition.” The latter tend to cite the fact that the first kind of *cognitio* is a cause of falsity (E2p41), whereas nothing worthy of the name “knowledge” should cause falsity. Since “cognition” is more epistemically neutral than “knowledge,” it better encompasses the first as well as the second and third kinds of *cognitio* (both of which contain only true ideas). However, if we consider that the first kind of *cognitio*, for Spinoza, can be understood as a part or fragment of a *true* idea in God's intellect, then it makes sense, in my mind, to consider it as a kind of *knowledge*, albeit a partial or fragmentary kind of knowledge. This line of reasoning is reinforced when we consider such passages as E5p38dem: “The Mind's essence consists in *cognitione* (by E2p11); therefore, the more the Mind *cogniscit* things by the second and third kind of *cognitionis*, the greater the part of it that remains [...]” As I see it, to translate *cognitio* and *cognoscere* here with the neutral “cognition” and “cognize” obscures the fact that the epistemic situation is not neutral. It is in our nature to *know*, and to the extent that we do not *know*, it is only because we are parts of God's infinite intellect, and thus *lack* knowledge. Whether our *cognitio* is inadequate (and thus a cause of falsity) or adequate (and thus true), then, it is a question of epistemically non-neutral *knowledge*, rather than neutral *cognition*. There is, however, another argument for translating *cognitio* as “cognition,” namely, it offers an easy solution for respecting the difference between *cognitio* and *scientia*. If *cognitio* is rendered as “cognition,” this leaves “knowledge” for *scientia*. I do not think this outweighs the disadvantages of the sterility of “cognition,” however, so I will use “knowledge” for both *cognitio* and *scientia*, despite the problem with this procedure. Some

in the unexamined ideas and beliefs we accumulate about ourselves and surroundings just by virtue of being born into the world and interacting with its various denizens. We imagine that fire is hot, dogs bark, our decisions are free, the world is full of beauty, but not without ugliness, eventually we shall die, and other such things. It is not as easy to say what reason consists in, so a statement of its content must wait. What is certain is that it represents a major advance beyond the haphazard opinions of the imagination to genuine understanding. Nevertheless, Spinoza often treats reason as having primarily subordinate significance, a stepping stone to an even higher form of knowledge: intuitive knowledge.² At the pinnacle of the hierarchy, intuitive knowledge is claimed to yield insight into the essences of things, notably our own.

Each Spinozan cognitive state is also an affective one. This explains why summing the ladder of knowledge comes with the promise of non-epistemic rewards as well: liberation from anger, fear, despair, and other destructive passions, as well as the enjoyment of the highest human blessedness and perfection, to name a few. The intrinsic connection that Spinoza sees between cognition and affection, and thus between knowledge and ethics, makes his theory of knowledge particularly attractive from a contemporary standpoint where epistemological and ethical issues are frequently siloed. Relative to its seventeenth-century context, too, Spinoza's epistemology is distinctive and distinctively compelling. Many of his contemporaries emphasized the material and technological fruits of knowledge. Descartes, for instance, memorably heralds the ascendance of modern, scientific humans as "the masters and possessors of Nature."³ While Spinoza by no means despises scientific and technological progress, its ultimate value, for him, lies chiefly in conditioning a more intellectual *summum bonum*: "the knowledge of the union that the mind has with the whole of Nature" (TIE 13/G 2:8). This is far from a renunciation of worldly striving à la Pascal. Knowledge is power, for Spinoza, to be sure,

have used "science" for *scientia* to solve this problem. While this might work in certain contexts, rendering *scientia intuitiva* (Spinoza's third kind of *cognitio*) as "intuitive science" sounds tortured to my ear. See Curley 2:637–38 for further discussion.

² See KV 2.4.9/G 1:61; KV 2.26.6/G 1:109; E5p28/G 2:297. The notion of reason as a stepping stone, which receives strong emphasis in the *Short Treatise*, is much less apparent in the *Ethics*, signaling, as I will suggest later, an elevation in the status of reason from the early works to the *Ethics*. Nevertheless, Spinoza consistently stresses the superiority of the third kind of knowledge, and its status as the pinnacle of human knowing throughout his works.

³ AT VI: 62.

but power conceived as psychological freedom, tranquility, and contentment.

These, then, are among the promises of Spinoza's epistemic program: knowledge of things as they truly are in themselves and the collateral achievement of human moral perfection. I have not even mentioned the prospective social benefits in the offing or Spinoza's eternity of mind doctrine. Suffice it to say that, with or without such additions, this all sounds very enticing. Can Spinoza actually deliver on any of these promises? This is a question that can be answered only by each student of Spinoza for themselves, but any serious assessment presupposes a careful study of Spinoza's epistemology. Remarkably, there have been very few books devoted to the topic.⁴ In part, this may be explained by the deep embeddedness of Spinoza's theory of knowledge in his broader metaphysical system, which makes it something of a challenge (one this book will attempt to meet) to treat his epistemology in any depth without also expounding a detailed interpretation of his metaphysics. But this does not account for the existence of important, influential books on Spinoza's philosophy in general with little or nothing to say about the three kinds of knowledge.⁵ The latter can only be explained (putting the predilections of commentators to the side) by the paucity and obscurity of what Spinoza says about the kinds of knowledge, especially the second and third kinds, despite their centrality to his philosophical project.

None of this means that Spinoza's epistemology is not worth studying in its own right. That the topic merits special focus is something that I hope will become increasingly apparent as we go along, if it is not so already. The endeavor faces significant challenges, however. The most significant are the two already indicated. First, it is necessary to respect the

⁴The only one of which I am aware, at least in English, is Parkinson 1954. A. Garrett 2003, which is more recent, should also be mentioned. Although it is devoted to Spinoza's method, there is significant overlap between methodology and epistemology in Spinoza, and Garrett's erudite study offers valuable insights into the latter. None of this is to say, of course, that there have not been many fine papers devoted to aspects of Spinoza's epistemology. I will have occasion to reference many of these over the course of this book.

⁵I have in mind, in particular, Bennett's *A Study of Spinoza's Ethics* (1984) and Della Rocca's *Spinoza* (2008). While addressing Spinoza's distinction between inadequate and adequate ideas, Della Rocca (2008) ignores the three kinds of knowledge altogether. Bennett, for his part, has some brief things to say about reason and imagination, but only condescends "reluctantly" to touch on intuitive knowledge in order to document its contribution to the "unmitigated and seemingly unmotivated disaster" that is, in Bennett's estimation, the second half of *Ethics* Part 5 (1984, 357).

embeddedness of Spinoza's theory of knowledge within the broader philosophical system, and the complex network of metaphysical underpinnings upon which a proper understanding of the epistemology rests, without losing sight of the epistemology to the metaphysics. Despite the connections to the metaphysics, as well as the ethics, which help to render Spinoza's epistemology especially compelling, the latter is more autonomous, nevertheless, in my view, than often thought. It will be relatively straightforward to meet this challenge, then, by supplying metaphysical background as the opportunity or need arises along the way of discussing the epistemological issues that are my concern.

Less straightforwardly, an interpretation of Spinoza's kinds of knowledge must be constructed with relatively scant, oftentimes seemingly contradictory textual materials. The primary problem is uncertainty regarding the nature and range of the content of Spinozan knowledge claims. It is not always apparent what knowledge is supposed to be *about* in Spinoza's system (beyond knowing that it must be about God, the one substance, one way or another). While this problem is formidable, it is not intractable, and I believe I have a way of mitigating the difficulty. A fruitful strategy, as I turn to explain, is to clarify the status of mathematical entities.⁶

The problem of uncertainty regarding the content of Spinozan knowledge claims is particularly acute in the case of mathematical content. Spinoza exhibits ambivalence about the epistemic status of mathematical ideas, as I will show, making it unclear whether knowledge of natural things includes mathematical knowledge or not. How this question is decided one way or another has far-reaching implications for the interpretation of Spinoza's epistemology and ontology. For this reason, my approach to Spinoza's epistemology will be based on an interpretation of the epistemic and ontological status of mathematical entities in Spinoza.

Generally speaking, the significance of mathematics for Spinoza's philosophy is well appreciated. The geometrical order in which Spinoza composed his masterwork, the *Ethics*, is the best-known and most outwardly striking way in which his philosophy bears the stamp of mathematical inspiration. Spinoza also frequently uses mathematical examples and analogies to illustrate key ideas and concepts of his philosophy. Notably, he compares the way in which things follow from God's infinite nature to the way in which it follows from the nature of a triangle that its three angles

⁶My article, "Geometrical Figures in Spinoza's Book of Nature" (Homan 2018b), is a forerunner of some of the interpretive ideas developed in greater detail here.

are equal to two right angles. In the early work, *Treatise on the Emendation of the Intellect*, moreover, Spinoza uses genetic ideas of mathematical objects to illustrate the formal properties of true ideas. For instance, he describes forming the concept of a sphere through the rotation of a semi-circle around a center, explaining that he knows this is a true idea regardless of whether any sphere has ever been formed in this way.

Such mathematical examples and analogies have signaled to a number of commentators that mathematics does more in Spinoza than provide a model for presenting philosophy *ordine geometrico*. Spinozan reality seems itself to be ordered geometrically. For Spinoza, says Gueroult:

Philosophy must take Geometry for its model, and will be true only if it manages to prove itself in the geometrical method. The geometrical method is, therefore, not just a borrowed garment, but Philosophy's inner spring, the necessary way in which it unfolds and advances as truth.⁷

More recently, Valtteri Viljanen argues that in Spinoza's "geometry-inspired ontology,"⁸ "each and every genuine thing is an entity of power endowed with an internal structure akin to that of geometrical objects."⁹ These interpretations emphasize what we might call the *formal* significance of mathematical examples.¹⁰ They highlight the way in which mathematics provides a model for Spinoza's philosophy, leaving open the question whether mathematics itself features in its *content*.

It is indisputable that Spinoza uses mathematical examples to illustrate formal or structural aspects of his philosophy. As I will discuss in detail in Chap. 2, the sphere conceived as the rotation of a semicircle illustrates the form that an idea must take if it is to be a true idea. Spinoza also contrasts three ways of solving a mathematical problem to illustrate formal differences between the aforementioned three kinds of knowledge. This example is of no small value for understanding the kinds of knowledge and will be treated accordingly in what follows. In this study, however, I will primarily be interested in a different question regarding mathematics' significance and place in Spinoza's thought. In particular, do mathematical

⁷ Gueroult 1974, 471, my translation.

⁸ Viljanen 2011, 21.

⁹ Viljanen 2011, 2.

¹⁰ Viljanen stresses *formal* causation in his interpretation of Spinoza's ontology. When I speak of formal in this context, I do not refer to formal causation, but the form-content distinction.

entities feature in the *content* of Spinozan knowledge and reality? This question can be asked more specifically about geometrical entities—spheres, triangles, and circles—and also about numbers. Do geometrical figures feature in the content of Spinozan knowledge and reality? Do numbers? Spinoza famously resolves to treat human behavior “as if it were a question of lines, planes, or bodies.”¹¹ What about lines, planes, and bodies themselves?¹² These questions provide a useful lens through which to interpret Spinoza’s epistemology. So, at least, I hope to show.

1.1 THE QUESTION OF MATHEMATIZATION

In the case of Descartes, Spinoza’s most important philosophical influence (and foil), the answer to the question concerning the reality of geometrical entities is most certainly, yes. Descartes stated as much explicitly: “I recognize no matter in corporeal things apart from that which the geometers call quantity, and take as the object of their demonstrations, i.e., that to which every kind of division, shape and motion is applicable.”¹³ Descartes’ comment complements the following celebrated passage in Galileo’s *Assayer*:

Philosophy is written in this all-encompassing book that is constantly open before our eyes, that is the universe; but it cannot be understood unless one first learns to understand the language and knows the characters in which it is written. It is written in mathematical language, and its characters are triangles, circles, and other geometrical figures; without these it is humanly impossible to understand a word of it, and one wanders around pointlessly in a dark labyrinth.¹⁴

¹¹ E3pref/G 2:138. Curley renders “*de lineis, planis, aut de corporibus*” as “lines, planes, and bodies” (my emphasis). I have opted for the more literal translation of “*aut*” here.

¹² One of the questions to be taken up in this study (especially in Chap. 3) is whether Spinoza conceives bodies in geometrical terms or in some non-geometrical fashion. Spinoza’s phrase, “lines, planes, or bodies,” provides *prima facie* evidence for the geometrical interpretation that I will defend, since the association with “lines” and “planes” suggests that by “bodies” he means geometrical solids.

¹³ CSM 1:247.

¹⁴ Galileo 2008, 183.

The view of nature as constituted by mathematical entities, particularly, geometrical ones, as expressed in the passages just quoted, has come to be spoken of in terms of the *mathematization* of nature.¹⁵

The mathematization of nature is often used to describe what is thought by many to be among the most distinctive changes brought about by the scientific revolution over the course of the sixteenth and seventeenth centuries. Koyré writes,

I believe that the intellectual attitude of classical science can be characterized by the following two changes, which are moreover intimately related: geometrization of space and dissolution of the Cosmos, that is to say the disappearance from within scientific reasoning of the Cosmos as a presupposition and the substitution for the concrete space of pre-Galilean physics of the abstract space of Euclidean geometry.¹⁶

The “grand narrative of mathematization”¹⁷ has been criticized for oversimplification,¹⁸ but I think it provides a useful heuristic for approaching Spinoza, nevertheless. Before I put the question of mathematization

¹⁵For a discussion of Descartes’ project of mathematization, see Gaukroger 1980. See, especially, Gaukroger 1980, 123–35, for a useful comparative analysis of the respective Cartesian and Galilean projects of mathematization. Cf. Ariew 2016. Ariew argues against associating this passage with the mathematization thesis on the grounds that Descartes’ physics is not founded on mathematics per se, but on the metaphysics of clear and distinct ideas; it is simply a coincidence, according to Ariew, that “mathematicians rely on some of the same clear and distinct ideas as natural philosophers do” (2016, 121). It is not clear to me, however, why it should matter to the validity of the mathematization thesis whether the overlap between physics and mathematics is coincidental or not. Even if it is due ultimately to a shared metaphysics of clarity and distinctness, the principles of physics end up being mathematical either way.

¹⁶Koyré 1978, 2–3.

¹⁷Gorham et al. 2016, 5.

¹⁸The charge of oversimplification is the guiding thesis of the recent volume of essays, *The Language of Nature: Reassessing the Mathematization of Natural Philosophy in the Seventeenth Century* (2016), edited by Gorham et al. The authors of the volume’s introduction point out how the notion of mathematization glosses over important differences between types of mathematization. Intuitive geometrical models contrasted with less intuitive algebraic methods, for instance, and seventeenth-century figures debated the respective merits of both. While the idea of mathematization had a great deal of power in the seventeenth-century imagination, this was not always matched with the success of mathematization efforts in practice. A number of fields resisted mathematization while even in physics, many philosophers, such as Descartes, failed to articulate basic laws of nature in mathematical terms. Many prominent early moderns, moreover, such as Gassendi and Locke, who did much to advance

directly to our protagonist, let me say something about my understanding of the mathematization thesis itself. First, I understand it more specifically as a thesis about *geometrization*. The focus on geometry is evident in the passages from Descartes and Galileo (and Koyré) quoted above. I will have something to say about number later on, but my primary focus will be on figures.

Second, I see the mathematization thesis as consisting in the claim that all finite bodies in nature are geometrical inasmuch as they have some figure—whether circular, triangular, or what have you—just by virtue of being spatially extended. To accept this claim is to be a realist about mathematization. Very generally, mathematical realism affirms the mind-independent existence of mathematical entities. Within this general categorization, we can distinguish two strains. A stronger strain, typically associated with Plato (in particular, his doctrine of forms), holds that mathematical entities exist in and through themselves, independently of bodies.¹⁹ To express this, I will sometimes attach a “per se” qualifier to talk of numbers or figures. (Thus, a sphere per se is a sphere conceived as existing in and through itself, independently of being physically instantiated by a body.) A weaker strain holds that mathematical entities exist independently of minds, but only insofar as they exist as the properties of bodies, not independently of the latter.²⁰ For the purposes of this study, I will assume that the stronger strain entails the weaker one, but not vice versa. Whether or not Descartes, for instance, believes that geometrical figures

“modern” thought, showed relatively little interest in mathematics. For further discussion of the oversimplification charge, see Gorham, Hill, and Slowik 2016, 1–28.

¹⁹For discussion of Platonism in contemporary philosophy of mathematics, see Balaguer 2009.

²⁰Some philosophers of mathematics associate this form of non-Platonist realism with Aristotle. (See Franklin 2009.) Since Aristotle’s philosophy of mathematics is a matter of scholarly dispute and since Aristotelianism is freighted with myriad connotations in the context of discussing early modern philosophy, I avoid this terminology here. I will touch upon Aristotle’s philosophy of mathematics as background for considering Descartes’ and Spinoza’s in Chap. 3. “Psychologism” is considered by some philosophers of mathematics to be another form of non-Platonist realism. (See Balaguer 2009, 38.) Since psychologism is the view that mathematical entities exist as mental entities, this realist categorization is potentially misleading, since in this study the view that figures exist only as mental entities is categorized as a form of *antirealism*. My categorization hews more closely to the terminological landscape in the philosophical discussion of the problem of universals (which overlaps with, but is distinct from, the discussion of the ontology of mathematical entities in philosophy of mathematics).

exist independently of bodies, he believes at minimum that bodies must have one kind of shape or another. Since my main question is whether natural bodies have mathematical properties, and, as a corollary, whether mathematics contributes to the knowledge of natural bodies, when I speak about mathematical realism (without qualification), I mean to encompass the weak no less than the strong strain. In the case of both Descartes and Galileo, scholars have debated the extent to which they might be interpreted as Platonists (in the sense outlined above).²¹ I will not have anything to say on this question here.²² It suffices for my purposes that Descartes and Galileo are both at least *weak* mathematical realists.

The umbrella of weak mathematical realism is broad enough to encompass even anti-rationalist philosophers, such as Gassendi and Hobbes.²³ To be sure, there are differences between Gassendi and Hobbes, on the one hand, and Descartes, on the other, in regard to philosophy of mathematics, as witnessed by Gassendi's and Hobbes' respective objections to Descartes' *Meditations*.²⁴ Both philosophers object, on similar grounds, to Descartes' claim in Meditation Five to know the true and immutable nature of a triangle regardless of whether any triangle exists mind-independently in nature or has ever so existed. What they take issue with is the notion that triangles have natures independent of physical instantiation. Hobbes argues, "A triangle in the mind arises from a triangle we have seen, or else it is constructed out of things we have seen."²⁵ Gassendi similarly writes, "It is the intellect alone which, after seeing material triangles, has formed this nature and made it a common nature."²⁶ As these quotations show, both Gassendi and Hobbes hold that the notion of a triangle comes from encounters with material triangles (or similar things from which the notion is constructed) and thus cannot exist, as Descartes claims, regardless of whether any triangle exists mind-independently in nature.

²¹ For discussion (and criticism) of Platonist readings of Galileo, see Palmerino 2016. For discussion of Platonist readings of Descartes, see Nolan 1997.

²² I touch on the question of Descartes' Platonism in Chap. 3.

²³ In dubbing Hobbes an anti-rationalist, I mean to highlight primarily his hostility to innate ideas, as exhibited in his objections to Descartes' *Meditations*. (The same goes for Gassendi, too.)

²⁴ This is not to say, of course, that there are no differences between Gassendi and Hobbes, too. See n. 27.

²⁵ CSM 2:135.

²⁶ CSM 2: 223.

It would be a mistake to assume, however, that Gassendi's and Hobbes' rejection of Descartes' true and immutable natures doctrine entails mathematical antirealism *tout court*. Although Gassendi and Hobbes are known for their respective commitments to "nominalism" about universals (like "triangle"),²⁷ this does not entail a rejection of the weak mathematical realism outlined above, but only of the stronger (Platonist) strain. Whether or not there are such things as mathematical universals (existing independently of minds and bodies), there may well be *particular* mathematical entities (or at least material things with particular mathematical properties), as shown by Hobbes' and Gassendi's respective talk of "a triangle we have seen" and "material triangles" in the above quotations.

Another potential mistake that we must guard against is thinking that limits on our *knowledge* of the mathematical properties of physical things say anything in and of itself about the *existence* of such mathematical properties. If there is one thing on which all the "modern" philosophers agree, it is surely that sensation cannot be innocently taken as a reliable guide to the true nature of the physical world. Since it is widely agreed that sensation must be relied upon to gain knowledge of physical nature, at least with respect to its particular details, it is also widely agreed that knowledge of the particular details of physical nature poses a serious epistemic challenge. This is true for Descartes and Galileo no less than for Gassendi and Hobbes (and, as we will see, Spinoza). Since Gassendi and Hobbes believe that sensation is the *only* access we have to physical reality, they tend to be more pessimistic about our prospects for mathematical knowledge of nature than Descartes, who believes that some things about the physical world are knowable a priori. Once again, however, this difference does not

²⁷A difference between Gassendi and Hobbes is that whereas Gassendi appears to recognize the existence of general *concepts*, Hobbes generally appears not to do so. Gassendi's talk of a common nature formed in the intellect in the passage quoted in the previous paragraph exhibits this recognition. The view that universals exist only as concepts in the mind is often called "conceptualism." In this case, "nominalism" would represent the stronger view that universals do not even exist as concepts. This terminological division can be seen in, for instance, Di Bella and Schmaltz 2017, 4–7. According to this terminology, then, Gassendi, along with many other early modern philosophers (including, arguably, Descartes), is a conceptualist while Hobbes is a nominalist. Usage of these terms is quite inconsistent, however. LoLordo (2017) depicts Gassendi as recognizing universal concepts, but characterizes him as a nominalist. Leibniz (1989, 128), notably, characterizes the mainstream early modern view as "nominalist," reserving the term "super-nominalist" for Hobbes. I am following Leibniz and LoLordo in using "nominalist" here in the broad sense that encompasses "conceptualism."

affect the basic question concerning the mind-independent *existence* of mathematical entities (or properties).

Let me add a further related clarification that is part terminological and part substantive. It is not uncommon for scholars to use the labels “constructivist” and “instrumentalist” in contrast with “realist” when discussing views of mathematization and mathematical entities.²⁸ In my view, a natural way to interpret the notions of mathematical constructivism and instrumentalism is entirely neutral with regard to the question of the mind-independent existence of mathematical entities. Instrumentalism suggests that mathematical calculations can be used to make predictions, while constructivism suggests that mathematical conceptions can be artificially crafted, like linguistic conventions, for human use and convenience. Hobbes, for instance, describes constructing the conception of a circle through a rotating line.²⁹ (Likely influenced by Hobbes, Spinoza uses the same example, as I will discuss later on.) For Hobbes, since this conception provides a cause of the circle, it allows for the deduction (or prediction) of effects. It should be clear, however, that the fact that this conception is artificially constructed and can be used (instrumentally) for deductive and predictive purposes says nothing about whether circles exist in nature that correspond to the conception or not. It is true that both “constructivism” and “instrumentalism” may carry antirealist connotations, but this is because they are often associated with independent reasons for rejecting the mind-independent existence of mathematical entities. (When this is the case, though, it is important to remember, the rejection is, at least in the seventeenth century, more clearly of the strong, Platonist, mathematical realism discussed above, not necessarily the weaker kind.) In the case of neither notion, however, is this association necessary. Hence, I find it potentially confusing to use these notions in contrast with realism and will instead deploy the starker “antirealism” for this purpose.

²⁸ Sepkoski’s monograph *Nominalism and Constructivism in Seventeenth-Century Mathematical Philosophy* (2007) is notable for its association of the term “constructivism” with the nominalism (and antirealism) of such figures as Gassendi, Hobbes, and Berkeley. Sepkoski defines constructivism in explicitly antirealist terms as “the belief that mathematical objects are not mind-independent entities or abstractions from physical reality, but rather are artificial ‘constructions’ produced by the mind that serve as tools in mathematical demonstration” (2007, 129). For usage of the term “constructivism” in relation to Spinoza, see Hübner 2016, 59. Gorham et al., by contrast, deploy the term “instrumentalism” in contrast with “realism” in discussing attitudes toward mathematization (2016, 3).

²⁹ See Hobbes 2005, 6.

While I will at times have occasion to speak of constructivism or mathematical construction, I will take these terms to be neutral as to the question of the mind-independent existence of mathematical entities, for the reasons just given.

From the foregoing sketch, I think it is safe to say that while seventeenth-century thinking about mathematization was far from monolithic, there were also some common points of agreement, even among otherwise quite disparate figures. This agreement centered around the idea that material things are extended and that extended matter has the properties of shape, size, and mobility. Even among figures generally pessimistic about the prospects for successfully applying mathematics to the study of natural phenomena, there is broad, implicit consensus that a material world exists outside the mind with the properties mentioned. Thus, the mathematization of nature, in at least a weak sense, can be considered something of a received (if not universally agreed upon) opinion among the generation of moderns that directly preceded Spinoza.³⁰

Against this backdrop, let us turn to Spinoza. Should he be seen as a mathematical realist as well? An affirmative answer to this question was long taken more or less for granted, as is perhaps unsurprising, given what has just been said. This tendency is succinctly represented by Jonathan Bennett's remark, "Being a child of his time, Spinoza [...] assumed space to be Euclidean and infinite in all directions."³¹ However, some recent scholars have cast doubt on Spinoza's acceptance of the mathematization thesis, arguing that numbers and figures are, for him, nothing more than mental abstractions or "beings of reason." According to Eric Schliesser, for instance, "Spinoza sided with those who criticized the aspirations of the physico-mathematicians such as Galileo, Huygens, Wallis, and Wren who thought the application of mathematics to nature was the way to

³⁰ Francis Bacon is perhaps an exception here, though even in his case, recent scholars have found him friendlier to mathematics and quantification than traditionally thought. See Jalobeanu 2016.

³¹ Bennett 1984, 21. See also Curley 1988, 33; Allison 1987, 25; Lecrivain 1986, 15–24; and Lachterman 1978, 75–80. In defending an alignment of Spinoza with the modern mechanistic philosophy of Descartes and Hobbes, Lachterman (1978, 76–7) takes himself to be departing from the previous, romantic, and idealist interpretations of Spinoza, which downplayed or ignored the scientific dimensions of his thought. In this light, I certainly do not suggest that the interpretation of Spinoza as a realist about mathematics was *always* standard.

make progress.”³² Reflecting this scholarly trend, the editors of a recent volume on mathematization in the seventeenth century allege that Spinoza has a “metaphysical program that is quite unfriendly to mathematization.”³³ If numbers and figures are beings of reason, then Spinoza’s book of nature is not written in mathematical language like Galileo’s, Spinoza’s material universe is not geometrical like Descartes’, and mathematics can be no help in understanding nature. If this is right, it means that Spinoza broke even more dramatically from Descartes than usually thought.

Is it right? An answer to this question will be developed over the course of the book, but some stage-setting remarks are in order. One thing is clear (and will become clearer below): Spinoza’s comments on mathematical entities are deeply ambivalent. As a result, the question of mathematization in Spinoza poses a genuine interpretive conundrum, and it is to the credit of recent antirealist interpretations of Spinoza that they have forced argument on the issue. As I suggested above, since the ambivalence about mathematical entities creates ambiguities that vex the interpretation of Spinoza’s epistemology, clarifying the status of mathematical entities holds out hope of interpretive remedy. To interpret Spinoza’s persistent treatment of mathematical entities as beings of reason to mean that Spinozan nature is not mathematical (per the antirealist interpretation) is certainly one way of clarifying the status of mathematical entities in Spinoza. However, the antirealist interpretation generates a dilemma. If Spinoza rejects mathematical entities (and properties) as part of his physical ontology, then, given that he affirms the existence of finite bodies, what, for him, are such things like?³⁴ They cannot be spheres or spherical, trapezoids

³²Schliesser 2014, 2. Other recent interpreters who have raised doubts about ascribing the mathematization thesis to Spinoza include Melamed 2000; Pterman 2015; and Manning 2016. A less recent detractor is Deleuze 1990, 21–2, 278.

³³Gorham et al. 2016, 6. It should be noted, however, that the paper on Spinoza included in the volume (Goldenbaum 2016) takes the standard view of Spinoza as a realist about mathematization for granted. See Goldenbaum 2016, 277.

³⁴I assume that Spinoza *has* a physical ontology, and, thus, that Spinoza is not an idealist. I recognize that some commentators have read Spinoza as an idealist, and thus my assumption that he is not might be deemed question-begging. To this charge I would say the following. First, although I accept that there are viable grounds for an idealist reading of Spinoza (notably, Spinoza’s definition of attribute in E1d4), it is nevertheless the case that the vast majority of textual evidence tends in the opposite direction. I have in mind Spinoza’s affirmation of a seemingly self-sufficient attribute of extension and his ubiquitous talk of extended bodies and their motions. Second, although I will not engage directly with the arguments for the idealist reading (i.e., I will not discuss the controversy surrounding E1d4

or trapezoidal. Either he has (or holds out for) a positive, non-mathematized conception of finite bodies, or he disclaims the possibility of knowing anything about them (beyond the most general metaphysical knowledge, such as, for instance, that they are modes of extension, defined by capacities for motion and rest).

Neither horn of this interpretive dilemma is without difficulty. The second, skeptical reading comports with Spinoza's skepticism about sensory cognition, upon which knowledge of particular finite bodies would have to rely. Such skepticism was a point of widespread consensus among seventeenth-century moderns, as noted above, but it did not lead everyone to deny the possibility of knowledge of particulars. Recognition of the inaccuracy of naïve experience instead prompted attention to scientific methodology. While it is possible that Spinoza thought our knowledge of concrete particulars was restricted to the most general metaphysical claims, his own attention to scientific methodology indicates otherwise. If Spinoza did think knowledge of particulars could provide at least the target for a scientific program, we are pushed to the other interpretive alternative: that Spinoza envisioned a non-mathematized scientific knowledge of bodies. The problem with this is that it is far from clear what such a non-mathematized conception of finite bodies might look like, for Spinoza, especially in light of the already intimated predominance of mathematization in the early modern imagination. Both antirealist interpretive options tend toward mystification of the Spinozan natural world. Recall Galileo's remark that without geometry "one wanders around pointlessly in a dark labyrinth."

Perhaps Spinozan nature is a dark labyrinth. Before we acquiesce to this conclusion, however, it is worth exploring the possibilities for a mathematical realist interpretation. In my opinion, a realist interpretation of geometrical figures in Spinoza is much more plausible than has been appreciated in the recent literature. Spinoza's treatment of mathematical

at any length), I will present arguments on behalf of geometrical figures as the determinations of finite bodies in Chap. 3. Inasmuch as these arguments help make the case for the realist reading of physical nature in Spinoza, my interpretation does not beg the question. Admittedly, Spinoza's affirmation of finite bodies is not without well-known problems, even if mind-independent physical reality is assumed. I touch on some of these issues in Chap. 3. For an overview of idealist readings of Spinoza, see Newlands 2011. For a recent defense of a realist reading of the attributes, see Melamed 2018, 90–5. See also, my paper, Homan 2016, in which I argue for the parity of thought and extension qua attributes, thereby countering a major motivation for the idealist reading.

entities as beings of reason must be dealt with, but I think this can be done within a realist interpretive framework. In short, I will argue that even if geometrical figures per se are beings of reason, they exist mind-independently nevertheless as the determinations of finite bodies. In the taxonomy sketched above, Spinoza is a weak mathematical realist (at least with regard to geometrical figures).

Like its antirealist counterpart, the realist interpretation serves to clarify an important question concerning the scope and content of Spinozan knowledge of the physical world. But the realist interpretation has a significant advantage: in delivering a positive verdict for geometrical figures, it licenses the development of mathematical examples for illustrating Spinozan knowledge claims. This is a highly valuable result, especially in the case of interpreting Spinoza's second and third kinds of knowledge, where the objects of these modes of knowing, and the content of Spinoza's definitions of them, are far from clear. The result also helps to illuminate related interpretive matters regarding Spinoza's philosophy of science, including his conception of scientific method.

1.2 OUTLINE OF CHAPTERS

This, then, is the overarching strategy of the book. I will develop the case for attributing a realist view of mathematization to Spinoza. Then, on this basis, I will use Spinoza's mathematical examples to help illuminate the content of Spinozan knowledge claims. Doing so will help to answer a number of interpretive conundrums in Spinoza's epistemology. I do not pretend to provide a comprehensive exploration of every nook and cranny of Spinoza's epistemology.³⁵ Nor will I be steered by contemporary

³⁵ It is perhaps futile to attempt to list the epistemological issues I will *not* take up, since there are indefinitely many that could be identified, but I want to mention three notable omissions. (1) One interesting question outside the scope of this study pertains to Spinoza's theory of error as privation: what happens to an imaginative, inadequate conception of X when we come to achieve an intellectual, adequate understanding of X? Is the former radically transformed (perhaps *eliminated*) or do we go on experiencing the world as we did prior to gaining adequate understanding, albeit with the *addition* of adequate ideas? For a thought-provoking discussion of this question, see Cook 1998. (2) Another issue is the question of whether Spinoza's theory of epistemic justification is foundationalist or coherentist. Since God is the epistemic foundation in Spinoza's system, and since God is, in a sense, everything, it would not be wrong to say that to know anything one must know everything. Nevertheless, in my view, it is God as foundation that is doing the epistemic work, not God as everything. While I do not argue for this point explicitly, what I say in Chap. 2 should help

epistemological concerns. Instead, I intend to follow a thread from Spinoza's early engagement with skepticism in the TIE to the culmination of his epistemology in *Ethics* Part 5 and the link between intuitive knowledge and the highest human blessedness. The thread will be guided by the question of mathematization and issue in an interpretation of the major elements of Spinoza's epistemology, especially the hierarchical trio of kinds of knowledge.

I begin, in Chap. 2, with Spinoza's response to skepticism. I argue that a due consideration of the nature of true mathematical ideas and the use to which Spinoza puts them against skeptical disputation suggest that his philosophical methodology is more Cartesian than has often been appreciated. The discussion of this chapter allows me to introduce a number of the major concepts and themes that scaffold the discussion of ensuing chapters, in particular, the distinction between intrinsic and extrinsic features of ideas, the key epistemic notions of adequacy and truth, the foundational role of God in Spinoza's system, and the question of the reality of mathematical objects.

In Chap. 3, I address the question of the ontological status of mathematical entities, particularly geometrical figures (though I also touch on numbers). I discuss the status of mathematical entities as beings of reason and mount a case against mathematical antirealism. Despite the fact that geometrical figures per se are beings of reason, I argue for a realist interpretation of geometrical figures as the determinations of finite bodies. Advancing this argument requires me to examine Spinoza's discussions of

to motivate, and partially justify, my view, if only indirectly. For discussion of this issue and defense of a coherentist reading, see Steinberg 1998. (3) Finally, I do not explicitly take up the question of Spinoza's commitment to the principle of sufficient reason. The PSR is emphasized in Michael Della Rocca's highly influential interpretation of Spinoza (especially in Della Rocca, 2008) and, as a result, has recently been much discussed by Spinoza scholars. (For critical discussions of Della Rocca's PSR-focused reading of Spinoza, see Laerke 2011, Garber 2015, and Lin 2019, 164–81.) There is no doubt that the PSR is relevant to Spinoza's epistemology. As I will emphasize and discuss in more detail below, to know X, for Spinoza, is to know the cause of X. Inasmuch as this suggests a commitment to the PSR, the PSR looms large over any study of Spinoza's epistemology. For Della Rocca, however, the PSR is an Ur-principle that governs all aspects of Spinoza's system, thus transcending epistemological matters (at least as narrowly conceived). Indeed, perhaps somewhat ironically, one of the few areas of Spinoza's philosophy that Della Rocca has relatively little to say about are the three kinds of knowledge themselves (especially the second and third kinds). To take up the PSR as understood by Della Rocca in any systematic manner, then, calls for a very different kind of study than what is proposed here.

physical individuals in the *Ethics* and elsewhere. In addition to marshaling textual evidence for my interpretation, I address some questions regarding the property ontology of geometrical figures.

Chapters 4 and 5 are focused on Spinoza's scientific methodology and philosophy of science. They follow up an implication of the findings of Chap. 3: if figures feature among the determinations of finite bodies, then geometry, as the science of figure, should have a role to play in the scientific investigation of finite bodies. In Chap. 4, I provide an interpretation of Spinoza's scientific method and discuss the interaction of reason and imagination in Spinozan science. I address a number of interpretive issues pertaining to reason especially, including the nature, origin, and adequacy of common notions. I argue for a hypothetico-deductive interpretation of Spinoza's scientific method, stressing the role of hypotheses in bridging the epistemic gap between nature's most general laws and singular things.

Chapter 5 is devoted, in part, to developing an example of Spinozan science in practice and exhibiting the role of geometry therein. In this regard, I offer a reading of Spinoza's epistolary writings on optics and his treatment of a question of optimal lens shape. I also address a further objection to my realist interpretation of geometrical figures stemming from Letter 12, as well as the difficulty raised by the incompleteness of Spinoza's thinking about physics for any interpretation of Spinozan science.

The topic of Chap. 6 is Spinoza's notion (or, more accurately, notions) of essence. This is the most metaphysical discussion of the book. It provides the necessary background for approaching Spinoza's conception of intuitive knowledge, which he characterizes in terms of knowledge of the essences of things. I argue for a spectrum interpretation of essences in Spinoza, distinguishing between common essences at the level of attribute and infinite mode at one extreme, individual essences at the level of finite individuals at the other extreme, and species essences in the middle (which, I argue, exist only as beings of reason). I also address the sense in which the essences of finite things exist non-durationally and are themselves finite (as opposed to infinite modes).

Chapter 7 attempts finally to make sense of Spinoza's obscure conception of intuitive knowledge. I stake out a number of interpretive claims on this issue. First, I argue that adequate knowledge of the singular essences of things is impossible for finite intellects, which means that intuitive knowledge can only aspire to adequate knowledge of common essences or species essences of things. Second, with the help of a geometrical example

that is modeled on, but more suggestive than, Spinoza's fourth proportional example, I argue for a "method interpretation" of the distinction between reason and intuitive knowledge. According to a method interpretation, in general, the second and third kinds of knowledge do not differ in terms of their respective knowledge contents, but only in their respective methods of arriving at the same knowledge content. According to my particular version of the method interpretation, intuitive knowledge is best understood as the perfection of reason. This means that the extent to which intuitive knowledge and reason are seen as different kinds of knowledge or merely different grades of a single kind is a question more of emphasis than substance. The burden of any method interpretation is to explain why Spinoza puts such an emphasis on the superiority of intuitive knowledge, linking it alone to the intellectual love of God and the highest human blessedness. I argue that my method interpretation offers an especially cogent explanation of this superiority in terms of affective differences between the kinds of knowledge. At the end of the chapter, I discuss the role that intuitive knowledge might play in Spinozan science.

In the final, concluding chapter, I reflect upon the portrait of Spinoza's epistemology that emerges over the course of the book, defend its sanguine, Cartesian cast, and highlight, in closing, an important epistemological contrast between Spinoza and Descartes.

1.3 A NOTE ON TEXTS

Before getting underway, let me add a remark about my use of Spinoza's texts in developing my interpretation. Since this is not a study of any particular text, but of Spinoza's epistemological thinking in general, I intend to make full use of the writings comprising Spinoza's corpus insofar as they are relevant to the matters in question. While this is generally standard practice and unproblematic, the authority of a few texts is sometimes the subject of scholarly doubts. This is true, in particular, in the case of the TIE and Spinoza's geometric exposition of *Descartes' Principles of Philosophy*, both of which, especially the former, will feature prominently in my interpretation. Doubts about the authority of the TIE attach to the fact that it was an early work (perhaps Spinoza's first) that Spinoza never completed. Whether or not Spinoza ever seriously intended to go back and finish the TIE after having initially set it aside, however, it seems to

have continued to satisfy him “in the main,” as Curley acknowledges,³⁶ throughout his life, as evidenced by occasional, scattered references in later writings. Of course, there are points of philosophical substance on which Spinoza’s thinking evolved or changed from the TIE to later works. In such cases (one or two of which I will discuss below), I will generally defer to the authority of later works, especially the *Ethics*. Insofar as the TIE is not contradicted by later doctrine, however, I will feel free to conscript it into my interpretation. The extent to which this proves illuminating will, I hope, amply justify the practice.

Being an exposition of Descartes’ philosophy, not his own, Spinoza’s DPP raises a different set of issues. In using it, I will look not just for lack of contradiction, but explicit confirmation, by explicit doctrine elsewhere in Spinoza’s oeuvre. When such confirmation is available, I will use the DPP as a welcome supplement to other texts. Although the *Cogitata Metaphysica* (Spinoza’s appendix to the DPP) seems to represent Spinoza’s own views to a greater extent than the main text of the DPP, I will adopt a similar, cautious approach to this text as well.

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³⁶ Curley 1: 5.



Mathematics and Methodology: Spinoza *Contra* Skepticism

Mathematical ideas have a vital role to play at the logical beginning of Spinoza's philosophy, in establishing the viability of his rationalist system against skepticism (by which I mean radical, all-knowledge-denying skepticism of the Pyrrhonian sort). Simple mathematical ideas display the formal features of true ideas, while failing to be true in the full sense. They show how to ground the system, while not themselves sufficing to do so. This ambivalent epistemic status of mathematical ideas will form a major theme of this study. Its significance for explaining Spinoza's response to skepticism has generally been overlooked. The latter is widely thought to run roughly as follows: "True ideas signal their own truth. So, in order to remove any skeptical doubts about our capacity for knowledge, all we need is a true idea; we do not need any guarantee of truth beyond the apprehension of the idea itself. But we do have a true idea, etc." This contra-skeptical response is dismissive and un-Cartesian. Dismissive, because, in renouncing any need for an external sign of truth, Spinoza rejects the very premise of the skeptical challenge—that it is possible for an idea to appear true, even on the closest inspection, but in fact be false. Un-Cartesian, since Descartes concedes just the possibility that Spinoza rejects. I take issue with this standard reading.¹ As I will argue, due

¹Examples of the dismissive, un-Cartesian interpretation of Spinoza's response to skepticism include Bolton 1985; D. Garrett 1990; Mason 1993; Wilson 1996; Della Rocca 1994, 2007, 2008; Perler 2017. An interpretation that paints a more Cartesian portrait of Spinoza's

consideration of mathematical ideas bedevils the dismissive, un-Cartesian interpretation of Spinoza's response to skepticism, which, for reasons to be explained below, I call the "dogmatic response." In my view, Spinoza is more indulgent of the skeptic's concerns (even if unsympathetic) and, indeed, more Cartesian in his response to them than generally appreciated.

Admittedly, the dogmatic response plays an important role in Spinoza's response to skepticism, and there is truth to interpretations that stress it. (For brevity's sake, I will sometimes refer to such interpretations as "dogmatic interpretations" without meaning that the interpretations themselves are dogmatic. My own interpretation incorporates elements of the dogmatic interpretation.) However, the dogmatic response cannot be all there is to Spinoza's response to skepticism, since there are ideas—most conspicuously, genetic, or causally constructed mathematical ideas—that have the intrinsic features of true ideas, but are not true in the sense of corresponding with a real object in nature. Spinoza recognizes this, describing such ideas as abstractions that exhibit only "the form of truth" (TIE 105/G 2:38). Not only do we have such ideas, but they are the most plausible candidates for the "given true idea" with which Spinoza claims the method, and, indeed, philosophy as a whole, must begin (TIE 38/G 2:16). Thus, Spinoza must show that his system does not traffic in abstractions, but is grounded in an idea that is true in the fullest sense. He does this by way of the idea of God. Given its overt parallels with Descartes' well-known appeal to God to counter skepticism, I call it the "Cartesian response." Ultimately, the dogmatic and Cartesian responses are interdependent, functioning as elements of a single contra-skeptical stratagem. Let us call it the "Cartesio-dogmatic response."

Exploring the ambivalent epistemic status of mathematical ideas in the context of Spinoza's response to skepticism provides an opportunity to introduce many of the concepts that will feature prominently in subsequent chapters, especially Spinoza's notion of adequate ideas, and to construct the theoretical framework within which the rest of the discussion will unfold. Fixing the role of God in grounding the system of knowledge, moreover, allows me to sketch the elementary metaphysical framework of Spinoza's philosophy. Finally, the chapter provides a first glimpse at the

response to skepticism is Doney 1975. See also Schneider 2016. Schneider's focus is not on Spinoza's response to skepticism, but his argument for a Cartesian-friendly interpretation of Spinoza's epistemological foundations is complementary, in part, with the Cartesian-friendly interpretation of Spinoza's response to skepticism that I defend in this chapter.

interpretive dividends to be gained through critical scrutiny of Spinoza's use of mathematical examples. I will begin by laying out the basis for the dogmatic interpretation of Spinoza's response to skepticism. It offers a useful backdrop against which to consider the methodological significance of the mathematical ideas that Spinoza puts forward as epistemic exemplars.

2.1 THE DOGMATIC RESPONSE

Spinoza's most sustained engagement with skepticism occurs in the *Treatise on the Emendation of the Intellect*, his early and unfinished methodological work. (As we will see, his discussion of skepticism is intertwined with discussion of philosophical method.) In TIE 30, Spinoza sets out to find "the Way and Method" for attaining knowledge, and immediately anticipates a skeptical objection:

To do this, the first thing we must consider is that there is no infinite regress here. That is, to find the best Method of seeking the truth, there is no need of another Method to seek the Method of seeking the truth, or of a third Method to seek the second, and so on, to infinity. For in that way we would never arrive at knowledge of the truth, or indeed at any knowledge. (TIE 30/G 2:13)

The skeptical objection that he anticipates here is a form of the classic problem of the criterion.² The problem is that you have to have criteria for distinguishing the true from the false, if you make claims to truth. But how do you know that the criteria are legitimate? You would seem to need second-order criteria to guarantee the first-order criteria, and third-order criteria to guarantee the second-order criteria, and so on, leading to an infinite regress. Here Spinoza frames the problem in terms of a regress of methods. In response, he goes on to give his well-known tools analogy. A hammer is needed to forge iron. One might think that in order to make a hammer, you first need a hammer with which to make it, and another hammer to have made that hammer, and so on. Reasoning in this way, one might conclude that human beings have no power to forge iron (TIE 31/G 2:13–14). The answer to this sophistical line of reasoning, Spinoza

²For a discussion of the issue as it emerged in the Hellenistic period, see Striker 1990. For discussions of the criterion problem in more recent epistemology, see Chisholm 1973 and Williams 1999. See Schneider 2016 for critical discussion of Chisholm's reading of Spinoza. For a defense of Chisholm's reading, see Delahunty 1985, 15–24.

explains, is to recognize that humans did not have to begin with hammers. They could begin with simpler tools with which they were born, such as hands. If we are simply born with hands, which are a kind of tool, we do not get ensnared in a regress of having needed tools with which to build tools and so on. By analogy, we have certain inborn intellectual tools that we can use to build up a system of knowledge without having needed other intellectual tools with which to fashion the initial intellectual tools. We do not need a method for arriving at a true idea, a method for finding that method, and so on, if “we have a true idea” (TIE 33/G 2:14) at the outset. So, the solution to the regress of methods objection is to recognize our possession of innate resources.

At this point, the skeptic rejoins: but how do you know that your alleged inborn true idea is in fact true? There must be a sign that the idea is true, and in order to be certain that the sign is the right sign, there is a need for another sign, and so on. We are back to the problem of the criterion with its attendant regress. So, how does Spinoza answer this more dogged version? How is he certain that an idea that he thinks is true in fact *is* true? Spinoza’s initial answer appeals to the relationship between an idea and its object. He says, “A true idea (for we have a true idea) is something different from its object. For a circle is one thing and an idea of the circle another – the idea of the circle is not something which has a circumference and a center, as the circle does” (TIE 33/G 2:14). He also gives another example: “Peter, for example, is something real; but a true idea of Peter is an objective essence of Peter, and something real in itself, and altogether different from Peter himself” (TIE 34/G 2:14). To have an idea of Peter, to know Peter, then, Spinoza goes on to explain, is something different from having an idea of an idea of Peter, from knowing an idea of Peter. Finally, he reasons: “From this it is clear that certainty is nothing but the objective essence itself, i.e., the mode by which we are aware of the formal essence [i.e., the essence of the object of the idea] is certainty itself. And from this, again, it is clear that, for the certainty of truth, no other sign is needed than having a true idea” (TIE 35/G 2:15).

So, no sign is needed to be certain that one has a true idea beyond the true idea itself. This, ultimately, is his answer to the regress objection. But how is Spinoza’s reasoning supposed to work here? How does he get from his initial distinction between idea and object (illustrated by the examples of the circle and Peter) to the claim that certainty “requires no sign,” as he puts it in the next paragraph (TIE 36/G 2:15)? I take it that Spinoza’s assertion that a true idea is something different from its object means that

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