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Step into the World of Mathematics

Math Is Beautiful and Belongs to All of Us

Translated by Lauri Snellman

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My Adventures in the World of Mathematics

I'll Reveal It Straight Away I'm thoroughly absorbed by mathematics. I feel that I'm a lamb leg that has been marinading for decades. The garlic of the marinade has been replaced with logarithms, and the olive oil with fractions.

In this chapter I'll tell you about my mathematical background, so that you'll get a better idea of the world I'm inviting you into.

1.1 Primary School

As a small boy, I was a natural science nerd living in Töölö. I did electric experiments on the differences between direct and alternating currents and on voltages. A little electric motor from a slide projector went faster on a 9 volt battery than on a 4.5 volt one. Adding a small adjustable resistor or a potentiometer made possible a smooth adjustment of the speed. Great! Replacing the battery with a 230 volt mains current didn't give an even faster speed, but burnt the fuse and broke the engine.

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On the other hand, the engine of the slide projector didn't work at all on batteries. It required a mains current. Did adding a potentiometer to this circuit slow down the rotation? In a way yes, because the fan stopped, burning the fuse and Mum's nerves with it.

I got a chemistry set for Christmas. After I've gone through the tricks in the manual, it was time to improvise. Adding baking soda to vinegar led to nice bubbles, which I tried to stop by putting a bottle stopper on the test tube. I cleaned up broken glass from my room after the explosion, and Mum wasn't happy on this time either.

My other experiments included putting together a rocket boat out of fuel torn from sparkler sticks and dropping rotating "paper landers" from a seventh-floor balcony. The result was a half a metre flame spouting from toy a boat standing still and parachute strings that were tangled by rotation.

Kids and youngsters: be careful with electricity and fire! It was just luck that I didn't hurt myself with my experiments.

My physics and mathematics experiments taught me a lot of things that don't work. My tests in the field of mathematics gave me many experiences of success, and maybe therefore math became my calling.

As a primary school pupil, I found somewhere a method for calculating square roots with a pen and paper. To my great delight, I determined the square root of 2 more accurately than Dad's calculator (that is, with 13 decimals 14,142,135,623,731). Dad worked at Taivallahti primary school, which was my school. He brought old math textbooks home from there. I solved all their exercises out of the sheer joy of calculating. I felt great happiness when I was able to study power and logarithmic functions years before they were taught in class.

1.2 High School

I developed warm feelings towards sines, cosines, tangents, secants and a certain Maija in junior high. It's a pity that I rarely met Maija. I could always meet the trigonometric functions though. They are still important in my life, but not quite as important as my dear wife Airi.

I still feel that the way sines have been treated has been unfair. In a unit circle, the cosine gives the x coordinate and the sine gives the y-coordinate. The sine should come before the *co*-sine, and x is before y in the alphabet. It's the wrong way around!

My interest of physics also grew, when the science magazine Tiede 2000 came out in the 1980s. I'm grateful to the pioneers of particle physics in Finland, who shared and popularized their knowledge in the magazine while doing their research. I was especially fascinated by the fact that there are only four fundamental forces in the universe.

Gravity was the most familiar to me out of all the four. I had often studied it, like next to a gravel ditch in Hyvinkää when I was 9. My plan for the experiment was based on this argument: since ski jumpers can come down smoothly to a downward slope, a small boy gliding like Superman from the top of the hill will make a soft landing to the sand. My experimental study showed this assumption false. Falling to the ground chest first knocked the wind out of me even though I was going downhill. Gasping for breath gave me a concrete idea of the gravitational force that pulls masses together.

I also got a tangible understanding of the *electric force* with the circuits of the Philips EE series and through the motor experiments I've told you about above.

I was annoyed that my junior high teacher wasn't able to fully explain the two other forces, the *weak* and *strong* nuclear force. Dear Raija Winterhalter: it wasn't your fault. Explaining the nuclear forces to a teenager is difficult even to a professional physicist. You were a great teacher!

My enthusiasm for physics led me to contact Helsinki University when I had a short practical work training at school in 1985. I checked out the address for the physics department from the phonebook and went right to the department at Siltavuorenpenger. I marched in to the High Energy Physics Research Institute without thinking more about my choice, because its door had a name that included the word "physics".

Choosing a job for the training with this partially random method worked out great. I got a good reception, and I got to use the department office's brand new Compact Macintosh computer. I drew works of art that were inspired by the humor magazine Pahkasika with the MacPaint program. My own Commodore 64 couldn't do that!

I had a great time at the HEP, and I got to know the exhilarating atmosphere and computers of the academic world. The bubble chamber photos were mysterious: elementary particles had drawn strange tracks in them. I also got to solder silver wires to the particle detectors in the experiment room, which were used as a base for the real research equipment at CERN. Fortunately my electronics hobby had developed my soldering skills.

I collected feedback papers during my job training for my school. A job description for university assistants stuck into my mind: "opening the door for the professor". Today I'm a professor myself, but I open the doors with my own hands. There are no assistants any more.

Although physics was still a strong competitor when I studied at Linnankoski senior high in Porvoo, I was better at mathematics and more interested in it. Chemistry couldn't beat maths either, even though my teacher Peter von Bagh made great experiments in class.

So I decided to apply for mathematics studies. I felt a strong calling for the general mathematics school at Helsinki University, but Dad's recommendation to go to the Helsinki University of Technology in Espoo was even stronger. I checked it out: HUT offered maths too, so I agreed to pick it. The school was called technical physics, but it was possible to specialize in maths after 2 years of common studies.

1.3 Engineering Studies at HUT

My first year at university was a shock. I had passed my courses at school with little work, but I had to study hard for the exams at HUT. Or I should have, because the duties of student life, that is parties, didn't always leave me enough time for that.

During the spring of my freshman year I found myself a suitable style of study. A differential equations exam was coming, and I couldn't turn the lecture notes into my own understanding at all. Fortunately I found an old and dusty book on the topic in the physics department. It helped me realize, how to solve the problems by assembling a suitable solution out of the general solutions of the equations. The exam went well.

I realized two important things. Once you have found some mathematics, it never changes. One should look for information outside the teachers' course material too.

After this turning point, I was able to balance my studies with an active participation in student activities, including parties. The obligatory physics courses were hard, but I was able to concentrate on the wonder of mathematics once I had passed them. Professor Rainer Salomaa, I'm sorry for the stupid question I made on the course Modern physics II: "Is quantum mechanics nonsense?" I already knew that it's not.

The rugged beauty of abstract mathematics drew me into its depths. I sank deeper and deeper into the whirlpool of theoretical structures that are difficult to explain to people who do not know them. Hard work with books that are abstract and difficult to understand brought into view masses of knowledge, where great crystal mountains and celestial cogwheels were attached to each other like pieces of puzzle lying in inner product space.

For some time I thought that the less the math I study has to do with life, the better. The world of odd calculations and of shapes curved in impossible ways hooked me like a computer game. I visited Helsinki University for extra courses on topology and algebra—and of course on algebraic topology too! Oh the happiness of an intellectual feast!

I started to specialize in applied mathematics already when doing my engineer's thesis in 1993–1994. Although Olavi Nevanlinna, Jukka Tuomela and Olli-Pekka Piirilä sketched a quite theoretical topic for my thesis, I was happy to notice that it had connections to designing the movements of robots.

In 1994–1995 I did my alternative national service in the Rolf Nevanlinna Institute (RNI) of the University of Helsinki, which is a cradle of industrial mathematics. I got to know two men, who charted my scientific path: Erkki Somersalo, the great storyteller and a grand master of applied mathematics, and Matti Lassas, the most versatile thinker I know and since then my good friend.

My national service included teaching a special course on applied mathematics and mathematical models for mathematically trained people who were left unemployed by the big recession in Finland in the 1990s. It went great, because 14 out of the 20 participants found work after it.

An immortal conversation on the philosophy of science was left in my mind during my time at RNI. It was between the theorist Matti Lassas and the coding wizard Seppo Järvenpää. It went as follows:

Matti: How do you know what your program is calculating, unless you have proven your formulas?
Seppo: How can I trust your theoretical results, unless I've tested them on a computer?

Since then, my guiding principle in science has been: build computer programs that solve the user's problem efficiently and that are based on proven mathematical theorizing.

1.4 Thesis and Medical Imaging Research

For centuries, mathematics and the natural sciences were one. Only in the century of theoretical and conceptual mathematics, the twentieth century, they got badly separated from each other. I felt at once that the enthusiasm for physics in my youth and my technical experiments belonged together seamlessly.

I started writing my thesis with Erkki Somersalo as my supervisor at the Helsinki University of Technology in the January of 1996. The topic was not clear at the beginning, but there were many alternatives. In the end, a new breakthrough article by the Rochester University professor Adrian Nachman got chosen.

Nachman's work was on electrical impedance tomography. In this imaging technique, ECG electrodes are attached to the skin. If one wants to observe the functioning of the heart and the lungs, 32 electrodes are girded across the chest like a necklace of pearls. Harmless and painless electric currents are input through the electrodes according to well planned forms. Electricity flows into the body from some electrodes and out of it through others. The tomography machine measures the voltages on the electrodes.

Different human tissues conduct electricity differently. Electricity goes effortlessly through the heart as it is filled with blood, but electricity also readily bypasses the lungs that are full of insulating air. The currents seek their way from an electrode to another through the easiest path, and therefore inhaling and exhaling give totally different voltage readings. In principle, this makes it possible to make a picture of the divisions of conductivity in the body. The picture changes with heartbeats and breathing, and this gives the doctor a picture of the patient's internal organs.

Interpreting the electric currents into a picture is a hugely challenging mathematical task. The electric currents in the body don't move along known paths, but seek the easiest way through the organs. And it is the functioning of the organs that we want to know, so we can't use that knowledge in the calculation!

Nachman's article was the first to present a method for calculating an impedance tomographic image. But it's a terribly complex and technical paper. It's just the right kind of paper for over-eager young mathematicians to study.

I started reading in the spring of 1996. How bad could that article be? It's only 49 pages! Getting to know the introduction went quite well, as it gave an excellent overview of impedance tomography and earlier mathematical inventions on the topic. When I got into the first main section, my reading quickly stopped in a sentence on the top of page 15: "Here we are using the continuity of the Riesz transforms." What are these mysterious Riesz transforms? I used several months to find it out. Wading through the article and understanding it took me 2 years. Mathematical texts can be that dense and difficult to understand!

Nachman's work brought together a great amount of earlier research in surprising ways. Some of his techniques were developed for quantum physics to determine the colliding particles out of the splinters left by particle collisions. Some formulae were developed to explain the structures of tsunami waves, others to describe phenomena on the surface of charged particles, and a few to divide sound signals into pure oscillating wave forms. Only in mathematics can such different pieces of puzzle attach to each other so seamlessly and determine a way of imaging the internal organs of a patient with electrical currents.

I had steadfastly decided to realize Nachman's ideas in the form of hospital equipment. But the task was huge. Nachman had assumed that the measurements were made with an infinite number of microscopic electrodes that are perfectly accurate. I had to turn the theory into a computational computer program that is able to make images of the data collected through 32 electrodes, and the data necessarily contains errors that diminish its accuracy.

I got lucky. Marko Vauhkonen wrote a thesis on medical physics in Kuopio in 1997. I took a train there together with Erkki Somersalo and Marko's opponent Margaret Chaney. During the trip, Margaret told me about Jennifer Mueller, a fellow in her research group. Jennifer was interested in developing a new imaging method for impedance tomography. We were able to invite Jennifer to visit Finland with money from Erkki's project grant in the June of 1997.

Finland is not always an easy destination for international visitors. I took Jennifer to the HUT guest room at the Otaniemi campus. I showed her the way to the University main building, agreed to meet her at 9 o'clock in the morning and went home. At 10 o'clock I got a very angry phone call. "Where is everybody?" Jennifer asked. "I tried to go to the university, but the doors are locked." Poor Jennifer was confused by jet lag and had woken up after an hour's nap, thinking that the bright summer evening is the morning. She had been wondering about tech students having a barbeque, but had assumed that it had something to do with odd Finnish breakfast customs.

The visit had a bad start, but it led into a friendship and a scientific cooperation that are still ongoing. Over the years, we have developed a new imaging method for impedance tomography step by step. This has required modifying and expanding on Nachman's ideas, and realizing them through computing. One of the high points of our long project was a 2004 paper, where we showed that the model works for data collected at a lab. Another was a paper in 2006, where we reported that we had used the method to make medical images of a real person for the first time. Before that, we had only tested our methods on computational models. Oh the great feeling, when I saw a video of the heart and lungs of a living person, which was calculated using methods developed by our group!

We're still continuing the development work. Jennifer has built an apparatus for measuring people in her lab in Colorado. She concentrates on imaging cystic fibrosis, especially in children. They have mucus in their lungs that makes it difficult to breathe for them. Therefore it's important that doctors are able to estimate how much air gets in the lungs. One can use a spirometer to do the measurement with adults, but small kids don't know how to use one. Impedance tomography offers an easy way to find out, how well the lungs are.

I've brought together a Finnish research group to investigate the use of impedance tomography to image cerebrovascular diseases. There are two main types of diseases: a thrombosis or a blood clot in the brain prevents the flow of blood into a part of the brain, and a haemorrhage. Both have similar symptoms (a smile on the other side of the face, confused speech, an inability to raise the other hand on the top of the head), but they have completely different treatments. A thrombosis patient should usually be given a thrombolysis as soon as possible to dissolve the clot, but thrombolysis is dangerous for a person with a haemorrhage. Usually a haemorrhage can be found or ruled out with an X-ray tomography, but it takes precious time to get to the imaging device. With impedance tomography, one could find the cause of the stroke right in the ambulance.

1.5 As an Industrial Developer

After my doctoral thesis, I was hired by Instrumentarium Imaging to develop medical technology. I worked on three-dimensional X-ray imaging. I'll tell you about it later. Here I look back at joining the brisk Instrumentarium team in the corporate orienteering league.

Orienteering too is an adventure in the world of mathematics. A map is an image of the real world. Bodies of water, roads, buildings, rivers and other significant locations are drawn into it in miniature. The geometric accuracy of a map makes it a mathematical model: from my home it's 1 km to the station, and in the map (1: 10,000) my home and the station are drawn 10 cm away from each other.

My colleagues coaxed me into an orienteering competition despite my warnings. According to my wife, one should walk in the opposite direction to the one I'm proposing when visiting an unknown city. My friends didn't support me either, when I tried to deny such charges in the social court of my friends. I was no star player to aid the orienteering team. Except in the sense that a minimum headcount was required of the team, and without me it would not have been reached.