The BEST WRITING on MATHEMATICS 2013

Mircea Pitici, Editor

FOREWORD BY
ROGER PENROSE

The **BEST** **WRITING** on **MATHEMATICS**

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Foreword

Roger Penrose

Although I did not expect to become a mathematician when I was growing up—my first desire had been to be a train driver, and later it was (secretly) to be a brain surgeon—mathematics had intrigued and excited me from a young age. My father was not a professional mathematician, but he used mathematics in original ways in his statistical work in human genetics. He clearly enjoyed mathematics for its own sake, and he would often engage me and my two brothers with mathematical puzzles and with aspects of the physical and biological world that had a mathematical character. To me, it had become clear that mathematics was something to be enjoyed. It was evidently also something that played an important part in the workings of the world, and one could see this basic role not only in physics and astronomy, but also in many aspects of biology.

I learned much of the beauties of geometry from him, and together we constructed from cardboard not only the five Platonic solids but also many of their Archimedean and rhombic cousins. This activity arose from one occasion when, at some quite early age, I had been studying a floor or table surface, tiled with a repeating pattern of ceramic regular hexagons. I had wondered, somewhat doubtfully, whether they might, if continued far enough into the distance, be able to cover an entire spherical surface. My father assured me that they could not but told me that regular pentagons, on the other hand, would do so. Perhaps there was some seed of a thought of a possible converse to this, planted early in my mind, about a possibility of using regular pentagons in a tiling of the plane, that found itself realized about one third of a century later!

My earliest encounter with algebra came about also at an early age, when, having long been intrigued by the identity $2 + 2 = 2 \times 2$, I had hit upon $1\frac{1}{2} + 3 = 1\frac{1}{2} \times 3$. Wondering whether there might be other examples, and using some geometrical consideration concerning squares and rectangles, or something—I had never done any algebra—I hit upon some rather too-elaborate formula for what I had guessed might be a general expression for the solution to this problem. Upon my showing this to my older brother Oliver, he immediately showed me how my formula could be reduced to $\frac{1}{a} + \frac{1}{b} = 1$, and he explained to me how this formula indeed provided the general solution to $a + b = a \times b$. I was amazed by this power of simple algebra to transform and simplify expressions, and this basic demonstration opened my eyes to the wonders of the world of algebra.

Much later, when I was about 15, I told my father that my mathematics teacher had informed us that we would be starting calculus on the following day. Upon hearing this, a desperate expression came over his face, and he immediately took me aside to explain calculus to me, which he did very well. I could see that he had almost deprived himself of the opportunity to be the first to introduce to me the joy and the magic of calculus. I think that almost as great as my immediate fascination with this wonderful subject was my father's passionate need to relate to me in this mathematically important way. This method (and through other intellectual pursuits such as biology, art, music, puzzles, and games) seems to have been his only emotional route to his sons. To try to communicate with him on personal matters was a virtual impossibility. It was with this background that I had grown up to be comfortable with mathematics and to regard it as a friend and as a recreation, and not something to be frightened or deterred by.

Yet there was an irony in store for me. Both my parents had been medically trained and had decided that of their three sons, I was the one to take over the family concerns with medicine (and, after all, I had my secret ambition to be a brain surgeon). This possibility went by the wayside, however, because of a decision that I had had to make at school that entailed my giving up biology in favor of mathematics, much to the displeasure of my parents. (It was my much younger sister Shirley who eventually took up the banner of medicine, eventually becoming a professor of cancer genetics.) My father was even less keen when I later expressed the desire to study mathematics, pure and simple, at university.

He seemed to be of the opinion that to do *just* mathematics, without necessarily applying it to some other scientific area of study, one had to be a strange, introverted sort of person, with no other interests but mathematics itself. In his desires and ambitions for his sons, my father was, indeed, an emotionally complicated individual!

In fact, I think that initially mathematics alone was my true main interest, with no necessity for it to relate to any other science or to any aspect of the external physical world. Nor had I any great desire to communicate my mathematical understandings to others. Yet, as things developed, I began to feel a greater and greater need to relate my mathematical interests to the workings of the outside world and also, eventually, to communicate what understandings I had acquired to the general public. I have, indeed, come to recognize the importance of trying to convey to others an appreciation of not only the unique value of mathematics but also its remarkable aesthetic qualities. Few other people have had the kind of advantages that I have myself had, with regard to mathematics, arising from my own curiously distinctive mathematical background.

This volume serves such a purpose, providing accounts of many of the achievements of mathematics. It is the fourth of a series of compendia of previously published articles, aimed at introducing to the general public various areas of mathematics and its multifarious applications. It is, in addition, aimed also at other mathematicians, who may themselves work in areas other than the ones being described here. With regard to this latter purpose, I can vouch for its success. For in my own case, I write as a mathematician whose professional interests lie in areas almost entirely outside those described here, and upon reading these articles I have certainly had my perspectives broadened in several ways that I had not expected.

The breadth of the ideas that we find here is considerable, ranging over many areas, such as the philosophy of mathematics, the issue of why mathematics is so important in education and society, and whether its public perception has changed in recent years; perhaps it should now be taught fundamentally differently, and there is the issue of the extent to which the modern technological world might even have thoroughly changed the very nature of our subject. We also find fascinating historical accounts, from achievements made a thousand years or so before the ancient Greeks, to the deep insights and occasional surprising errors made in more modern historical times, and of the wonderful mathematical instruments that played important roles in their societies. We find unexpected connections with geometry, both simple and highly sophisticated, in artistic creations of imposing magnitude and to the fashionable adornment of individual human beings. We learn of the roles of symmetry in animals and in art, and of the use of art in illustrating the value of mathematical rigor. There is much here on the role of randomness and how it is treated by statistics, which is a subject of ubiquitous importance throughout science and of importance also in everyday life.

Yet I was somewhat surprised that, throughout this great breadth of mathematical application, I find no mention of that particular area of the roles of mathematics that I myself find so extraordinarily remarkable, and to which I have devoted so much of my own mathematical energies. This area is the application of sophisticated mathematics to the inner workings of the physical world wherein we find ourselves. It is true that with many situations in physics, very large numbers of elementary constituents (e.g., particles) are involved. This truth applies to the thermodynamic concepts of heat and entropy and to the detailed behavior of gases and other fluids. Many of the relevant concepts are then governed by the laws of large numbers—that is, by the principles of statistics—and this issue is indeed addressed here from various different perspectives in several of these articles.

However, it is often the case that such statistical considerations leave us very far from what is needed, and a proper understanding of the underlying laws themselves is fundamentally needed. Indeed, in appropriate circumstances (i.e., when the physical behavior is in sufficiently "clean" systems), a precision is found that is extraordinary between the observed physical behavior and the calculated behavior that is expected from the known physical laws. This precision already exists, for example, in modern treatments that use powerful computer techniques for the ancient 17th century laws of Isaac Newton. But in appropriate circumstances, the agreement can be far more impressive when the appropriate mathematically sophisticated laws of general relativity or quantum field theory are brought into play.

These matters are often hard to explain in the general terms that could meet the criteria of this collection, and one can understand the omission here of such extraordinary achievements of mathematics. It must be admitted, also, that there is much current activity of considerable mathematical sophistication that, though it is ostensibly concerned with the workings of the actual physical world, has little, if any, direct observational connection with it. Accordingly, despite the considerable mathematical work that is currently being done in these areas—much of it of admittedly great interest with regard to the mathematics itself—this work may be considered from the physical point of view to be somewhat dubious, or tenuous at best because it has no observational support as things stand now. Nevertheless, it is within physics, and its related areas, such as chemistry, metallurgy, and astronomy, that we are beginning to witness the deep and overreaching command of mathematics, when it is aided by the computational power of modern electronic devices.

Introduction

Mircea Pitici

In the fourth annual volume of *The Best Writing on Mathematics* series, we present once again a collection of recent articles on various aspects related to mathematics. With few exceptions, these pieces were published in 2012. The relevant literature I surveyed to compile the selection is vast and spread over many publishing venues; strict limitation to the time frame offered by the calendar is not only unrealistic but also undesirable.

I thought up this series for the first time about nine years ago. Quite by chance, in a fancy, and convinced that such a series existed, I asked in a bookstore for the latest volume of *The Best Writing on Mathematics.* To my puzzlement, I learned that the book I wanted did not exist—and the remote idea that I might do it one day was born timidly in my mind, only to be overwhelmed, over the ensuing few years, by unusual adversity, hardship, and misadventures. But when the right opportunity appeared, I was prepared for it; the result is in your hands.

Mathematicians are mavericks—inventors and explorers of sorts; they create new things and discover novel ways of looking at old things; they believe things hard to believe, and question what seems to be obvious. Mathematicians also disrupt patterns of entrenched thinking; their work concerns vast streams of physical and mental phenomena from which they pick the proportions that make up a customized blend of abstractions, glued by tight reasoning and augmented with clues glanced from the natural universe. This amalgam differs from one mathematician to another; it is "purer" or "less pure," depending on how little or how much "application" it contains; it is also changeable, flexible, and adaptable, reflecting (or reacting to) the social intercourse of ideas that influences each of us.

When we talk about mathematics, or when we teach it, or when we write about it, many of us feign detachment. It is almost a cultural universal to pretend that mathematics is "out there," independent of our whims and oddities. But doing mathematics and talking or writing about it are activities neither neutral nor innocent; we can only do them if we are engaged, and the engagement marks not only us (as thinkers and experimenters) but also those who watch us, listen to us, and think with us. Thus mathematics always requires full participation; without genuine involvement, there is no mathematics.

Mathematicians are also tinkerers—as all innovators are. They try, and often succeed, in creating effective tools that guide our minds in the search for certainties. Or they err; they make judgment mishaps or are seduced by the illusion of infallibility. Sometimes mathematicians detect the errors themselves; occasionally, others point out the problems. In any case, the edifice mathematicians build should be up for scrutiny, either by peers or by outsiders.

And here comes a peculiar aspect that distinguishes mathematics among other intellectual domains: Mathematicians seek validation inside their discipline and community but feel little need (if any) for validation coming from outside. This professional chasm surrounding much of the mathematics profession is inevitable up to a point because of the nature of the discipline. It is a Janus-faced curse of the ivory tower, and it is unfortunate if we ignore it. On the contrary, I believe that we should address it. Seeking the meaning and the palpable reasoning underlying every piece of mathematics, as well as conveying them in natural language, are means that bridge at least part of the gap that separates theoretical mathematics from the general public; such means demythologize the widespread belief that higher mathematics is, by its epistemic status and technical difficulty, inaccessible to the layperson.

Mathematics and its applications are scrutable only as far as mathematicians are explicit with their own assumptions, claims, results, and interpretations. When these elements of openness are missing mathematicians not only fail to disrupt patterns of entrenched thinking but also run the risk of digging themselves new trenches. Writing about mathematics offers freedoms of explanation that complement the dense texture of meaning captured by mathematical symbols.

Talking plainly about mathematics also has inestimable educational and social value. A sign of a mature mind is the ability to hold at the same time opposite ideas and to juggle with them, analyze them, refine them, corroborate them, compromise with them, and choose among them. Such intellectual dexterity has moral and practical consequences, for the lives of the individuals as well as for the life of society. Mathematical thinking is eminently endowed to prepare the mind for these habits, but we almost never pay attention to this aspect, we rarely notice it, and we seldom talk about it. We use contradiction as a trick of the (mathematical) trade and as a routine method of proof. Yet opposing ideas, contrasts, and complementary qualities are intimately interwoven into the texture of mathematics, from definitions and elementary notions to highly specialized mathematical practice; we use them implicitly, tacitly, all the time.

As a reader of this book, you will have a rewarding task in identifying in the contributions some of the virtues I attribute to writing about mathematics—and perhaps many others. With each volume in this series, I put together a book I like to read, the book I did not find in the bookstore years ago. If I include a contribution here, it does not mean that I necessarily concur in the opinions expressed in it. Whether we agree or disagree with other people's views, our polemics gain in substance if we aim to comprehend and address the highest quality of the opposing arguments.

Overview of the Volume

In a sweeping panoramic view of the likely future trajectory of mathematics, Philip Davis asks pertinent questions that illuminate some of the myriad links connecting mathematics to its applications and to other practical domains and offers informed speculations on the multifaceted mathematical imprint on our ever more digitized world.

Ian Stewart explains recent attempts made by mathematicians to refine and reaffirm a theory first formulated by Alan Turing, stating, in its most general formulation, that pattern formation is a consequence of symmetry breaking.

Terence Tao observes that many complex systems seem to be governed by universal principles independent of the laws that govern the interaction between their components; he then surveys various aspects of several well-studied mathematical laws that characterize phenomena as diverse as spectral lines of chemical elements, political elections, celestial mechanics, economic changes, phase transitions of physical materials, the distribution of prime numbers, and others.

A diversity of contexts, with primary focus on social networking, is also Gregory Goth's object of attention; in his article he examines the "small-world" problem—the quest to determine the likelihood that two individuals randomly chosen from a large group know each other.

Charles Seife argues that humans' evolutionary heritage, cultural mores, and acquired preconceptions equip us poorly for expecting and experiencing randomness; yet, in an echo of Tao's contribution, Seife observes that in aggregate, randomness of many independent events does obey immutable mathematical rules.

Writing from experience, Donald Knuth shows that the deliberate and methodical coopting of randomness into creative acts enhances the beauty and the originality of the result.

Soren Johnson discusses the advantages and the pitfalls of using chance in designing games and gives examples illustrative of this perspective.

John Pavlus details the history, the meaning, and the implications of the P versus NP problem that underpins the foundations of computational complexity theory and mentions the many interdisciplinary areas that are connected through it.

Renan Gross analyzes the geometry of the Jerusalem Chords Bridge and relates it to the mathematics used half a century ago by the French mathematical engineer Pierre Bézier in car designs—an elegantly simple subject that has many other applications.

Daniel Silver presents Albrecht Dürer's *Painter's Manual* as a precursor work to projective geometry and astronomy; he details some of the mathematics in the treaty and in Dürer's artworks, as well as the biographical elements that contributed to Dürer's interaction with mathematics.

Kelly Delp writes about the late William Thurston's little-known but intensely absorbing collaboration with the fashion designer Dai Fujiwara and his Issey Mayake team; she describes the topological notions that, surprisingly, turned out to be at the confluence of intellectual passions harbored by two people so different in background and living on opposite sides of the world.

Fiona and William Ross tell the brief history of Jordan's curve theorem, hint at some tricky cases that defy the simplistic intuition behind it, and, most remarkably, illustrate the nonobvious character of the theorem with arresting drawings penned by Fiona Ross.

To answer pressing questions about the need for widespread mathematics education, Anna Sfard sees mathematics as a narrative means to comprehend the world, which we humans developed for our convenience; she follows up on this perspective by arguing that the story we tell (and teach) with mathematics needs to change, in sync with the unprecedented changes of our world.

Erin A. Maloney and Sian L. Beilock examine the practical and psychological consequences of states of anxiety toward mathematical activities. They contend that such feelings appear early in schooling and tend to recur in subsequent years, have a dual cognitive and social basis, and negatively affect cognitive performance. The authors affirm that the negative effect of mathematical anxiety can be alleviated by certain deliberate practices, for instance by writing about the emotions that cause anxiety.

David R. Lloyd reviews arguments put forward by proponents of the idea that the five regular polyhedral shapes were well known, as mathematical objects, perhaps one millennium before Plato—in Scotland, where objects of similar configurations and markings have been discovered. He concludes that the objects, genuine and valuable aesthetically and anthropologically, do not substantiate the revisionist claims at least as far as in the mathematical knowledge they reveal.

In the interaction between the material culture of mathematical instruments and the mathematics that underlined it from the 16th to the 18th centuries in Western Europe, Jim Bennett decodes subtle reciprocal influences that put mathematics in a nodal position of a network of applied sciences, scientific practices, institutional academics, entrepreneurship, and commerce.

Frank Quinn contrasts the main features of mathematics before and after the profound transformations that took place at the core of the discipline roughly around the turn of the 19th century; he contends that the unrecognized magnitude of those changes led to some current fault lines (for instance between teaching and research needs in universities, between school mathematics and higher level mathematics, and others) and in the future might even marginalize mathematics.

Prakash Gorroochurn surveys a collection of chance and statistics problems that confused some of the brilliant mathematical minds of the past few centuries and initially were given erroneous solutions—but had an important historical role in clarifying fine distinctions between the theoretical notions involved.

Elie Ayache makes the logical-philosophical case that the prices reached by traders in the marketplace take precedence over the intellectual speculations intended to justify and to predict them; therefore, the contingency of number prices supersedes the calculus of probabilities meant to model it, not the other way around.

Finally, Kevin Hartnett reports on recent developments related to the *abc* conjecture, a number theory result that, if indeed proven, will have widespread implications for several branches of mathematics.

Other Notable Writings

This section of the introduction is intended for readers who want to read more about mathematics. Most of the recent books I mention here are nontechnical. I offer leads that can easily become paths to research on various aspects of mathematics. The list that follows is not exhaustive, of course—and I omit titles that appear elsewhere in this volume.

Every year I start by mentioning an outstanding recent work; this time is no exception. The reader interested in the multitude of modalities available for conveying data (using graphs, charts, and other visual means) may relish the monumental encyclopedic work *Information Graphics* by Sandra Rendgen and Julius Wiedemann.

An intriguing collection of essays on connections between mathematics and the narrative is *Circles Disturbed*, edited by Apostolos Doxiadis and Barry Mazur. Another collection of essays, more technical but still accessible in part to a general readership, is *Math Unlimited*, edited by R. Sujatha and colleagues; it explores the relationship of mathematics with some of its many applications.

Among the ever more numerous popular books on mathematics I mention Steven Strogatz's *The Joy of X*, Ian Stewart's *In Pursuit of the Unknown*, Dana Mackenzie's *The Universe in Zero Words*, Jeffrey Bennett's *Math for Life*, Lawrence Weinstein's *Guesstimation 2.0*, Norbert Hermann's *The Beauty of Everyday Mathematics*, Keith Devlin's *Introduction to Mathematical Thinking* and Leonard Wapner's *Unexpected Expectations*. Two successful older books that see new editions are *Damned Lies and Statistics* by Joel Best and *News and Numbers* by Victor Cohn and

Lewis Cope. An eminently readable introduction to irrational numbers is appropriately called *The Irrationals,* authored by Julian Havil. A much needed book on mathematics on the wide screen is *Math Goes to the Movies* by Burkard Polster and Marty Ross.

Two venerable philosophers of science and mathematics have their decades-long collections of short pieces republished in anthologies: Hilary Putnam in *Philosophy in an Age of Science* and Philip Kitcher in *Preludes to Pragmatism.* Other recent books in the philosophy of mathematics are *Logic and Knowledge* edited by Carlo Cellucci, Emily Grosholz, and Emiliano Ippoloti; *Introduction to Mathematical Thinking* by Keith Devlin; *From Foundations to the Philosophy of Mathematics* by Joan Roselló; *Geometric Possibility* by Gordon Belot; and *Mathematics and Scientific Representation* by Christopher Pincock. Among many works of broader philosophical scope that take mnemonic inspiration from mathematics, I mention *Spinoza's Geometry of Power* by Valtteri Viljanen, *The Geometry of Desert* by Shelly Kagan, and *The Politics of Logic* by Paul Livingston. Two volumes commemorating past logicians are *Gödel's Way* by Gregori Chaitin and his collaborators, and *Hao Wang* edited by Charles Persons and Montgomery Link. And a compendious third edition of Michael Clark's *Paradoxes from A to Z* has just become available.

In the history of mathematics, a few books focus on particular epochs—for instance, the massive *History of Mathematical Proof in Ancient Traditions* edited by Karine Chemla and the concise *History of the History of Mathematics* edited by Benjamin Wardhaugh; on the works and the biography of important mathematicians, as in *Henri Poincaré* by Jeremy Gray, *Interpreting Newton,* edited by Andrew Janiak and Eric Schliesser, *The King of Infinite Space* [Euclid] by David Berlinski, and *The Cult of Pythagoras* by Alberto Martínez; on branches of mathematics, for example *The Tangled Origins of the Leibnizian Calculus* by Richard Brown, *Calculus and Its Origins* by Richard Perkins, and *Elliptic Tales* by Avner Ash and Robert Gross; or on mathematical word problems, as in *Mathematical Expeditions* by Frank Swetz. An anthology rich in examples of popular writings on mathematics chosen from several centuries is *Wealth in Numbers* edited by Benjamin Wardhaugh. Two collections of insightful historical episodes are Israel Kleiner's *Excursions in the History of Mathematics* and Alexander Ostermann's *Geometry by Its History.* And an intriguing attempt to induce mathematical rigor in historical-religious controversies is *Proving History* by Richard Carrier.

The books on mathematics education published every year are too many to attempt a comprehensive survey; I only mention the few that came to my attention. In the excellent series *Developing Essential Understanding* of the National Council of Teachers of Mathematics, two recent substantial brochures written by Nathalie Sinclair, David Pimm, and Melanie Skelin focus on middle school and high school geometry. Also at the NCTM are *Strength in Numbers* by Ilana Horn; an anthology of articles previously published in *Mathematics Teacher* edited by Sarah Kasten and Jill Newton; and *Teaching Mathematics for Social Justice,* edited by Anita Wager and David Stinson. Other books on equity are *Building Mathematics Learning Communities* by Erika Walker and *Towards Equity in Mathematics Education* edited by Helen Forgasz and Ferdinand Rivera. In a niche of self-help books I would include Danica McKellar's *Girls Get Curves* and Colin Pask's *Math for the Frightened.* A detailed ethnomathematics study is Geoffrey Saxe's *Cultural Development of Mathematical Ideas* [in Papua New Guinea]. And a second volume on *The Mathematics Education of Teachers* was recently issued jointly by the American Mathematical Society and the Mathematical Association of America.

In a group I would loosely call applications of mathematics and connections with other disciplines, notable are *Fractal Architecture* by James Harris, *Mathematical Excursions to the World's Great Buildings* by Alexander Hahn, *Proving Darwin* by Gregory Chaitin, *Visualizing Time* by Graham Wills, *Evolution by the Numbers* by James Wynn, and *Mathematics and Modern Art* edited by Claude Bruter. Slightly technical but still widely accessible are *Introduction to Mathematical Sociology* by Phillip Bonacich and Philip Lu, *The Essentials of Statistics* [for social research] by Joseph Healey, and *Ways of Thinking, Ways of Seeing* edited by Chris Bissell and Chris Dillon. More technical are *The Science of Cities and Regions* by Alan Wills, and *Optimization* by Jan Brinkhuis and Vladimir Tikhomirov.

I conclude by suggesting a few interesting websites. "Videos about numbers & stuff" is the deceptively self-deprecating subtitle of the Numberphile (http://www.numberphile.com/) page, hosting many instructive short videos on simple and not-so-simple mathematical topics. A blog that keeps current with technology that helps teaching mathematics is Mathematics and Multimedia (http://mathandmultimedia.com/); also a well-done educational site is Enrich Mathematics (http://nrich.maths.org/frontpage) hosted by Cambridge University.

Ostermann, Alexander. *Geometry by Its History.* Heidelberg, Germany: Springer Verlag, 2012.

Parsons, Charles, and Montgomery Link. (Eds.) *Hao Wang: Logician and Philosopher.* London, UK: College Publications, 2011.

Pask, Colin. *Math for the Frightened: Facing Scary Symbols and Everything Else That Freaks You Out about Mathematics.* Amherst, NY: Prometheus Books, 2012.

Perkins, David. *Calculus and Its Origins.* Washington, DC: Mathematical Association of America, 2012.

Pincock, Christopher. *Mathematics and Scientific Representation.* Oxford, UK: Oxford University Press, 2012.

Polster, Burkard, and Marty Ross. *Mathematics Goes to the Movies.* Baltimore, MD: The Johns Hopkins University Press, 2012.

Putnam, Hilary. *Philosophy in an Age of Science: Physics, Mathematics, and Skepticism.* Cambridge, MA: Harvard University Press, 2012.

Rendgen, Sandra, and Julius Wiedemann. *Information Graphics.* Cologne, Germany: Taschen, 2012.

Roselló, Joan. *From Foundations to Philosophy of Mathematics: An Historical Account of their Development in XX Century and Beyond.* Cambridge, UK: Cambridge University Press, 2012.

Saxe, Geoffrey B. *Cultural Development of Mathematical Ideas: Papua New Guinea Studies.* New York: Cambridge University Press, 2012.

Sinclair, Nathalie, David Pimm, and Melanie Skelin. *Developing Essential Understanding of Geometry for Teaching Mathematics in Grades 6–8.* Reston, VA: The National Council of Teachers of Mathematics, 2012.

Sinclair, Nathalie, David Pimm, and Melanie Skelin. *Developing Essential Understanding of Geometry for Teaching Mathematics in Grades 9–12.* Reston, VA: The National Council of Teachers of Mathematics, 2012.

Stewart, Ian. *In Pursuit of the Unknown: 17 Equations that Changed the World.* New York: Basic Books, 2012.

Strogatz, Steven. *The Joy of X: A Guided Tour of Math, from One to Infinity.* Boston: Houghton Mifflin Harcourt, 2012.

Sujatha, R., H. M. Ramaswamy, and C. S. Yogananda. (Eds.) *Math Unlimited: Essays in Mathematics.* St. Helier, British Channel Islands: Science Publishers, 2012.

Swetz, Frank J. *Mathematical Expeditions: Exploring Word Problems across the Ages.* Baltimore, MD: The Johns Hopkins University Press, 2012.

Viljanen, Valtteri. *Spinoza's Geometry of Power.* Cambridge, UK: Cambridge University Press, 2012.

Wager, Anita A., and David W. Stinson. (Eds.) *Teaching Mathematics for Social Justice: Conversations with Educators.* Reston, VA: The National Council of Teachers of Mathematics, 2012.

Walker, Erica N. *Building Mathematics Learning Communities: Improving Outcomes in Urban High Schools.* New York: Teachers College, Columbia University Press, 2012.

Wapner, Leonard M. *Unexpected Expectations: The Curiosities of a Mathematical Crystal Ball.* New York: Taylor and Francis Group, 2012.

Wardhaugh, Benjamin. (Ed.) *Wealth of Numbers: An Anthology of 500 Years of Popular Mathematics.* Princeton, NJ: Princeton University Press, 2012.

Wardhaugh, Benjamin. (Ed.) *The History of the History of Mathematics: Case Studies for the Seventeenth, Eighteenth, and Nineteenth Centuries.* Bern, Switzerland: Peter Lang, 2012.

Weinstein, Lawrence. *Guesstimation 2.0: Solving Today's Problems on the Back of a Napkin.* Princeton, NJ: Princeton University Press, 2012.

Wills, Graham. *Visualizing Time*. Heidelberg, Germany: Springer Verlag, 2012.
Wilson, Alan. *The Science of Cities and Regions: Lectures on Mathematical Model Design.* New York: Springer Science+Business Media, 2012.
Wynn, James. *Evolution by the Numbers: The Origins of Mathematical Argument in Biology*. Anderson, SC: Parlor Press, 2012.

The BEST WRITING on MATHEMATICS

2013

The Prospects for Mathematics in a Multimedia Civilization

Philip J. Davis

I. Multimedia Mathematics

First let me explain my use of the phrase "multimedia civilization."* I mean it in two senses. In my first usage, it is simply a synonym for our contemporary digital world, our click-click world, our "press 1, 2, or 3 world", a world with a diminishing number of flesh-and-blood servers to talk to. This is our world, now and for the indefinite future. It is a world that in some tiny measure most of us have helped make and foster.

In my second usage, I refer to the widespread and increasing use of computers, fax, e-mail, the Internet, CD-ROMs, iPods, search engines, PowerPoint, and YouTube—in all mixtures. I mean the phrase to designate the cyberworld that embraces such terms as interface design, cybercash, cyberlaw, virtual-reality games, assisted learning , virtual medical procedures, cyberfeminism, teleimmersion, interactive literature, cinema, and animation, 3D conferencing, and spam, as well as certain nasty excrescences that are excused by the term "unforeseeable developments." The word (and combining form) "cyber" was introduced in the late 1940s by Norbert Wiener in the sense of feedback and control. Searching on the prefix "cyber" resulted in 304,000,000 hits, which, paradoxically, strikes me as a lack of control.

I personally cannot do without my word processor, my mathematical software, and yes, I must admit it, my search engines. I find I can check conjectures quickly and find phenomena accidentally. (It is also

* This article is an expanded and newly updated version of a Urania Theater talk given as part of the International Congress of Mathematicians, Berlin, Germany, Aug. 21, 1998.

to his book on scientific computation, "The object of computation is not numbers but insight." Insight into a variety of physical and social processes, of course. But I perceive (40 years after Hamming's book and with a somewhat cynical eye) that the real object of computation, commercial and otherwise, is often neither numbers nor insight nor solutions to pressing problems, but worked on by physicists and mathematicians, to perfect money-making products. Often computer usages are then authorized by project managers who have little technical knowledge. The Descartesian precept *cogito ergo sum* has been replaced by *producto ergo sum*.

If, by chance, humanity benefits from this activity, then so much the better; everybody is happy. And if humanity suffers, the neo-Luddites will cry out and form chat groups on the Web, or the hackers will attack computer systems or humans. The techno-utopians will explain that you can't make omelets without breaking a few eggs. And pure mathematicians will follow along, moving closer to applications while justifying the purity of their pursuits to the administrators, politicians, and the public with considerable truth that one never knows in advance what products of pure imagination can be turned to society's benefit. The application of the theory of numbers to cryptography and the (Johann) Radon transform and its application to tomography have been displayed as shining examples of this. Using that most weasel of rhetorical expressions, "in principle," in principle, all mathematics is potentially useful.

I could use all my space describing many applications that seem now to be hot and are growing hotter. I will mention several and comment briefly on but a few of them. In selecting these few, I have ignored "pure" fields out of personal incompetence. I simply do not have the knowledge or authority to single out from a hundred expanding subfields the ones with particularly significant potential and how they have fared via multimedia. For more comprehensive and authoritative presentations, I recommend *Mathematics: Frontiers and Perspectives* and *Mathematics Unlimited—2001 and Beyond.*

Mathematics and the Physical and Engineering Sciences

These have been around since Galileo, but Newton's work was the great breakthrough. However, only in the past hundred years or so has theoretical mathematics been of any great use to technology. The pursuit of physical and engineering sciences is today unthinkable without significant computational power. The practice of aerodynamic design has altered significantly, but the "digital wind tunnel" has not yet arrived, and some have said it may never. Theories of turbulence are not yet in satisfactory shape—how to deal with widely differing simultaneous scales continues to perplex. Newtonians who deal with differential-integral systems must learn to share the stage with a host of probabilists with their stochastic equations. Withal, hurricane and tornado predictions continue to improve, perhaps more because of improvements in hardware (e.g., real-time data from aircraft or sondes and from nanocomputers) than to the numerical algorithms used to deal with the numerous models that are in use. Predictions of earthquakes or of global warming are controversial and need work. Wavelet, chaos, and fractal theorists and multiresolution analysts are hard at work hoping to improve their predictions.

Mathematics and the Life Sciences

Mathematical biology and medicine are booming. There are automatic diagnoses. There are many models around in computational biology; most are untested. One of my old Ph.D. students has worked in biomolecular mathematics and designer drugs. He and numerous others are now attempting to model strokes via differential equations. Good luck!

I visited a large hospital recently and was struck by the extent that the aisles were absolutely clogged with specialized computers. Later, as a patient, I was all wired up and plugged into such equipment with discrete data and continuous waveforms displayed bedside and at the nurses' stations. Many areas of medical and psychological practice have gone or are going virtual. There is no doubt that we are now our own digital avatars and we are all living longer and healthier lives. In this development, mathematics, though way in the background and though not really understood by the resident physicians or nurses, has played a significant role.

Work on determining and analyzing the human genome sequences, with a variety of goals in mind and using essentially combinatorial and probabilistic methods, is a hot field. In the past decade, the cost of DNA sequencing has come down dramatically.

Genetic engineering on crops goes forward but has raised hackles and doomsday scenarios.

Mathematics and the Military Sciences

If mathematics contributes significantly to the life sciences, there is also mathematics that contributes to the "death sciences": war, both defensive and offensive. For the past 75 years, military problems have been a tremendous engine, supplying money and pulling both pure and applied formulations to new achievements: interior and exterior ballistics, creating bombs, missiles, rockets, antirocket rockets, drones, satellites, war-gaming strategies, and combat training in the form of realistic computer games. The use of mathematics in the service of war and defense will be around as long as aggression is a form of human and governmental behavior. Some authorities have claimed that aggression is built into the human brain. In any case, the psychology of aggression is an open field of study.

Mathematics and Entertainment

There is mathematics and entertainment through animation, simulation, and computer graphics. The ex-executive of Silicon Graphics opined some years ago that the future of the United States lay not in manufacturing nor in the production of food, but in producing a steady flow of entertainment for the rest of the world. Imagine this: a future president of the United States may have to warn us against the media-entertainment complex as Eisenhower did with the military- industrial complex.

But there is more! Through animation and simulation, the world of defense joins up with the world of entertainment and the world of medical technology. These worlds find common problems and can often share computer software. (There have been conferences on this topic.) Mickey Mouse flies the stealth bomber, and virtual surgery can be performed via the same sort of software products. There are now university departments devoted to the design of new video and simulation games, using humans as players, and with a wide variety of applications, including pure research. Young mathematicians have begun to offer their talents not just to university departments of mathematics but also to Hollywood and TV producers.

Mathematics and Money

Marriages of business and mathematics are booming. Property, business, and trade have always been tremendous consumers of low-level mathematics. In deep antiquity, they were probably the generators of such mathematics. But now it is no longer low-level. Zebra stripes (i.e., product identification, or UPC codes) have an interesting mathematical basis. Mathematics and business is a possible major in numerous universities, with professorial chairs in the subject. Software is sold for portfolio management and to automate income tax returns. Wall Street is totally computerized, nanotrading is almost continuous, and millions are playing the market using clever statistical strategies often of their own personal devising. The practice of arbitrage has generated theorems, textbooks, and software.

Mathematics and the Graphic Arts

Graphic art is being revolutionized along mathematical lines, a tendency—would you believe it?—that was present 3,000 years ago in the art of Egypt when some of their art was pixelized. Computer art and op art are now commonplace on gallery walls if a bit ho-hum. Is such art a kind of "soft mathematics"? When I consider the progress that computer art has made from the theoretical approximation theory of my research years, from the elementary paint programs developed in such places as the University of Utah a generation and a half ago, to the sophisticated productions of today's Pixar Animation Studios, my mind boggles. Three-dimensional printing, which makes a solid object from a digital model, advances the older programmed lathes. Two, three, and higher dimensional graphical presentations are more prosaic as art but are also important scientifically; they play an increasing role in presentation and interpretation.

Mathematics, Law, Legislation, and Politics

Law is just beginning to feel the effect of mathematization. Leibniz and Christian Wolff talked about this three centuries ago. Nicholas Bernoulli talked about it. Read Bernoulli's 1709 inaugural dissertation *On the Use of Probability (Artis conjectandi) in Law.*

There are now DNA identifications. The conjectured (or proved) liaison between Thomas Jefferson and Sally Hemings made the front pages. But can statisticians be trusted? "Experts" are often found testifying on both sides of a question.

Statistics are more and more entering the courts as evidence, and courts may soon require a resident statistician to interpret things for the judges, even as my university department has a full-time computer maven to resolve the questions and glitches that arise constantly. There