

“An engaging, personal, and highly user-friendly voyage into
some of the great mysteries and wonders of our world.”

—ALAN LIGHTMAN

THE GREAT UNKNOWN

SEVEN JOURNEYS
TO THE FRONTIERS
OF SCIENCE



MARCUS
DU SAUTOY

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ZERO

**THE KNOWN
UNKNOWNNS**

O

Everyone by nature desires to know.

—Aristotle, *Metaphysics*

EVERY WEEK, headlines announce new breakthroughs in our understanding of the universe, new technologies that will transform our environment, new medical advances that will extend our lives. Science is giving us unprecedented insights into some of the big questions that have challenged humanity ever since we've been able to formulate them. Where did we come from? What is the ultimate destiny of the universe? What are the building blocks of the physical world? How does a collection of cells become conscious?

In the last ten years alone we've landed a spaceship on a comet, built robots that can create their own language, used stem cells to repair the pancreas of diabetic patients, discovered how to use the power of thought to manipulate a robotic arm, and sequenced the DNA of a 50,000-year-old cave girl. Science magazines are bursting with the latest breakthroughs emerging from the world's laboratories. We know so much.

Science is our best weapon in our fight against fate. Instead of giving in to the ravages of disease and natural disaster, we have created vaccines to combat deadly viruses like polio and Ebola. As the world's population continues to escalate, scientific advances provide the best hope of feeding the 9.6 billion people who are projected to be alive in 2050. Science warns us about the deadly impact we are having on our environment and gives us the chance to do something about it before it is too late. An asteroid might have wiped out the dinosaurs, but science is our best shield against any future direct hits. In the human race's constant battle with death, science is its best ally.

Science is king not only when it comes to our fight for survival but also in improving our quality of life. We are able to communicate with friends and family across vast distances. We have created virtual worlds to which we can escape in our leisure time and we can re-create in our living rooms the great performances of Mozart, Miles, and Metallica at the press of a button.

The desire to know is programmed into the human psyche. Early humans with a thirst for knowledge were the ones who survived to transform their environment. Those not driven by that craving were left behind. Evolution has favored the mind that wants to know the secrets of how the universe works. The adrenaline rush that accompanies the discovery of new knowledge is nature's way of telling us that the desire to know is as important as the drive to reproduce. As Aristotle suggested in the opening line of *Metaphysics*, understanding how the world works is a basic human need.

When I was a schoolkid, science very quickly captivated me. I fell in love with its extraordinary power to reveal the

workings of the universe. The fantastic stories that my science teachers told me seemed even more fanciful than the fiction I'd been reading at home. I persuaded my parents to buy me a subscription to *New Scientist* and devoured *Scientific American* in our local library. I hogged the television each week to watch episodes of *Horizon* and *Tomorrow's World*. I was enthralled by Jacob Bronowski's *Ascent of Man*, Carl Sagan's *Cosmos*, and Jonathan Miller's *Body in Question*. Every Christmas, the Royal Institution Christmas Lectures provided a dollop of science alongside our family turkey. My stocking was stuffed with books by George Gamow and Richard Feynman. It was a heady time, with new breakthroughs announced each week.

Alongside these stories of discovery, I began to get fired up by the untold tales. What we knew lay in the past but we didn't yet know the future, my future. I became obsessed with the puzzle books of Martin Gardner that my math teacher gave me. The excitement of wrestling with a conundrum and the sudden release of euphoria as I cracked each puzzle got me addicted to the drug of discovery. Those puzzles were my training ground for the greater challenge of tackling questions that didn't have an answer in the back of the book. It was the unanswered questions, the mathematical mysteries and scientific puzzles that no one had cracked, that would become the fuel for my life as a scientist.

It is quite extraordinary how much more we have understood about the universe even in the half century that I've been alive. Technology has extended our senses so we can see things that were beyond the conception of the scientists who excited me as a kid. A new range of telescopes that look out at the night sky enabled us to discover planets like Earth

that could be home to intelligent life. They have revealed the amazing fact that three quarters of the way into the lifetime of our universe, its expansion started to accelerate. I remember reading as a kid that we were in for a big crunch, but now it seems that we have a completely different future awaiting us.

Particle colliders like the Large Hadron Collider at CERN have allowed us to penetrate the inner workings of matter itself, revealing new particles—like the top quark discovered in 1994 and the Higgs boson discovered in 2012—that were bits of speculative mathematics when I was reading my *New Scientist* at school. And since the early '90s the fMRI scanner has allowed us to look inside the brain and discover things that were not even considered part of the remit of science when I was a kid back in the '70s. The brain was the preserve of philosophers and theologians, but today technology can reveal when you are thinking about Jennifer Aniston or predict what you are going to do next even before you know it yourself.

Biology has seen an explosion of breakthroughs. In 2003 it was announced that scientists had mapped an entire human DNA sequence consisting of 3 billion letters of genetic code. In 2011 the complete neuronal network of the *C. elegans* worm was published, providing a complete picture of how the 302 neurons in the worm are connected. Chemists, too, have been breaking new territory. A totally new form of carbon was discovered in 1985, which binds together like a football; and chemists surprised us again in 2003 by creating the first examples of graphene, showing how carbon can form a honeycomb lattice one atom thick.

We stand on the shoulders of giants, as Newton famously declared. And so my own journey to the frontiers of knowledge has pushed me to explore how others have articulated their work, to listen to lectures and seminars by those immersed in the fields I'm trying to understand, and to talk to those pushing the boundaries of what is known, questioning contradictory stories and consulting the evidence recorded in scientific journals. How much can you trust any of these stories? Just because the scientific community accepts a story as the current best fit doesn't mean it is true. Time and again, history reveals the opposite to be the case, and this must always act as a warning that current scientific knowledge is provisional. Mathematics has a slightly different quality, as a proof provides the chance to establish a more permanent state of knowledge. But even when I am creating a new proof, I will often quote results by fellow mathematicians whose proofs I haven't checked myself. To do so would mean running in order to keep still.

For any scientist the real challenge is not to stay within the secure garden of the known but to venture out into the wilds of the unknown. That is the challenge at the heart of this book.

WHAT WE DON'T KNOW

Despite all the breakthroughs made over the last centuries, there are still lots of deep mysteries waiting out there for us to solve. Things we don't know. The knowledge of what we don't know seems to expand faster than our catalog of breakthroughs. The known unknowns outstrip the known

knowns. And it is those unknowns that drive science. A scientist is more interested in the things he or she can't understand than in telling all the stories we already know the answers to. Science is a living, breathing subject because of all those questions we can't answer.

For example, the stuff that makes up the physical universe we interact with seems to account for only 4.9 percent of the total matter content of our universe. So what is the other 95.1 percent of so-called dark matter and dark energy made up of? If our universe's expansion is accelerating, where is all the energy coming from that fuels that acceleration?

Is our universe infinite? Are there infinitely many other infinite universes parallel to our own? If there are, do they have different laws of physics? Were there other universes before our universe emerged from the Big Bang? Did time exist before the Big Bang? Does time exist at all, or does it emerge as a consequence of more fundamental concepts?

How can we unify Einstein's theory of general relativity, the physics of the very large, with quantum physics, the physics of the very small? This is the search for something called quantum gravity, an absolute necessity if we are ever going to understand the Big Bang.

And what of the understanding of our human body, something so complex that it makes quantum physics look like a high school exercise? We are still trying to come to grips with the complex interaction between gene expression and our environment. Can we find a cure for cancer? Is it possible to beat aging? Could there be someone alive today who will live to be a thousand years old?

And what about where humans came from? Evolution is a process of random mutations, so would a different roll of the evolutionary dice still produce organisms with eyes? If we rewound evolution and pressed “play,” would we still get intelligent life, or are we the result of a lucky roll of the dice? Is there intelligent life elsewhere in our universe? And what of the technology we are creating? Can a computer ever attain consciousness? Will I eventually be able to download my consciousness so that my mind can survive the death of my body?

Mathematics, too, is far from finished. Despite popular belief, Fermat’s Last Theorem was not the last theorem. Mathematical unknowns abound. Are there any patterns in prime numbers, or are they outwardly random? Will we be able to solve the mathematical equations for turbulence? Will we ever understand how to factorize large numbers efficiently?

Despite so much that is still unknown, scientists are optimistic that these questions won’t remain unanswered forever. The last few decades give us reason to believe that we are in a golden age of science. The rate of discoveries in science appears to grow exponentially. In 2014 the science journal *Nature* reported that the number of scientific papers has been doubling every nine years since the end of World War II. Computers are also developing at an extraordinary rate. Moore’s Law has it that computer processing power will double every two years. Ray Kurzweil believes that the same applies to technological progress: that the rate of change over the next hundred years will be comparable to what we’ve experienced in the last 20,000 years.

Can scientific discovery really sustain this growth? Kurzweil talks about the Singularity, a moment when the intelligence of our technology will exceed human intelligence. Is scientific progress destined for its own singularity, a moment when we know it all? Surely at some point we might actually discover the underlying equations that explain how the universe works. We will discover the final particles that make up the building blocks of the physical universe and how they interact with each other. Some scientists believe that the current rate of scientific progress will lead to a moment when we might discover a theory of everything. They even give it a name: ToE.

As Stephen Hawking declared in *A Brief History of Time*, “I believe there are grounds for cautious optimism that we may be near the end of the search for the ultimate laws of nature.” He concludes dramatically with the provocative statement that then “we would know the mind of God.”

Is such a thing possible? To know everything? Would we want to know everything? Scientists have a strangely ambivalent relationship with the unknown. On the one hand, what we don't know is what intrigues and fascinates us, and yet the mark of success as a scientist is resolution and knowledge, to make the unknown known.

Are there limits to what we can discover? Are there quests that will never be resolved? Are some regions beyond the predictive powers of science and mathematics—like time before the Big Bang? Are there ideas so complex that they exceed the conception of our finite human brains? Can brains really investigate themselves, or does the analysis enter an infinite loop from which it is impossible to rescue itself? Are

there mathematical conjectures that can never be proved true?

It seems defeatist, even dangerous, to acknowledge such questions. While the unknown is the driving force for doing science, the unknowable is science's nemesis. As a fully signed-up member of the scientific community, I hope that we can ultimately answer the big open questions. So it seems important to know whether the expedition I've joined will hit boundaries beyond which we cannot proceed. Are there in fact any questions that won't ever get closure?

That is the challenge I've set myself in this book. I want to know whether there are things that, by their very nature, we will never know. Are there things that will always be beyond the limits of knowledge? Despite the marauding pace of scientific advances, are there things that will remain beyond the reach of even the greatest scientists? Mysteries that will forever remain part of the great unknown?

It is, of course, very risky at any point in history to try to articulate the Things We Cannot Know. How can you know what new insights will suddenly pull the unknown into the knowable? This is partly why it is useful to look at the history of how we came to know the things we know, because it reveals how often we've been at a point where we think we have reached the frontier, only to find a greater landscape beyond.

Take the statement made by French philosopher Auguste Comte in 1835 about the stars: "We shall never be able to study, by any method, their chemical composition or their mineralogical structure." An absolutely fair statement given that this knowledge seemed to depend on our visiting the star. What Comte hadn't considered was the possibility that

by those with political power. This is the domain of delusion. Repressed thoughts. The Freudian unconscious.

I would love to tell you about the unknown unknowns, but then they'd be known! Nassim Taleb, author of *The Black Swan*, believes that the emergence of unknowns is responsible for the biggest changes in society. For Kelvin, relativity and quantum physics turned out to be the great unknown unknown that he was unable to imagine. My hope in this book is to articulate the known unknowns and ask whether any will remain forever unknown.

I have called these unknowns "Edges." There are seven of them, and each one represents the horizon beyond which we cannot see. My journey to the Seven Edges of knowledge will pass through the known knowns, to demonstrate how we have traveled beyond what we previously thought were the limits of knowledge. This journey will also test my own ability to grasp what is known, because it's becoming increasingly challenging as a scientist to know even the knowns.

As much as this book is about what we cannot know, it is also important to understand what we do know and how we know it. My journey to the frontiers of knowledge will take me through the terrain that scientists have already mapped, to the very limits of today's breakthroughs. On the way I will stop to consider those moments when scientists thought they had hit a wall beyond which progress was no longer possible, only for the next generation to find a way. This will give us an important perspective on those problems that we might think are unknowable today. By the end of our journey, I hope this book will provide a comprehensive survey not just of what we cannot know but also of the things we do know.

To help me through these areas of science that are outside my comfort zone, I have enlisted the help of experts to guide me as I reach each science's Edge and to test whether it is my own limitations or limitations inherent in the questions I am tackling that make these questions unknowable.

What happens then if we encounter a question that cannot be answered? How does one cope with not knowing? Dare I admit to myself that some things will forever remain beyond my reach? How do we cope with not knowing? That challenge has elicited some interesting responses from humans across the millennia, not least the creation of an idea called God.

TRANSCENDENCE

There is another reason why I have been driven to investigate the unknowable, which is also related to my new job. The previous incumbent of the chair for the Public Understanding of Science was a certain Richard Dawkins. When I took over the position from Dawkins I braced myself for the onslaught of questions that I would get, not about science, but about religion. The publication of *The God Delusion* and his feisty debates with creationists resulted in Dawkins spending the later years of his tenure debating questions of religion and God.

So it was inevitable that when I took up the chair people would be interested in my stance on religion. My initial reaction was to distance myself from the debate about God. My job was to promote scientific progress and to engage the public in the breakthroughs happening around them. I was

keen to move the debate back to questions of science rather than religion.

In an urban environment like London, football has taken over the role that religion played in society of binding a community together, providing rituals that they can share. For me, the science that I began to learn as a teenager did a pretty good job of pushing out any vaguely religious thoughts I had as a kid. I sang in my local church choir, which exposed me to the ideas that Christianity had to offer for understanding the universe. School education in the 1970s in the United Kingdom was infused with mildly religious overtones: renditions of “All Things Bright and Beautiful” and the Lord’s Prayer in assemblies. Religion was dished up as something too simplistic to survive the sophisticated and powerful stories that I would learn in the science labs at my secondary school. Religion was quickly pushed out. Science . . . and football . . . were much more attractive.

Inevitably the questions about my stance on religion would not be fobbed off with such a flippant answer. I remember that during one radio interview on a Sunday morning on BBC Northern Ireland I was gradually sucked into considering the question of the existence of God. I guess I should have seen the warning signs. On a Sunday morning in Northern Ireland, God isn’t far from the minds of many listeners.

As a mathematician I am often faced with the challenge of proving the existence of new structures or coming up with arguments to show why such structures cannot exist. The power of the mathematical language to produce logical arguments has led a number of philosophers throughout the ages to resort to mathematics as a way of proving the

existence of God. But I always have a problem with such an approach. If you are going to prove existence or otherwise in mathematics, you need a very clear definition of what it is that you are trying to prove exists.

So after some badgering by the interviewer about my stance on the existence of God, I pushed him to try to define what God meant for him so that I could engage my mathematical mind. “It is something which transcends human understanding.” At first I thought: what a cop-out. You have just defined it as something that by its very nature I can’t get a handle on. But I became intrigued by this definition. Perhaps it wasn’t such a cop-out after all.

What if you *define* God as the things we cannot know. The gods in many ancient cultures were often placeholders for the things people couldn’t explain or understand. Our ancestors found volcanic eruptions or eclipses so mysterious that they became acts of gods. As science has explained such phenomena, these gods have retreated.

This definition has some things in common with a God commonly called the “God of the gaps.” This phrase was generally used as a derogatory term by religious thinkers who could see that this God was shrinking in the face of the onslaught of scientific knowledge, and a call went out to reject this kind of God. The phrase “God of the gaps” was coined by the Oxford mathematician and Methodist church leader Charles Coulson, when he declared: “There is no ‘God of the gaps’ to take over at those strategic places where science fails.”

But the phrase is also associated with a fallacious argument for the existence of God, one that Richard Dawkins spends some time shooting down in *The God Delusion*: if there

are things that we can't explain or know, there must be a God at work filling the gap. I am more interested not in the existence of a God to fill the gap, but in equating God with the abstract idea of the things we cannot know. Not in the things we currently don't know, but the things that by their nature we can never know—the things that will always remain transcendent.

Religion is more complex than the simple stereotype often offered up by modern society. For many ancient cultures in India, China, and the Middle East, religion was not about worshiping a supernatural intelligence so much as it was an attempt to appreciate the limits of our understanding and of language. As the theologian Herbert McCabe declared, "To assert the existence of God is to claim that there is an unanswered question about the universe." Science has pushed hard at those limits. So is there anything left? Will anything always be beyond the limit? Does McCabe's God exist?

This is the quest at the heart of this book. But first we need to know if, in fact, anything will remain unanswered about the universe. Is there really anything we cannot know?

knowing how it will land. Dice are the ultimate symbol of the unknowable. The future seems knowable only when it becomes the past.

I have always been extremely unsettled by things that I cannot work out. I don't mind not knowing something, provided there is some way ultimately to calculate the answer—with enough time. Is the fate of this perfect Las Vegas die truly unknowable? Or with enough information can I actually deduce its next move? Surely it's just a matter of applying the right laws of physics and solving the appropriate mathematical equations. Surely this is something I can figure out. Or is it?

My subject, mathematics, was invented to give people a glimpse of what's out there, to look into the future—to become masters of fate, not its servants. Mathematics is the science of patterns. Being able to spot a pattern is a powerful tool in the evolutionary fight for survival. The pattern of the sun means that I can rely on its rising in the sky tomorrow or on the moon running through twenty-eight sunrises before it becomes full again. The caves in Lascaux show how counting thirteen quarters of the moon from the first winter rising of the Pleiades will bring you to a time in the year when the horses are pregnant and easy to hunt. Being able to predict the future is the key to survival.

But some things appear to have no pattern or appear to have patterns so complex that they are beyond our ability to spot them. An individual roll of the dice is not like the rising of the sun. There seems to be no way of knowing which of the six faces will be pointing upward once the die finally comes to rest. This is why dice have been used since antiquity as a way to decide disputes, to play games, to wager money.

On a recent trip to Israel I took my children to an archeological dig at Beit Guvrin. It was such a popular settlement in ancient times that the site consists of layer upon layer of cities built on top of one another. There is so much stuff in the ground that the archeologists are happy to enlist amateurs like me and my kids to help excavate the site, even if a few pots are broken along the way. Sure enough, we pulled out lots of pottery shards, but we also kept unearthing animal bones. We thought they were the remains of dinner, but our guide explained that in fact they were the earliest form of dice.

Archeological digs of settlements dating back to Neolithic times have revealed a disproportionately high density of heel bones of sheep and other animals among the shattered pottery and flints that are usually found in such sites. These bones are, in fact, the ancestors of my casino dice. When thrown, the bones naturally land on one of four sides. Often there are letters or numbers carved into each side. These early dice are thought to have been used for divination, connecting the outcome of the roll of the dice to the will of the gods. Knowledge of how the dice would land was believed to transcend human understanding.

Over time, dice assumed a more prosaic place as part of our world of leisure. The first cube-shaped dice like the one on my desk were found around Harappa in what is now northeast Pakistan, where one of the first urban civilizations evolved, dating back to the third millennium BC. At the same time, you find four-faced pyramid dice appearing in a game that was discovered in the city of Ur, in ancient Mesopotamia. The Romans and Greeks were addicts of games of dice, as were medieval soldiers, who returned from the Crusades with

a new game called hazard, derived from the Arabic word for dice: *al-zahr*. It was an early version of craps, the game that is being played in the casinos in Vegas.

If I could predict the roll of the dice, the many games that depend on them would never have caught on. The excitement of backgammon or craps comes from not knowing what number you will throw. So perhaps gamers won't thank me as I try to break the mystery and predict the roll of my dice.

For centuries no one even thought that such a feat was possible. The ancient Greeks, who were among the first to develop mathematics as a tool to navigate their environment, didn't have any clue as to how to tackle such a dynamic problem. Their mathematics was a static, rigid world of geometry, not one that could cope with things tumbling across the floor. They could produce formulas to describe the contours of a cube, but once the cube started moving they were lost.

Aristotle believed that events could essentially be classified into three categories: "certain events" that happen by necessity following the laws of nature; "probable events" that happen in most cases but could have a few exceptions; and finally "unknowable events" that happened by pure chance. Aristotle put the roll of dice firmly in the last category.

As Christian theology made its impact on philosophy, matters worsened. Since the roll of the dice was in the hands of God, it was not something that humans could aspire to know. As St. Augustine put it, "We say that those causes that are said to be by chance are not non-existent but are hidden, and we attribute them to the will of the true God."

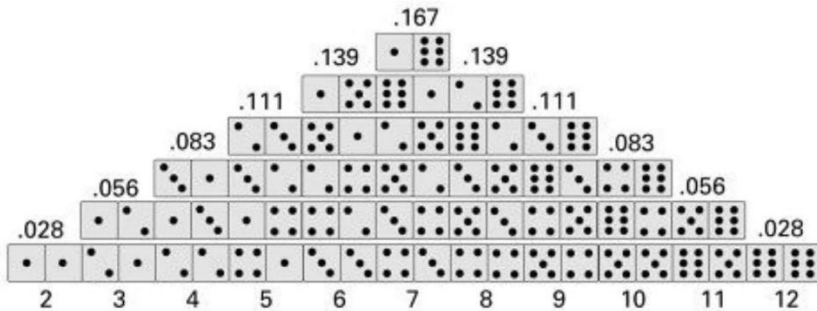
There was no such thing as chance, no free will. The unknowable was known to God, who determined the outcome. Any attempt to predict the roll of the dice was the work of a heretic, someone who dared to think they could know the mind of God. King Louis XI of France even went as far as to prohibit the manufacture of dice, believing that games of chance were ungodly. It wasn't until the sixteenth century that dice were wrestled out of the hands of God and put into the hands, and minds, of humans.

I've put two more dice next to my beautiful Las Vegas die. So here's a question: If I throw all three dice, is it better to bet on a score of 9 or a score of 10 coming up? Prior to the sixteenth century no tools were available to answer such a simple question. And yet anyone who had played for long enough would know that if I were throwing only two dice then it would be wise to bet on 9 rather than 10. Experience would tell you before too long that, on average, you get 9 a third more often than you get 10. With three dice it is harder to get a feel for which way to bet, because 9 and 10 seem to occur equally often. But is that really true?

In Italy at the beginning of the sixteenth century, an inveterate gambler by the name of Girolamo Cardano first realized that there are patterns that can be exploited in a game of dice. They weren't patterns that could be used on an individual throw. Rather, they emerged over the long run, patterns that a gambler like Cardano, who spent many hours throwing dice, could use to his advantage. So addicted was he to gambling that on one occasion he even sold his wife's possessions to raise the funds for the table stakes.

Cardano had the clever idea of counting how many different futures the dice could have. If I throw two dice,

there are thirty-six different futures. They are depicted in the following diagram.



Only three of them total 10, while four give you a score of 9. So Cardano reasoned that if you are throwing two dice, it makes sense to bet on 9 rather than 10. It did not help in any individual game, but in the long run it meant that Cardano would come out on top. Unfortunately, while he was a disciplined mathematician, he wasn't very disciplined when it came to gambling. He managed to lose all of his father's inheritance and would regularly get into knife fights with his opponents when the dice went against him.

He was nevertheless determined to get one prophecy correct. He had predicted the date of his death: September 21, 1576. To make sure he got this bet right he took matters into his own hands and committed suicide when the date finally struck. As much as I crave knowledge, I think this is going a little far. Indeed, knowing the date of your death is something that most people would prefer to opt out of. But Cardano was determined to win, even when he was dicing with Death.

Before taking his life, he wrote what many consider to be the first book that made inroads into predicting the behavior of dice as they roll across the table. Although written around

different future scenarios and divide the spoils according to which version of the future favored which player.

It is easy to get fooled here. There seem to be three scenarios. Fermat wins the next round and pockets sixty-four francs. Pascal wins the next round, resulting in a final round that either man wins. Fermat wins in two out of these three scenarios, so perhaps he should get two thirds of the winnings. This was the trap that de Méré fell into. Pascal argued that this wasn't correct: "The Chevalier de Méré is very talented but he is not a mathematician; this is, as you know, a great fault." A great fault, indeed!

Pascal argued that the spoils should be divided differently. There was a 50:50 chance that Fermat would win in one round, in which case he would get sixty-four francs. But if Pascal won the next round, then the two friends were equally likely to win the final round, so could divide the spoils thirty-two francs each. In either case, Fermat is guaranteed thirty-two francs. So the other thirty-two francs should be split equally, giving Fermat forty-eight francs in total.

Fermat, writing from his home near Toulouse, concurred with Pascal's analysis: "You can now see that the truth is the same in Toulouse as in Paris."

Pascal and Fermat's analysis of the game of points could be applied to much more complex scenarios. Pascal discovered that the secret to deciding the division of the spoils is hidden inside something now known as Pascal's triangle.

| | | | | | | |
|---|---|----|----|---|---|--|
| | | | 1 | | | |
| | | | 1 | 1 | | |
| | | 1 | 2 | 1 | | |
| | 1 | 3 | 3 | 1 | | |
| 1 | 4 | 6 | 4 | 1 | | |
| 1 | 5 | 10 | 10 | 5 | 1 | |

The triangle is constructed in such a way that each number is the sum of the two numbers immediately above it. The numbers you get are key to dividing the spoils in any interrupted game of points. For example, if Fermat needs two points for a win while Pascal needs four, then you consult the $2 + 4 = 6$ th row of the triangle and add the first four numbers together and the last two. This is the proportion in which you should divide the spoils. In this case it's a $1 + 5 + 10 + 10 = 26$ to $1 + 5 = 6$ division. So Fermat gets $26/32 \times 64 = \text{F}52$ and Pascal gets $6/32 \times 64 = \text{F}12$. In general, a game where Fermat needs n points to Pascal's m points can be decided by consulting the $(n + m)$ th row of Pascal's triangle.

The French may have been beaten by several millennia to the discovery that this triangle is connected to the outcome of games of chance. The Chinese were inveterate users of dice and other methods like the *I Ching* to try to predict the future. The text of the *I Ching* dates back some three thousand years and contains precisely the same table that Pascal produced to analyze the outcomes of tossing coins, but today the triangle is attributed to Pascal rather than the Chinese.

Pascal wasn't interested only in dice. He famously applied his new mathematics of probability to one of the great unknowns: the existence of God.

“God is, or He is not.” But to which side shall we incline? Reason can decide nothing here. There is an infinite chaos which separated us. A game is being played at the extremity of this infinite distance where heads or tails will turn up. . . . Which will you choose then? Let us see. Since you must choose, let us see which interests you least. You have two things to lose, the true and the good; and two things to stake, your reason and your will, your knowledge and your happiness; and your nature has two things to shun, error and misery. Your reason is no more shocked in choosing one rather than the other, since you must of necessity choose. . . . But your happiness? Let us weigh the gain and the loss in wagering that God is. . . . If you gain, you gain all; if you lose, you lose nothing. Wager, then, without hesitation that He is.

Called Pascal’s wager, the argument is hardly compelling. It hinges on the belief that the payout would be much greater if one opted for a belief in God. You lose little if you are wrong and win eternal life if correct. On the other hand, wager against the existence of God and losing results in eternal damnation, while winning gains you nothing beyond the knowledge that there is no God. The argument falls to pieces if the probability of God existing is actually zero. Even if it isn’t, the cost of belief might be too high when set against the probability of God’s existence.

The probabilistic techniques developed for dealing with uncertainty by mathematicians like Fermat and Pascal were incredibly powerful. Phenomena that were regarded as the expression of the gods were beginning to be within reach of the minds of men. Today these probabilistic methods are our best weapon for trying to navigate everything from the behavior of particles in a gas to the ups and downs of the stock market. Indeed, the very nature of matter itself seems to be at the mercy of the mathematics of probability, as we

shall discover in the Third Edge, when we apply quantum physics to predict what fundamental particles are going to do when we observe them. But for someone searching for certainty, these probabilistic methods represent a frustrating compromise.

I certainly appreciate the great intellectual breakthrough that Fermat, Pascal, and others made, but it doesn't help me to know the outcome when I throw my dice. As much as I've studied the mathematics of probability, it has always left me with a feeling of dissatisfaction. The one thing any course on probability drums into you is that it doesn't matter how many times in a row you get a 6: this has no influence on what will happen on the next throw.

So is there some way of knowing how my dice will land? Or is that knowledge always going to be out of reach? Not according to the revelations of a scientist in England.

THE MATHEMATICS OF NATURE

Isaac Newton is my all-time hero in the fight against the unknowable. The idea that I could possibly know everything about the universe has its origins in Newton's revolutionary work *Philosophiae Naturalis Principia Mathematica*. First published in 1687, the book is dedicated to developing a new mathematical language that promised to unlock how the universe behaves. It was a dramatically new model of how to do science. The work "spread the light of mathematics on a science which up to then had remained in the darkness of conjectures and hypotheses," declared the French physicist Alexis Clairaut in 1747.

The *Principia Mathematica* is also an attempt to unify, to create a theory that describes the celestial and the earthly, the big and the small. Johannes Kepler had come up with laws that described the motions of the planets, laws he'd developed empirically by looking at data and trying to come up with equations that would explain the data. Galileo had described the trajectory of a ball flying through the air. It was Newton's genius to understand that these were two examples of a single phenomenon: gravity.

Born on Christmas Day in 1643 in the Lincolnshire town of Woolsthorpe, Newton was always trying to tame the physical world. He made clocks and sundials, constructed miniature mills powered by mice, sketched countless plans for buildings and ships, and drew elaborate illustrations of animals. The family cat disappeared one day, carried away by a hot-air balloon that Newton had made. His school reports, however, did not anticipate a great future, describing him as "inattentive and idle."

Idleness is not necessarily such a bad trait in a mathematician. It can be a powerful incentive to look for some clever shortcut to solve a problem rather than relying on hard labor. But it's not generally a quality that teachers appreciate. Newton was doing so badly at school that his mother decided the whole thing was a waste of time and that he'd be better off learning how to manage the family farm in Woolsthorpe. Unfortunately, Newton was equally hopeless at managing the family estate, so he was sent back to school. Although probably apocryphal, it is said that Newton's sudden academic transformation coincided with a blow to the head that he received from the school bully. Whether true or not, his academic transformation saw him suddenly excelling

take an even smaller snapshot. What about halving the window of time again:

$$(10.25 \times 10.25 - 10 \times 10)/0.25 = 20.25 \text{ meters per second.}$$

I hope the mathematician in you has spotted the pattern. If I take a window of time that is x seconds, the average speed over this time will be $20 + x$ meters per second. The speed as I take smaller and smaller windows of time is getting closer and closer to twenty meters per second. So, although to calculate the speed at 10 seconds looks like I have to figure out the calculation $0/0$, the calculus makes sense of what this should mean.

Everything around us is in a state of flux, so it was perhaps no wonder that his discovery would be so influential. But for Newton calculus was a means to an end, a personal tool that helped him reach the scientific conclusions that he documents in the *Principia*, the great treatise published in 1687 that describes his ideas on gravity and the laws of motion.

Writing in the third person, he explained that his calculus was key to the scientific discoveries contained inside: “By the help of this new Analysis Mr. Newton found out most of the propositions in the *Principia*.” And yet no account of the “new analysis” is published. He privately circulated his ideas among friends, but he felt no urge to publish them for others to appreciate.

Fortunately calculus is now widely taught. It is a language that I spent years learning and gaining fluency in as a mathematical apprentice. To know my dice, I will need to mix Newton’s mathematical breakthrough with his great contribution to physics: the famous laws of motion with which he opens his *Principia*.

A THEORY OF EVERYTHING

Newton outlines in the *Principia* three simple laws from which so much of the dynamics of the universe evolve.

Newton's First Law of Motion: *A body will continue in a state of rest or uniform motion in a straight line unless it is compelled to change that state by forces acting on it.*

This sounds obvious, but it was not so obvious to the likes of Aristotle. If you roll a ball along a flat surface it comes to a rest. It looks like you need a force to keep it moving. There is, however, a hidden force that is changing its speed: friction. If I throw my dice in outer space, away from any gravitational fields, then they will carry on flying in a straight line at constant speed ad infinitum.

In order to change an object's speed or direction you need a force to act against it. Newton's second law explained how that force would change its motion. Calculus has already allowed me to calculate the speed of my die as it falls down toward the table. The rate of change in that speed may be deduced by applying calculus again. Newton's second law holds that there is a direct relationship between the force being applied and the rate of change in speed.

Newton's Second Law of Motion: *The rate of change of motion, or acceleration, is proportional to the force that is acting on it and inversely proportional to its mass.*

To understand the speed of my cascading dice, I need to understand the forces acting on them. Newton's Universal Law of Gravitation identified one of the principal forces affecting everything from the fall of an apple to the movement of a planet through the solar system. The law

states that the force acting on a body of mass m_1 by another body of mass m_2 that is a distance of r away is equal to

$$\frac{G \times m_1 \times m_2}{r^2}$$

where G is an empirical physical constant that controls how strong gravity is in our universe.

With these laws I can now describe the trajectory of a ball flying through the air, or a planet traveling through the solar system, or of dice falling from my hand. The next problem occurs when the dice hit the table. What happens then? Newton has a third law, which provides a clue.

Newton's Third Law of Motion: When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction to that of the first body.

Newton himself used these laws to deduce an extraordinary string of observations about the solar system. As he wrote, "I now demonstrate the system of the World." To apply his ideas to the trajectory of the planets, he began by reducing each planet to a point located at the center of mass and assumed that all the planet's mass was concentrated at this point. Then, by applying his laws of motion and his new mathematics, he successfully deduced Kepler's laws of planetary motion.

He was now able to calculate the relative masses of the large planets, the Earth and the sun, and to explain a number of the curious irregularities in the motion of the moon due to the pull of the sun. He deduced that the Earth is not a perfect sphere and suggested it must be squashed between the poles

due to its rotation, causing a centrifugal force. The French thought the opposite was true: that the Earth should be pointy in the direction of the poles. An expedition set out in 1733 that proved Newton—and the power of mathematics—correct.

It was an extraordinary feat. Newton's three laws meant that all motion of particles in the universe could potentially be deduced. He had come up with the seeds of a Theory of Everything. It took other scientists to grow these seeds and apply them to more complex settings. In their original form, Newton's laws were not suited to describing the motion of less rigid bodies or bodies that deform. It was the great eighteenth-century Swiss mathematician Leonhard Euler who would provide equations that generalized Newton's laws. Euler's equations could be applied more generally to something like a vibrating string or a swinging pendulum.

After Newton, more equations were developed that explained various natural phenomena. Euler produced equations for nonviscous fluids. At the beginning of the nineteenth century, French mathematician Joseph Fourier found equations to describe heat flow. Compatriots Pierre-Simon Laplace and Siméon-Denis Poisson took Newton's equations to produce more generalized equations for gravitation, which were then seen to control other phenomena like hydrodynamics and electrostatics. The behaviors of viscous fluids were described by the Navier-Stokes equations, and electromagnetism by James Clerk Maxwell's equations.

With the discovery of calculus and the laws of motion, it seemed that Newton had turned the universe into a deterministic clockwork mechanism controlled by

mathematical equations. Scientists believed they had indeed discovered the Theory of Everything. In his *Philosophical Essay on Probabilities* published in 1812, the mathematician Pierre-Simon Laplace summed up most scientists' belief in the extraordinary power of mathematics to explain everything about the physical universe: "We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed; if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes."

This view that the universe was knowable, both past, present and future, became dominant among scientists in the centuries following Newton's great opus. It seemed as if any thought of God acting in the world had been completely removed. God might be responsible for getting things up and running, but from that point on, mathematics and physics took over.

So what of my lowly die? Surely with the laws of motion at hand I can simply combine my knowledge of its geometry with information on its motion and subsequent interactions with the table to predict the outcome. I've written out the equations on my notepad, and they look pretty daunting.

Newton, as it turns out, also contemplated the problem of how to predict the role of dice. His interest was prompted by a letter he received from Samuel Pepys, who wanted

just needs to apply Newton's second law to map out the course of the planets into the distant future. The trouble was that the math was extremely tricky.

Newton had solved the behavior of two planets (or a planet and a sun). They would follow elliptical paths, with their common focal point being the common center of gravity. This would repeat itself periodically to the end of time. But Newton was stumped when he introduced a third planet. Trying to calculate the behavior of a solar system consisting of the sun, the Earth, and the moon seemed simple enough, but already you are facing an equation with eighteen variables: nine for the positions and nine for the speeds of the three planets. Newton conceded that "to consider simultaneously all these causes of motion and to define these motions by exact laws admitting of easy calculation exceeds, if I am not mistaken, the force of any human mind."

King Oscar II of Norway and Sweden decided to mark his sixtieth birthday in 1889 by offering a prize for solving a problem in mathematics. There are not many monarchs around the world who would choose math problems to celebrate their birthdays, but Oscar had enjoyed the subject ever since he had excelled at it as a student at Uppsala University.

His majesty Oscar II, wishing to give a fresh proof of his interest in the advancement of mathematical science, has resolved to award a prize on January 21, 1889, to an important discovery in the field of higher mathematical analysis. The prize will consist of a gold medal of the eighteenth size bearing his majesty's image and having a value of a thousand francs, together with the sum of two thousand five hundred crowns.

Three eminent mathematicians convened to choose a number of suitable challenges and to judge the entries. One of the questions they posed was to establish mathematically whether the solar system was stable. Would it continue to turn like clockwork, or, at some point in the future, might the Earth spiral off into space and disappear from our solar system?

To answer this question it would be necessary to solve the equation that had stumped Newton. Poincaré believed that he had the skills to win the prize. One of the common tricks used by mathematicians is to attempt a simplified version of the problem first, to see if it is tractable. So Poincaré started with three bodies. This was still far too difficult, so he decided to simplify the problem further. Instead of the sun, Earth, and moon, why not try to understand two planets and a speck of dust? The two planets won't be affected by the dust particle, so he could assume, thanks to Newton's solution, that they just repeated ellipses around each other. The speck of dust, on the other hand, would experience the gravitational force of the two planets. Poincaré set about trying to describe the path traced by the speck of dust. Some understanding of its trajectory would form an interesting contribution to the problem.

Although he couldn't crack the problem completely, the paper he submitted was more than good enough to secure King Oscar's prize. He'd managed to prove the existence of an interesting class of paths that would repeat themselves, so-called periodic paths. Periodic orbits were by their nature stable because they would repeat themselves over and over, like the ellipses that two planets would be guaranteed to execute.

The French authorities were very excited that the award had gone to one of their own. The nineteenth century had seen Germany steal a march on French mathematics, so the French academicians excitedly heralded Poincaré's win as proof of a resurgence of French mathematics. Gaston Darboux, the permanent secretary of the French Academy of Sciences, declared, "From that moment on the name of Henri Poincaré became known to the public, who then became accustomed to regarding our colleague no longer as a mathematician of particular promise but as a great scholar of whom France has the right to be proud."

Preparations began for the publication of Poincaré's solution in a special edition of the Royal Swedish Academy of Science's journal *Acta Mathematica*. Then came the moment every mathematician dreads. Poincaré thought his work was safe. He'd checked every step in the proof. But just before publication, one of the editors of the journal raised a question over one of the steps in his mathematical argument.

Poincaré had assumed that a small change in the positions of the planets, a little rounding up or down here or there, was acceptable, as it would result in only a small change in their predicted orbits. It seemed a fair assumption. But there was no justification given for why this would be so. And in a mathematical proof, every step, every assumption, must be backed up by rigorous mathematical logic.

The editor wrote to Poincaré for some clarification on this gap in the proof. But as Poincaré tried to justify this step, he realized he'd made a serious mistake. He wrote to Gösta Mittag-Leffler, the head of the prize committee, hoping to limit the damage to his reputation:

The consequences of this error are more serious than I first thought. I will not conceal from you the distress this discovery has caused me. . . . I do not know if you will still think that the results which remain deserve the great reward you have given them. (In any case, I can do no more than to confess my confusion to a friend as loyal as you.) I will write to you at length when I can see things more clearly.

Mittag-Leffler decided he needed to inform the other judges. He did so in a letter:

Poincaré's memoir is of such a rare depth and power of invention, it will certainly open up a new scientific era from the point of view of analysis and its consequences for astronomy. But greatly extended explanations will be necessary and at the moment I am asking the distinguished author to enlighten me on several important points.

As Poincaré struggled away he soon saw that he was simply mistaken. Even a small change in the initial conditions could result in wildly different orbits. He couldn't make the approximation that he'd proposed. His assumption was wrong.

Poincaré telegraphed Mittag-Leffler to break the bad news and tried to stop the paper from being printed. Embarrassed, he wrote:

It may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible.

Mittag-Leffler was "extremely perplexed" to hear the news:

It is not that I doubt that your memoir will be in any case regarded as a work of genius by the majority of geometers and that it will be the departure point for all future efforts in celestial mechanics. Don't therefore think that I regret the prize. . . . But here is the worst of it. Your letter arrived too late and the memoir has already been distributed.

Mittag-Leffler's reputation was on the line for not having picked up the error before they'd publicly awarded Poincaré the prize. This was not the way to celebrate his monarch's birthday! "Please don't say a word of this lamentable story to anyone. I'll give you all the details tomorrow."

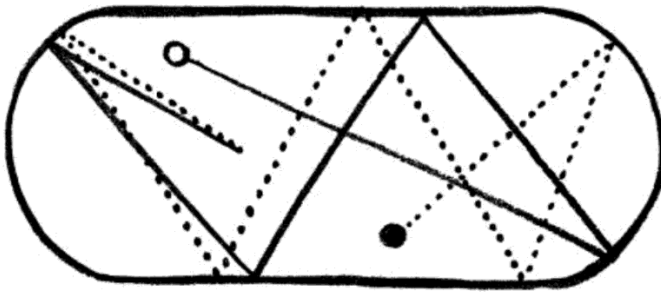
The next few weeks were spent trying to retrieve the printed copies without raising suspicion. Mittag-Leffler suggested that Poincaré should pay for the printing of the original version. Poincaré, who was mortified, agreed, even though the bill came to over 3,500 crowns, 1,000 crowns more than the prize he'd originally won.

In an attempt to rectify the situation, Poincaré set about trying to sort out his mistake, to understand where and why he had gone wrong. In 1890 Poincaré wrote a second, extended paper explaining his belief that very small changes could cause an apparently stable system suddenly to fly apart.

What Poincaré discovered, thanks to his error, led to one of the most important mathematical concepts of the twentieth century: chaos theory. It was a discovery that placed huge limits on what we humans could ever hope to know. I may have written down all the equations for my die, but what if it behaves like the planets in the solar system? According to Poincaré's discovery, if I make just one small error in recording its starting location, that error could expand into a large difference in the outcome by the time the

trajectory it won't alter the course of the planet much. But the solar system seems to be playing a slightly more interesting game of billiards than the ones I played as a student.

Rather surprisingly, if you change the shape of the billiard table this intuition turns out to be wrong. If you shoot balls around a billiard table shaped like a stadium with semicircular ends but straight sides, the paths can diverge dramatically even if the balls started off heading in almost exactly the same direction. This is the signature of chaos theory: sensitivity to very small changes in the initial conditions.



Two quickly diverging paths taken by a billiard ball around a stadium-shaped billiard table

So the challenge is to determine whether the fall of my dice can be predictable, like a conventional game of billiards, or whether we are playing a giant game of chaotic billiards.

Poincaré is generally credited as the father of chaos theory, but the sensitivity of dynamic systems to small changes was not very well known for decades into the twentieth century. It really took the rediscovery of the phenomenon by scientist Edward Lorenz, who like Poincaré

thought he'd made some mistake, before the ideas of chaos theory became more widely known.

While working as a meteorologist at the Massachusetts Institute of Technology in 1963, Lorenz had been running equations for the change of temperature in a dynamic fluid on his computer when he decided he needed to rerun one of his models for longer. So he took some of the data that had been output earlier in the run and re-entered it, expecting to be able to restart the model from that point.

When he returned from coffee, he discovered to his dismay that the computer hadn't reproduced the previous data and that it had very quickly generated a wildly divergent prediction. At first he couldn't understand what was happening. If you input exactly the same numbers into an equation, you don't expect to get a different answer at the other end. It took him a while to realize what was going on: he hadn't input the same numbers. The computer printout of the data he'd used had only printed the numbers to three decimal places, while it had been calculating using the numbers to six decimal places.

Even though the numbers were different, they differed only in the fourth decimal place. You wouldn't expect this to make that big a difference, but Lorenz was struck by the impact of such a small difference on the resulting data. Here are two graphs created using the same equation, where the data that are put into the equations differ very slightly. One graph uses the input data 0.506127 and the second graph approximates this to 0.506. Although the graphs start out following similar paths, they very quickly behave completely differently.



The model that Lorenz was running was a simplification of weather models that analyzed how the flow of air behaves when subjected to differences in temperature. His rediscovery of how small changes in starting conditions can have such a big impact on future outcomes would have huge implications for our attempts to use mathematical equations to predict the future. As Lorenz wrote, “Two states that were imperceptibly different could evolve into two considerably different states. Any error in the observation of the present state—and in a real system, this appears to be inevitable—may render an acceptable prediction of the state in the distant future impossible.”

When Lorenz sought to explain his findings to a colleague, he was told, “Edward, if your theory is correct, one flap of a seagull’s wings could alter the course of history forever.” The seagull would eventually be replaced by the now famous butterfly when Lorenz presented his findings in 1972 at the American Association for the Advancement of Science in a paper titled “Does the Flap of a Butterfly’s Wings in Brazil Set off a Tornado in Texas?”

Curiously, both the seagull and the butterfly might have been preempted by the grasshopper. It seems that already in 1898 Professor W. S. Franklin had realized the devastating

effect that the insect community could have on the weather. In a book review, he posited, “An infinitesimal cause may produce a finite effect. Long-range detailed weather prediction is therefore impossible, and the only detailed prediction which is possible is the inference of the ultimate trend and character of a storm from observations of its early stages; and the accuracy of this prediction is subject to the condition that the flight of a grasshopper in Montana may turn a storm aside from Philadelphia to New York!”

This is an extraordinary position to be in. Science offers a completely deterministic description of the evolution of many dynamic systems like the weather. And yet in many cases we are denied access to its predictions, as any measurement of the location or wind speed of a particle is inevitably going to be an approximation of its true conditions.

The National Weather Service, when making weather predictions, takes the data recorded by weather stations dotted across a region and then, instead of running equations on these data, the meteorologists do several thousand runs, varying the data over a range of values. The predictions stay close for a while, but by about five days into the future the results have often diverged so wildly that one set of data predicts a heat wave while a few changes in the decimal places of the data result in drenching rain.

“There is a maxim which is often quoted, that ‘The same causes will always produce the same effects,’” wrote the great Scottish scientist James Clerk Maxwell in his book *Matter and Motion*, published in 1877. “There is another maxim which must not be confounded with this, which asserts that ‘Like causes produce like effects.’ This is only true when small

variations in the initial circumstances produce only small variations in the final state of the system.” The discovery of chaos theory in the twentieth century revealed this maxim to be false.

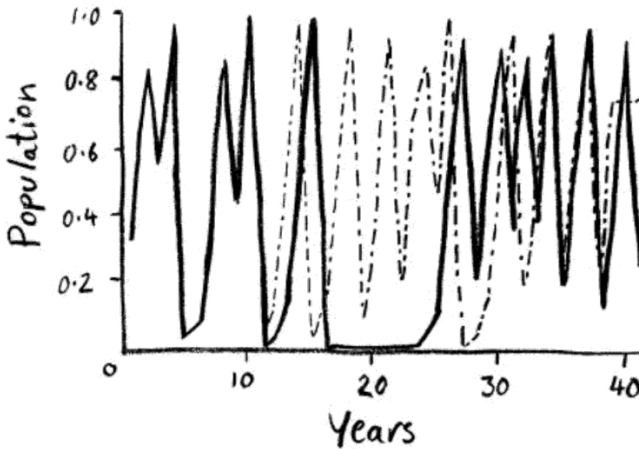
Of course, there are times when small changes don't alter the course of the equations dramatically, like the paths in the classic billiard table. So how can you know the point when you can't know what is going to happen next? Robert May discovered a beautiful example when he analyzed the equations for population growth.

Born in Australia in 1938, May had originally trained as a physicist working on superconductivity, but his academic work took a dramatic turn when he was exposed in the late 1960s to the newly formed movement for social responsibility in science. His attention shifted from the behavior of collections of electrons to the more pressing questions of the behavior of population dynamics in animals. Biology, at the time, was not a natural environment for the mathematically minded, but following May's work that would change. This fusion of the hardcore mathematical training he'd received as a physicist with a new interest in biology led to his great breakthrough.

In a paper in *Nature* called “Simple Mathematical Models with Very Complicated Dynamics,” published in 1976, May explored the dynamics of a mathematical equation describing population growth from one season to the next. He revealed how even a quite basic equation can produce extraordinarily complex results. His equation for population dynamics wasn't some complicated differential equation but a simple, discrete feedback equation that anyone with a calculator could explore.

was going on in this higher region. And it was what he called chaos.

Beyond $r = 3.56995$ (or, more precisely, the limit point of a system of equations of increasing degree), the result becomes very sensitive to what the initial population looks like. Change the initial number of animals by a minute amount and a totally different result can ensue.



Two populations with $r = 4$ that start off with a difference of just one animal in a thousand. Although they start behaving similarly, by year 15 they are demonstrating very different behaviors.

As I turn up the dial on r , there can still be pockets of regular behavior, as Jim Yorke had discovered. For example, take $r = 3.627$ and the population becomes periodic again, bouncing around between six different values. Keep dialing r up and the six changes to twelve, which becomes twenty-four, doubling each time until chaos strikes again.

Bob May recognized just what a warning shot such a simple system was to anyone who thought they knew it all: “Not only in research, but in the everyday world of politics

and economics, we would be better off if more people realized that simple systems do not necessarily possess simple dynamic properties.”

THE POLITICS OF CHAOS

Bob May is currently practicing what he preaches. Or perhaps I should say Lord May of Oxford, as I was corrected by a man in a top hat who greeted me at the door to the entrance of the House of Lords. May has in recent years combined his scientific endeavors with energetic political activism. He now sits as a cross-party member of the House of Lords, where I popped in for lunch to find out how he was faring in his mission to alert politicians to the impact of chaotic systems on society.

Ushered through the entrance to the Lords by the man in the top hat and policemen with machine guns, I found May waiting for me on the other side of metal detectors and X-ray machines. May has no truck with all these formal titles and, in his earthy Australian manner, still insists on being called Bob. “I’m afraid I messed up and already ate lunch but I’ll come and eat cake while you get some lunch,” he said with a guileless smile. As I ate my fish he consumed an enormous piece of chocolate cake. At seventy-nine, May is as energetic and engaged as ever, and he was rushing off after his second lunch to a select committee discussing the impact of a new rail link between London and northwest England. Before joining the Lords, May was chief scientific adviser both to John Major’s Conservative government and Tony Blair’s Labor government. I asked how tricky a balancing act such a

political position is for a man who generally is not scared to tell it like it is.

“At the interview I was told that there would be occasions where I would be called upon to defend the decisions of a minister and how would I feel about that? I said that I would never under any circumstances deny a fact. On the other hand, I’m fairly good at the kind of debating competition where you’re given a topic and according to a flip of a coin you’ve got to argue for either side of the debate. So I said I’d be happy explaining why the minister’s choice was arrived at. I simply wouldn’t agree to endorse it if it wasn’t right.”

A typical mathematician’s response. Set up the minister’s axioms and then demonstrate the proof that led to the conclusion—a judgment-free approach. That’s not to say that May isn’t opinionated and prepared to give his own views on the subject at hand.

I was curious as to how governments deal with the problems that chaos theory creates when trying to make policy decisions. How do politicians cope with the challenges of predicting or manipulating the future, given that we can have only partial knowledge of the systems being analyzed?

“I think that’s rather a flattering account of what goes on here,” he said. “With some notable exceptions it’s mostly a bunch of very egotistical people, very ambitious people, who are primarily interested in their own careers.”

What about May personally? What impact did the discoveries he’d made have on his view of science’s role in society?

“It was weird. It was the end of the Newtonian dream. When I was a graduate student it was thought that with better and better computer power we would get better and

better weather predictions because we knew the equations and we could make more realistic models of the Earth.” But May is cautious not to let the climate change deniers use chaos theory as a way to undermine the debate.

“Not believing in climate change because you can’t trust weather reports is a bit like saying that because you can’t tell when the next wave is going to break on Bondi beach you don’t believe in tides.”

May likes to quote a passage from Tom Stoppard’s play *Arcadia* to illustrate the strange tension that exists between the power of science to know some things with extraordinary accuracy and chaos theory, which denies us knowledge of many parts of the natural world. One of the protagonists, Valentine, declares, “We’re better at predicting events at the edge of the galaxy or inside the nucleus of an atom than whether it’ll rain on auntie’s garden party three Sundays from now.” May jokes that his most cited works are not the high-profile academic papers he’s published in prestigious scientific journals like *Nature*, but the program notes he wrote for Stoppard’s play when it was first staged at the National Theater in London. “It makes a mockery of all these citation indexes as a way of measuring the impact of scientific research.”

So what are the big open questions of science that May would like to know the answer to? Consciousness? An infinite universe?

“I think I’d look at it in a less grand way, so I’d look at it more in terms of the things I am working on at the moment. Largely by accident I’ve been drawn into questions about banking.”

That was a surprise. The question of how to create a stable banking system seemed very parochial, but May has recently been applying his models of the spread of infectious diseases and the dynamics of ecological food webs to understanding the banking crisis of 2008. Working with Andrew Haldane at the Bank of England, he has been considering the financial network as if it were an ecosystem. Their research has revealed how financial instruments intended to optimize returns with seemingly minimal risk can cause instability in the system as a whole.

May believes that the problem isn't necessarily the mechanics of the market itself. It's the way small things in the market are amplified and perverted by how humans interact with them. For him, the most worrying thing about the banking mess is getting a better handle on this contagious spreading of worry.

“The challenge is: How do you put human behavior into the model? I don't think human psychology is mathematizable. Here we are throwing dice with our future. But if you're trying to predict the throw of the dice, then you want to know the circumstance of who owns the dice.”

That was something I hadn't taken into account. Perhaps I should factor in who sold me the casino die in the first place.

“I think many of the major problems facing society are outside the realm of science and mathematics,” he said. “It's the behavioral sciences that are the ones we are going to have to depend on to save us.”

Looking around the canteen at the House of Lords, you could see the sheer range and complexity of human behavior at work. It makes the challenge of mathematizing even the interactions in this tiny microcosm of the human population

throw of the evolutionary dice. But there is a second important strand to Darwin's proposal, which is the idea of natural selection. Some of those random changes will give offspring an increased chance of survival, while others will result in a disadvantage. The point of evolution by natural selection is that the organism with the advantageous change will be more likely to survive long enough to reproduce.

Suppose, for example, that I start with a population of giraffes with short necks. The environment of the giraffes changes such that there is more food in the trees, so that any giraffe born with a longer neck is going to have a better chance of survival. Let's suppose that I throw my Vegas die to determine the chance of a mutation for each giraffe born in the next generation following this environmental change. A roll of a 1, 2, 3, 4, or 5 condemns the giraffe to a neck of the same size or shorter, while a throw of a 6 corresponds to a chance mutation that causes a longer neck. The lucky longer-necked giraffes get the food, and the shorter-necked giraffes don't survive to reproduce. So it is just the longer-necked giraffes that get the chance to pass on their DNA.

In the next generation the same thing happens. Roll a 1, 2, 3, 4, or 5 on the die and the giraffe doesn't grow any taller than its parents. But another 6 and the giraffe's neck grows a bit more. The taller giraffes survive again. The environment favors the giraffes that have thrown a 6. Each generation ends up a bit taller than the last generation until there comes a point when it is no longer an advantage to grow any further.

The combination of chance and natural selection results in our seeing more giraffes with ancestors that all threw 6s. In retrospect it looks like amazing serendipity that you would see so many 6s in a row. But the point is that you don't see

any of the other rolls of the dice because they don't survive. What looks like a rigged game is just the result of the combination of chance and natural selection. There is no grand design at work. The run of consecutive 6s isn't a lucky streak; it is the only thing we would expect to see from such a model.

It's a beautifully simple model, but, given the complexity of the changes in the environment and the range of mutations that can occur, this simple model can produce extraordinary complexity, which is borne out by the sheer variety of species that exist on Earth. Although the model is simple, it is frustratingly inadequate at making predictions. One of the reasons I never really fell in love with biology is that there seemed to be no way to explain why we got cats and zebras out of this evolutionary model and not some other strange selection of animals. It all seemed so arbitrary, so random. But is that really right?

There is an interesting debate going on in evolutionary biology about how much chance there is in the outcomes we are seeing. If we rewind the story of life on Earth to some point in the past and threw the dice again, would we see very similar animals appearing or could we get something completely different? This is the question that May raised at the end of our lunch.

It does appear that some parts of our evolutionary process seem inevitable. It is striking that throughout evolutionary history, the eye evolved independently fifty to a hundred times. This is strong evidence for the fact that the different rolls of the dice that have occurred across different species seem to have produced species with eyes regardless of what is going on around them. Lots of other examples illustrate how

some features, if they are advantageous, seem to rise to the top of the evolutionary swamp. This is illustrated every time you see the same feature appearing more than once in different parts of the animal kingdom. Dolphins and bats, for example, use echolocation, but they evolved this trait independently at very different points on the evolutionary tree.

But it isn't clear how far these outcomes are guaranteed by the model. If there is life on another planet, will it look anything like the life that has evolved here on Earth? This is one of the big open questions in evolutionary biology. As difficult as it may be to answer, I don't believe it qualifies as something we can never know. It may remain something we will never know, but there is nothing by its nature that makes it unanswerable.

Are there other great unsolved questions of evolutionary biology that might be contenders for things we can never know? For example, why, 542 million years ago, at the beginning of the Cambrian period, was there an explosion in the diversity of life on Earth? Before this moment life consisted of single cells that collected into colonies. But over the next twenty-five million years, a relatively short period on the scale of evolution, there is a rapid diversification of multicellular life that ends up resembling the diversity that we see today. An explanation for this exceptionally fast pace of evolution is still missing. This is in part due to lack of data from that period. Can we ever recover that information, or could this always remain a mystery?

Chaos theory is usually a limiting factor in what we can know about the future. But it can also imply limits on what we can know about the past. We see the results, but deducing

the cause means running the equations backward. Without complete data, the same principle applies backward as forward. We might find ourselves at two very divergent starting points that can explain very similar outcomes. But we'll never know which of those origins was ours.

One of the big mysteries in evolutionary biology is how life got going in the first place. The game of life may favor runs of 6s on the roll of the evolutionary dice, but how did the game itself evolve? Various estimates have been proposed for the chances of everything lining up to produce molecules that replicate themselves. In some models, the origin of life is equivalent to nature having thrown thirty-six dice and getting them all to land on 6. For some, this is proof of the existence of God or of some form of a grand designer to rig the game. But this is to misunderstand the huge time scale that we are working on.

Miracles do happen . . . given enough time. Indeed, it would be more striking if we didn't get these anomalies. The point is that the anomalies stick out. They get noticed, while the less exciting rolls of the dice are ignored.

The lottery is a perfect test bed for the occurrence of miracles in a random process. On September 6, 2009, the following six numbers were the winning numbers in the Bulgarian state lottery: 4, 15, 23, 24, 35, 42. Four days later, the same six numbers came up again. Incredible, you might think. The government in Bulgaria certainly thought so and ordered an immediate investigation into the possibility of corruption. But what it failed to take into account is that each week, across the planet, different lotteries are being run. They have been running for decades. If you do the math, it

would be more surprising not to see such a seemingly anomalous result.

The same principle applies to the conditions for producing self-replicating molecules in the primeval soup that made up the Earth before life emerged. Mix together plenty of hydrogen, water, carbon dioxide, and some other organic gases and subject them to lightning strikes and electromagnetic radiation, and already experiments in the lab show the emergence of organic material found only in living things. No one has managed to spontaneously generate anything as extraordinary as DNA in the lab. The chances of that are very small.

But that's the point. Given the billion billion or so possible planets available in the universe on which to try out this experiment, together with a billion or so years to let the experiment run, it would be more striking if that outside chance of creating something like DNA didn't happen. Keep rolling thirty-six dice on a billion billion different planets for a billion years and you'd probably get one roll with all thirty-six dice showing 6. Once you have a self-replicating molecule, it has the means to propagate itself, so you only need to get lucky once to kick off evolution.

THE FRACTAL TREE OF LIFE

Our problem as humans is that we have not evolved minds able to navigate very large numbers. Probability is something we have little intuition for. But it's not only the mathematics of probability that is at work in evolution. The evolutionary

outcomes of the mechanism of evolution by speeding up time. But the model will only be as good as our hypotheses. If we've got the model wrong, it won't tell us what is really happening in nature.

Computer models such as these hold the key to answering the question Poincaré first tackled when he discovered chaos: Will there even be a stable Earth orbiting the sun for evolution to continue playing its game of dice? How safe is our planet from the vagaries of chaos? Is our solar system stable and periodic, or do I have to worry about a grasshopper disrupting our orbit around the sun?

A BUTTERFLY CALLED MERCURY

Poincaré wasn't able to answer the King of Sweden's question about the solar system, that is, whether it would remain in a stable equilibrium or fly apart in a catastrophic exhibition of chaotic motion. His discovery that some dynamic systems can be sensitive to small changes in data opened up the possibility that we may never know the precise fate of the solar system much in advance of any potentially devastating scenario unfolding.

It is possible that the solar system is in a safe, predictable region of activity, but the evidence suggests we can't console ourselves with this comforting mathematical hope. Recent computer modeling has provided us with new insights, which reveal that the solar system is indeed within a region dominated by the mathematics of chaos. We can now measure how big an effect a small change will have on the outcome of a closed system using something called the Lyapunov

exponent. If the Lyapunov exponent is positive, it means that if I make a small change in the initial conditions then the distance between the paths will diverge exponentially.

Using this new equation, several groups of scientists have confirmed that our solar system is indeed chaotic. They have calculated that the distance between two initially close orbital solutions increases by a factor of ten every ten million years. This is certainly on a different timescale to our inability to predict the weather. Nevertheless, it means that I can have no definite way of knowing what will happen to the solar system over the next five billion years.

If you're wondering in despair whether we can know anything about the future, then take heart in the fact that mathematics isn't completely hopeless at making predictions. There is an event that the equations guarantee will occur if we make it to five billion years from now, but it's not good news. At this point, the sun will run out of fuel and evolve into a red giant that will engulf Earth and the other planets in our solar system. But until this solar blowout, I am faced with trying to solve chaotic equations if I want to know which planets will still be around to see that red giant.

If I want to know what will happen, I have no choice but to run simulations in which I vary the precise locations and speeds of the planets. The forecast is in some cases rather frightening. In 2009 French astronomers Jacques Laskar and Mickael Gastineau ran several thousand models of the future evolution of our solar system. Their experiments have identified a potential butterfly: Mercury.

The simulations start by feeding in the records we have of the positions and velocities of the planets to date, but it is difficult to know these with one hundred percent accuracy.

So each time they run the simulation they make small changes to the data. Because of the effects of chaos theory, a small change could result in a large deviation in the outcomes.

For example, astronomers know the dimensions of the ellipse of Mercury's orbit to an accuracy of several meters. Laskar and Gastineau ran 2,501 simulations varying these dimensions over a range of less than a centimeter. Even this small perturbation resulted in startlingly different outcomes for our solar system.

If the solar system were to be ripped apart, you might expect one of the big planets, like Jupiter or Saturn, would be the culprit. But the orbits of the gas giants are extremely stable. It's the rocky terrestrial planets that are the troublemakers. In one percent of simulations, Laskar and Gastineau found that tiny Mercury posed the biggest risk. The models show that Mercury's orbit could start to extend due to a certain resonance with Jupiter, with the possibility that Mercury could collide with its closest neighbor, Venus. In one simulation, a close miss was enough to throw Venus out of kilter, with the result that Venus collides with Earth. Even close encounters with the other planets would be enough to cause such tidal disruption that the effect would be disastrous for life on our planet.

This isn't simply a case of abstract mathematical speculation. Evidence of such collisions has been observed in the planets orbiting the binary star Upsilon Andromedae. Their current strange orbits can be explained only by the ejection of an unlucky planet sometime in the star's past. But before we head for the hills, the simulations reveal that it will

take several billion years before Mercury might start to misbehave.

INFINITE COMPLEXITY

What of my more modest goal of predicting the throw of my die? Laplace would have said that provided I can know its dimensions, the distribution of its atoms, the speed at which it is launched, and its relationship to its surrounding environment, theoretically the calculation is possible. But the discoveries of Poincaré and those who followed have revealed that just a few decimal places could be the difference between a 6 or a 2. The die is designed to have only six different outcomes, yet the input data range over a potentially continuous spectrum of values. The question is whether the dynamics of the die are truly chaotic—or could they be simpler than one might expect? If I vary the angle at which the die leaves my hand, is there a moment when the outcome flips from a 6 to a 2, or is it much more sensitive to small changes?

A Polish research team recently analyzed the throw of a die mathematically, and by combining this with the use of high-speed cameras they have revealed that my die may not be as chaotic and unpredictable as I feared. It just depends on the conditions of the table onto which you are throwing your die. The research group consists of a father-and-son team, Tomasz and Marcin Kapitaniak, together with Jarosław Strzalko and Juliusz Grabski, and they are based in Łódź. The model they considered, published in a paper in the journal *Chaos* in 2012, assumes that the die is perfectly balanced, like