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The Language of Mathematics

Telling Mathematical Tales

Mathematics Education Library

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CLUDE: MAORI MATHEMATICS CABULARY

t: The Maori language was adapted for mathematical discourse during the 1980s. Several issues arose from this intensive time of specific language development. The story of this development, with examples of difficulties is outlined.

rds: bilingual mathematics, Maori language, mathematical discourse

87. New Zealand. A warm, stuffy room in an old school ng. A group of mathematics teachers have been working for a discussing mathematics education for the indigenous Maori e. They have been developing mathematical vocabulary in the i language, and this evening they are working on statistical . They are trying to explain the difference between continuous iscrete data to a Maori elder. Examples are given: heights and sizes; temperatures and football scores; time and money. The pt is grasped easily enough, but the elder must put forward stions for Maori vocabulary for use in mathematics classes. He not transliterate to produce Maori sounding versions of the sh words: for example, he might have tried *konitinu* for uous or *tihikiriti* for discrete. He does try existing words for of the examples that are given: *ikeike (height)*, and *tae (score)*— ese terms are not representative enough for the mathematicians room, and are rejected. Then he begins to try metaphors. At attempt a short discussion amongst those mathematics teachers know the Maori language quickly reaches consensus that the hor suggested will not do. Then he suggests *rere* and *arawhata*. of us in the room with only a little Maori understand the non meanings of these words as 'flying' and 'ladder'. It does not good enough for us. But the eyes of the good Maori speakers up. They know that these words as a pair refer to the way a

nation taken from a smooth stream of possible measurements, iscrete data is information that can only have particular values. New technical vocabulary is born.

though I became aware of the importance of language in ematics education while working in Swaziland in the late 1970s, rst serious involvement in this area was as part of this group of ers developing vocabulary and grammar so that mathematics be taught in the Maori language to the end of secondary tion.

aori is a Polynesian language brought to New Zealand by the first s over 1000 years ago. It was an oral language, and was not n down until European traders and missionaries came to New nd around 1800. As happened in other places in the world, icant European settlement signalled the start of a decline in the f the indigenous language through familiar colonial processes. ver, in the 1970s, a Maori cultural renaissance began. As part of bilingual primary schools were established, although mathematics cience were still mainly taught in English (Nathan, Trinick, Tobin, rton, 1993). Bilingual secondary schools developed during the , but Maori children remained alienated from mathematics and ce. One response was the call for mathematics and science ction in Maori (Fairhall, 1993; Ohia, 1993), and a small group gathered together by the Department of Education to develop mathematical language for this purpose (Barton, Fairhall & k, 1995a). The group included teachers, mathematicians, mathe- s educators, linguists, Maori elders, and Maori language experts. rked under strict guidelines laid down by the Maori Language nission, (these guidelines included a ban on the use of trans- ions), and an imperative to ensure that any new language retained grammatical structures.

is was a very exciting time for those involved. It felt as though ere in a crucible of language development, and we were all nged both linguistically and mathematically. Linguistically the nge was to produce vocabulary and grammar that had new uses r as Maori was concerned) but that was recognisably Maori in its ure, denotations, and connotations. There was a lot of use of phor, for example using *kauwhata* for a graphical framework or f axes. *Kauwhata* refers to a rectilinear frame used for drying Another vocabulary creation technique was to use standard i grammatical constructions, for example using standard suffixes

n opportunity to resurrect old Maori words that had gone out of
vith new (but related) technical meanings. The word *wariu* for
e' had been used for many years, but was rejected as a trans-
ion. It was replaced by an old word, *uara*, that had fallen out of
ut meant the value or standing of someone.

athematically, those of us with expertise in the subject were
nged to accurately explain the meanings and functions of many
ematical terms and concepts. This proved more difficult than
be expected, particularly for the very basic concepts. For
ple, words like 'number' and 'graph' have meanings that shift in
ent contexts and at different stages of development of mathe-
al understanding. We were prompted to construct a genealogy of
ematical terminology that showed which words were base words
thematical discourse and how other words could or should be
ed from them. For example, 'multiple' is a child of 'number' and
iply'. This genealogical tree was not always obvious, nor is it
e.

e whole process was characterised by a cycle of collecting the
being used in existing bilingual and immersion classrooms,
the words and phrases back to Maori communities for their
ent, writing up the results, and presenting this material to the
Language Commission for their decisions and ratification. The
was repeated three times over fifteen years, and the process and
sulting vocabulary and grammar have been published in a series
ers and dictionaries (Barton, Fairhall & Trinick, 1995a, 1995b,
NZ Ministry of Education, 1991, 1994, 1995). It happened that
lowing' and 'waterfall' metaphors described above as words for
ete' and 'continuous' were eventually rejected in this process
eplaced by words based on the Maori word *motumotu*—which
s divided into isolated parts as islands are upon the sea.

, was the Maori language successfully adapted to the teaching of
ematics? The answer is yes, ... and no. There is evidence that
taught mathematics in Maori are doing well (Aspin, 1995).
students have been taught mathematics in Maori up to Year 13
inal year of secondary school), but difficulties continue to exist
ding suitably qualified teachers (that is, those who are fluent in
Maori and mathematics), especially at senior levels.

owever, those of us involved in the Maori mathematics language
opment had become increasingly uncomfortable with some
ts of our work. Somehow the mathematical discourse that had

oped did not feel completely right, but we were unable to put our r on why. We came to talk about this as the "Trojan Horse" omenon: mathematics education seemed to be a vehicle that led subtle corruption of the ethos of the Maori language (Barton, all & Trinick, 1998).

n example of grammatical corruption had happened during the ulary development process. It had been difficult to translate the pts of positive and negative numbers. At the first meeting with aori Language Commission a discussion had resulted in a very greement on the part of the Commission to alter the grammar of nguage and use the direction-indicating adverbs *ake* (up) and *iho* n) as adjectives for the noun *tau* (number). *Ake* and *iho* should modify verbs, as in *heke iho* (fall down). But the adjectival uses *ke* (literally 'upwards number' for positive number) and *tau iho* lly 'downwards number' for negative number) were to be per- d. Four years later, at the second meeting with the Commission, nember demanded that this decision be rescinded. She had heard children in a school playground extend this grammatical misuse eir everyday discourse. A child had been heard to say "*korero* (literally 'upwards talk') to refer to praise. *Ake* should not be n this way as an adjective in correct Maori language. Under her imperative, an alternative formulation for positive and negative ers was immediately found.

r feeling that we had more fundamentally permanently changed ature of the language was finally confirmed several years later. xample that epitomised the problem was that of the grammatical f numbers. Classroom discourse that had developed during the used numbers grammatically very much as they are used in sh. However, in Maori as it was spoken before European contact, ers were verbal in their grammatical role (Trinick, 1999; w, 2001; Waite, 1990).

hat does "numbers were verbal in their grammatical role" mean? re not familiar with numbers as verbs. A number does not seem an action. However it can be. In English there are verbal forms e numbers 1 to 4: I can *single* someone out. I can *double* my bet. *triple* my earnings—well actually I can't, but someone else be able to. A new school may even *quadruple* its enrolment a few years. However, these forms are not the basis of our standing of number. In everyday talk, numbers are usually used djectives. There are three bottles on the table. I have five fingers. there might be green bottles on the table, and I have long s. (Technically, however, numbers are not adjectives. They are ally considered to have their own grammatical form).

Maori, prior to European contact, numbers in everyday talk like actions. The grammatical construction used would have like saying that "the bottles are three-ing on the table", or that ingers five". Just as the bottles are standing on the table, or my s wiggle.

r awareness of this old Maori grammar of number suddenly ened when we tried to negate sentences that used numbers. The uction that 'sounded right' was not the same as the construction should logically follow from the classroom mathematics rse.

t us look at this in detail. To negate a verb in Maori the word is used:

We are going to the house.	=	*E haere tatou ki te whare.*
e not going to the house, we are returning.	=	*Kaore tatou e haere kit e whare, e hoki mai ke.*

like English, where negating both verbs and adjectives requires ord 'not', in Maori to negate an adjective a different word is *ehara*:

This is a big house.	=	*He whare nui tenei.*
his is not a big house, it is a small house.	=	*Ehara tenei I te whare nui, he whare iti ke.*

Maori, negating number uses the verbal form, *kaore*:

There are four hills.	=	*E wha nga puke.*
e are not four hills, there are three.	=	*Kaore e wha nga puke, e toru ke.*

re was evidence that the classroom discourse that had been oped was against the original ethos of the Maori language. ers had been changed to become adjectival. While constructing ctionaries and glossaries of mathematics vocabulary, the verbal of numbers was ignored, and a classroom discourse that d numbers as they are in English was perpetuated. Thus the matics vocabulary process contributed to changes in Maori age use.

ss, and I was concerned about the consequences for bilingual
ltilingual mathematics education. But also, as a mathematician,
curious about the mathematical concepts inherent in the original
usage of number. Would mathematics have developed diffe-
if it had developed through languages in which numbers were
l? More generally, I became curious about the way that mathe-
al ideas are presented differently in other languages.
began a search for other examples, and an investigation into the
ematical consequences and the implications for mathematics
tion. I soon discovered that this material was not 'lost'. Many
people—linguists, anthropologists, mathematics educators, ethno-
ematicians—had recorded and discussed unexpected ways of
ssing mathematical thinking in many different languages.
ver these examples had not previously been considered from a
ematical point of view, and only briefly had educational conse-
es been considered (E.g. Pinxten, van Dooren, & Harvey, 1983,
5). I quickly came to believe that there were important mathe-
al ideas to be found, and I began to change some of my views
mathematics itself. In addition, some of my thinking about
ematics education was being turned around. This book is the
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RODUCTION

t: An outline of the structure of the book is presented, making the argument that the language we use for everyday mathematical ideas presents us with valuable evidence and insights into the nature of mathematics.

rds: mathematical discourse, nature of mathematics

egin the book by looking at the way people speaking different ages talk about mathematical ideas in their everyday conversation. up questioning some common beliefs about mathematics, its y, and its pedagogy.

e way we (English speakers) use numbers, the way we give ions, the way we express relationships, are all so commonplace t is hard to imagine any other way of expressing these ideas. ke for granted the structures of the following sentences:

There are four people in the room.
The book costs forty-five dollars.
Two and three are five.
Turn left.
Go straight on.
The sun rises in the east.
A dog is a mammal.
He is not my father.
I will either go shopping or read my book this afternoon.

t apparently simple English language statements turn out to be ssed quite differently in some other languages—so differently is often difficult to write in one language the equivalent of what ng said in another. Even when quantity is expressed in the est way—when we count—it is done in fundamentally different in different languages, as has been illustrated in the Preface. We ot talking about just different vocabulary. Nor is it a matter of y occurs in the way languages express numbers, the grammar of ematical discourse.

e first part of this book explores these differences. In order to r explore how other languages construct mathematical talk, I igated languages as different as possible from my own first age of English. Distant languages are most likely to have niliar structures. Unfamiliar structures are good clues in a search fferent mathematical conceptions. Therefore most of the examples bed are from indigenous languages rather than Indo-European ages: the Polynesian languages Maori, Hawaiian, and Tahitian; uskera language of the Basque people; Kankana-ey from the llera region of The Philippines; Dhivehi from the Maldives; e from Liberia, and First Nation languages from North America.

e first part also includes some mathematical flights of fancy g from the way various languages discuss numbers and shapes. maginings illustrate the possibility of different mathematical s. However the main point of this section is to lay down the nce of language difference with respect to mathematical talk. onstrate the congruence between mathematics as we know it and nglish language. Other languages are not so congruent.

rt II discusses what all this means for mathematics. Does it mean nathematics as an academic discipline with very powerful cal applications is somehow different in different parts of the ? A bridge designed using mathematical theory surely stands (or in the same way independently of the country it is built in, or of nguage of the person who solved the equations of its design? y 1 + 1 = 2 in Alaska, Nigeria, Tahiti, and Singapore? I argue for ative answers to conventional questions about mathematics— e it comes from, how it develops, what it does, what it means. lenge the idea that mathematics is the same for everyone, that it expression of universal human thought—and explain the ons about the bridge and 1 + 1 posed incredulously above.

nother issue concerns the relationship between language and ematical thought. Does the language we speak limit what we can lo, and think mathematically? If this is so, we can infer serious quences for mathematics if one language comes to dominate ematical discourse, as English is doing within the international rch arena. The question is wrongly posed. We probably do not to focus on the limitations created by languages—languages are iently creative as living structures to describe whatever we want vity embedded in other languages. New mathematical ideas (or eas given new roles) lie hidden in minority languages.

e third part of the book briefly discusses the consequences for ay we learn and teach mathematics. Can these linguistic insights nathematics tell us anything about how we gain mathematical standing? I make two fundamental suggestions. We should do abstract activity, both in the early stages of learning mathe-s, and when students are having difficulty. However, in saying he nature of useful abstract activity needs to be reconsidered. econd major suggestion is that undirected mathematical play is a thing at all levels of education from early childhood to graduate

oes a better awareness of the links between mathematics and age lead us to practical strategies in mathematics classrooms? tors have known for some time about the importance of talking, e need for formal language development within the mathematics ulum. And yet mathematics teachers do not universally use age activities. We re-examine the argument for these roles for age, and give some examples. In addition a plea is made for the tance of teaching about the nature of mathematics.

hat about classrooms where more than one language is spoken, hat do the conclusions of Part I mean for students who learn matics in an unfamiliar language? Much writing on multilingual ooms characterises such environments as full of problems. out denying the complexity of the situation, the ideas in this book st that these classes have, rather, an abundance of resources. The on is how teachers can best utilise the linguistic potential n.

nally, having started with evidence collected from many lang- of indigenous groups around the world, I end with a consi-on of the particular issues faced by these groups with respect to matics education. A proper understanding of the link between age and mathematics may be the key to finally throwing off hadow of imperialism and colonisation that continues to haunt tion for indigenous groups in a modern world of international ages and global curricula.

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r some time now, I have felt that many debates in mathematics tion have been dominated by ideologies and theories, rather than on a hunger strike, and people leapt into political action and ed with little regard for critical argument or evidence. I think that matter as important and deep-seated as this, there should be nce of a more permanent kind that can clarify some of the e. This book can be read as an attempt to interpret the evidence language with respect to mathematics and mathematics education. evidence presented here seems to me to support a weakly vist philosophical position in that mathematics might have been d otherwise, and a social constructivist mathematics education on in that we develop mathematics in conjunction with our age. However readers would be mistaken to think that arguing positions is what the book is about. The evidence is presented nterpreted.

fore we start, a short statement about what I mean by mathe- s, and a few caveats. Mathematics is a tricky word, loaded, for nany non-mathematicians amongst us, with thoughts of school- ers and textbooks and homework exercises. For mathematicians eaning is richer, although there is considerable disagreement over act reference (Davis & Hersch, 1981). The problem for this book t I wish to talk about mathematical things in general, and in xts in which formal mathematics has no part. For example, as far m aware, in pre-European Maori culture, there was no area of ledge or discourse equivalent to mathematics as understood . How then can I talk about aspects of that culture being ematical? The problem is circumvented in this book by mentally cing the words 'mathematics' (or 'mathematical') with the phrase cerning) a system for dealing with quantitative, relational, or l aspects of human experience", or "QRS-system" for short. any system that helps us deal with quantity or measurement, or lationships between things or ideas, or space, shapes or patterns, be regarded as mathematics. My translation allows the word ematical' to be used much more widely than just to refer to s in mathematics texts or journals. If I want to talk about the er, formal, conventional world of academic mathematics as it is plified in schools and universities all over the world, then I will e words "near-universal, conventional mathematics", or "NUC- ematics" to refer to it. As an aside, I am told by sailing friends NUC means "not under control" and refers to ships that have been oned at sea. Elements of this idea in NUC-mathematics will be nated in the following pages.

pter 1

CE: POINTS OF REFERENCE

t: The way in which we talk about positions and locations is explored through several languages. The different way of talking in the Tahitian language is extrapolated into a geometrical system. The chapter concludes with a discussion of possible social origins of geometry as it is usually taught.

rds: geometry, space, coordinate systems, Tahitian, Navajo

e quest to find new mathematical ideas in other languages took rst to Tahiti. The Maori and Tahitian languages are very close Tahitian is still the first language of most Tahitians (unlike). I was interested to find out whether the verbal grammatical f number that we had found in Maori (see Preface) was the same nitian.

fact the verbal nature of numbers is well-preserved in Tahitian. her words, the Tahitian language is linguistically more conser-, meaning that it has changed less under the influence of contact other languages. It has been suggested that Tahitian has better ed its original syntax because King Pomare II had helped with rst translation (of the Bible), that is, a native speaker was ved. The first Maori translation, on the other hand, was a com-on of translations by various English missionaries. Foreign ators are likely to miss grammatical differences that are not part eir own linguistic landscape. An alternative explanation is that an language has undergone less change compared with Maori se the colonial policies of the French in Tahiti were more atist than the assimilation policies of the English in New Zealand le, 2003).

an example of the difference between Maori and Tahitian, ers are used with all the verbal particles in Tahitian. In both languages, verbs are preceded by particles that indicate the tense te of the action: *i* (indefinite past), *kua* (perfect or completed), *e* ure), and *kia* (intentional). In Maori, *e*, *ka*, and *kia* are all used numbers, although *e* is by far the most common. There is some ment about other particles despite recent grammars giving exam- Biggs, 1969; Harlow, 2001; Trinick, 1999, p. 106-11). In Tahi- *e*, *ka*, *kia* and *kua* (in Tahitian the 'k' is replaced with a glottal are all in standard usage (Académie Tahitienne, 1986).

t while investigating Tahitian another feature of Polynesian ages struck my mathematical imagination: the way in which on is described. There was a feature of the way one might talk the position of something that was quite unusual to my English- age experience. A Tahitian speaker tends to use both himself (or f) and the person being spoken to as reference points.

fore we explore this further, let us look at how location is bed from a purely linguistic point of view, and then look at it a mathematical point of view. Finally we will bring these two s together, and explore the implications of this Tahitian language e.

WAYS OF LOCATING: LINGUISTIC FEATURES

, how do we talk about location? The language we use depends e situation. In English, in small scale situations such as descri- people seated around a table, we tend to use phrases like "John is site Peter", or "John is a little way to the left", or "John is sitting long from Peter". As the scale gets larger, for example when ling by car, then we use the north, south, east, west compass , "he lives ten kilometres north of the city". We also sometimes nother kind of reference, the position of something along a path, ample, "the house is on the road to the beach", or "the town is -river from here". The use of these different methods of location igation is discussed later.

cus on the directional aspect of location for a moment. Different ages, and different cultural groups, use the various methods in ions that are unlike English usage, and some languages have systems that are not used in English. Australian Aboriginal e, for example, use the north/south/east/west system in very local ions, such as describing the position of people in a room, or e a picture might be placed on a wall (Harris, 1991). At very g ages, even before they can speak, Aboriginal children are aware any Oceanic languages use a geographic direction-reference n (Senft, 1997). This is a response to the dominance of some ular geographic features. For example, if you live on an island, *nland* and *seaward* have universal application in a way that they t have in the interior of Mongolia. Rivers may provide another rsally applicable reference, and, if you travel by foot, then *uphill lownhill* become significant when describing the location of ations. For example, in the Solomon Islands language Longgu 1997), there are two axes of orientation, one is East/West ed from the rising and setting of the sun), and the other is inland eawards (since most Longgu speakers are coastal dwellers). In nguage, as in others, the geographical references are sometimes on very small scales, such as describing the position of two s relative to each other on the table. They can also be used in al locations, such as describing the position of lizards on a wall.

owing the direction of something is not usually sufficient to its location; its distance is also needed. There are many different of expressing distance, for example the formalised measures es or inches), localised units (arms-length or a street block), time, (a day's walk or five minutes' drive), or volume (a fuel- distance).

e direction and distance of an object is still not enough to fy its position. We also need to say from where the direction and ce applies. For example, the reference point could be the speaker n is sitting on *my* left"), or it could be the person who is being n to ("John lives just round the corner from *you*"), or it could be er person or object known to both the speaker and the listener nada is four hours drive south of *Madrid*"). Most languages use ree types of reference, although, as for directions, the area of ation of the different forms are not always the same.

lynesian languages, including Maori and Tahitian, have matical forms that make distinctions that are not present in sh. In English we refer to *this* tree, to indicate that the tree is near , the speaker, or *that* tree, to indicate that the tree is at a distance me, the speaker. In Maori and Tahitian, we can refer to this tree *rakau*), or that tree near to you, the listener (*tena rakau*), or that istant from us both (*tera rakau*). In general, reference is much gocentric in Polynesian languages compared with English, and much more account of the point of view of the listener as well as peaker. This occurs to the extent that acknowledgement of the

WAYS OF LOCATING: MATHEMATICAL SYSTEMS

ow let us leave language aside for a moment, and turn to mathe-s. The position of an object in two dimensions (that is, on a ce) is generally defined using the Cartesian coordinate system, so d after Rene Descartes (1596 -1650), the French philosopher and ematician who first used it in an algebraic way. (Coordinate ns of this kind were known earlier than this: Archimedes and onius both used versions of this system in 200BC). From a origin, two reference lines, or axes, are drawn at right angles. osition of a point is determined by two measurements: the first urement is the distance along the horizontal line, and the second distance along the vertical one. The distance is positive if it is to ght or upwards, and negative if it is to the left or downwards (see -1).

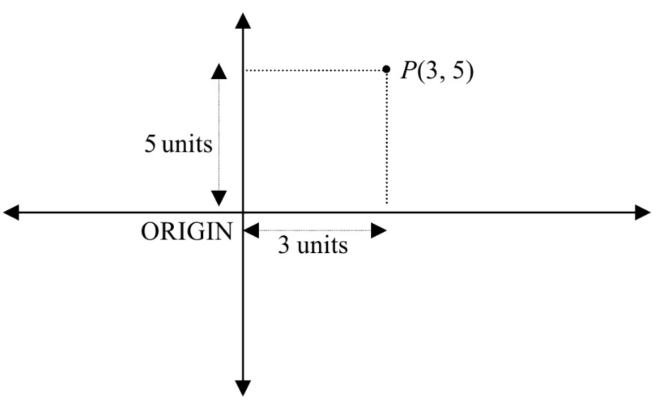

Figure 1-1. Cartesian Coordinate System

e second common way that position is determined, the Polar inate system, also uses a single origin, but only one reference The development of this system is usually attributed to Newton ernoulli, but some version of it is present in the work of Kepler.

e position of a point in this system is also determined by two urements: one is the distance of the point from the origin, the is the angle between the reference line and the line joining the and the origin. The angle is positive if it is in an anticlockwise