THE MAN FROM THE FUTURE

The Visionary Life of John von Neumann

By ANANYO BHATTACHARYA

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About the Author

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To geeks and nerds everywhere,
but especially for the three closest to me.

‘If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is.’

John von Neumann

INTRODUCTION:
Who Was John von Neumann?

'Von Neumann would carry on a conversation with my three-year-old son, and the two of them would talk as equals, and I sometimes wondered if he used the same principle when he talked to the rest of us.'

Edward Teller, 1966

Call me Johnny, he urged the Americans invited to the wild parties he threw at his grand house in Princeton. Though he never shed a Hungarian accent that made him sound like horror-film legend Bela Lugosi, von Neumann felt that János – his real name – sounded altogether too foreign in his new home. Beneath the bonhomie and the sharp suit was a mind of unimaginable brilliance.

At the Institute for Advanced Study in Princeton, where he was based from 1933 to his death in 1957, von Neumann enjoyed annoying distinguished neighbours such as Albert Einstein and Kurt Gödel by playing German marching tunes at top volume on his office gramophone. Einstein revolutionized our understanding of time, space and gravity. Gödel, while no celebrity, was equally revolutionary in the field of formal logic. But those who knew all three concluded that von Neumann had by far the sharpest intellect. His colleagues even joked that von Neumann was descended from a superior species but had made a detailed study of human beings so he could imitate them perfectly.

As a child, von Neumann absorbed Ancient Greek and Latin, and spoke French, German and English as well as his native Hungarian. He devoured a forty-five-volume history of the world and was able to recite whole chapters verbatim decades later. A professor of Byzantine history who was invited to one of von Neumann's parties said he would come only if it was agreed they would not discuss the subject. 'Everybody thinks I am the world's greatest expert in it,' he told von Neumann's wife, 'and I want them to keep on thinking that.'

The principal focus of von Neumann's incredible brain, however, was neither linguistics nor history but mathematics. Mathematicians often describe what they do as a sort of noble game, the object of which is to prove theorems, divorced from any real application. That is often true. But maths is also the language of the sciences – the most powerful tool we have for understanding the universe. 'How can it be that mathematics,' asked Einstein, 'being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality?'[1] No one has come up with a definitive answer to that question. Since antiquity, however, mathematicians with a talent for its application have, like von Neumann, understood that they have a path to wealth, influence and the power to transform the world. Archimedes spent time on otherworldly pursuits such as finding a new way to approximate the number pi. But the war machines he designed to exacting mathematical principles, such as a giant claw that could pluck ships from the sea, frustrated for a time the Roman army.

The mathematical contributions von Neumann made in the mid-twentieth century now appear more eerily prescient with every passing year. To fully understand the intellectual currents running through our century – from politics to economics, technology to psychology – one has to understand von Neumann's life and work in the last. His thinking is so pertinent to the challenges we face today that it is tempting to wonder if he was a time traveller, quietly seeding ideas that he knew would be needed to shape the Earth's future.

Born in 1903, von Neumann was just twenty-two years old when he helped to lay the mathematical foundations of quantum mechanics. He moved to America in 1930 and, realizing early on that war was looming, studied the mathematics of ballistics and explosions. He lent his expertise to the American armed forces and the Manhattan Project: among the scientists at Los Alamos who developed the atomic bomb, it was von Neumann who determined the arrangement of explosives that would be required to detonate the more powerful 'Fat Man' device by compressing its plutonium core.

The same year he joined the Manhattan Project, von Neumann was finishing, with the economist Oskar Morgenstern, a 640-page treatise on game theory – a field of mathematics devoted to understanding conflict and cooperation. That book would change economics, make game theory integral to fields as disparate as political science, psychology and evolutionary biology and help military strategists to think about when leaders should – and should not – push the nuclear button. With his unearthly intelligence and his unflinching attitude to matters of life and death, von Neumann was one of a handful of scientists who inspired the iconic Stanley Kubrick character Dr Strangelove.

After the atom bombs he helped to design were dropped on Hiroshima and Nagasaki, von Neumann turned his efforts to building possibly the world's first programmable electronic digital computer, the ENIAC. Initially his aim was to calculate whether or not it would be possible to build a more powerful bomb – the hydrogen bomb. He then led the team that produced the first computerized weather forecast. Not content with computers that merely calculated, von Neumann showed during a lecture in 1948 that information-processing machines could, under certain circumstances, reproduce, grow and evolve. His automata theory inspired generations of scientists to try and build self-replicating machines. Later, his musings on the parallels between the workings of brains and computers helped to trigger the birth of artificial intelligence and influenced the development of neuroscience.

Von Neumann was a pure mathematician of extraordinary ability. He established, for example, a new branch of mathematics, now named after him, that was richly productive: half a century later, Vaughan Jones won the Fields Medal – often called the Nobel Prize of maths – for his work exploring one tiny aspect of it. But mere intellectual puzzles, no matter how profound, were not enough for him. Von Neumann constantly sought new practical fields to which he could apply his mathematical genius, and he seemed to choose each one with an unerring sense of its potential to revolutionize human affairs. 'As he moved from pure mathematics to physics to economics to engineering, he became steadily less deep and steadily more important,' observed von Neumann's former colleague, mathematical physicist Freeman Dyson.[2]

When he died, aged just fifty-three, von Neumann was as famous as it is possible for a mathematician to be. The writer William S. Burroughs claimed von Neumann's game theory inspired some of his bizarre literary experiments, and he is name-checked in the novels of Philip K. Dick and Kurt Vonnegut. Since then, however, von Neumann has, compared with his august Princeton associates, faded from view.

Caricatured as the coldest of cold warriors, and with wide-ranging contributions that are almost impossible to summarize, when von Neumann is remembered it is largely for his legendary feats of mental gymnastics. Yet his legacy is omnipresent in our lives today. His views and ideas, taken up by scientists, inventors, intellectuals and politicians, now inform how we think about who we are as a species, our social and economic interactions with each other and the machines that could elevate us to unimaginable heights or destroy us completely. Look around you and you will see Johnny's fingerprints everywhere.

1
Made in Budapest
A genius is born and bred

'Von Neumann was addicted to thinking, and in particular to thinking about mathematics.'

Peter Lax, 1990

The scientists and technicians working on America's secret atom bomb project at Los Alamos during the 1940s called them the 'Martians'. The joke was that with their strange accents and exceptional intellects, the Hungarians among them were from some other planet.

The Martians themselves differed on why one small country should churn out so many brilliant mathematicians and scientists. But there was one fact upon which they were all agreed. If they came from Mars, then one of their number came from another galaxy altogether. When the Nobel Prize-winning physicist and Martian Eugene Wigner was asked to give his thoughts on the 'Hungarian phenomenon', he replied there was no such thing. There was only one phenomenon that required any explanation. There was only one Johnny von Neumann.

Neumann János Lajos (in English, John Louis Neumann – the surname comes first in Hungarian) was born in sparkling Belle Epoque Budapest on 28 December 1903. Created when the old capital of Buda was merged with the nearby cities of Óbuda and Pest in 1873, Budapest was thriving. The Hungarian parliament building on the banks of the Danube was the largest in the world and the grand Beaux Arts stock exchange palace was unrivalled in Europe. Beneath Andrássy Avenue, a glorious boulevard lined with neo-renaissance mansions, ran one of the world's first electrified underground railway lines. Intellectuals flocked to the coffee houses (the city boasted more than 600), and the acoustics of the opera house, also built around this time, are still considered to be among Europe's finest.

Johnny – his family and friends in Hungary called him Jancsi (pronounced *yan-shi*), a diminutive version of János – was the first of three sons born to Miksa (often translated to Max) and Margit (Margaret), educated, well-to-do parents plugged into the Hungarian capital's dazzling intellectual and artistic life. His brother Mihály (Michael) followed in 1907, and Miklós (Nicholas) in 1911. The family lived in an eighteen-room apartment on the top floor of 62 Vaczi Boulevard.[1]

The ground floor of the home was occupied by the sprawling salesrooms of Kann-Heller, a hardware firm founded by Margaret's father, Jacob Kann, and his partner.

Kann-Heller had sold farm machinery, then successfully pioneered catalogue sales in Hungary, much as Sears had done earlier in the United States. The Heller family had the run of the whole first floor. The second and third floors were occupied by Kann's four daughters and their families. Today, on the corner of the building, next to the entrance to the offices of an insurance firm, is a plaque honouring 'one of the most outstanding mathematicians of the 20th century'.

In 1910, a quarter of Budapest's population, and more than half of its doctors, lawyers and bankers, were Jewish, as were many of those involved in the city's thriving cultural scene. In that success, some sought to find conspiracy. The supposed domination of the city by Jews led Karl Lueger, the firebrand populist mayor of Vienna, to call Austria-Hungary's twin capital 'Judapest'. Lueger's racist rhetoric would inspire a young homeless Adolf Hitler, kicking around Vienna after being denied admission to the city's Academy of Fine Arts.

The brunt of Jewish immigration to Hungary had taken place in the last two decades of the nineteenth century. Many, in search of jobs, had settled in rapidly growing Budapest. Jews did not face pogroms as they did in Russia, and the anti-Semitism that for generations has run deep and strong through Europe was, if not altogether absent, at least not usually sanctioned by the government. 'Respectable opinion, including most of the aristocracy and gentry, rejected anti-Semitism,' notes Hungarian-American historian John Lukacs.[2]

Still, for all their prosperity and happiness, the Neumanns were, like many Jews in the Austro-Hungarian Empire, haunted by the anxiety that the good times would not last. Though the dozens of ethnic groups living within its borders were ostensibly united under the popular emperor in Vienna, and by an economic logic that allowed the free movement of goods and services across a huge swathe of southeastern Europe, differences sometimes came to the fore. Robert Musil, one of the empire's many great writers, said that the numerous internecine conflicts 'were so violent that they several times a year caused the machinery of State to jam and come to a dead stop. But between whiles, in the breathing-spaces between government and government, everyone got on excellently with everyone else and behaved as though nothing had ever been the matter'.[3]

Despite the febrile atmosphere of Austria-Hungary, it would not be internal divisions but the First World War that would precipitate the empire's downfall. By 1910, Max had sensed the darkening mood in Europe and wanted his sons' education to prepare them for the worst. Children did not then start school in Hungary until the age of ten, but affluent Budapestian families had no problems finding nursemaids, governesses or tutors. Max emphasized foreign languages, reasoning that his sons would then be able to make themselves understood no matter where they were or who happened to be in charge. So six-year-old Jancsi learned French from Mademoiselle Grosjean and Italian from Signora Puglia. Between 1914 and 1918, the brothers were also taught English by Mr Thompson and Mr Blythe. Though held as enemy aliens in Vienna at the start of the war, Max, a man of influence, had 'no difficulties in having their place of "internment" officially moved to Budapest'.[4] Max also insisted the boys learn Ancient Greek and Latin. 'Father', Nicholas recalled in his memoirs, 'believed in the life of the mind.'[5]

Jancsi was a formidable mental calculator even as a child.[6] Some sources suggest that he could multiply two eight-digit numbers together in his head when he was six.[7] These abilities, remarkable enough to astonish his early tutors, may have been partly inherited from his maternal grandfather. Though Jacob Kann had no formal education beyond secondary school, he could add or multiply numbers into the millions. Von Neumann would recall his twinkly-eyed grandfather's mental gymnastics with pride when he was older, but he admitted he was never quite able to match them himself.

The eldest of the von Neumann brothers did not, however, shine in all things. He never, for instance, mastered a musical instrument. Puzzled that young Jancsi only ever played scales on the cello, his family investigated to find that the five-year-old had taken to propping up books on his music stand so he could read while 'practising'. At chess, a game often associated with mathematical ability, he was middling.[8] Despite developing various 'systems' that he thought would inevitably lead to a win, he lost consistently against his father even as a teenager.

Equally, von Neumann had no interest in sport and, barring long walks (always in a business suit), he would avoid any form of vigorous physical exercise for the rest of his life. When his second wife, Klári, tried to persuade him to ski, he offered her a divorce. 'If being married to a woman, no matter who she was, would mean he had to slide around on two pieces of wood on some slick mountainside,' she explained, 'he would definitely prefer to live alone and take his daily exercise, as he put it, "by getting in and out of a pleasantly warm bathtub".'[9]

Intellectually, life at home was as stimulating as any child prodigy could wish. Max, a doctor of law turned investment banker, bought a library from the estate of a wealthy family when the boys were young. He converted a room in the apartment to house the collection, with shelves from floor to ceiling. This was where Jancsi would make his way through the library's centrepiece, the *Allgemeine Geschichte*, a massive history of the world edited by the German historian Wilhelm Oncken, which began in Ancient Egypt and concluded with a biography of Wilhelm I, the first German emperor, commissioned by the Kaiser himself. When von Neumann became embroiled in American politics after he emigrated, he would sometimes avoid arguments that were threatening to become too heated by citing (sometimes word for word) the outcome of some obscurely related affair in antiquity that he had read about in Oncken as a child.

The children's education often continued over lunch and dinner, when they were encouraged to present a particular topic that had caught their attention earlier in the day. Once, for example, Nicholas read up on the poetry of Heinrich Heine, sparking a discussion on how anti-Semitism would affect them in future. Heine was born into a Jewish family but reluctantly converted to Christianity, 'the ticket of admission into European culture', in an effort to boost his career. Frank debates such as this may have helped Jancsi recognize early on the dangers posed by National Socialism.

Jancsi in a sailor's suit, aged seven. Courtesy of Marina von Neumann Whitman.

Jancsi's mealtime seminars were often on scientific subjects. He noted that babies of different nationalities learned their native languages in about the same length of time. What then, he asked, is the brain's primary language? How does the brain communicate with itself? This is a question that he would continue to wrestle with, even on his deathbed. On another occasion, he wondered whether the spiral cavity of the inner ear known as the cochlea was sensitive only to the component frequencies of sound (and their respective volumes) or the shape of the sound wave as a whole.[10]

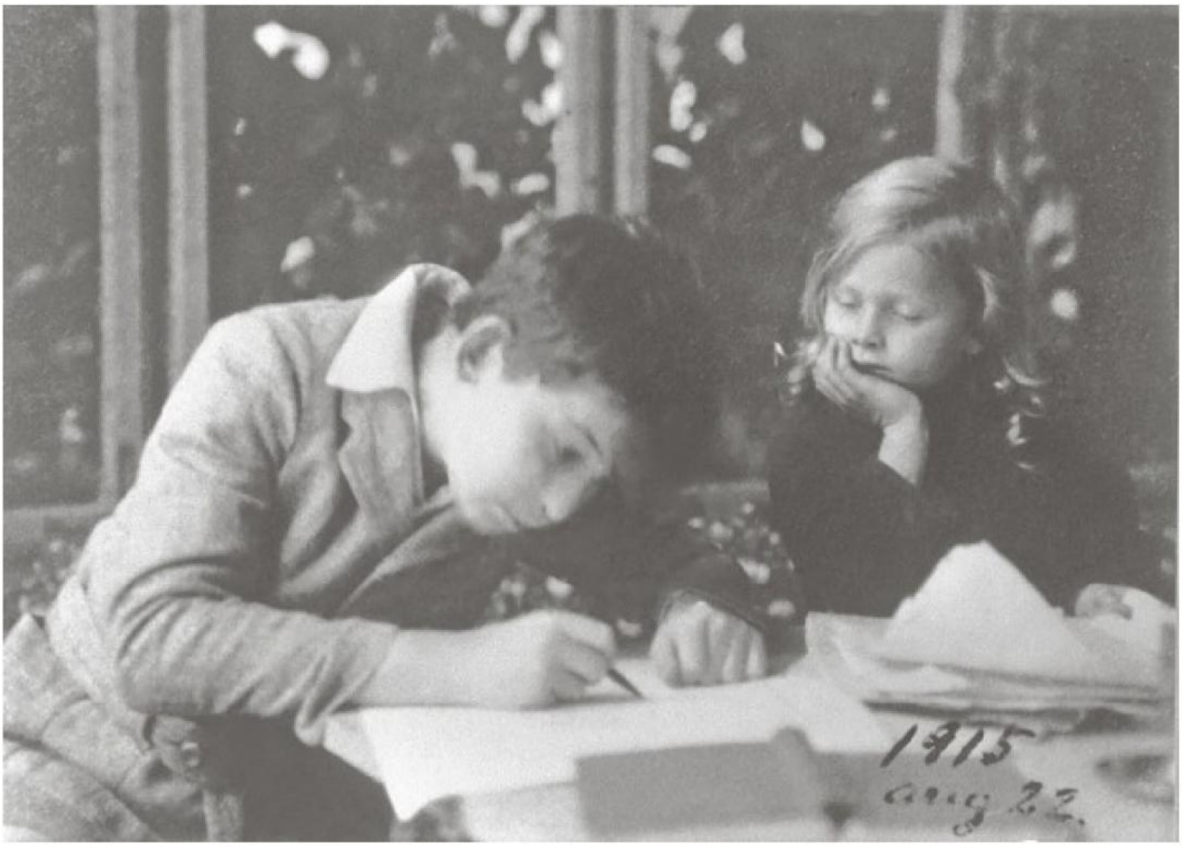
Max, who would lunch at home before returning to the office in the afternoon, would share his investment decisions and ask his sons' opinions. Occasionally, Max would bring home more tangible evidence of the companies in which he was investing. When he financed a newspaper business, he brought back pieces of metal type, and the ensuing discussion centred on the printing press. Another venture to win Max's support was the Hungaria Jacquard Textile Weaving Factory, an importer of automated looms.[11] Invented in the early nineteenth century by the Frenchman Joseph Marie Charles (known as 'Jacquard'), these devices could be 'programmed' with punched cards. 'It probably does not take much imagination to trace this experience to John's later interest in punched cards!' notes Nicholas.[12]

Guests at the Neumanns' table were also contributing to the prodigy's academic development. Businessmen from all over Europe would find themselves politely pummelled by questions from Max's sons, who were allowed to sit in on working dinners. Other regular visitors included the psychoanalyst Sándor Ferenczi, a close associate of Sigmund Freud, whose conversation may have helped to form Johnny's later thoughts on the parallels between computers and brains. Physicist Rudolf Ortvay would call, fresh from studies at the University of Göttingen, the world's leading centre of mathematics and soon to be pivotal in the development of the new quantum mechanics. Ortvay would keep up a correspondence with Jancsi throughout his life. Another frequent guest, Lipót (Leopold) Fejér, occupied a chair of mathematics at the University of Budapest. He was soon to be one of several inspiring professors charged with giving the boy extracurricular maths lessons.

After 1910, Max became an economic adviser to the Hungarian government, a role that would rapidly propel him into the highest echelons of Budapest society. Three years later, a forty-three-year-old Max was rewarded with a hereditary title from the Austrian emperor Franz Joseph I for 'meritorious services in the financial field'. Max, a romantic, chose Margitta (then in Hungary, now in Romania) as the town that was to be associated with his title, traditionally the location of the family seat. Max's only connection to the place, however, was that the patron saint of the local church had the same name, Margit, as his wife. So the Neumanns became margittai Neumann (Neumann of Margitta) in Hungarian, and Max chose three marguerites (a type of daisy) for the insignia on the family's coat of arms. Many of the wealthy Jewish families ennobled during that period (more than 200 between 1900 and 1914) changed their names to more Germanic- or Hungarian-sounding ones to assimilate and often changed their faith too. Proud Max, though never particularly observant, did neither. Jancsi, who rather enjoyed the trappings of nobility when he was older, would adopt the Germanicized version of the name, first becoming Johann Neumann von Margitta while studying in Switzerland, then losing the place name to become simply 'von Neumann' in Germany.[13] After Max's death in 1928, his three boys converted to Catholicism for reasons similar to those of Heine.

In the same year that the Neumanns joined the European aristocracy, preparations were being made for Jancsi to start school. In much of Europe, a 'gymnasium' is a school that prepares students for further study at a university. Nearly all the Martians went to one of three elite fee-paying gymnasia in Budapest.

The foremost of these was the Minta or Model *gimnázium*, founded in 1872 by Mór von Kármán, one of Hungary's leading education experts and, like Max, an ennobled Jew. The Minta was a test bed for von Kármán's educational theories, largely imported from Germany. Discipline and rigour were central, and education was based on problem-solving rather than rote-learning. 'At no time did we memorize rules from the book,' says von Kármán's son, Theodore, who attended the school. 'Instead we sought to develop them ourselves. In my case the Minta gave me a thorough grounding in inductive reasoning, that is, deriving general rules from specific examples – an approach that remained with me throughout my life.'[14] The younger von Kármán was to become the twentieth century's leading expert on aerodynamics and would shape the aircraft designs of both the German Luftwaffe (inadvertently) and the US Air Force.



Von Neumann doing mathematics, aged eleven, with his cousin Katalin (Lili) Alcsuti. Courtesy of Marina von Neumann Whitman.

The Minta's methods were successful enough to be widely copied by other schools including the older 'Fasori' Lutheran gymnasium, regarded as second only to the Minta itself. The Lutheran school was open to boys (there were scant educational opportunities for girls) of all faiths. Because the professional classes in Budapest were dominated by Jews, most of the students at the Lutheran school were in fact of Jewish descent.

The third option was a Real school (pronounced Re-Al, in the German fashion). These *reáliskola* provided a technical education, usually teaching no Greek and little Latin. 'The *reáliskola* was not at all inferior to the *gimnázium*, just different in scope and somewhat more practical then the "gentlemanly" *gimnázium*,' according to one historian and 'boasted extraordinary students in mathematics and the sciences'.[15] Among them were Fejér, Leo Szilard, who first conceived of the nuclear chain reaction that powers reactors and bombs, and Dennis Gabor, who won the Nobel Prize in physics in 1971 for inventing the hologram. One Real school in particular, located in District VI of Budapest, was considered to be on a par with the two gymnasia. From these three, Max chose the Lutheran gymnasium. The Minta's methods were too new-fangled to be trustworthy. The Real, on the other hand, lacked the classical education that he prized.

Some suggest these apparent genius factories were responsible for the great outpouring of Hungarian brilliance between 1880 and 1920. However, not all their ex-pupils concurred. Szilard, who attended the thoroughly modern and well-equipped Real school in District VI, found the maths classes 'intolerably boring' and, in an interview, called his teacher 'a complete idiot'.[16] Another of the Martians, Edward (Ede) Teller, joined the Minta in 1917, almost twenty years after von Kármán had left, and found his time there a trial. The maths classes 'set me back several years', he complained in his memoirs. 'Challenging students to explore ideas was not a common aim at the Minta.'[17]

Others believe the 'Hungarian phenomenon' was driven by two apparently contradictory elements of Hungarian society at the time: liberalism and feudalism. It was easier for Jews to rise to prominence in Austria-Hungary than in many of its less liberal European neighbours but the levers of power, in particular the civil service and military, were almost entirely in the hands of the Hungarian upper classes. Known derisively as the 'sandaled' nobility, this often-impoverished aristocracy was suspicious of the many non-Hungarians in their sprawling country whose loyalty to the old order was questionable. They allowed the new Jewish émigrés to prosper in professions including banking and medicine that they regarded as beneath them and granted the most successful – like Max – hereditary titles as a means of cementing their loyalty. Of the Martians, all were from Jewish backgrounds, all were wealthy, and two were titled.

Von Neumann himself attributed his generation's success to 'a coincidence of some cultural factors' that produced 'a feeling of extreme insecurity in the individuals, and the necessity to produce the unusual or face extinction'.[18] In other words, their recognition that the tolerant climate of Hungary might change overnight propelled some to preternatural efforts to succeed. Physics and mathematics were safe choices for Jews who wished to excel: an academic career could be pursued in many countries, and the subjects were viewed – in the early twentieth century, at least – as relatively harmless. Moreover, one could reasonably hope that good work in these fields would be fairly rewarded.[19] The truth of general relativity was established through experiment and was not contingent on whether the person who developed the theory was Jew or Gentile.

Whatever the relative contributions of schooling, upbringing and Hungarian society, in von Neumann's case everything propitiously aligned to produce a mathematical mind of rare ability. Jancsi started at the Lutheran in 1914, where he was about to prove he was no ordinary schoolboy. The foundations of mathematics were being shaken by the discovery of paradoxes that threatened to bring down the entire edifice. Some contended that hundred-year-old theorems that did not pass rigorous new standards of proof should be banished altogether. A battle for the soul of mathematics would soon follow. The very notion of truth was at stake. The seventeen-year-old von Neumann stepped in to put things right.

2

To Infinity and Beyond

A teenager tackles a crisis in mathematics

> *'Mathematics is the foundation of all exact knowledge of natural phenomena.'*
>
> *David Hilbert, 1900*

Von Neumann's unique talents were spotted as soon as he started school. He attracted the attention of the Lutheran school's legendary maths teacher, László Rátz, who is so venerated in Hungary that a street in Budapest is named after him. Rátz quickly concluded that von Neumann would benefit from a mathematical education far more advanced than he could provide. He arranged to see Max and offered to organize extracurricular tuition for Jancsi at the University of Budapest. Rátz promised his pupil would continue to receive the full benefits of the Lutheran's classical education by attending all classes (including, somewhat redundantly, maths). Max, well aware of his son's mathematical talents, agreed. Rátz refused to accept any money for his services. The privilege of teaching Jancsi, he felt, was sufficient.

The young von Neumann made an instant impact on his new tutors. His first mentor, Gábor Szego″, who would later lead Stanford University's maths department,[1] was moved to tears after their first meeting. The most influential of von Neumann's tutors was Lipót Fejér, a seminal figure in Hungarian mathematics who drew many of the country's most talented stars, including, earlier, Szego″ himself. 'There was hardly an intelligent, let alone a gifted, student who could exempt himself from the magic of his lectures,' says fellow Hungarian mathematician George Pólya. 'They could not resist imitating his stress patterns and gestures, such was his personal impact upon them.'[2] Fejér's interest in his young charges went well beyond the call of duty. 'What does little Johnny Neumann do?' he would write to Szego″ years later. 'Please let me know what impact you notice so far of his Berlin stay.'[3] Szego″ was by then teaching at the University of Berlin, where 'little Johnny' was ostensibly boning up on undergraduate chemistry. (He was mostly creaming off all he could from the best at the university's prestigious mathematics department.)

Fejér and Michael (Mihály) Fekete, another of Fejér's former students, took on the brunt of von Neumann's education during his teenage years. All three of these tutors – Szego″, Fejér and Fekete – shared an interest in orthogonal polynomials, so it was natural that they were the subject of von Neumann's first paper. Orthogonal polynomials are sets of independent mathematical functions that can be added together to make any other. The complicated heaving and swaying of a ship at sea, for example, can be broken down into a simpler sum of these functions (a process known as harmonic analysis) and plugged into a computer to simulate the vessel's motion. This facility for making messy real-world data more manageable is why such polynomials are often used in physics and engineering.

For mathematicians, a key characteristic of polynomials is their 'zeroes'; that is, the question of where they cross the *x*-axis on a graph. Von Neumann's first paper,[4] written with Fekete, investigated the zeroes of Chebyshev polynomials, discovered as a result of a Russian mathematician's obsession with the problem of how to most efficiently turn the up and down motion of a steam engine piston into the circular motion of a wheel.[5]

This was von Neumann's introduction to the norms and conventions of academic maths. He was just seventeen when the completed manuscript was sent away for publication. Mathematicians have their own style much as novelists do. Von Neumann's voice appears here for the first time, more or less fully formed. 'Johnny's unique gift as a mathematician was to transform problems in all areas of mathematics into problems of logic,' says Freeman Dyson.

> He was able to see intuitively the logical essence of problems and then to use the simple rules of logic to solve the problems. His first paper is a fine example of his style of thinking. A theorem which appears to belong to geometry, restricting the possible positions of points where some function of a complex variable is equal to zero, is transformed into a statement of pure logic. All the geometrical complications disappear, and the proof of the theorem becomes short and easy.[6]

While von Neumann would never again mention this treatise in any future work, Fekete, inspired by the prodigy, would devote most of his remaining career to the subject.

Hungary, by this time, had fought in and lost a world war. But Budapest was never close to the front, and life for the wealthy denizens of Vaczi Boulevard had continued largely as before. Their lives were temporarily disrupted by a coup and the establishment in 1918 of Europe's first communist government (after Russia), led by Béla Kun, a non-practising Hungarian Jew who had become a convert to the revolutionary cause as a Russian prisoner of war. Kun was officially the new administration's foreign minister, but his popularity ensured that he held the reins of power. 'My personal influence in the Revolutionary Governing Council is such that the dictatorship of the proletariat is firmly established,' he told Lenin, 'since the masses are backing me.'[7]

As armed party enforcers in leather jackets known as the 'Lenin Boys' stalked the streets of Budapest, the Neumann family packed their bags and left for a holiday home on the Adriatic – but not before Max had secured their Budapest dwelling from being requisitioned by the new government. 'Under the guiding principle of equal facilities to all, the big apartments were broken up,' said Nicholas, who was seven at the time. But the party members charged with the task were soon convinced to forget the matter. 'On the piano under a weight, my father put a bundle of British pound notes, I don't know how much. The Communist official with the red armband promptly went there, took it, and the committee left and we remained in the apartment.'[8]

After Kun waged an ill-conceived war to re-establish Hungary's pre-First World War borders, the Romanian army marched into Budapest to topple his 133-day-old Hungarian Soviet Republic. Kun fled, eventually to live in exile in Russia, where he was executed in 1937 after being branded a Trotskyist and enemy of the people. The chaos of the Kun regime would remain etched on von Neumann's mind. 'My opinions have been violently opposed to Marxism ever since I remember,' he told a congressional confirmation hearing in 1955, after he was nominated to the Atomic Energy Commission, 'and quite in particular since I had about a three-month taste of it in Hungary in 1919.'[9]

The von Neumanns visiting an army artillery post ca.1915. John is sitting on the gun barrel. The three children on the gun carriage are (from top left to bottom right) brother Michael, cousin Lili, and brother Nicholas (in a dress). Courtesy of Marina von Neumann Whitman.

Max had meanwhile left Budapest for Vienna to contact Admiral Miklós Horthy, a war hero who was to assume control of the counter-revolutionary forces amassing against Kun. In the chaos that followed the downfall of Kun's government, Horthy's forces ranged through Hungary exacting revenge on anyone they felt had sympathized with the communists. Jews had occupied prominent positions in Kun's short-lived government and soon became the focus of their ire. The Lenin Boys and their ilk had despatched 500 souls during the Red Terror. In the ensuing White Terror, Horthy's officers would kill around 5,000. Rape, torture and public hangings were commonplace. Bodies were mutilated as a warning to others. Horthy mildly rebuked one of his most brutal lieutenants

> for the many Jewish corpses found in the various parts of the country ... This, [Horthy] emphasized, gave the foreign press extra ammunitions against us ... in vain, I tried to convince him that the liberal papers would be against us anyway, and it did not matter that we killed only one Jew or we killed them all.[10]

mathematics, was of 'a gloriously fertile originality,' said Carl Friedrich Gauss, the most famous mathematician of the age. While Bolyai and Lobachevsky pictured planes that curved through space, Riemann's surfaces often twisted and contorted in ways that could barely be imagined at all. His mathematics could describe space with any number of dimensions – hyperspace – just as easily as the three familiar spatial ones. More than half a century later, Riemann's geometry would prove 'admirably appropriate' for describing the warped four-dimensional spacetime of Einstein's general relativity.

The hyperbolic surfaces of a wood ear mushroom.

By the late nineteenth century, many other assumptions and proofs in Euclid's work were being questioned. Some argued that it was time to start from scratch by rebuilding geometry on new foundations. The task was taken up by David Hilbert, who was to become the most influential mathematician of the early twentieth century. His resulting book, *Grundlagen der Geometrie* (*The Foundations of Geometry*), published in 1899, is now recognized in its clarity of reasoning as the rightful heir of Euclid's *Elements*. A mathematical best-seller, its impact was immediate.

Hilbert's aim was no less than to distil, from the works of his predecessors, trustworthy ways to reason about *any* elementary geometry. To avoid relying on the intuition of his readers, the familiar terms of school geometry (points, lines, planes, etc.) were emptied of their meaning. In his book these words were simply convenient labels for mathematical objects that would be rigorously defined by the mathematical relationships between them.[15] 'One must be able to say at all times – instead of points, straight lines, and planes – tables, chairs and beer mugs,' Hilbert had explained years earlier.[16] The advantage of this incredibly abstract approach to geometry was that his findings would be valid for *any* set of objects as long as they obeyed the rules he had carefully concocted.

Hilbert defined his axioms with a rigour that went well beyond Euclid's. His improved axiomatic method would come to define the way mathematics was done in the twentieth century. Hilbert's book on geometry's foundations cemented his reputation as a great mathematician. Not yet forty and secure in his chair at the University of Göttingen, he set about proving himself to be a master administrator by attracting funding and talent. By 1920, there was no mathematics department in the world to rival that of Göttingen.

In his position as the discipline's foremost spokesperson, Hilbert now demanded that all of mathematics – and indeed the sciences – be made as bulletproof as his new geometry. In 1880, the famous physiologist Emil du Bois-Reymond had declared there were some questions (he called them 'world riddles'), such as the ultimate nature of matter and force, that science could never answer. '*Ignoramus et ignorabimus*,' as he put it: 'we do not know and will not know'.

Hilbert was having none of it. In 1900, he sounded his opposition to du Bois-Reymond's pessimism, denying that there were any such limits to knowledge. Every question had a definite answer, he argued, even if what that answer showed was that answering the original question was impossible. At the International Congress of Mathematicians in Paris that year, he posed twenty-three problems that would shape mathematics in the twentieth century. 'For us there is no *ignorabimus*, and in my opinion none whatever in natural science,' Hilbert was to thunder even thirty years later. 'In opposition to the foolish *ignorabimus* our slogan shall be: *Wir müssen wissen – wir werden wissen*.' We must know – we will know. Many rallied to Hilbert's side, eager to make mathematics (and so, it followed by Hilbert's logic, the sciences) unassailable. But his project ran into the sand almost as soon as it was conceived.

In 1901, the British philosopher and logician Bertrand Russell found a paradox at the heart of set theory, a branch of mathematics pioneered by Georg Cantor a quarter of a century earlier. Cantor, a brilliant, deeply religious Russian-born German Protestant, was the first mathematician to see that there are a multitude of different infinities, and some infinities are demonstrably larger than others. To the greatest of his infinities, the 'Absolute', Cantor assigned the Greek capital omega, Ω. Only in the mind of God, he said, could Ω be truly contemplated in all its glory. Realizing the controversial nature of the findings, he called his new infinities 'transfinite numbers' in order to distinguish them from the old concept of infinity. His insights, he said, came directly from God.

Not everyone agreed. 'God made the natural numbers; all else is the work of man,' growled Leopold Kronecker, a contemporary grandee of German mathematics who found Cantor's juggling with infinities suspicious and distasteful. He called Cantor a 'charlatan' and 'corrupter of youth' and squashed his hopes of moving from Halle University to a chair at the much more prestigious University of Berlin. Cantor was ill-equipped emotionally to deal with the vitriolic response to his transfinite numbers. Kronecker's attacks precipitated a bout of depression and the first of many visits to a sanatorium.

By the time Russell began his work, set theory was considered rather more brilliant than suspicious. Mathematics does not ultimately deal with finite groups of numbers. If, for example, a mathematician wants to prove something about prime numbers then, usually, the goal is to find a theorem that applies equally well to *all* prime numbers – of which there are infinitely many. Mathematicians had embraced Cantor's theory as a powerful tool for manipulating and proving theorems about such sets of infinite size.

Russell's paradox, however, threatened to deal a far more serious blow to set theory than earlier ideological objections. The problem was this: consider a set of objects – all possible types of cheesecake, say. This set may include any number of different cheesecakes (New York cheesecake, German *Käsekuchen*, lemon ricotta, etc.) but, because a set is not literally a cheesecake, the set of all cheesecakes is not a member of itself. The set of all things that are *not* cheesecakes, on the other hand, *is* a member of itself.

But what, Russell wondered, about the set of *all* sets that are not members of themselves. If this is *not* a member of itself, then, by definition, it should be (because its members do not include itself). Conversely, if it *is* a member of itself, then it should not be (because it does). This was Russell's paradox in a nutshell. His analysis of the paradox revealed it to be similar in form to several others, including the liar's paradox ('this statement is a lie'). 'It seemed unworthy of a grown man to spend time on such trivialities,' he complained, desperate for a solution, 'but what was I to do?'[17]

Russell had begun a huge effort to describe precisely the logical basis of all of mathematics, but was plunged into despair by his discovery. Unable to progress with his work, he spent the next several years trying, without success, to resolve the contradiction he had found. 'Every morning I would sit down before a blank sheet of paper,' he said. 'Throughout the day, with a brief interval for lunch, I would stare at the blank sheet. Often when evening came it was still empty ... it seemed quite likely that the whole rest of my life might be consumed in looking at that blank sheet of paper.'[18]

Russell's paradox and others like it threatened to kick away a cornerstone of mathematics and with it Hilbert's programme of re-establishing mathematics on more rigorous grounds. Alarmed, Hilbert called for mathematicians to resolve the crisis Russell's discovery had triggered. 'No one', he vowed, 'will drive us out of this paradise that Cantor has created for us.'[19]

Not all mathematicians were as resolved to save Cantor as Hilbert was. One group, the 'intuitionists', led by the pugilistic and highly strung young Dutch mathematician L. E. J. 'Bertus' Brouwer, argued Russell's paradox showed that mathematics was bumping up against the limits of the human mind. Brouwer was wary of transfinite numbers. There was no reason to believe, he argued, that the rules of logic could be applied to everything in mathematics and, in particular, not to Cantor's dubious infinite sets. The law of the excluded middle, for example, states that either a proposition or its negation is true. So the sentence 'I am a dog' is either true or false but cannot be both. Brouwer contended that to prove this satisfactorily for a set, each of its members must be inspected to establish whether the proposition did or did not hold. To do this for the members of an infinitely large set is, of course, impossible. Playing fast and loose with infinite sets, Brouwer claimed, had led to paradoxes of the sort that so troubled Russell.

In Göttingen, Hilbert reacted angrily. Whereas once he had supported Brouwer's application for a chair at the University of Amsterdam, he now campaigned for him to be removed from the board of editors of *Mathematische Annalen*, one of the field's most prestigious journals. Einstein, one of the editors, dismissed the feud as a completely overblown *Froschmäusekrieg* (literally a war of frogs and mice, a German phrase describing a bitter but unimportant altercation). For Hilbert, however, this was not merely some trivial spat about mathematical niceties. There was much more at stake. 'If mathematical thinking is defective,' he asked, 'where are we to find truth and certitude?'

For any ambitious young mathematician determined to prove his mettle, the idea of saving mathematics from itself was irresistible. Despite his tender years, von Neumann was well equipped for the task. He had lectured Wigner enthusiastically

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