THE MATH OF LIFE & DEATH

7 MATHEMATICAL PRINCIPLES THAT SHAPE OUR LIVES

KIT YATES

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INTRODUCTION

ALMOST EVERYTHING

My four-year-old son loves playing out in the garden. His favorite activity is digging up and inspecting creepy crawlies, especially snails. If he is patient enough, after the initial shock of being uprooted, they will emerge cautiously from the safety of their shells and start to glide over his little hands, leaving viscid trails of mucus. Eventually, when he tires of them, he will discard them, somewhat callously, in the compost heap or on the woodpile behind the shed.

Late last September, after a particularly busy session in which he had unearthed and disposed of five or six large specimens, he came to me as I was sawing up wood for the fire and asked, "Daddy, how many snails is [*sic*] there in the garden?" A deceptively simple question for which I had no good answer. It could have been one hundred or it could have been one thousand. He would not have comprehended the difference. Nevertheless, his question piqued an interest in me. How could we figure this out together?

We decided to conduct an experiment. The next weekend, on Saturday morning, we went out to collect snails. After ten minutes, we had a total of 23 of the gastropods. I took a Sharpie from my back pocket and placed a subtle cross on the back of each. Once they were all marked up, we tipped up the bucket and released the snails back into the garden.

A week later we went back out for another round. This time, our ten-minute scavenge brought us just 18 snails. When we inspected them closely, we found that 3 of them had the cross on their shells, while the other 15 were unblemished. This was all the information we needed to make the calculation.

The idea is as follows: The number of snails we captured on the first day, 23, is a given proportion of the total population of the garden, which we want to get a handle on. If we can work out this proportion, then we can scale up from the number of snails we caught to find the total population of the garden. So we use a second sample (the one we took the following Saturday). The proportion of marked individuals in this sample, 3/18, should be representative of the proportion of marked individuals in the garden as a whole. When we simplify this proportion, we find that the marked snails make up one in every six individuals in the population at large (you can see this illustrated in figure 1). Thus we scale up the number of marked individuals caught on the first day, 23, by a factor of six to find an estimate for the total number of snails in the garden, which is 138.

After finishing this mental calculation I turned to my son, who had been "looking after" the snails we had collected. What did he make of it when I told him that we had roughly 138 snails living in our garden? "Daddy"—he looked down at the fragments of shell still clinging to his fingers—"I made it dead." Make that 137.

FIGURE 1: The ratio of snails recaptured (marked ⊗) to the total captured (marked ○) on day 2 is 3:18, which should be the same as the ratio of snails captured on day 1 (marked ×) to all snails in the garden, 23:138.

This simple mathematical method, known as capture-recapture, comes from ecology, where it is used to estimate animal population sizes. You can use the technique yourself: take two independent samples and compare the overlap between them. Perhaps you want to estimate the number of raffle tickets that were sold at the local fair or to estimate the attendance at a football match using ticket stubs rather than having to do an arduous head count.

Capture-recapture is used in serious scientific projects as well. It can, for example, give vital information on the fluctuating numbers of an endangered species. By providing an estimate of the number of fish in a lake, it might allow fisheries to determine how many permits to issue. Such is the effectiveness of the technique that its use has evolved beyond ecology to provide accurate estimates on everything from the number of drug addicts in a population to the number of war dead in Kosovo. This is the pragmatic power that simple mathematical ideas can wield. These are the sorts of concepts that we will explore throughout this book and that I use routinely in my day job as a mathematical biologist.

When I tell people I am a mathematical biologist, I usually get a polite nodding of the head accompanied by an awkward silence, as if I were about to test them on their recall of the quadratic formula or Pythagoras's theorem. More than simply being daunted, people struggle to understand how a subject such as math, which they perceive as being abstract, pure, and ethereal, can have anything to do with a subject such as biology, which is typically thought of as being practical, messy, and pragmatic. This artificial dichotomy is often first encountered at school: If you liked science but you weren't so hot on algebra, then you were pushed down the life sciences route. If, like me, you enjoyed science but you weren't into cutting up dead things (I fainted once, at the start of a dissection class, when I walked into the lab and saw a fish head sitting at my bench space), then you were guided toward the physical sciences. Never the twain shall meet.

This happened to me. I dropped biology at sixth form and took A levels in math, further math, physics, and chemistry. When it came to university, I had to further streamline my subjects and felt sad that I had to leave biology behind forever: a subject that I thought had incredible power to change lives for the better. I was hugely excited about the opportunity to plunge myself into the world of mathematics, but I couldn't help worrying that I was taking on a subject that seemed to have few practical applications. I couldn't have been more wrong.

While I plodded through the pure math we were taught at university, memorizing the proof of the intermediate value theorem or the definition of a vector space, I lived for the applied-math courses. I listened to lecturers as they demonstrated the math that engineers use to build bridges so that they don't resonate and collapse in the wind, or to design wings that ensure planes don't fall out of the sky. I learned the quantum mechanics that physicists use to understand the strange goings-on at subatomic scales, and the theory of special relativity, which explores the strange consequences of the invariance of the speed of light. I took courses explaining the ways in which we use mathematics in chemistry, in finance, and in economics. I read about how we use mathematics in sports to enhance the performance of our top athletes, and how we use mathematics in the movies to create computer-generated images of scenes that couldn't exist in reality. In short, I learned that mathematics can be used to describe almost everything.

In the third year of my degree I was fortunate enough to take a course in mathematical biology. The lecturer was Philip Maini, an engaging Northern Irish professor in his forties. Not only was he the preeminent figure in his field (he would later be elected to the Fellowship of the Royal Society), but he clearly loved his subject, and his enthusiasm spread to the students in his lecture theater.

More than just mathematical biology, Philip taught me that mathematicians are human beings with feelings, not the one-dimensional automatons that they are often portrayed to be. A mathematician is more than just, as the Hungarian probabilist Alfréd Rényi once put it, "a machine for turning coffee into theorems." As I sat in Philip's office awaiting the start of the interview for a PhD place, I saw, framed on the walls, the numerous rejection letters he had received from the Premier League

I genuinely believe that math is for everyone and that we can all appreciate the beautiful mathematics at the heart of the complicated phenomena we experience daily. As we will see in the following chapters, math is the false alarms that play on our minds and the false confidence that helps us sleep at night; the stories pushed at us on social media and the memes that spread through it. Math is the loopholes in the law and the needle that closes them; the technology that saves lives and the mistakes that put them at risk; the outbreak of a deadly disease and the strategies to control it. It is the best hope we have of answering the most fundamental questions about the enigmas of the cosmos and the mysteries of our own species. It leads us on the myriad paths of our lives and lies in wait, just beyond the veil, to stare back at us as we draw our final breaths.

CHAPTER 1

THINKING EXPONENTIALLY

The Sobering Limits of Power

Darren Caddick is a driving instructor from a small town in South Wales. In 2009, he was approached by a friend with a lucrative offer. By contributing just £3,000 to a local investment syndicate and recruiting two more people to do the same, he would see a return of £23,000 in just a couple of weeks. Initially, thinking it was too good to be true, Caddick resisted the temptation. Eventually, though, his friends convinced him that "nobody would lose, because the scheme would just keep going and going and going," so he decided to throw in his lot.

Unwittingly, Caddick had found himself at the bottom of a pyramid scheme that couldn't "just keep going." Initiated in 2008, the Give and Take scheme ran out of new investors and collapsed in less than a year, but not before sucking in £21 million from over ten thousand investors across the UK, 90 percent of whom lost their £3,000 stake. Investment schemes that rely on investors recruiting multiple others to realize their payout are doomed to failure. The number of new investors needed at each level increases in proportion to the number of people in the scheme. After fifteen rounds of recruitment, there would be over ten thousand people in a pyramid scheme of this sort. Although that sounds like a large number, it was easily achieved by Give and Take. Fifteen rounds further on, however, and one in every seven people on the planet would need to invest to keep the scheme going. This rapid growth phenomenon, which led to an inevitable lack of new recruits and the eventual collapse of the scheme, is known as exponential growth.

No Use Crying over Spoiled Milk

Something grows exponentially when it increases in proportion to its current size. Imagine, when you open your pint of milk in the morning, a single cell of the bacteria *Streptococcus faecalis* finds its way into the bottle before you put the lid back on. *Strep f.* is one of the bacteria responsible for the souring and curdling of milk, but one cell is no big deal, right? Maybe it's more worrying when you find out that, in milk, *Strep f.* cells can divide to produce two daughter cells every hour. At each generation, the number of cells increases in proportion to the current number of cells, so their numbers grow exponentially.

The curve that describes how an exponentially growing quantity increases is reminiscent of a quarter-pipe ramp used by skaters, skateboarders, and BMXers. Initially, the gradient of the ramp is very low—the curve is extremely shallow and gains height only gradually (as you can see from the first curve in figure 2). After two hours four *Strep f.* cells are in your milk, and after four hours there are still only sixteen, which doesn't sound like too much of a problem. As with the quarter pipe, though, the height of the exponential curve and its steepness rapidly increase. Quantities that grow exponentially might appear to grow slowly at first, but they can take off quickly in a way that seems unexpected. If you leave your milk out on the side for forty-eight hours, and the exponential increase of *Strep f.* cells continues, when you pour it on your cereal again, there could be almost a thousand trillion cells in the bottle—enough to make your blood curdle, let alone the milk. At this point the cells would outnumber the people on our planet forty thousand to one. Exponential curves are sometimes referred to as J-shaped as they almost mimic the letter J's steep curve. As the bacteria use up the nutrients in the milk and change its pH, the growth conditions deteriorate, and the exponential increase is only sustained for a relatively short time. Indeed, in almost every real-world scenario, long-term exponential growth is unsustainable, and in many cases pathological, as the subject of the growth uses up resources in an unviable manner. Sustained exponential growth of cells in the body, for example, is a typical hallmark of cancer.

FIGURE 2: J-shaped exponential growth (*left*) and decay (*right*) curves.

Another example of an exponential curve is a free-fall waterslide, so called because the slide is initially so steep that the rider feels the sensation of free fall. This time, as we travel down the slide, we are surfing an exponential *decay* curve, rather than a *growth* curve (you can see an example of such a graph in the second image of figure 2). Exponential decay occurs when a quantity *decreases* in proportion to its current size. Imagine opening a huge bag of M&M's, pouring them out onto the table, and eating all the sweets that land with the *M*-side facing upward. Put the rest back in the bag for tomorrow. The next day give the bag a shake and pour out the M&M's. Again, eat the *M*-up sweets and put the rest back in the bag. Each time you pour the sweets out of the bag, you get to eat roughly half of those that remain, irrespective of the number you start with. The number of sweets decreases in proportion to the number left in the bag, leading to exponential decay in the number of sweets. In the same way, the exponential waterslide starts high up and

Having received a payout, the pilot then drops out of the scheme, and the two copilots are promoted to pilot, awaiting the recruitment of eight new passengers at the bottom of their trees. Airplane schemes are particularly seductive for investors, as new participants need only recruit two other people to multiply their investment by a factor of eight (although these two are required to recruit two more and so on). Other, flatter, schemes require far more recruitment effort per individual for the same returns. The steep four-level structure of Give and Take meant that crew members never took money directly from the passengers they recruited. Since new recruits are likely to be friends and relatives of the crew members, this ensures that money never travels directly between close acquaintances. This separation of the passengers from the pilots, whose payouts they fund, renders recruitment easier and reprisals less likely, making for a more attractive investment opportunity and thus facilitating the recruitment of thousands of investors to the scheme.

In the same way, many investors in the Give and Take pyramid scheme were given the confidence to invest by stories of successful payouts that had previously been made, and in some cases by even witnessing these payouts firsthand. The scheme's organizers, Fox and Chalmers, hosted lavish private parties at the Somerset hotel owned by Chalmers. Flyers handed out at the parties included pictures of the scheme's members, sprawled on cash-covered beds or waving fists of fifties at the camera. To each of these parties the organizers also invited some of the scheme's "brides"—those people (mainly women) who had made it to the position of pilot of their pyramid cell and were due to receive their payouts. The brides would be asked a series of four simple questions—such as "What part of Pinocchio grows when he lies?"—in front of an audience of two hundred to three hundred potential investors.

This "quiz" aspect of the scheme was supposed to exploit a loophole in the law, which Fox and Chalmers believed allowed for such investments if an element of "skill" was involved. In mobile-phone footage of one such event, Fox can be heard shouting, "We are gambling in our own homes and that's what makes it legal." She was wrong. Miles Bennet, the lawyer prosecuting the case, explained, "The quiz was so easy that there were never any people in the payout position who didn't get their money. They could even get a friend or a committee member to help with the questions, and the committee knew what the answers were!"

This didn't stop Fox and Chalmers from using these prize-giving parties as inoculants in their low-tech viral-marketing campaign. Upon seeing the brides presented with their £23,000 checks, many of the invited guests would invest and encourage their friends and family to do the same, forming the pyramid beneath them. Providing each new investor passed the baton to two or more others, the scheme would continue indefinitely. When Fox and Chalmers started the scheme, back in the spring of 2008, they were the only two pilots. By recruiting friends to invest and indeed help organize the scheme, the pair quickly brought four more people on board. These four recruited eight more and then sixteen and so on. This exponential doubling of the number of new recruits in the scheme closely mimics the doubling of the number of cells in a growing embryo.

The Exponential Embryo

When my wife was pregnant with our first child, we were obsessed, like many first-time parents-to-be, by trying to find out what was going on inside my wife's midriff. We borrowed an ultrasound heart monitor to listen to our baby's heartbeat; we signed up for clinical trials to get extra scans; and we read website after website describing what was going on with our daughter as she grew and continued to make my wife sick every day. Among our "favorites" were the "How big is your baby?"–type websites, which compare, for each week of gestation, the size of an unborn baby to a common fruit, vegetable, or other appropriately sized foodstuff. They give substance to prospective parents' unborn fetuses with epigrams such as "Weighing about one and a half ounces and measuring about three and a half inches, your little angel is roughly the size of a lemon" or "Your precious little turnip now weighs about five ounces and is approximately five inches long from head to bottom."

What struck me about these websites' comparisons was how quickly the sizes changed from week to week. At week four, your baby is roughly the size of a poppy seed, but by week five, she has ballooned to the size of a sesame seed! This represents an increase in volume of roughly sixteen times in a week.

Perhaps, though, this rapid increase in size shouldn't be so surprising. When the egg is initially fertilized by the sperm, the resulting zygote undergoes sequential rounds of cell division, called cleavage, which allow the number of cells in the developing embryo to increase rapidly. First, it divides into two. Eight hours later these two further subdivide into four, and after eight more hours, four become eight, which soon turn into sixteen, and so on—just like the number of new investors at each level of the pyramid scheme. Subsequent divisions occur almost synchronously every eight hours. Thus, the number of cells grows in proportion to the quantity of cells in the embryo at a given time: the more cells there are, the more new cells are created at the subsequent division. In this case, since each cell creates exactly one daughter cell at each division, the factor by which the number of cells in the embryo increases is two; in other words, the size of the embryo doubles every generation.

During human gestation the period in which the embryo grows exponentially is, thankfully, relatively short. If the embryo were to carry on growing at the same exponential rate for the whole pregnancy, the 840 synchronous cell divisions would result in a superbaby comprising roughly 10^{253} cells. To put that into context, if every atom in the universe were itself a copy of our universe, then the total number of atoms in all these universes would be roughly equivalent to the number of superbaby's cells. Naturally, cell division becomes less rapid as more complex events in the life of the embryo are choreographed. In reality the number of cells an average newborn baby comprises can be approximated at a relatively modest 2 trillion. This number of cells could be achieved in fewer than forty-one synchronous division events.

The Destroyer of Worlds

Exponential growth is vital for the rapid expansion in the number of cells necessary for the creation of a new life. However the astonishing and terrifying power of exponential growth also led nuclear physicist J. Robert Oppenheimer to proclaim, "Now I am become Death, the destroyer of worlds." This growth was not the growth of cells, nor even of individual organisms, but of energy created by the splitting of atomic nuclei.

During World War Two, Oppenheimer was the head of the Los Alamos laboratory, where the Manhattan Project—to develop the atomic bomb—was based. The splitting of the nucleus (tightly bound protons and neutrons) of a heavy atom into smaller constitutive parts had been discovered by German chemists in 1938. Named nuclear fission in analogy to the binary fission, or splitting, of one living cell into two, as occurs to such great effect in the developing embryo. Fission was found either to occur naturally, as radioactive decay of unstable chemical isotopes, or to be induced artificially by bombarding the nucleus of one atom with subatomic particles in a so-called nuclear reaction. In either case, the splitting of the nucleus into two smaller nuclei, or fission products, was concurrent with the release of large amounts of energy in the form of electromagnetic radiation, as well as the energy associated with the movement of the fission products. It was quickly recognized that these moving fission products, created by a first nuclear reaction, could be used to impact further nuclei, splitting more atoms and releasing yet more energy: a so-called nuclear chain reaction. If each nuclear fission produced, on average, more than one product that could be used to split subsequent atoms, then, in theory, each fission could trigger multiple other splitting events. If continued, the number of reaction events would increase exponentially, producing energy on an unprecedented scale. If a material could be found that would permit this unchecked nuclear chain reaction, the exponential increase in energy emitted over the short timescale of the reactions would potentially allow such a *fissile* material to be weaponized.

In April 1939, on the eve of the outbreak of war across Europe, French physicist Frédéric Joliot-Curie (son-in-law of Marie and Pierre and also a Nobel Prize winner in collaboration with his wife) made a crucial discovery. He published in the journal *Nature* evidence that, upon fission caused by a single neutron, atoms of the uranium isotope U-235 emitted on average 3.5 (later revised down to 2.5) high-energy neutrons. This was precisely the material required to drive the exponentially growing chain of nuclear reactions. The "race for the bomb" was on.

With Nobel Prize–winner Werner Heisenberg and other celebrated German physicists working for the Nazis' parallel bomb project, Oppenheimer knew he had his work cut out at Los Alamos. His main challenge was to create the conditions that would facilitate an exponentially growing nuclear chain reaction allowing the almost instantaneous release of the huge amounts of energy required for an atom bomb. To produce this self-sustaining and sufficiently rapid chain reaction, he needed to ensure that enough of the neutrons emitted by a fissioning U-235 atom were reabsorbed by the nuclei of other U-235 atoms, causing them to split in turn. He found that, in naturally occurring uranium, too many of the emitted neutrons are absorbed by U-238 atoms (the other significant isotope, which makes up 99.3 percent of naturally occurring uranium), meaning that any chain reaction dies out exponentially instead of growing. To produce an exponentially growing chain reaction, Oppenheimer needed to refine extremely pure U-235 by removing as much of the U-238 in the ore as possible.

These considerations gave rise to the idea of the *critical mass* of the fissile material. The critical mass of uranium is the minimum amount required to generate a self-sustaining nuclear chain reaction. It depends on a variety of factors. Perhaps most crucial is the purity of the U-235. Even with 20 percent U-235 (compared to the naturally occurring 0.7 percent), the critical mass is still over four hundred kilograms, making high purity essential for a feasible bomb. Even when he had refined sufficiently pure uranium to achieve supercriticality, Oppenheimer was left with the challenge of the delivery of the bomb itself. Clearly he couldn't just package up a critical mass of uranium in a bomb and hope it didn't explode. A single, naturally occurring decay in the material would trigger the chain reaction and initiate the exponential explosion.

With the specter of the Nazi bomb-developers constantly at their backs, Oppenheimer and his team came up with a hastily developed idea for the delivery of the atomic bomb. In their "gun-type" method, one

Infrared radiation burned exposed skin for miles in every direction. People on the ground close to the bomb's hypocenter were instantly vaporized or charred to cinders.

Akiko was sheltered from the worst of the bomb's blast by the earthquake-proof bank. When she regained consciousness, she staggered out onto the street. As she emerged, she found that the clear blue morning skies had gone. The second sun over Hiroshima had set almost as quickly as it had risen. The streets were dark and choked with dust and smoke. Bodies lay where they had fallen for as far as the eye could see. Only 260 meters from the hypocenter, Akiko was one of the closest to it to survive the terrible exponential blast.

The bomb itself and the resulting firestorms that spread across the city are estimated to have killed around seventy thousand people, fifty thousand of whom were civilians. The majority of the city's buildings were also completely destroyed. Oppenheimer's prophetic musings had come true. The justification for the bombings of both Hiroshima and, three days later, Nagasaki, as necessary measures to end the war, is still debated to this day.

The Nuclear Option

Whatever the rights and wrongs of the atomic bomb, the greater understanding of the exponential chain reactions caused by nuclear fission that was developed as part of the Manhattan Project gave us the technology required to generate clean, safe, low-carbon energy through nuclear power. One kilogram of U-235 can release roughly 3 million times more energy than burning the same amount of coal. Despite evidence to the contrary, nuclear energy suffers from a poor reputation for safety and environmental impact. In part, exponential growth is to blame.

On the evening of April 25, 1986, Alexander Akimov checked in for the night shift at the power plant in which he was shift supervisor. An experiment, designed to stress-test the cooling-pump system, was to get underway in a couple hours. As he initiated the experiment, he could have been forgiven for thinking how lucky he was—at a time when the

Soviet Union was collapsing and 20 percent of its citizens were living in poverty—to have a stable job at the Chernobyl nuclear power station.

At around 11:00 p.m., to reduce for the purposes of the test the power output to around 20 percent of normal operating capacity, Akimov remotely inserted a number of control rods between the uranium fuel rods in the reactor core. The control rods absorbed some of the neutrons released by atomic fission, so that these neutrons didn't cause too many other atoms to split. This put a break on the rapid growth of the chain reaction that would be allowed to run exponentially out of control in a nuclear bomb. However, Akimov accidentally inserted too many rods, causing the power output of the plant to drop significantly. He knew that this would cause reactor poisoning—the creation of material, like the control rods, that would further slow the reactor and decrease the temperature, which would lead to more poisoning and further cooling in a self-reinforcing feedback loop. Panicking now, he overrode the safety systems, placing over 90 percent of the control rods under manual supervision and removing them from the core to prevent the debilitating total shutdown of the reactor.

As he watched the needles on the indicator gauges rise as the power output slowly increased, Akimov's heart rate gradually returned to normal. Having averted the crisis, he moved to the next stage of the test, shutting down the pumps. Unbeknownst to Akimov, backup systems were not pumping coolant water as fast as they should have been. Although it was initially undetectable, the slow-flowing coolant water had vaporized, impairing its ability both to absorb neutrons and to reduce the heat of the core. Increased heat and power output led to more water flash-boiling into steam, allowing more power to be produced: another, altogether more deadly, positive feedback loop. The few remaining control rods that Akimov did not have under his manual supervision were automatically reinserted to rein in the increased heat generation, but they weren't enough. Upon realizing the power output was increasing too rapidly, Akimov pressed the emergency shutdown button designed to insert all the control rods and power down the core, but it was too late. As the rods plunged into the reactor, they caused a short but significant spike in power output, leading to an overheated core, fracturing some of the fuel rods and blocking further insertion of the control rods. As the heat energy rose exponentially, the power output increased to over ten times the usual operating level. Coolant water rapidly turned to steam, causing two massive pressure explosions, destroying the core and spreading the fissile radioactive material far and wide.

Refusing to believe reports of the core's explosion, Akimov relayed incorrect information about the reactor's state, delaying vital containment efforts. Upon eventually realizing the full extent of the destruction, he worked, unprotected, with his crew to pump water into the shattered reactor. As they worked, crew members received doses of two hundred grays per hour. A typical fatal dose is around ten grays, meaning that these unprotected workers received fatal doses in less than five minutes. Akimov died two weeks after the accident from acute radiation poisoning.

The official Soviet death toll from the Chernobyl disaster was just thirty-one, although some estimates that include individuals who helped in the large-scale cleanup are significantly higher. This is not to mention the deaths caused by the dispersal of radioactive material outside the immediate vicinity of the power plant. A fire that ignited in the shattered reactor core burned for nine days. The fire threw into the atmosphere hundreds of times more radioactive material than had been released during the bombing of Hiroshima, causing widespread environmental consequences for almost all of Europe.

On the weekend of May 2, 1986, for example, unseasonably heavy rainfall lashed the highlands of the UK. Within the falling raindrops were the radioactive products of the fallout from the explosion—strontium-90, cesium-137, and iodine-131. In total, around 1 percent of the radiation released from the Chernobyl reactor fell on the UK. These radioisotopes were absorbed by the soil, incorporated by the growing grass, and then eaten by grazing sheep. The result—radioactive meat.

The Ministry of Agriculture immediately placed restrictions on the sale and movement of sheep in the affected areas, with implications for nearly nine thousand farms and over 4 million sheep. Lake District sheep farmer David Elwood struggled to believe what was happening.

The cloud carrying the invisible, almost undetectable, radioisotopes cast a long shadow over his livelihood. Every time he wanted to sell sheep, he had to isolate them and call in a government inspector to check their radiation levels. Each time the inspectors came they would tell him restrictions would only last another year or so. Elwood lived under this cloud for over twenty-five years, until the restrictions were finally lifted in 2012.

It should, however, have been much easier for the government to inform Elwood and other farmers when radiation levels would be safe enough for them to sell their sheep freely. Radiation levels are remarkably predictable, thanks to the phenomenon of exponential *decay*.

The Science of Dating

Exponential decay, in direct analogy to exponential growth, describes any quantity that *decreases* with a rate proportional to its current value—remember the reduction in the number of M&M's each day and the waterslide curve that described their decline. Exponential decay describes phenomena as diverse as the elimination of drugs in the body and the rate of decrease of the head on a pint of beer. In particular, it does an excellent job of describing the rate at which the levels of radiation emitted by a radioactive substance decrease over time.

Unstable atoms of radioactive materials will spontaneously emit energy as radiation, even without an external trigger, in a process known as radioactive decay. At the level of an individual atom, the decay process is random—quantum theory implies that it is impossible to predict when a given atom will decay. However, in a material comprising huge numbers of atoms, the decrease in radioactivity is a predictable exponential decay. The number of atoms decreases in proportion to the number remaining. Each atom decays independently of the others. The rate of decay can be characterized by the half-life of a material—the time it takes for half of the unstable atoms to decay. Because the decay is exponential, no matter how much of the radioactive material is present to start with, the time for its radioactivity to decrease by half will always be the same. Pouring

M&M's out on the table each day and eating the *M*-up sweets leads to a half-life of one day—we expect to eat half of the sweets each time we pour them out of the bag.

The phenomenon of exponential decay of radioactive atoms is the basis of radiometric dating, the method used to date materials by their levels of radioactivity. By comparing the abundance of radioactive atoms to that of their known decay products, we can theoretically establish the age of any material emitting atomic radiation. Radiometric dating has well-known uses, including approximating the age of the Earth and determining the age of ancient artifacts such as the Dead Sea Scrolls. If you ever wondered how on earth they knew that archaeopteryx was 150 million years old or that Ötzi the iceman died fifty-three hundred years ago, the chances are that radiometric dating was involved.

Recently, more accurate measurement techniques have facilitated the use of radiometric dating in "forensic archaeology"—the use of exponential decay of radioisotopes (among other archaeological techniques) to solve crimes. In 2017, radiocarbon dating exposed the world's most expensive whiskey as a fraud. The bottle, labeled as an 1878 Macallan single malt, was proved to be a cheap blend from the 1970s, much to the chagrin of the Swiss hotel that sold a single shot of it for $10,000. In December 2018, in a follow-up investigation, the same lab found that over a third of "vintage" Scotch whiskeys they tested were also fakes. But perhaps the most high-profile use of radiometric dating is in verification of the age of historical artworks.

Before World War Two, only thirty-five paintings by Dutch Old Master Johannes Vermeer were known to exist. In 1937, a remarkable new work was discovered in France. Lauded by art critics as one of Vermeer's greatest works, *The Supper at Emmaus* was quickly procured at great expense for the Museum Boijmans Van Beuningen in Rotterdam. Over the next few years several more, hitherto unknown, Vermeers surfaced. These were quickly appropriated by wealthy Dutchmen, in part in an attempt to prevent the loss of important cultural property to the Nazis.

spreads between people through a social network, just like a virus. Richard Dawkins coined the word *meme* in his 1976 book, *The Selfish Gene*, to explain the way in which cultural information spreads. He defined memes as units of cultural transmission. In analogy to genes, the units of heritable transmission, he proposed that memes could self-replicate and mutate. The examples he gave of memes included tunes, catchphrases, and, in a wonderfully innocent indication of the times in which he wrote the book, ways of making pots or building arches. Of course, in 1976, Dawkins had not come across the internet in its current form, which has allowed the spread of once unimaginable (and arguably pointless) memes including #thedress, rickrolling, and lolcats.

One of the most successful, and perhaps genuinely organic, examples of a viral marketing campaign was the ALS ice bucket challenge. During the summer of 2014, videoing yourself having a bucket of cold water thrown over your head and then nominating others to do the same, while possibly donating to charity, was the thing to do in the northern hemisphere. Even I caught the bug.

Adhering to the classic format of the ice bucket challenge, after being thoroughly soaked I nominated two other people in my video, whom I later tagged when I uploaded it to social media. As with the neutrons in a nuclear reactor, as long as, on average, at least one person takes up the challenge for every video posted, the meme becomes self-sustaining, leading to an exponentially increasing chain reaction.

In some variants of the meme, those nominated could either undertake the challenge and donate a small amount to the amyotrophic lateral sclerosis (ALS) association or another charity of their choice, or choose to shirk the challenge and donate significantly more in reparation. In addition to increasing the pressure on nominated individuals to participate in the meme, the association with charity had the added bonus of making people feel good about themselves by raising awareness, and promoting a positive image of themselves as altruistic. This self-congratulatory aspect increased the infectiousness of the meme. By the start of September 2014, the ALS association reported receiving over $100 million in additional funding from over 3 million donors. As a result of the funding received during the challenge, researchers discovered a third gene responsible for ALS, demonstrating the viral campaign's far-reaching impact.

In common with some extremely infective viruses such as flu, the ice bucket challenge was also highly seasonal (an important phenomenon, in which the rate of disease spread varies throughout the year, and that we will meet again in chapter 7). As autumn approached and colder weather hit the northern hemisphere, getting doused in ice-cold water suddenly seemed like less fun, even for a good cause. By September, the craze had largely died off. Just like the seasonal flu, though, it returned the next summer and the summer after in similar formats, but to a largely saturated population. In 2015, the challenge raised less than 1 percent of the previous year's total for the ALS association. People exposed to the virus in 2014 had typically built up a strong immunity, even to slightly mutated strains (different substances in the bucket, for example). Tempered by the immunity of apathy, each new outbreak soon died out as each new participant failed, on average, to pass on the virus to at least one other.

Is the Future Exponential?

A parable of exponential growth is told to French children to illustrate the dangers of procrastination. One day, it is noted that an extremely small algal colony has formed on the surface of the local lake. Over the next few days, the colony is found to be doubling its coverage of the surface of the lake each day. It will continue to grow like this until it covers the lake unless something is done. If left unchecked, it will take sixty days to cover the surface of the lake, poisoning its waters. Since the algal coverage is initially so small, with no immediate threat, the algae is left to grow until it covers half the surface of the lake, when it will more easily be removed. The question is then asked, "On which day will the algae cover half of the lake?"

A common answer that many people give to this riddle, without thinking, is thirty days. But, since the colony doubles in size each day, if the lake is half-covered one day, it will be completely covered the next day. The perhaps surprising answer, therefore, is that the algae will cover half the surface of the lake on the fifty-ninth day, leaving only one day to save the lake. At thirty days the algae takes up less than a billionth of the capacity of the lake. If you were an algal cell in the lake, when would you realize you were running out of space? Without understanding exponential growth, if someone told you on the fifty-fifth day, when the algae covered only 3 percent of the surface, that the lake would be completely choked in five days' time, would you believe it? Probably not.

This highlights the way in which we, as humans, have been conditioned to think. Typically, for our forebears, the experiences of one generation were much like those of the last: they did the same jobs, used the same tools, and lived in the same places as their ancestors. They expected their descendants to do the same. However, the growth of technology and social change is now occurring so rapidly that noticeable differences occur within single generations. Some theoreticians believe that the rate of technological advancement is itself increasing exponentially.

Computer scientist Vernor Vinge encapsulated just such ideas in a series of science fiction novels and essays, in which successive technological advancements arrive with increasing frequency until new technology outstrips human comprehension. The explosion in artificial intelligence ultimately leads to a "technological singularity" and the emergence of an omnipotent, all-powerful superintelligence. American futurist Ray Kurzweil attempted to take Vinge's ideas out of the realm of science fiction and apply them to the real world. In 1999, in his book *The Age of Spiritual Machines*, Kurzweil hypothesized "the law of accelerating returns." He suggested that the evolution of a wide range of systems—including our own biological evolution—occurs at an exponential pace. He even went so far as to pin the date of Vinge's "technological singularity"—the point at which we will experience, as Kurzweil describes it, "technological change so rapid and profound it represents a rupture in the fabric of human history"—to around 2045. Among the implications of the singularity, Kurzweil lists "the merger of biological and nonbiological intelligence, immortal software-based humans, and ultra-high levels of intelligence that expand outward in the universe at the speed of light." While these extreme, outlandish predictions should probably have been confined to the realm of science fiction, some technological advances really have sustained exponential growth over long periods.

Moore's Law—the observation that the number of components on computer circuits seems to double every two years—is a well-cited example of exponential growth of technology. Unlike Newton's laws of motion, Moore's Law is not a physical or natural law, so there is no reason to suppose it will continue to hold forever. However, between 1970 and 2016 the law has held remarkably steady. Moore's Law is implicated in the wider acceleration of digital technology, which in turn contributed significantly to economic growth in the years surrounding the turn of the last century.

In 1990, when scientists undertook to map all 3 billion letters of the human genome, critics scoffed at the scale of the project, suggesting that it would take thousands of years to complete at the current rate. But sequencing technology improved at an exponential pace. The complete "Book of Life" was delivered in 2003, ahead of schedule and within its $1 billion budget. Today, sequencing an individual's whole genetic code takes under an hour and costs less than $1,000.

Population Explosion

The story of the algae in the lake highlights that our failure to think exponentially can be responsible for the collapse of ecosystems and populations. One species on the endangered list, despite clear and persistent warning signs, is, of course, our own.

Between 1346 and 1353, the Black Death, one of the most devastating pandemics in human history (we will investigate infectious-disease spread in more detail in chapter 7), swept through Europe, killing 60 percent of its population. The total population of the world was reduced to around 370 million. Since then the global population has increased constantly without abating. By 1800, the human population had almost reached its first billion. The perceived rapid increase in population at that time prompted the English mathematician Thomas Malthus to suggest that the human population grows at a rate that is proportional to its current size. As with the cells in the early embryo or the money left untouched in a bank account, this simple rule suggests exponential growth of the human population on an already-crowded planet.

A trope of many science fiction novels and films (take the recent blockbusters *Interstellar* and *Passengers*, for example) is to solve the problems of the world's growing population through space exploration. Typically, a suitable Earth-like planet is discovered and prepared for habitation for the overspilling human race. Far from being a purely fictional fix, in 2017 eminent scientist Stephen Hawking gave credibility to the proposition of extraterrestrial colonization. He warned that humans should start leaving Earth within the next thirty years, to colonize Mars or the Moon, if our species is to survive the threat of extinction presented by overpopulation and associated climate change. Disappointingly, though, if our growth rate continued unchecked, even shipping half of Earth's population to a new Earth-like planet would only buy us another sixty-three years until the human population doubled again and both planets reached saturation point. Malthus forecast that exponential growth would render the idea of interplanetary colonization futile when he wrote, "The germs of existence contained in this spot of earth, with ample food, and ample room to expand in, would fill millions of worlds in the course of a few thousand years."

However, as we have already found (remember the bacteria *Strep f.* growing in the milk bottle at the start of this chapter), exponential growth cannot be sustained forever. Typically, as a population increases, the resources of the environment that sustains it become more sparsely distributed, and the net rate of growth (the difference between the birth rate and the death rate) naturally drops. The environment is said to have a "carrying capacity" for a particular species—an inherent maximum sustainable population limit. Darwin recognized that environmental limitations would cause a "struggle for existence" as individuals "compete for their places in the economy of nature." The simplest mathematical model to capture the effects of competition for limited resources, within or between species, is known as the logistic growth model.

time. However, exponential behavior may make us, as individuals, feel as if we have less time left than we think.

Time Flies When You're Getting Old

Do you remember, when you were younger, that summer holidays seemed to last an eternity? For my children, who are four and six, the wait between consecutive Christmases seems like an inconceivable stretch of time. In contrast, as I get older, time appears to pass at an alarming rate, with days blending into weeks and then into months, all disappearing into the bottomless sinkhole of the past. When I chat weekly with my septuagenarian parents, they give me the impression that they barely have time to take my call, so busy are they with the other activities in their packed schedules. When I ask them how they fill their week, however, it often seems as if their unrelenting travails might comprise the work of just a single day for me. But then what would I know about competing time pressures? I just have two kids, a full-time job, and a book to write.

I should not be too caustic with my parents, though, because it seems that perceived time really does run more quickly the older we get, fueling our increasing feelings of overburdened time-poverty. In an experiment carried out in 1996, a group of younger people (nineteen to twenty-four) and a group of older people (sixty to eighty) were asked to count out three minutes in their heads. On average, the younger group clocked an almost-perfect three minutes and three seconds of real time, but the older group didn't call a halt until a staggering three minutes and forty seconds, on average. In other related experiments, participants were asked to estimate the length of a fixed period of time during which they had been undertaking a task. Older participants consistently gave shorter estimates for the length of time they had experienced than younger groups. For example, after two minutes of real time, the older group had, on average, clocked less than fifty seconds in their heads, leading them to question where the remaining minute and ten seconds had gone.

This acceleration in our perception of the passage of time has little to do with leaving behind those carefree days of youth and filling our calendars with adult responsibilities. A number of competing ideas explain why, as we age, our perception of time accelerates. One theory notes that our metabolism slows as we get older, matching the slowing of our heartbeats and our breathing. Just as with a stopwatch that is set to run fast, children's versions of these "biological clocks" tick more quickly. In a fixed period of time children experience more beats of these biological pacemakers (breaths or heartbeats, for example), making them feel as if a longer time has elapsed.

A competing theory suggests that our perception of time's passage depends upon the amount of new perceptual information we are subjected to from our environment. The more novel stimuli, the longer our brains take to process the information. The corresponding period of time seems, at least in retrospect, to last longer. This argument can explain the movie-like perception of events playing out in slow motion in the moments immediately preceding an accident. In these scenarios, so unfamiliar is the situation for the accident victim that the amount of novel perceptual information is correspondingly huge. It might be that rather than time actually slowing during the event, our recollection of the event is decelerated in hindsight, as our brain records more detailed memories based on the flood of data it receives. Experiments on subjects experiencing the unfamiliar sensation of free fall have demonstrated this.

This theory ties in nicely with the acceleration of perceived time. As we age, we tend to become more familiar with our environments and with life experiences. Our daily commutes, which might initially have appeared long and challenging, full of new sights and opportunities for wrong turns, now flash by as we navigate their familiar routes on autopilot.

It is different for children. Their worlds are often surprising places filled with unfamiliar experiences. Youngsters are constantly reconfiguring their models of the world around them, which takes mental effort and seems to make the sand run more slowly through their hourglasses than for routine-bound adults. The greater our acquaintance with the routines of everyday life, the quicker we perceive time to pass, and generally, as we age, this familiarity increases. This theory suggests that, to make our time last longer, we should fill our lives with new and varied experiences, eschewing the time-sapping routine of the everyday.

Neither of the above ideas explains the almost perfectly regular rate at which our perception of time seems to accelerate. That the length of a fixed period of time appears to reduce continually as we age suggests an "exponential scale" to time. We employ exponential scales instead of traditional linear scales when measuring quantities that vary over a huge range of different values. The most well-known examples are scales for energy waves such as sound (measured in decibels) or seismic activity. On the exponential Richter scale (for earthquakes), an increase from magnitude 10 to magnitude 11 would correspond to a tenfold increase in ground movement, rather than a 10 percent increase as it would do on a linear scale. At one end, the Richter scale captured the low-level tremor felt in Mexico City in June 2018 when Mexican football fans in the city celebrated their goal against Germany at the World Cup. At the other extreme, the scale recorded the 1960 Valdivia earthquake in Chile. The magnitude 9.6 quake released energy equivalent to over a quarter of a million of the atomic bombs dropped on Hiroshima.

If a period of time is judged in proportion to the time we have already been alive, then an exponential model of perceived time makes sense. As a thirty-four-year-old, a year accounts for just under 3 percent of my life. My birthdays seem to come around all too quickly these days. But to ten-year-olds, waiting 10 percent of their life for the next round of presents requires almost saintly patience. To my four-year-old son, the idea of having to wait a quarter of his life until he is the birthday boy again is almost intolerable. Under this exponential model, the proportional increase in age that a four-year-old experiences between birthdays is equivalent to a forty-year-old waiting until he or she turns fifty. When looked at from this relative perspective, it makes sense that time seems only to accelerate as we age.

We commonly categorize our lives into decades—our carefree twenties, our serious thirties, and so on—which suggests that each period should be afforded an equal weighting. However, if time does appear to speed up exponentially, chapters of our life spanning different lengths of time might feel as if they were of the same duration. Under the exponential model, the ages from five to ten, ten to twenty, twenty to forty, and even forty to eighty might all seem equally long (or short). Not to precipitate the frantic scribbling of too many bucket lists, but under this model the forty-year period between forty and eighty, encompassing much of middle and old age, might flash by as quickly as the five years between your fifth and tenth birthdays.

It should be some small compensation, then, for pensioners Fox and Chalmers, jailed for running the Give and Take pyramid scheme, that the routine of prison life, or just the exponentially increasing passage of perceived time, should make their sentences seem to pass very quickly indeed.

In total, nine women were sentenced for their part in the scheme. Although some were forced to pay back part of the money they had made from the scheme, little of the millions of pounds invested in it was recovered. None of this money made its way to the scheme's defrauded investors—the unsuspecting victims who lost everything because they underestimated the power of exponential growth.

From the explosion of a nuclear reactor to the explosion of the human population, and from the spread of a virus to the spread of a viral marketing campaign, exponential growth and decay can play an unseen, but often critical, role in the lives of normal people like you and me. The exploitation of exponential behavior has spawned branches of science that can convict criminals and others that can now, quite literally, destroy worlds. Failing to think exponentially means our decisions, like uncontrolled nuclear chain reactions, can have unexpected and exponentially far-reaching consequences. Among other innovations, the exponential pace of technological advancements has hastened in the era of personalized medicine, in which anyone can have his or her DNA sequenced for a relatively modest sum. This genomics revolution has the potential to lend unprecedented insight into our own health traits, but only, as we will examine in the next chapter, if the mathematics that underpins modern medicine is able to keep pace.