THE QUANTUM MATRIX

HENRY BAR'S PERILOUS STRUGGLE FOR QUANTUM COHERENCE

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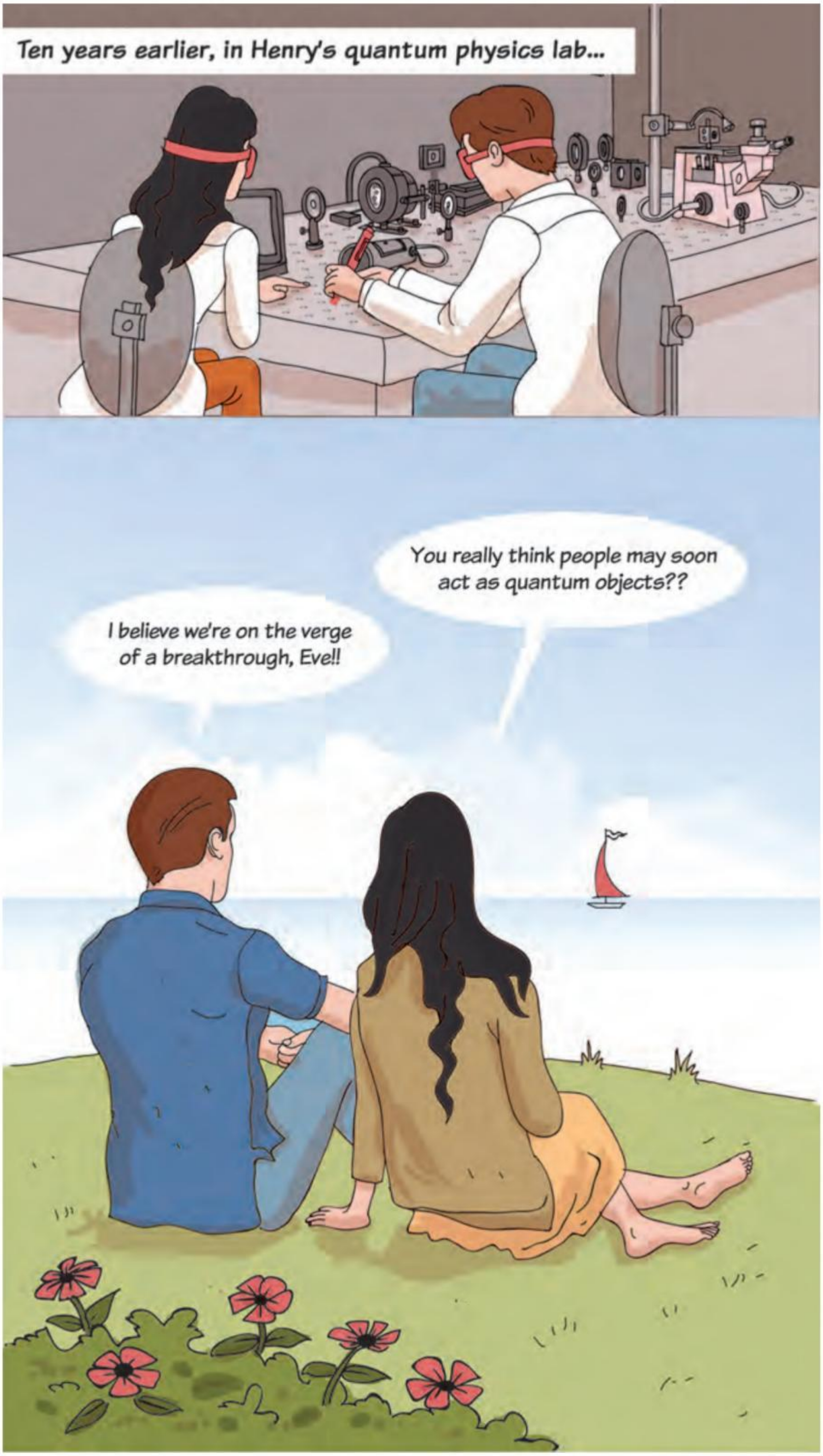
PART I

Basic concepts

Henry Bar's Debut as Quantum Superhero


Ten years earlier, in Henry's quantum physics lab...
I believe we're on the verge of a breakthrough, Eve!!
You really think people may soon act as quantum objects??


Back to the present..
So, the consensus is that quantum physics only holds for tiny objects??
I disagree!
EUREKA!!
I've crafted a Quantum Suit!! This proves quantum physics can rule large objects like myself!!
It's unfair! Henry had shared his idea with me before we drifted apart. I'm entitled to it more than his present friends, A&B! You'll all be sorry for that!!!

CHAPTER 1

What is Quantumness?

1.1 HOW SMALL IS A QUANTUM?

We have found Henry Bar at the turning point of his life, where he is on the verge of becoming the first quantum superhero, having discovered the incredible yet true principle that all things, large and small, are subject to the laws of quantum physics. He finds out that it may be possible, albeit extremely challenging, even for us humans to manifest our "quantumness". This principle underlies Henry's design and implementation of his fabulous quantum suit that allows him to act as a distinctly quantum object in his outlandish adventures.

Henry has made his dramatic discovery as a result of his pondering over deep questions that must be as troubling to the reader as they were to him:

- What is quantum physics?
- What are its laws?
- How do these laws fit into the overall framework of physics and science in general?

Let us recreate Henry's process of coping with these questions by proceeding from the general to the particular issues.

Henry has come to realize that physics is the most comprehensive and fundamental natural science in that it describes and explains the structure and dynamics of all known objects by a set of concise mathematical rules. These rules, when deemed sufficiently general, attain the status of laws of nature.

The diversity of disciplines that have become part of physics is staggering, to Henry's amazement: nowadays, the laws of physics are the key not only to understanding all phenomena that have long been described by branches of physics—e.g., mechanics, electromagnetism and heat—but also to chemistry, biological processes, planetary sciences and cosmology.

He has realized that physics is not only vast but also confidence-inspiring, because since the days of Galileo (in the early seventeenth century) it has followed the arduous yet safe path (at least in hindsight) prescribed by Francis Bacon, the Elizabethan prophet of science:

> ***Observation*** *(of natural phenomena) –> **hypothesis** (concerning their underlying principle or mechanism) –> **theory** (culminating in a **law**) –> **experimental test** of the theory.*

This path has eventually led to the birth of contemporary physics. Henry's awareness of the fierce scientific battles that marked the evolution of physics has not weakened his conviction that present-day physical theory has been incontestably validated, both by experiments conducted under stringent accuracy bounds and by the scrupulous criteria of mathematical and logical consistency.

Henry, as all of us, has only had first-hand experience of objects behaving according to classical physics. When you push an object, it accelerates and moves along a trajectory determined by the pushing force. Two objects that are so distant that there is no force acting between them behave totally independently.

But quantum objects defy all the foregoing notions: an object can be smeared over a large region of space, and, depending on how you push it, may become either fuzzier (more wavelike) or, conversely, more localized (particle-like). Two quantum objects that were in touch and then receded far apart are still "entangled"; that is, no longer independent. This is only a partial list of strange notions associated with "quantumness".

Now, here is the true spoiler for our recount of Henry's quest: he has come to the conclusion that there is a solid foundation for his astounding finding that **all physics is essentially quantum**. In order to understand and appreciate the boldness of this conclusion we have to pre-empt the historical narrative (Section 1.2) of the emergence of quantum physics in the early twentieth century as a revolutionary theory of radiation, atoms and subatomic particles. It defied the physical theories which reigned supreme at the time: Newton's physics of material bodies and the forces that act between them, or Maxwell's laws of electromagnetic forces and radiation (ranging from radio waves through light waves to gamma rays). Our narrative reveals how quantum physics (or quantum mechanics, as it was originally named) was recognized some ninety years ago to be the only viable theory of atomic and subatomic phenomena and of effects involving very feeble portions of light called photons or quanta. On the other hand, all things ranging from large molecules through living cells or dust particles to people, mountains or planets are still described by "classical"—that is, Newton's—physics.

Henry has become aware of a subtle division between classical and quantum electromagnetic phenomena. Since the mid-nineteenth century it has been known that electromagnetic forces (fields) and radiation—e.g., light—propagate in space as waves. The same is true of photons in quantum physics. The difference is that the energy carried by a single photon, a single "particle of light", cannot be divided into smaller portions, at least not by simple manipulations, so that a photon is a truly indivisible quantum carrier of energy.

For a while, Henry was deeply puzzled by this alleged rift between classical and quantum descriptions: does it mean that there are two (or more) unrelated kinds of physics? If so, should not physics be deprived of its status as a universal "science of everything"? And if, on the contrary, physics is one, how do classical and quantum physics connect?

In the course of his studies, Henry often came across the "correspondence principle": the statement that the application of the rules of quantum physics to objects much heavier or bulkier than an atom yields a "classical" result. Yet this principle did not appear to him compelling: why should the basic rules change as the mass or size grow? Furthermore, in his more advanced reading Henry encountered a remarkable quantum effect known as *superfluidity* whereby a liquid composed of a macroscopic number of helium-3 atoms may flow at very low temperatures as if it were a single quantum object! So, Henry triumphantly concluded that his hunch had been correct: not only mass or size matter when discerning a quantum object from a classical one. But then how to decide whether an object is quantum? And is such a decision always clear-cut?

Having acquired a deeper understanding of quantum physics, it has become clear to Henry that the key quantity in assessing quantumness is "**action**": the change in the energy of the object multiplied by the duration of the change. The essence of quantumness in nature is that action cannot be less than an elementary portion; a quantum in Latin. How small is this quantum of action? As explained in the appendix (Section 1.4), it is an exceedingly small number, common to all processes in nature (a universal constant). It is known as Planck's constant, which is denoted by the symbol $\hbar$. Henry Bar's name is a pun on this symbol, which is the "coat of arms" of our quantum superhero.

In order to appreciate the smallness of $\hbar$, Henry analyzed the action of a very lightweight and compact object: a microscopic pendulum which weighs one billionth of a gram and is attached to a micrometer-long cord. This calculation shows that a kick that would set this pendulum in motion would still be 1,000,000,000,000,000,000 times larger than $\hbar$; to observe a single quantum of action we would then need an accuracy of 1 part in 10^{18}!

This analysis has been an eye-opener for Henry: he has realized that sufficiently high accuracy of measurements or control can make any object, no matter how heavy or large, reveal its quantumness! This conclusion has been reinforced by Henry's further reading that nowadays quantum effects are being measured for clouds of millions of ultracold atoms or nanomechanical cantilevers and membranes, but as late as the 1990s such ultraprecise measurements would have been considered a pipedream (Chapters 2–4). The size, weight and complexity of objects that act quantum-mechanically keeps growing by the year. That is why experiments probing the quantumness of man-size objects such as Henry, although unfeasible today, cannot be ruled out in the future.

Henry has thus come to the conclusion that the boundary between quantum and classical objects is largely arbitrary. Current experiments are already capable of revealing the quantumness of objects visible to the naked eye; techniques whose germs already exist, as we will show, can push this boundary to the extent that even macroscopic objects may display quantum behavior.

Notwithstanding the formidable technical challenges that may prevent us from observing quantum features of various objects, a much more general insight transpires from the foregoing discussion: *the universality of quantum description*. Quantumness is lurking beneath classical phenomena, **physics is one**, and it is up to us to reveal its quantum face, should we wish so. As the Henry Odyssey unfolds, so will the narrative of the effects of quantum physics, from the simplest to the more advanced.

1.2 FROM ATOMISM TO QUANTUM MECHANICS

Let us trace the origins of Henry's initial view shared by many scientists and the lay public alike, which draws a line, crudely speaking, between the quantum description that is incontestable in explaining atomic or subatomic phenomena and the classical (Newton's, Einstein's, or Maxwell's, as the case may be) description of the macroscopic world. How did this view come about?

Its roots are in the notion of atomism that originated in ancient Greece and took more than two millennia to become a universal theory of the "true" reality beneath the realm of our everyday (macroscopic) experience. The evolution of atomism and the emergence of quantum physics from these roots will be sketched in what follows.

a) *The atom makes its appearance*. The idea of an atom as the ultimate, indivisible (a-tom in Greek) and immutable constituent of matter is attributed to Democritus (Figure 1.1), "the laughing philosopher", in the fifth century BC. This idea won many adepts in the Greco-Roman world among followers of Epicurean and Stoic philosophy. They were drawn by the blind, purposeless world view of atomism: atoms collide at random, combine to form an object, then randomly recede and disintegrate the object, then recombine, and so on and so forth, forever. Its opponents loathed this world view, either on religious grounds or because it defied their logic. It is unfortunate for the development of science that Aristotle in the fourth century BC ridiculed atomism because it contradicted his notion that the properties of objects are immutable, as opposed to the atomistic view that all objects incessantly change, disappear and are born again through atomic collisions and only the atoms are unchanged. The indisputable authority of Aristotle condemned atomism to oblivion. Still, this idea resurfaced in the late seventeenth century and acquired repute in the nineteenth century.

One discipline where atomism found a fertile ground was chemistry in the aftermath of the revolutionary discovery by the French lawyer-turned-scientist A. Lavoisier (shortly before his head was chopped off by a revolutionary tribunal in 1792) that in chemical reactions the mass (weight) ratios between the reacting substances are fixed. The British scientist J. Dalton (Figure 1.1) surmised in 1803 that Lavoisier's mysterious discovery could be explained by the supposition that all substances are composed of atoms with fixed specific masses. The Italian scientist A. Avogadro proposed in 1811 that vessels of the same volume should contain the same number of atoms regardless of the atomic mass, if we assume that the atoms are tightly packed in the vessel as tiny balls. A hundred years later this universal number was quantified, measured and named after Avogadro. Perhaps the greatest achievement of atomism in nineteenth-century

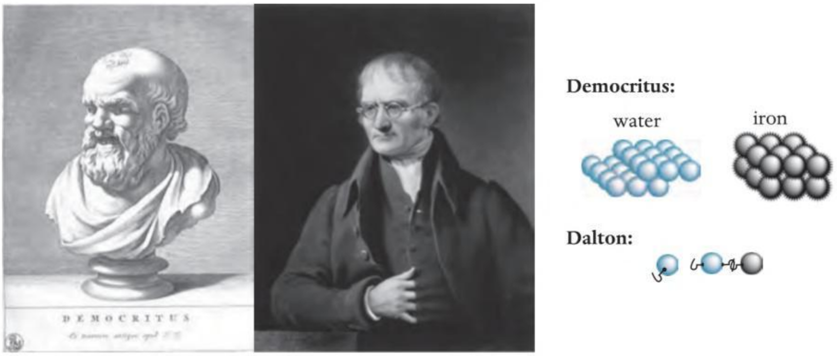


Fig. 1.1 The Fathers of atomism—Democritus (left) and Dalton (right)—and their ideas. Democritus posited that atoms of different shape, size and smoothness combine to form different substances. Dalton assumed that atoms bind to form molecules.

chemistry was the arrangement of all chemical elements in a periodic table by the Russian chemist D. Mendeleev, who arranged the elements in the table according to a mysterious index which he named the "atomic number". It was not until the quantum theory of atoms emerged in the twentieth century that the atomic number was understood to be determined by the inner structure of the atom, which had been completely unknown at the time of Mendeleev! Despite these tremendous nineteenth-century discoveries that relied on atomism, in the absence of direct evidence for the existence of atoms, their opponents remained unconvinced, as our tale will show.

Another discipline where a handful of bold scientists resorted to atomism was the physics of gases. In 1738 the Italian scientist D. Bernoulli gave an atomistic explanation to the seventeenth-century R. Boyle's law whereby gas pressure grows inversely with the volume of the container. According to Bernoulli, pressure was a force exerted on the container walls by colliding gas atoms, and Newtonian mechanics implied that this force would increase as the container became tighter. Unfortunately, Newton's own opposition to atomism hindered the progress of this idea. Oddly enough, Newton believed in the corpuscular (particle-like) nature of light, but not of matter.

It was the Scottish physicist J. C. Maxwell (Figure 1.2) who gave atomism a boost in the mid-nineteenth century by introducing the concept of random collisions among atoms in a gas: he maintained that the average velocity of many colliding atoms determines the gas temperature, but an individual atom may have velocity, of which we may know only the statistical probability. This atomic theory of gases was embraced and further developed by the Austrian physicist L. E. Boltzmann (Figure 1.2) and the American scientist J. W. Gibbs in the late nineteenth century. Yet it was met by vehement opposition. The influential physicist and philosopher from Prague, E. Mach, objected to atoms on "positivist" grounds: if you cannot see or detect an atom and cannot even ascribe a definite velocity or path to it, then it is fiction that does not represent "positive" data. F. W. Ostwald, the German founder of physical chemistry, was another objector to atoms on similar grounds, but mainly because he had an alternative theory of matter. Most of the attacks on atomism were directed against Boltzmann, who viewed the random, statistical description of atoms as the basis of heat exchange theory known as thermodynamics. Such statistical description was considered by many to be an abomination to the exact science of certainty that physics, including thermodynamics, was supposed to be. In the face of so much opposition, Boltzmann, who felt that his life's work had been rejected, killed himself in one of his depressive moods in 1905.

Fig. 1.2 Maxwell (left) and Boltzmann (right). The greatest nineteenth-century atomists posited that random collisions among many atoms in a vessel are the origin of thermodynamic laws.

Little did Boltzmann know that, within months of his death, Einstein's theory of the random motion of a macroscopic particle in a liquid (known as Brownian motion) would fully vindicate the reality of the atomic (or molecular) composition of liquids, and suggest a way of inferring the atoms' presence and number (Avogadro's number) from the rate at which the macroscopic particle diffuses in the liquid under random collisions by the atoms. A few years later, the influential French physicist J. Perrin concluded, on the basis of all available evidence, that atoms were no longer a hypothesis but scientific fact: the atom finally prevailed! The ancient atomists would have probably been thrilled at this acceptance of their notions of random collisions between indivisible entities as the key to observable phenomena. Mach, on the other hand, persisted in his denial of the existence of atoms till his death. Evidently, scientists may also have their biases.

b) *The atom splits*. Just as the status of the atom was elevated from fiction to fact, another conceptual change came about. Two landmark experiments convincingly demonstrated not just the existence of atoms but also their divisibility that contradicted their very name and original notion. J. J. Thomson (Britain) measured the fragments of gas atoms shattered by electric impulses. Upon applying an electric force to the gas, Thomson measured the curvature of the fragment trajectories and deduced from Newtonian mechanics the ratio of the electric charge to the mass of the fragments. One fragment, nicknamed the "nucleus", was found to be positively charged and thousands of times heavier than the other, negatively charged fragment, termed the *electron*. Subsequently, E. Rutherford (New Zealand/Britain) (Figure 1.3) measured by similar methods the charges and masses of fragments of the atomic nucleus that disintegrated by radioactive decay. The atom was now real and divisible.

Rutherford thought of the atom as a miniature planetary system: a heavy nucleus encircled by orbiting, much lighter electrons. Yet it was unclear what kept only certain electronic orbits stable, but not others. Data on radiation emission and absorption by atoms suggested that electrons in atoms changed their orbital motion in a peculiar, inexplicable manner. A new theory was acutely needed.

c) *Radiation goes quantum*. In 1900 the German physicist M. Planck (Figure 1.4) presented his astounding theory of the properties of radiation emanating from an almost closed box or cavity (oven). Planck's theory was meant to explain the experimentally observed dependence of the radiation frequency on the temperature of the oven (cavity) walls. Planck came to the momentous conclusion that no equilibrium between the absorption and emission of radiation by the oven walls is possible unless we suppose that the radiation can take up or give off energy only in tiny, discrete portions (quanta in Latin). There was no analog to this radical conclusion in the physics of the day, namely, in thermodynamics or in Maxwell's theory of electromagnetic radiation wherein radiation energy at a given frequency may have arbitrary, not just discrete, values. However, Planck noted that without the discreteness (quantum) assumption the existing theory would lead to the absurd conclusion that the total radiated intensity was infinite, which meant that radiation could not be in equilibrium with the walls, contrary

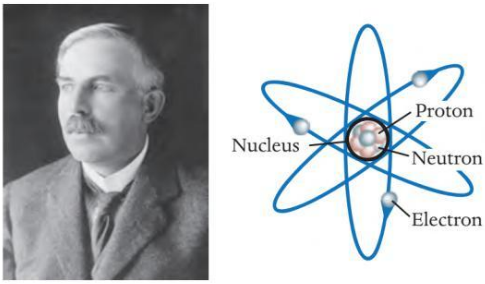


Fig. 1.3 Rutherford—the discoverer of atom splitting—and his planetary model of the atom

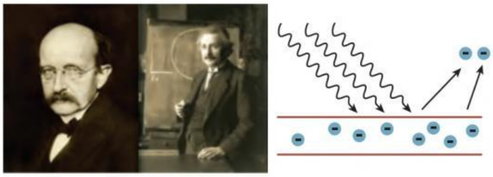


Fig. 1.4 The Fathers of radiation quanta—Planck (left) and Einstein (right)—and the photoelectric effect whereby light quanta knock out electrons from a metal.

to the experimental evidence! Yet Planck was reluctant to think of quanta as real objects. After years of attempts to reconcile this notion with Maxwell's electromagnetism and thermodynamics, to no avail, he was forced to recognize that, unintentionally, he had opened up a new era in physics: *quantum physics was born in 1900!*

d) *Quanta become real*. Five years later, in 1905, A. Einstein (Figure 1.4) took the notion of light quanta in earnest in his theory of P. Lenard's photoelectric effect; namely, the ejection of electrons from a metal surface by light. Einstein asserted that light quanta are essential for understanding the troubling experimental finding that the velocities of the ejected electrons are unaffected by the light intensity. This finding contradicted Maxwell's theory whereby light intensity corresponds to the force that accelerates the electrons and thus determines their velocities. Einstein's explanation of this contradiction was that indeed Maxwell's theory is inadequate here. Instead, one should take into account that a quantum of light must have sufficient energy, which is proportional to its frequency, to release an electron from its "trap" formed by the surrounding charges in the metal (the "work function"). There is thus a sharp threshold in the energy or frequency of the quanta above which electrons are ejected. However, an increase in the light intensity at a frequency above this threshold would merely increase the number of light quanta and therefore the number of ejected electrons but not their velocities, in agreement with experiment. Einstein's "exotic" explanation of the photoelectric effect, for which he was later awarded the Nobel Prize, convinced many physicists that the reality of quanta was plausible. Einstein's view was that light quanta (termed *photons*) represented the "granularity of the electromagnetic field in space and time". This granularity reminded Newton's corpuscular theory of light that was defeated by wave optics and Maxwell's theory of electromagnetic waves in the nineteenth century (see Section 2.2). Yet, photons were much more puzzling: they appeared to be both particles and waves! This bizarre duality called for further explanation.

e) *The atom goes quantum*. In 1913 N. Bohr (Denmark) (Figure 1.5) put forward a quantum model of the atom, which, unlike the earlier planetary model by Rutherford, explained the observed frequencies (or wavelengths) of radiation absorbed and emitted by atoms. The word "quantum" in this model, inspired by Einstein and Planck, referred to the discrete values of these frequencies ("spectral lines"). The great achievement of Bohr's quantum model was its ability to explain the puzzling *discreteness* of the solar spectrum; namely, the frequencies of the lines associated with light emission from hydrogen gas in the sun.

Implicit in Bohr's model (later expounded by A. Sommerfeld, Germany) was the idea that the electron bound to the atomic nucleus behaves as a wave: it circles

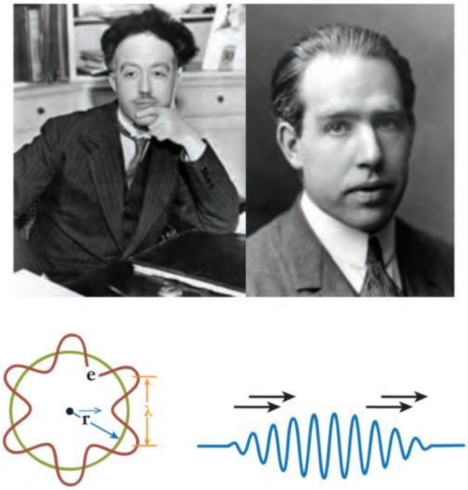


Fig. 1.5 Prince de Broglie and his matter waves (left); Bohr and his quantized (standing wave) electron orbitals in an atom (right).

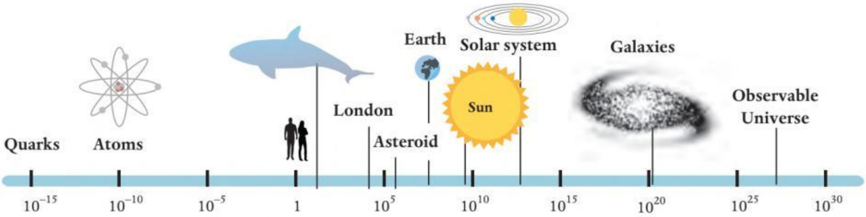


Fig. 1.6 Size scales (in cm) from the observable universe down to sub-nuclear particles (quarks).

around the nucleus in an orbit only if it forms a standing wave (Section 1.3). Hence, its energy is discrete (quantized) because successively larger orbits ("orbitals") are realized by standing waves that correspond to successive multiples ("levels") of the lowest orbital energy. Thus, the notion of quantum waves was introduced into theoretical physics.

The notion of quantum waves of matter, on which we shall elaborate in Chapter 2, quickly became the cornerstone of a new, universal description of microscopic objects, nicknamed "quantum mechanics" (QM). Its universality was based on the striking conclusion shared by Planck, Einstein, Bohr and de Broglie (Figure 1.5) that all quanta of energy, be it in light, atoms or free electrons, are multiples of the same tiny number: Planck's constant, denoted by $\hbar$ (see Section 1.4, Appendix). The smallness of this constant appeared to delineate the boundary between microscopic scales describable by QM and macroscopic scales ruled by classical mechanics and electromagnetism (Figure 1.6). However, a much clearer distinction between classical and quantum phenomena emerged after 1926 (Chapters 2–4).

1.3 FROM CLASSICAL TO QUANTUM WORLD VIEW: IS REALITY SIMPLE OR COMPLEX?

The inception of quantum physics or quantum mechanics (QM) was a landmark in the age-old quest for an answer to a key question of natural philosophy: is reality, although manifest by an endless variety of seemingly complex phenomena, reducible to a few simple constituents?

The human strive for simplicity was expressed with extreme poignancy in an aphorism of the first Greek philosopher Thales (seventh century BC): "Everything is water". Crude as it may appear, this aphorism expresses the first known attempt to construct a "theory of everything".

More elaborate but still simple was the idea of atomism introduced by Democritus (Section 1.2), who summarized it thus: "According to the senses—there are color, taste, smell; but according to reason—only atoms and void." The known quantum physicist Feynman expressed his admiration for the idea of atomism thus: "If . . . all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generations of creatures . . . it is . . . that all things are made of atoms".

Such reductionism of the complex reality to simple constituents characterized nineteenth-century atomism in chemistry and gas theory. Despite the discovery that the atom has structure, as was revealed by Thomson's and Rutherford's experiments, scientists insisted on reductionist simplicity in their thinking of the atom. This is evident in Rutherford's (planetary) model of the atom.

Early quantum theory by Planck, Einstein, Bohr and de Broglie pushed reductionism even further by introducing elementary units (quanta) of energy and thus reinforced the view of universal simplicity of atoms and their constituents.

Yet subsequent developments in QM have led to a revision of this view and culminated in a more elaborate scheme of reality, where many attributes are needed to classify the basic properties of atomic, subatomic or sub-nuclear particles that have by now become highly complex entities. Furthermore, atoms and their constituents are, in general, no longer immutable or completely stable: atoms may undergo transmutation and their electrons change their energies in an elaborate way. Subatomic particles are subject to even more drastic changes, and, with very few exceptions (such as the proton and the electron), do not last forever.

Not only do these modern notions undermine the tenets of original atomism, they also pose a serious challenge: can we really describe phenomena occurring on macroscopic scales, including material properties, by means of these not quite simple "building blocks" of matter that abide by QM? Even if conceptually this is the case, it can hardly be put to a practical test, as any attempt to use QM tools to calculate or predict macroscopic phenomena is doomed to fail: even the largest computing resources imaginable cannot cope with the complexity of calculating the evolution of large numbers of interacting atoms quantum-mechanically. Therefore, molecules comprised of tens, let alone hundreds, of atoms cannot be exactly analyzed by QM. Instead, M. Levitt, A. Warshel (Israel, later the USA) and M. Karplus (USA), who were awarded the Nobel Prize in Chemistry in 2013, combine QM and classical methods to calculate the structure and dynamics of such multi-atomic molecules.

At present, the only way to describe the intricate details of systems composed of more than a few atoms is to adopt approximate methods tailored to the level of complexity of the system at hand. This means that we effectively introduce *different physical rules* for different levels of complexity: this is the essence of modern chemistry, condensed-matter physics and other forms of many-body physics. Even more extreme are the approximations that underlie statistical physics or thermodynamics, which are powerful tools for predicting the average evolution of large collections of atoms but tell us practically nothing about individual atoms.

The moral is that constructing a description of highly complex systems from that of its elementary constituents may well be practically and even principally prohibitive. To stress this point, the term *emergent properties* has been coined: it denotes characteristics of complex systems that cannot be straightforwardly inferred from those of their ingredients. Reality may be conceptually simple, but this simplicity is only manifest when we isolate a few atoms, photons or subatomic particles from the rest of the world. S. Haroche (France) and D. Wineland (USA) were awarded the Nobel Prize in Physics in 2012 for having pursued this approach, which reveals QM in all its glory. A major challenge for science in the twenty-first century is to extend calculational and experimental QM tools to macroscopic systems, thereby merging the "bottom-up" (progressing from simple to complex phenomena) and "top-down" (progressing conversely) approaches to understanding reality.

Yet there is a deeper unresolved issue: how far up the complexity ladder can we push QM as a conceptual framework for explaining reality? In particular, is QM at all relevant to biological processes? There are currently many proponents of a discipline that has been termed *quantum biology*, but the evidence for truly quantum effects in the functioning of live organisms is still too flimsy to pass judgment on its validity. Even more far-fetched are the recently proposed applications of QM notions to the domains of human consciousness, psychology and social structure. However, those involved in such endeavors are not trying to unify physics and human sciences in a common framework, which may be futile (or not?), but rather borrow quantum tools for the construction of models that bear analogy to QM for whatever reason. QM is just too powerful a trick to pass by!

A Fuzzy World

When contemplating our world
The gist of it you should behold
And reach the strange but clear conclusion:
Its ruggedness is an illusion.
Its stuff is but a fuzzy cloud
Uncertain, flimsy and spread out.
But when perceived, it may be asked:
Can its wave-nature be unmasked?

1.4 APPENDIX: CONSTANTS AND VARIABLES IN QUANTUM PHYSICS

This appendix introduces the basic mathematical notions and notations that will serve in subsequent appendices to describe features and phenomena of quantum physics.

Mathematical methods of physics in general employ constants and variables. Constants are numbers that quantify unchanging physical relations. Usually, a constant is denoted by a designated letter or symbol, to avoid the need of repeating the entire number every time. Constants play such an important role in physics that they are the subject of a central field of research known as *metrology*. Numerous physicists in this field perform elaborate experiments trying to ascertain the *exact* value of these constants.

Let us consider several fundamental constants. The *speed of light* is one such constant. Its symbol is the letter c and its value is 299,792,458 meters per second. This means that light travels around 300,000 kilometers in one second. Thus, for example, it takes around 8 minutes for light from the sun to reach Earth.

The fundamental constant of quantum physics, and Henry Bar's superhero symbol, is $\hbar$, (hence the name "Henry Bar"). This constant is known as *the reduced Planck's constant*, where Planck's constant is designated simply by the letter h (without the bar) and the two are related by $\hbar = h/2\pi$. Why is Planck's constant so important that we named our quantum superhero after it? The full answer requires reading a good deal more of this book. Here, suffice it to say that h relates the frequency of a quantum of light to its energy: $E = hf$, where E is the energy and f is the frequency of the quantum. This means that h is the change in energy (whose unit is called Joule, designated J) produced by a change in frequency (whose unit is called Hertz, designated Hz) of the quantum. In other words, if a quantum of light increases its frequency by 1 Hz, its energy is increased by h J.

The value of Planck's reduced constant is $\hbar = 1.054\ 10^{-34}$ J/Hz. The meaning of 10^{-34} is 1 divided by 100,000,000,000,000,000,000,000,000,000,000,000. This is an *extremely* small number. The common notion that "quantumness is only manifest for small things" stems from Planck's constant, which appears in almost any formula in quantum physics, being so small. However, throughout this book we will come across phenomena that defy this notion, as does Henry.

A much more common type of mathematical objects that we shall encounter are letters and symbols that represent quantities that change; that is, they have *variable* values. An example is the position of an object, usually denoted by the letter x, that can acquire different values as the object moves. In the explanation of $\hbar$ above, E and f are variables denoting energy and frequency, respectively, that can change due to a dynamical process.

A ubiquitous mathematical notation used in physics is that of indices, which are usually designated by either superscripts or subscripts. For example, x_1 may denote the position of one thing and x_2 the position of another. If we have one hundred objects, labeling them x_1, x_2, x_3 . . . up to x_{100} would be very cumbersome. Instead, indices obey the following notation: x_{i} ($\mathrm{i} = 1, \ldots, 100$), which means that x_{i} represent the positions of all objects, from 1 to 100.

Let us introduce another extremely important variable, time, denoted by the letter t. In physics, time is measured in seconds and plays a pivotal role in describing change. Thus, for example, speed is the *change in position over the time interval* that this change takes, $v = (x_2 – x_1)/(t_2 – t_1)$, where x_1 and x_2 denote the position of the same object at two times, t_1 and t_2, respectively. We will introduce a more sophisticated way to represent changes, namely derivatives, in subsequent chapters.

Another common notation is that of summation. Say we have one hundred objects with different masses, measured in grams, g. The notation $M = m_1 + m_2 + \ldots + m_{100}$ for the sum of their masses is too cumbersome. Instead, the compact notation of a sum sign, $\sum$, can be introduced, which allows us to write: $M = \sum_{i=1}^{100} m_i$. Below the sum sign we have $i = 1$, which denotes the first value of the index of summation, here i. Above the sum sign we have 100, the last value of the summation index. After the sum sign comes the indexed variable that is the object of summation; here the mass, m_i.

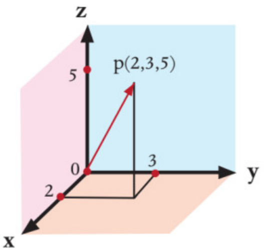


Fig. 1.7 Position along mutually perpendicular axes denoted by a three-dimensional vector.

As a more complex example, consider a beam of light made of light quanta, called photons, each with its own frequency. We have f_i $i = 1, \ldots, 10^3$, denoting the frequencies of all light quanta. Their *total energy*, as explained above, is $E_{total} = \sum_{i=1}^{10^3} E_i = \sum_{i=1}^{10^3} hf_i = h\sum_{i=1}^{10^3} f_i$. The first equality uses the summation symbol over the energies of all the quanta; the second equality uses the above relation between energy and frequency *for each light quantum*; and the last equality uses the fact that Planck's constant is the same for all quanta to rewrite the total sum as Planck's constant times the sum of the light-quanta frequencies.

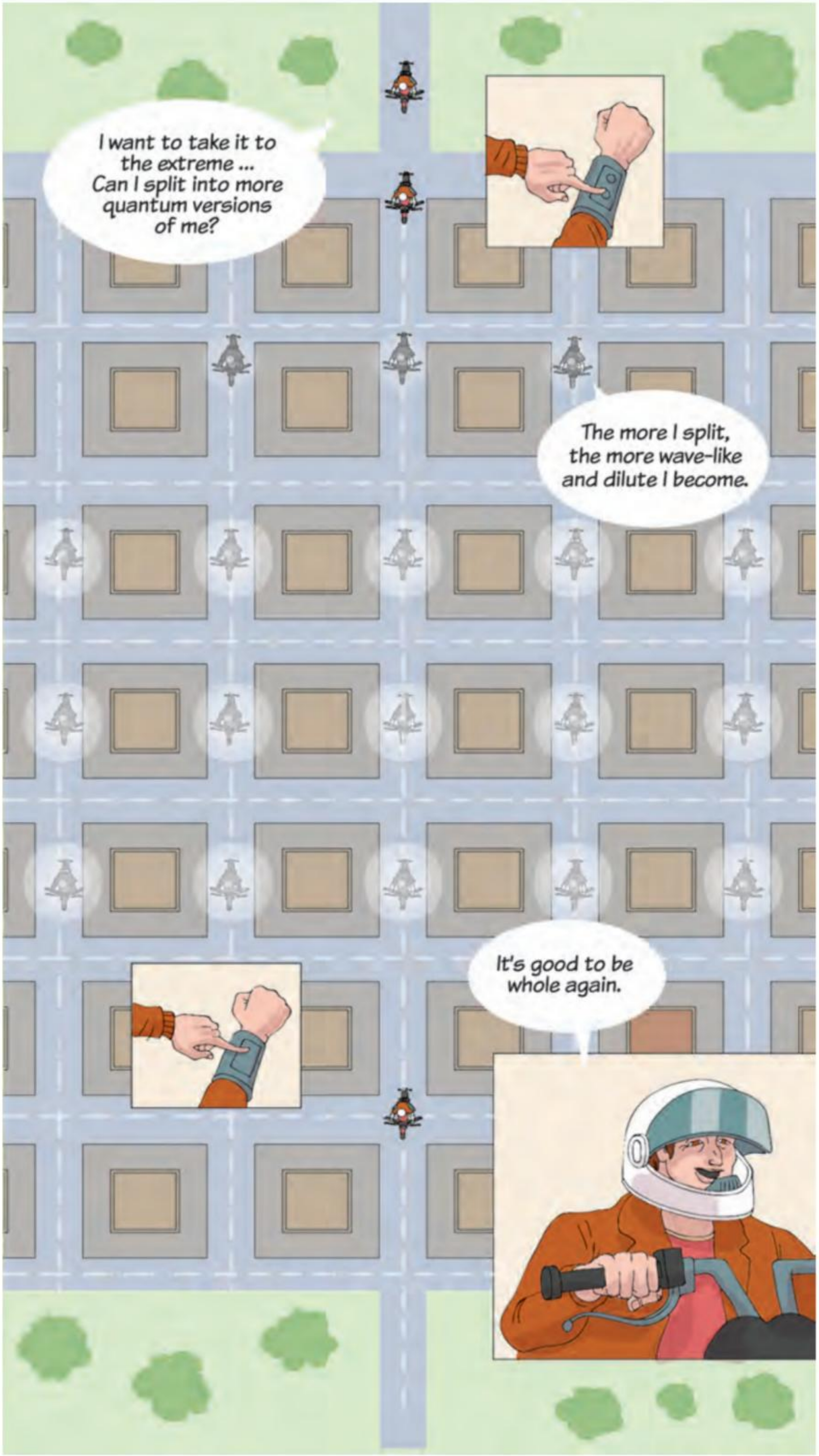
The last notation we wish to introduce is that of a *vector*: a collection of variables describing a specific entity. As an example we consider the three-dimensional (3D) position of an object; namely, its position along the mutually perpendicular *x*-, *y*- and *z*-axes. To be specific, let us take the *x*-axis to extend from the backward to the forward direction, the *y*-axis from the left to the right, and the *z*-axis to point from the bottom to the top (Figure 1.7). Instead of specifying an object's position by the variables x,y,z, we denote it by the vector $\underline{x} = (x,y,z)$.

One may perform simple operations on vectors. For example, vector addition, $\underline{x}_3 = \underline{x}_1 + \underline{x}_2$ signifies a vector that has the form $\underline{x}_3 = (x_1+x_2,\ y_1+y_2,\ z_1+z_2)$, wherein the values in each dimension are summed independently. The same applies to subtraction. Vector multiplication, denoted by a dot-product, $\underline{x}_3 = \underline{x}_1{}^{\star}{\cdot}\underline{x}_2$, obeys the rule $\underline{x}_3 = (x_1 {}^{\star} x_2, y_1 {}^{\star} y_2, z_1 {}^{\star} z_2)$.

To summarize, in this appendix we have introduced the notion of constants, dwelling on the speed of light and Planck's constant, and the notion of variables. The latter can be indexed to account for multiple objects or components and can be "grouped" into vectors, for which the summation notation and vector operations, such as summation and dot-product (vector multiplication) were defined. In subsequent chapters we will use these notations to describe some quantum-physical phenomena that Henry Bar encounters.

Henry Bar Splits and Recombines


How could I split and go through both doors?
It proves I was spread out like a wave!
Let's see what else I can do as a wave...


I want to take it to the extreme ... Can I split into more quantum versions of me?
The more I split, the more wave-like and dilute I become.
It's good to be whole again.

CHAPTER 2

What is a Quantum Superposition?

2.1 A SUPERHERO IN A SUPERPOSITION

Henry has just become the first quantum superhero, having overcome the tremendous technological obstacles en route to the implementation of his quantum suit. In the previous episode he successfully tested the first function of his quantum suit, the Split button. Congratulations are in order, as he has thereby freed himself and mankind from the constraints of our mundane "classical" existence by revealing our latent quantum features.

The feature that the Split button exposes is the ability of a quantum object to be *in several places at the same time.* More precisely, it is the ability to propagate along different paths simultaneously, which Henry experienced by walking through both the revolving door and the sliding door at once. How can such a bizarre phenomenon occur?

It certainly contradicts the behavior of a material object that is ruled by Newton's "classical" physics. Even if the object is extremely small, such as a typical molecule, it is expected to maintain its cohesiveness at all times, so that it does not blur or diffuse as it moves along a (unique) trajectory determined by the forces that act upon it. At each time instant, the position of the object is specified by a set of numbers—its coordinates. Instead, Henry's Split button replaces his instantaneous position by two positions, each specified by different coordinates—a so-called "*superposition*".

This behavior appears less bizarre if we think of Henry as a *wavelike* object. Consider a wave in a pond. It does not have a single position but instead is "smeared" over many positions throughout the pond at each time instant. Accordingly, it is described by a large set of coordinates simultaneously. As it propagates in the pond, it spreads out more and more.

Such wavelike properties have long been known in branches of "classical" physics that have evolved from Newton's mechanics: acoustics (the physics of sound waves in gases), hydrodynamics (the physics of waves in liquids), and in classical optics which is ruled by Maxwell's electromagnetic theory.

Yet Henry is not merely a wave either. He is aware of being a *single entity*—one Henry, even when he splits. He becomes a superposition of two lookalikes (versions), but they are not his clones or copies, as they share the mass of the original Henry (mass conservation is undisputed in this case). More strangely, as we shall elaborate in Chapter 4, any attempt to localize (pin down) one of the lookalikes will recreate the whole Henry again, not a fraction of him. The lookalikes only tell us where Henry can be found if localized, not where he actually is. *He can be anywhere before he localizes*.

This strange property distinguishes a quantum wave from the classical waves mentioned previously. As discussed in Chapter 1, quantum physics originated from the concept of light quanta, which propagate as waves, but maintain the indivisibility of their energy: a single photon cannot be divided by a splitting operation into half-photons that may emerge at different points of space and time, as we shall always find either one or zero photons. The same is true for electrons or other subatomic particles, as discussed in Section 2.2. Half-particles cannot be found, only whole particles or none at all. Counterintuitive as these properties may appear, they have been confirmed by a multitude of experiments over the past century.

This unique character of quantum waves is captured by a mathematical description known as a "wavefunction"—a set of numbers that are assigned to every position (a point in space) at a given time. These numbers are known as the "probability amplitude" of the wave at that point. In Henry's case his two lookalikes represent equal probability amplitudes of finding him at the revolving door or the sliding door, so that his chances (probability) of emerging (i.e., being localized) here or there are 50%/50%. Yet before he localizes (materializes) *we cannot tell where he is.*

The splitting phenomena described above are not restricted to a superposition of two quantum waves (or probability amplitudes), but actually apply to superpositions of any number of such waves. During Henry's ride through town, he pushes his Split button many times, each time becoming more spread out, until the streets are flooded with many of Henry's lookalikes. The probability amplitude of each lookalike is then small, and the probability (which is the probability amplitude squared) of finding him in any given street is still much smaller.

But why is Henry's splitting so outlandish that it can only be revealed by a futuristic contraption? Let us keep in mind that such splitting requires the object to become a superposition of wavelike components (probability amplitudes) that are spatially *distinguishable,* but partly overlap to ensure they are mutually *coherent*, i.e. stem from one wave—such as Henry's wavelike lookalikes that ride along different streets or pass through different doors, but still maintain their coherence ("oneness"). Both requirements are extremely harsh to fulfill by an energetic, massive object. The reason is that the wavelike "fuzziness" or smearing of a quantum object becomes smaller as its mass and energy grow, so that experiments capable of revealing this property become harder.

Let us make this discussion somewhat quantitative. As noted in Section 2.2, an electron with an energy obtained by running it through a tension of 100 Volts was first shown to be a wave by scattering it off atoms in a crystal that are 0.4 nanometers apart, giving rise to a superposition of scattered electron waves separated by roughly the extent of one such wave. Had the distance between atoms in the crystal been much larger, the scattered electron waves would not have exhibited coherence, i.e. they would not have "added up" (interfered) at the detector, as explained in Chapter 3. Much heavier objects, e.g. atoms or molecules, must be endowed with much less motional (kinetic) energy in order to observe their coherent splitting into wavelike components.

It then transpires that Henry's mass being so large, he must be given an exceedingly small kinetic energy to allow for his coherent splitting on the scale portrayed in our cartoons. Giving or receiving such tiny energy is an incredible challenge at present, but one which does not violate the known laws of physics.

Henry's quantum suit has been equipped not only with the Split function but also with its reverse: the Recombine function that reunites Henry's lookalikes into his original self. Such functions are not only figments of our imagination; they are readily available experimentally. For example, consider an electron bound to an atomic nucleus (Figure 2.1). Initially, the electron wavefunction has a well-defined amount ("level") of energy, as it (crudely speaking) occupies an orbital that encircles the nucleus. Now we shine a laser pulse at the atom. If we choose the pulse to have the right intensity, frequency and duration it will cause the electron wavefunction to split into an equal superposition of two wavefunctions corresponding to different energy levels or orbitals. An identical subsequent laser pulse will recombine the superposition and restore the electron wavefunction to its original form. This example, which will be revisited in Chapter 3, illustrates our ability to do and undo splitting quantum operations of the kind that Henry performed on himself.

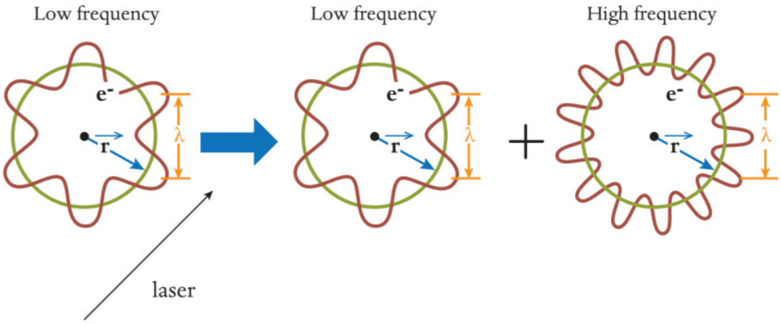


Fig. 2.1 Laser-induced splitting of an atomic-electron low-energy (or low-frequency) wavefunction into a superposition of energy levels (orbitals corresponding to low and high frequency or energy, respectively).

A superposition of the atom's electron orbitals is an example of a superposed internal degree of freedom; namely, the atom may propagate as a single object through space while its internal state is described by a superposition of two or more wavefunctions with different energies.

As our story unfolds, Henry gradually uncovers his quantum potentialities and takes advantage of them by adding more functionalities to his quantum suit, and taking advantage of other wavelike properties. He can now revise the battle song of the "*Scarlet Pimpernel*":

The elusive Henry Bar

She seeks him here, she seeks him there—
His foe—she seeks him everywhere.
Is he at the sliding or the revolving door?
At both! Such is the quantum-wave galore!

2.2 SUPERPOSITIONS: FROM LIGHT WAVES TO WAVEFUNCTIONS

In order to understand in depth Henry Bar's quantum wavelike behavior, let us review the emergence of the relevant concepts and their subsequent development which has crystallized quantum physics. Strikingly, the decisive stage in the formation of this revolutionary theory that rules physics to this day occurred in a span of few years, during the true heyday of quantum physics: 1923–27. However, it had taken more than a century for this revolution to come to fruition.

a) *Light waves emerge*. That light propagates as a wave was convincingly argued by the Dutch scientist C. Huygens (in the mid-seventeenth century), as opposed to Newton, who maintained that light consists of particles (corpuscles). Whereas Newton's corpuscles / particles propagate along straight lines known as rays, light waves, according to Huygens, scatter from obstacles or emanate from sources as expanding spheres. Huygens had no direct evidence for his theory of light, which was rejected and remained dormant for some 150 years. An experiment of the British physicist T. Young in London in 1805 demonstrated the ability of light to penetrate two slits in a correlated (coherent) fashion called "interference" (Chapter 3). A. J. Fresnel's experiments (*c*.1830) in France displayed wavelike light penetration through pinholes—an effect called "diffraction"—and revealed the conditions under which light propagates as concentric, spherical waves emanating from a point source, rather than straight-line rays. All of Fresnel's effects could be inferred from the comprehensive theory (developed in the 1860s) of the Scottish physicist J. C. Maxwell, which treated light as an electromagnetic wave; i.e., as a propagation of electric and magnetic fields that alternate in space and time. Maxwell's theory of electromagnetism was quickly recognized to be the pinnacle of our understanding of all forms of electromagnetic radiation, and it is still widely used today. Yet at the time it triggered a heated debate as to whether electromagnetic waves propagate in empty space or in a hypothetical medium called "ether". The ether theory was dispelled with an experiment by the American physicist A. A. Michelson (who nevertheless continued to believe in it). This experiment confirmed the special relativity theory of the Dutch physicist H. A. Lorentz and of Einstein (1905), whereby light-wave velocity is constant; i.e., independent of the light-source motion. Such motion would have affected light-velocity had light propagated in ether, depending on whether the light source (affixed to the Earth's surface) had receded from this ether or approached it.

b) *Quanta emerge*. The next revolution was the concept of light quanta introduced by Planck and Einstein (reviewed in Chapter 1). Remarkably, already in 1909, G. J. Taylor (Britain) experimentally tested the propagation properties of a single photon and came to the same conclusion as Einstein: that a photon propagates as a wave. His ingenious experiment consisted in the propagation of exceedingly feeble light—approximately one photon—through a pinhole, and photographing the resulting spatial pattern which consisted of concentric rings, just like Fresnel's pattern. To be able to record the effect of such a tiny amount of light on a photographic plate, Taylor had installed a slow-burning candle inside a photographic dark chamber, sealed the setup, placed a "do not touch" sign on top, and in the fashion of a gentleman-scientist of the day, went yachting for more than three months(!), which was the necessary exposure time for photographing a *single* photon. The recorded pattern (Figure 2.2) indicated that a single photon indeed propagates as a wave!

c) *From quantum waves to wave mechanics*. In 1923 a Parisian PhD student, Prince Louis de Broglie, published his revolutionary surmise that a material object, such as an electron, moves as a packet (superposition) of waves with different energies, alias a "wavepacket" (Figure 2.3). This meant that much like light waves, "matter waves" are delocalized—spread throughout space—so that they may be found at different locations at the same time.

In essence, de Broglie extended the notion of a quantum, introduced into electromagnetic theory by Planck and Einstein, to a matter wave—i.e. a massive object—following the hint contained in Bohr's theory of the atom, wherein the

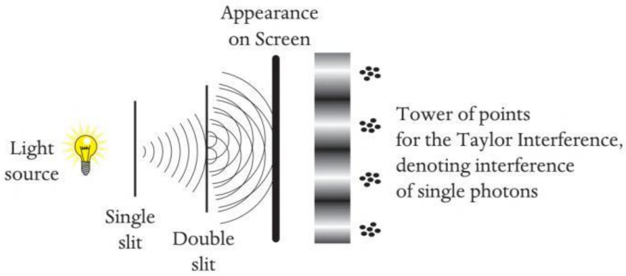


Fig. 2.2 Fresnel's pinhole diffraction (light passing one slit), Young's two-slit interference (periodic bright and dark stripes on the screen) and Taylor's single-quantum interference (periodic dot pattern on the screen, each dot representing a quantum or photon that strikes the screen).

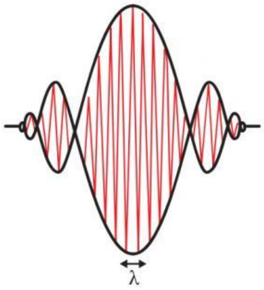


Fig. 2.3 A wavepacket describing the motion of a quantum particle governed by the Schrödinger equation. The amplitude of the interfering or superposed quantum waves forming the wavepacket oscillates in space on the scale of the de Broglie wavelength λ. The dotted contour denotes the amplitude variation in space between crests and troughs on much larger scales. This contour indicates the extent to which the position of a particle is smeared by its quantumness.