THE ROAD TO RELATIVITY

THE ROAD TO RELATIVITY

THE HISTORY AND MEANING OF EINSTEIN'S "THE FOUNDATION OF GENERAL RELATIVITY"

Featuring the Original Manuscript of Einstein's Masterpiece

HANOCH GUTFREUND
& JÜRGEN RENN

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A BRIEF NOTE ON THE PUBLICATION OF THIS WORK

We congratulate the authors of this book on their initiative in presenting to the nonprofessional reader the history and meaning of Albert Einstein's greatest intellectual achievement—his general theory of relativity. The book is the result of the scholarly effort of its authors, yet their institutional affiliation carries in this context an additional symbolic value. Albert Einstein, The Hebrew University of Jerusalem, and the Max Planck Society form a triangle of relations that deserves some attention.

Albert Einstein was a founder of the Hebrew University of Jerusalem. He served on its Board of Governors and as the first chairman of its Academic Committee. On the occasion of the opening of the university in 1925, he published a mission statement in which he wrote: "A university is a place where the universality of the human spirit manifests itself," and expressed the wish that "our University will develop speedily into a great spiritual center, which will evoke the respect of cultured mankind the world over." This vision has been amply fulfilled.

In 1950, Einstein gave profound expression to his lifelong commitment to the Hebrew University: he bequeathed his own true wealth—his personal papers and literary estate—to the university, making it the eternal home of his intellectual legacy. Today they make up the Albert Einstein Archives, which constitute a cultural asset of supreme importance to mankind. Its holdings are unique—they consist of numerous manuscripts, prolific correspondence, and a large variety of additional material about Einstein. The material in the archives sheds light on the multifaceted aspects of Einstein's scientific work, his political activities, and his private life. The documents have enabled scholars to trace the development of the ideas that led Einstein to his general theory of relativity. It is this intellectual journey that is the subject matter of the present book.

Einstein submitted his theory of general relativity to the Royal Prussian Academy of Sciences in November 1915. In 1917, he became the first director of the Kaiser Wilhelm Institute for Physics. After one of the predictions of the new theory was confirmed, the *Berliner Illustrirte Zeitung*, featuring Einstein's photo on the front page, proudly announced him as "a new celebrity in world history." All this acclaim ended tragically when the Nazis came to power, and Einstein, like many of his colleagues of Jewish heritage, became homeless in his own homeland. After the defeat of Nazi Germany, when the magnitude of suffering inflicted on nations, ethnic groups, and individuals by the Nazi policy and ideology became clearly evident, Einstein rejected numerous invitations and suggestions to return to Germany and to rejoin German scientific institutions. For instance, Einstein was invited to join the newly established Max Planck Society—successor to the Kaiser

Wilhelm Society—by its president Otto Hahn. His refusal was sharp and clear. In 1947, Einstein also refused to approve any publication of his writings in Germany. His stance changed in 1954, when he agreed to the publication of a new German edition of his popular book *The Special and General Theory of Relativity*.

In 1959, the Max Planck Society pioneered academic contacts with the Weizmann Institute in Israel, even before diplomatic relations between Germany and Israel had been established. These contacts were the beginning of a long and productive academic cooperation between the two countries. Currently, Max Planck researchers and their Israeli colleagues are working together on 88 joint projects. Nearly a quarter of these projects involve scientists from the Hebrew University, demonstrating how well our two scientific institutions complement each other. In the recently founded Max Planck–Hebrew University Center on Sensory Processing of the Brain in Action we have joined forces to shed light on the functional building blocks of the brain, the neural circuits.

Regarding the theme of this book, Einstein's theory of general relativity, its consequences, and its history are being explored at several Max Planck Institutes, including the Albert Einstein Institute in Golm and the Max Planck Institute for the History of Science in Berlin. Between 1999 and 2005, the Max Planck Society realized a large-scale historical research project investigating the involvement of its predecessor society in Nazi crimes. To mark the centennial of Einstein's "miraculous year," the authors of the present book collaborated in producing the 2005 Berlin exhibition *Albert Einstein—Chief Engineer of the Universe* on behalf of both the Max Planck Society and the Hebrew University of Jerusalem. The 100th anniversary of the discovery of general relativity has brought our institutions together once more and motivated the authors to produce this book. We are grateful for this enterprise.

PROFESSOR MENACHEM BEN-SASSON
President of the Hebrew University of Jerusalem

PROFESSOR MARTIN STRATMANN
President of the Max Planck Society

first step in an intellectual journey that is still ongoing. Like many other foundational papers, it functions as a nodal point, summing up the past and opening wide vistas for the future.

Even at the moment of his greatest triumph, Einstein never doubted this. In 1916, he wrote:

> It appears that the quantum theory would have to modify not only Maxwellian electrodynamics, but also the new theory of gravitation.[10]

There is a well-known tension between the methods of quantum field theory and the structure of general relativity. The methods of quantization of non-general-relativistic theories are based on the existence of a fixed kinematical background space-time structure, providing the *where* and *when* for all events. This space-time structure is needed both for the development of the formalism of the dynamical theory to be quantized and—equally important—for its physical interpretation: If a system prepared *here* and *now* is subject to some dynamical interactions, what will be the result of a measurement made on the system *there* and *then*?

General relativity does not fit this pattern. It is a background-independent theory with no fixed, nondynamical structures, and hence it has no kinematics independent of its dynamics. In such a theory, *here* and *now*, and *there* and *then*, are not part of the questions posed to a system but part of the answers given!

However there is hope: general relativity and special-relativistic quantum field theories do share one fundamental feature that often is not sufficiently stressed: the primacy of processes over states. The four-dimensional approach, emphasizing processes in regions of space-time, is basic to both. The ideal approach to quantum gravity would be a background-independent method of quantization that takes process as primary.[11]

The challenge of finding such an approach still awaits solution. But even if or when a satisfactory quantization of Einstein's gravitational field equations is found, that still will not be the end of the story, as Einstein always realized. Early in 1917, he wrote:

> But I do not doubt that sooner or later the day will come, when this way of conceiving [of gravitation] will have to give way to another that differs from it fundamentally, for reasons that today we cannot even imagine. I believe that this process of deepening of theory has no limit.[12]

JOHN STACHEL

NOTES

1. Einstein to Georg Jaffe, 19 January 1954, cited from John Stachel, *Einstein from 'B' to 'Z,'* p. 294.
2. *1912 Manuscript on the Special Theory of Relativity: A Facsimile* (New York: George Braziller, 1996).
3. Jürgen Renn, ed., *The Genesis of General Relativity*, 4-vol.set, in *Boston Studies in the Philosophy of Science*, vol. 250 (Dordrecht: Springer 2007); *Michel Janssen, John Norton, Jürgen Renn, Tilman Sauer, and John Stachel,* vol. 1: *Einstein's Zurich Notebook: Introduction and Source;* vol. 2: *Einstein's Zurich Notebook: Commentary and Essays.*

4. Albert Einstein, "Relativity and the Problem of Space," in *Relativity: The Special and General Theory, Appendix V* (New York: Crown 1952), p. 155.
5. See Einstein's "Odyssey: His Journey from Special to General Relativity," *The Sciences* 19 (1979): 14–15, 32–34; reprinted in *Einstein from 'B' to 'Z,'* pp. 225–232.
6. Criteria must be given for the selection of such an extension, or extensions, if these criteria do not lead to a unique selection.
7. John Stachel, "The Hole Argument," *Living Reviews in Relativity*," http://www.livingreviews.org/lrr-2014–1, sec. 1, "Why Should We Care?
8. Speech at the Lord Mayor's Day Luncheon at the Mansion House, London, 9 November 1942.
9. See Einstein's "Odyssey: His Journey from Special to General Relativity."
10. "Approximative Integration of the Field Equations of Gravitation," in CPAE, vol. 6, *The Berlin Years: Writings 1914–1917* (English translation supplement), tr. Alfred Engel (Princeton, NJ: Princeton University Press, 1997), 201–209; citation from p. 209.
11. For further discussion of this question, see "The Hole Argument," sec. 6.4, "The Problem of Quantum Gravity."
12. Einstein to Felix Klein, 4 April 1917, cited from CPAE, vol. 8A (Princeton, NJ: Princeton University Press, 1998), 431.

PREFACE

This book presents a facsimile of the manuscript of Albert Einstein's canonical 1916 paper on the general theory of relativity, which may be considered one of the most sophisticated intellectual achievements produced by a single human mind.

Each page of Einstein's manuscript is accompanied by brief essays to guide the nonspecialist through Einstein's arguments and to place this work in a broad intellectual and historical context. The explanatory texts refer to the topics on the specific page and to relevant historical backgrounds. The different kinds of commentaries are differentiated by their typographic styles. So as not to interfere with a fluent reading of the essays, the bibliographic information and suggestions for further reading pertaining to the content of each page are given at the end of the book.

The reproduction of the manuscript is preceded by a comprehensive historical introduction narrating the evolution of general relativity into a full-fledged theory. The introduction and the texts accompanying the manuscript tell essentially the same story but they do so in a different style, in a different format, and sometimes at a different level of exposition. It is hoped that this dual approach will help readers appreciate the development from different angles and will help them choose which track they would like to pursue.

The advantage of presenting this story on the background of Einstein's manuscript is explained in the prologue, "The Charm of a Manuscript." This prologue also explains how the manuscript moved from Berlin, where it was written, to its eternal home at the Hebrew University in Jerusalem, where Einstein's papers are preserved.

The manuscript is followed by a postscript describing the aftermath of the completion of the theory and its immediate cosmological implications. Einstein's 1916 publication does not in fact represent his final views on a number of issues relating to general relativity. A timeline is provided to help orient the reader in the developments between 1905 and 1932, covering the genesis and formative years of general relativity. For the benefit of the reader with a more advanced background in science, the English translation of Einstein's 1916 paper is appended.

Another helpful element is a glossary of scientists and philosophers relevant to Einstein's thinking, featuring their images and brief biographical sketches. This glossary explicitly demonstrates what is conveyed throughout the text: that Einstein maintained a broad network of connections and exchanges with friends and colleagues as he struggled with the challenge of creating his new theory of gravitation. We are grateful to Giuseppe Castagnetti for composing the biographical notes and to Beatrice Hilke for her assistance with the images.

The story outlined in this book is known to few but is of interest to many, concerning as it does one of the most important turning points in the history of science. This book is an attempt to make this development accessible to a broad audience.

We specifically chose to illustrate the text with drawings by Laurent Taudin in a cartoonlike style to add a light, anecdotal flavor. We are grateful to Laurent for his inventiveness but also for his striking capacity to grasp the essence of the subject matter.

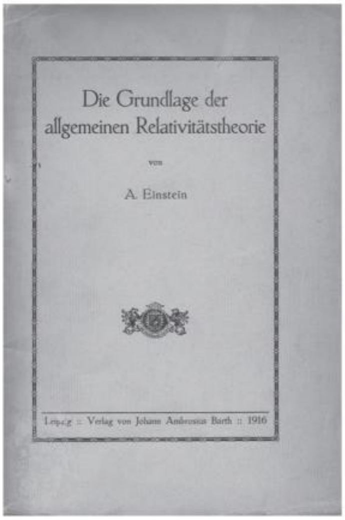
We are grateful to Ingrid Gnerlich from Princeton University Press for guiding us through different stages of this project and to the anonymous referees appointed by Princeton University Press for their suggestions, which we have followed. Special thanks are due to our colleagues and friends Jean Eisenstaedt, Robert Schulmann, and Bernard Schutz for critically reading earlier versions of the manuscript. Particular thanks go to our friends Michel Janssen and John Stachel, whose suggestions were very helpful in improving the text.

We are grateful to the staff of the Albert Einstein Archives at the Hebrew University for their assistance, specifically to director Roni Grosz, to Barbara Wolff, and to Chaya Becker. Our thanks also go to Diana Kormos-Buchwald, the general editor of the Einstein Papers Project, for allowing us to quote extensively from the published volumes of the Collected Papers of Albert Einstein, as well as for her personal encouragement with which she has accompanied our work.

This project owes a special debt to two institutions that were directly and indirectly involved. The Hebrew University allowed us to use the manuscript and other archival material, and the Max Planck Institute for the History of Science became the venue where this project was created. We are therefore grateful for the support of both institutions.

Finally, we acknowledge with appreciation and gratitude the invaluable editorial assistance and professional support of Lindy Divarci.

THE ROAD TO RELATIVITY



Die Grundlage der
allgemeinen Relativitätstheorie

von

A. Einstein

Leipzig :: Verlag von Johann Ambrosius Barth :: 1916

MPIWG Library

did gradually lead closer to the objectives. That is why now finally the basic formulas are good, but the derivations abominable; this deficiency must still be eliminated."[5] Without eliminating what appeared to him as an avoidable complexity, Einstein submitted the manuscript for publication to Wilhelm Wien, the editor of *Annalen der Physik*, the leading journal in physics at the time, on March 19, 1916. In the submission letter, Einstein informed the editor that he had also discussed, with the publisher of the journal, an additional publication of this manuscript as a separate booklet. The article "Foundation of General Relativity" was published on May 11th in *Annalen der Physik* and also separately.

The general relativity manuscript is now part of the Albert Einstein Archives at the Hebrew University of Jerusalem. How it got there is a complex story, the details of which are not completely known. Apparently, Einstein gave the manuscript to his friend the physicist astronomer Erwin Freundlich, with whom he had an ongoing dialogue on possible observational tests of phenomena predicted by the new relativistic theory of gravitation. In 1920, Freundlich was one of the founders of the Einstein Donation Fund, which supported the construction of the Einstein Tower in Potsdam, where such tests were to be conducted. We do not know when and why Einstein gave the manuscript to Freundlich. The nature of this "gift" later became a point of dispute between them. By the end of December 1921, the relationship between the two colleagues and friends had deteriorated. Einstein resigned from the board of trustees of the fund and demanded that Freundlich return the manuscript. In an angry letter to Freundlich he wrote:

> As concerns my manuscript, I ask you to arrange to have it handed over to me immediately, without wasting another word on it. I had requested that you send it back to me in the summer. You promised in writing to send it back immediately upon your return from your summer trip. When you did not follow through with it then, my wife wrote you a letter in this regard, to which you did not respond. Now

Erich Mendelsohn:
Sketches for the design of the Einstein tower, 1918.
bpk / Kunstbibliothek, Staatliche Museen zu Berlin

> you retrospectively contend I had given the manuscript to you, for which there was absolutely no reason. As if this were not enough, you took steps behind my back to sell the manuscript abroad, as you yourself told me. I hope now that you will do your duty without my having to admonish you again. [6]

Einstein retold the story of the manuscript in a letter to Arnold Berliner,[7] editor of the journal *Naturwissenschaften*, who tried to mediate this dispute. Einstein concluded: "I find Freundlich's conduct such that I want nothing to do with him. . . . It no longer concerns the manuscript but the man, whom I cannot trust anymore." The handwritten draft of this letter contains a sentence that Einstein crossed out: "Auf das Manuscript verzichte ich hiermit; mit Freude daran." (I am happy to do without the manuscript.)

Freundlich returned the manuscript, and in April 1922, Einstein entrusted the industrialist and philosopher of science Paul Oppenheim with selling it, giving the following instructions: "The Jewish University of Jerusalem shall be given half of the proceeds; of the remaining half you may dispose as your conscience tells you."[8] Thus, Einstein left it to Oppenheim's discretion to decide on Freundlich's claim to rightful ownership of the manuscript, although in a postscript Einstein stated that he was deeply convinced that Freundlich had no right to it and that his behavior was deceitful. Oppenheim was a friend of both adversaries and did not want to serve as a moral judge between them. Rather, he wished to restore their friendship.

In July 1923, Einstein took another course of action. He asked Heinrich Loewe, a prominent member of "The Preparatory Board of the Hebrew University and the Jewish National Library in Jerusalem" to sell the manuscript. This time the instructions concerning the allocation of the proceeds were very specific: They were to be distributed in equal parts among the library in Jerusalem, the Einstein Donation Fund, the fund securing Mrs. Freundlich's pension, and Einstein himself, who would then donate his share to charity. These instructions were confirmed in a letter from Loewe to Einstein.[9]

The manuscript was not sold, and its fate is revealed in correspondence between Einstein and his wife Elsa when in 1925 he spent two months in South America. Only his letters to Elsa survive; we do not know what she wrote to him. On April 15, in a postscript, he wrote: "Do not give away the manuscript, dear Elsa. . . . The time is not good for selling it. Better after my death."[10] Einstein did not know that on March 19th, Leo Kohn had already received the manuscript from Elsa on behalf of the Board of Trustees of the University of Jerusalem. The document,[11] signed by Kohn, that confirms this transaction stipulates that it be returned "without delay to Professor Einstein, in case any inconvenience be caused to him by the University's acceptance of the manuscript." This document also states that Mrs. Einstein should receive 2000Mk, to be transferred to the Einstein Fund in Potsdam for the use of Prof. Dr. Freundlich, and 400Mk should be given to Mrs. Einstein for her charities.

When Einstein learned that the manuscript was on its way to Jerusalem, he wrote to Elsa, on April 23rd, with relief: "I am glad that I now got rid of the manuscript and thank you for doing me this favor of love (*Liebesdienst*); better than burned or sold."[12]

The general relativity manuscript has been in the possession of the Hebrew University since its opening on April 1, 1925, and is cherished as one of the university's most precious treasures. The manuscript was displayed for the first time in its entirety at an exhibition marking the 50th anniversary of the Israeli Academy of Science. Each one of its 46 pages was enclosed in a box with controlled illumination and microclimate. Like its 1912 predecessor, the manuscript attracted crowds of interested and excited visitors.

In 2013, the European Space Agency launched an Automated Transfer Vehicle (ATV-4), named "Albert Einstein," carrying supplies and equipment to the International Space Station (ISS). The cargo of ATV-4 contained the first page of the manuscript described in this book, which astronaut Luca Parmitano signed on board the ISS as a symbolic gesture

acknowledging the importance of this manuscript and of what it represents in the history of mankind.

This is the story of a single albeit very important manuscript. The Albert Einstein Archives at the Hebrew University contain many such manuscripts, all of which constitute inspiring chapters in the history of physics. They are being edited and explored by historians of science at the Einstein Papers Project at the California Institute of Technology and elsewhere. All shed light on how science was done in the formative years of modern physics.

NOTES

1. Walter Benjamin, *The Work of Art in the Age of Mechanical Reproduction* (London: Penguin, 2008).
2. Albert Einstein, "On the Electrodynamics of Moving Bodies" (1905), in CPAE vol. 2, Doc. 23, pp. 140–171.
3. The facsimile copy of this manuscript was published by George Braziller as *Einstein's 1912 Manuscript on the Special Theory of Relativity* (New York: Braziller, 1996).
4. It has been analyzed in detail in Michel Janssen, "Of Pots and Holes: Einstein's Bumpy Road to General Relativity," *Annalen der Physik* 14 (2005), Supplement: 58–85; and in Tilman Sauer, "Einstein's Review Paper on General Relativity Theory," in *Landmark Writings in Western Mathematics*, 1640–1940, ed. I. Grattan-Guiness (Amsterdam: Elsevier, 2005), 802–822.
5. Einstein to H. A. Lorentz, 17 January 1916, CPAE vol. 8, Doc. 183.
6. Einstein to Erwin Freundlich, 20 December 1921, CPAE vol. 12, Doc. 330, AEA 11–314.
7. Einstein to Arnold Berliner, 24 December 1921, vol. 12, Doc. 339, AEA 11–318, AEA 11–319.
8. Einstein to Paul Oppenheim, 15 April 1922, CPAE vol. 13, Doc. 146, AEA 11–323.
9. Heinrich Loewe to AE, 30 July 1923, AEA 36–860.
10. Einstein to Elsa Einstein, 15 April 1925, AEA 143–186.
11. Leo Kohn, 19 March 1925, AEA 36–863.
12. Einstein to Elsa Einstein, 23 April 1925, AEA 143–187.

"In Prague, I found the necessary concentration for developing the basic idea of the general theory of relativity."

potential was represented by a single function—the space-dependent speed of light—and the theory he developed was restricted to a static gravitational field.

It is interesting to note that Einstein's work on gravitation in Prague was done to a large extent within the context of a controversy with the physicist Max Abraham, famous for his contributions to electrodynamics and electron theory. Abraham was the first to publish, in January 1912, a complete theory of the gravitational field formulated within the framework of Minkowski's four-dimensional spacetime.[5] At first, Einstein was impressed but then reacted skeptically. To his friend Besso he wrote: "At first (for 14 days) I too was completely bluffed by the beauty and simplicity of his formulas."[6] Yet, in the ensuing controversy both Abraham and Einstein developed important insights.

In a foreword to the Czech edition of 1923 of his famous little popular book "About the Special and General Theory of Relativity in Plain Terms," Einstein refers to his work in Prague:[7]

> I am pleased that this small book . . . should now appear in the native language of the country in which I found the necessary concentration for developing the basic idea of the general theory of relativity which I had already conceived in 1908 [he must have meant 1907]. In the quiet rooms of the Institute of Theoretical Physics of Prague's German University in Vinicna Street, I discovered that the principle of equivalence implies the deflection of light rays near the Sun by an observable amount. . . . In Prague I also discovered the shift of spectral lines towards the red. . . . However, the decisive idea of the analogy between the mathematical formulation of the theory and the Gaussian theory of surfaces came to me only in 1912 after my

> return to Zurich, without being aware at that time of the work of Riemann, Ricci, and Levi-Civita. This was first brought to my attention by my friend Grossmann.

ZURICH In 1911, Marcel Grossmann was appointed dean of the mathematics-physics department of the Swiss Federal Institute of Technology (ETH). One of his first initiatives as dean was to write to Einstein asking if he would be interested in returning to Zurich to join the ETH. Einstein agreed, declining an earlier offer from Utrecht as well as an opportunity to go to Leiden, both of which would have been enticing given the proximity of colleagues such as H. A. Lorentz. Whatever the reasons Einstein had for preferring Zurich over Utrecht or Leiden, at that time it was the right decision. A short time after returning to Zurich in August 1912, he began an intensive and fruitful collaboration with Grossmann that became a landmark in the development of general relativity.

During the Zurich period, Einstein produced three documents that played a significant role in the search for a theory of general relativity: the Zurich Notebook, the Einstein-Grossmann *Entwurf* paper, and the Einstein-Besso manuscript. We shall discuss the contents and significance of these documents in the relevant sections of this account of Einstein's roadmap to general relativity, so we only briefly describe them now.

The Zurich Notebook contains Einstein's notes from the intermediate phase of his search for a relativistic theory of gravitation, when he was exploring, with the help of Grossmann, the concepts and methods of tensor calculus and Riemannian geometry. The notebook consists of 96 pages, not all of them devoted to relativity. Einstein nevertheless gave it the title "Relativität." The notes were written between mid-1912 and the beginning of 1913. Einstein used the notebook from both the front and the back, and his entries meet upside down about a quarter way through. This notebook constitutes a very

"With the help of a mathematical friend [Marcel Grossmann] here [in Zurich], I will overcome all difficulties."

important document in the history of science and is of pivotal importance for our understanding of the origins of the general theory of relativity.[8]

The Zurich Notebook essentially contains the blueprint for the generally covariant theory, but owing to a yet immature physical understanding to be described shortly, Einstein abandoned this theory. Instead, he and Grossmann published the "Outline of a Generalized Theory of Relativity and of a Theory of Gravitation," which has since been termed the *Entwurf* theory from its German title, which means outline.[9] Although this theory did not meet Einstein's initial requirement of general covariance, he convinced himself that this was the best that could be done, and despite this and other shortcomings of the theory, he expressed satisfaction with it until the summer of 1915.

The so-called Einstein-Besso manuscript is a collection of about fifty pages of calculations, about half of them in Einstein's handwriting and the other half in Besso's. These pages contain a calculation of the precession of the perihelion of Mercury based on the field equation of the *Entwurf* theory and a calculation of the metric tensor in a rotating frame of reference.[10]

The Swiss Department of the Interior approved the request of ETH for a full professorship for Einstein. However, it lasted only three semesters. Einstein was in great demand, and the next offer he could not refuse came from Berlin.

Berlin In 1913, Max Planck was elected secretary of the Royal Prussian Academy of Sciences. Shortly after his election, Planck launched a campaign to elect Einstein to the academy. In July 1913, Planck went to Zurich with Walther Nernst to present to Einstein a tempting three-part proposal: election to the academy with generous financial support,

"She [Else Löwenthal] was the main reason for my coming to Berlin, you know."

directorship of the Kaiser Wilhelm Institute of Physics without a real administrative burden, and a professorship at the University of Berlin without teaching obligations.

Einstein accepted the offer, giving different reasons to different people in justifying his decision. To Lorentz he wrote: "I could not resist the temptation to accept a position in which I am relieved of all responsibilities so that I can give myself over completely to rumination."[11] But to his good friend Heinrich Zangger he admitted that the main reason for accepting this offer was that this would bring him close to his cousin Elsa, whom he was passionately courting at that time and who would later become his second wife: "Despite being in Berlin, I am living in tolerable solitude. But here I have something that makes for a warmer life, namely, a woman whom I feel closely attached to. . . . She was the main reason for my coming to Berlin, you know."[12]

In November 1913, His Imperial and Royal Majesty Wilhelm II confirmed Einstein's election as a regular member of the physics-mathematics section of the academy. Thus, at the age of 34, he became the youngest-ever member of the academy.

Shortly after Einstein's arrival in Berlin, World War I broke out. Confronted with the realities of war, he eventually left the ivory tower of science to become a political opponent of Germany's involvement in the war. In Berlin, Einstein encountered the phenomenon of anti-Semitism and became aware, more than ever before, of his Jewish identity.[13] In Berlin, his relations with Mileva deteriorated to the point of separation—Mileva and the children returned to Zurich. In the midst of all this, Einstein ardently pursued his scientific work and, according to his own testimony, worked harder than ever.

Einstein continued to work on his and Grossmann's *Entwurf* theory of gravitation and suggested new arguments to support its validity. His satisfaction with the *Entwurf* theory solidified to the point that he was ready in October 1914 to summarize it in a review article, "The Formal Foundation of the General Theory of Relativity,"[14] which he published in the meeting reports of the Royal Prussian Academy of Sciences. It took him less than a year to regret it.

Einstein's doubts concerning the *Entwurf* theory began to build in the summer of 1915. He finally abandoned the theory and, in an outburst of creativity and hard work, completed in November of that year his general theory of relativity.

Einstein had joined Max Planck, Walther Nernst, and many others in Berlin, which at the time was the world capital of physics. Even during the hardships of the war years, the city maintained an inspiring atmosphere and work routine in the physics community. Gerald Holton, a pioneer of Einstein scholarship in its historical and philosophical context, addressed the question,[15] "How much did these facts contribute to Einstein's unique ability to develop, between 1915 and late 1917, his general relativity theory in Berlin? Could he have done so if he had accepted a grand offer from a city in another country?" Holton's clear answer is, "No other man than Einstein could have produced General Relativity, and in no other city than in Berlin," albeit not without help from his friends in Zurich!

THE CHALLENGE OF GRAVITATION

The 1905 theory of relativity had established a new understanding of space and time, and all physical interactions needed henceforth to fit within its framework. In addition, the theory had combined the laws of conservation of energy and momentum into a single

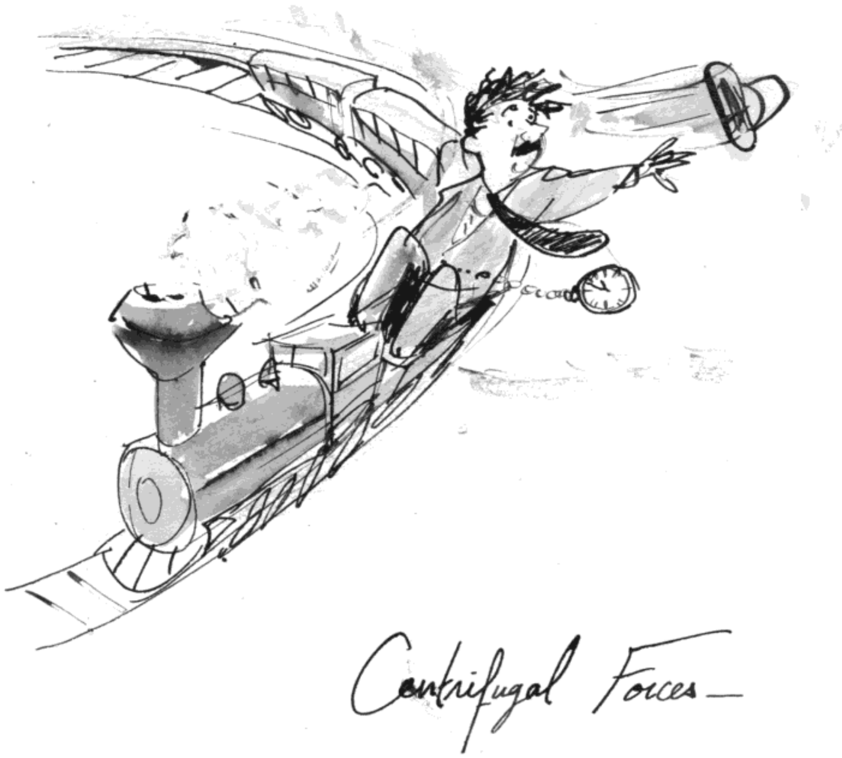
Even when the train is moving, the coffee does not miss the cup. This is the classical principle of relativity.

law and it had demonstrated that mass is a form of energy. The consequences of this theory could be conveniently described in the framework of a new mathematical formalism developed by Herman Minkowski, Einstein's former teacher at the ETH in Zurich. This formalism[16] combines space and time into one entity—spacetime—and assigns a geometric distance between any two physical events that occur at different positions and different times. One usually refers to points in spacetime as events because they are characterized by location and time of occurrence. The square of this distance is simply the square of the time separation between the two events minus the square of their spatial separation. Observers moving at constant velocity with respect to each other may compute this value using their respective positions and time measurements, and they will get the same result. In other words, Minkowski's four-dimensional spacetime is equipped with a "metric" instruction that is employed to measure the distance between events. This may be compared with the familiar metric instruction to measure the distance between two points in three-dimensional space: sum the squares of the Cartesian coordinate separations.

It was not difficult to adapt the domain of electromagnetism to the new spacetime framework of the theory of special relativity, which had actually been inspired by Maxwell's electrodynamics. But gravitation, that is, the force of gravity between two masses, presented problems in this respect. Because Newton's law of gravity assumes an instantaneous action at a distance, this law in its classical form was not directly compatible with the special theory of relativity. One of the consequences of this theory is that no physical effect can propagate with a speed exceeding that of light in a vacuum. Thus, a new gravitational theory was needed, but it was not clear how such a theory should look, what heuristic assumptions could be made, and even what specific criteria it should satisfy.

with respect to each other. Galileo's principle suggested that this might hold true even for reference frames accelerated with respect to each other, because all bodies in such reference frames behave in the same way; namely, they fall in the same time. But to compare an accelerated reference frame with a reference frame at rest and to claim that they are somehow equivalent, one must introduce an additional assumption. In an accelerated reference frame somewhere in empty space, like a spaceship far from Earth, bodies will fall to the ground because of the acceleration. In a reference frame at rest on Earth, bodies will fall to the ground because of Earth's gravitation. If the behavior is the same in both cases, gravitation and the apparent forces in the rocket due to its accelerated motion—also known as inertial forces—must be equivalent. This is Einstein's famous equivalence principle, one of the most important heuristic clues in constructing a generalized theory of relativity. In retrospect, he referred to this idea as "the happiest thought" of his life.[22] The equivalence principle states that the gravitational field has only a relative existence, because for an observer falling freely from the roof of a house there temporarily exists, at least in his or her immediate vicinity, no gravitational field. In particular, all physical processes in a uniform and homogeneous gravitational field are equivalent to those that occur in a uniformly accelerated system of reference without a gravitational field. This concept can be illustrated either by an accelerated spaceship or by the thought experiment of a falling elevator.

Including inertial forces in the attempt to construct a new theory of gravitation had far-reaching consequences. Inertial forces are fictitious forces acting on masses in



A "fictitious" force makes it difficult for Einstein to keep his hat on.

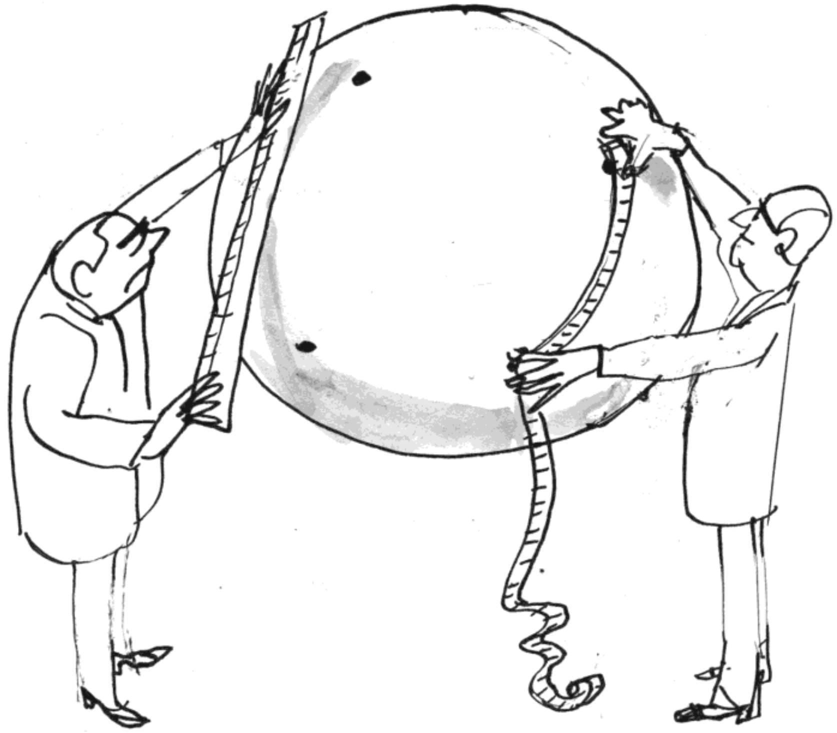
accelerated frames of reference, like the centrifugal force experienced on a merry-go-around. Einstein used different types of inertial forces as test cases for the new theory, for example, those in an accelerating spaceship. He also considered inertial forces that act in a rotating system of reference, such as the force shaping the surface of a liquid in a rotating bucket, which Newton used to demonstrate the concept of absolute motion. These "fictitious" forces are actually real forces, but their origin has remained enigmatic in classical physics because they have been ascribed to a mysterious property of absolute space. Considering such inertial forces on a par with the well-known Newtonian force, Einstein could then draw qualitative conclusions as well as derive requirements for the mathematical apparatus of his new theory.

One crucial conceptual insight provided by his thought experiments concerns the bending of light in a gravitational field and the nature of time. Einstein inferred the bending of light rays in a gravitational field from the argument that the path of a light ray in an accelerated laboratory must be curved due to the superposition of the motion of the laboratory and the motion of the light. The conclusion that this result must also be valid in a gravitational field was in agreement with the assumption that energy has not only inertial but also gravitational mass, so that light should be subject to attraction by gravity. The deflection of light in a gravitational field suggests that the speed of light should no longer be assumed to be constant, contrary to special relativity. This qualitative conclusion was supported by an analysis of time synchronization in an accelerated reference frame, as described by Einstein in an article written in 1907 (see note 4). His analysis implied that accelerating clocks at different locations run at different rates. He reached the same conclusion by comparing the rate of clocks located at different positions on a rotating disk.

What causes the curvature
of the surface of water
in a rotating bucket?

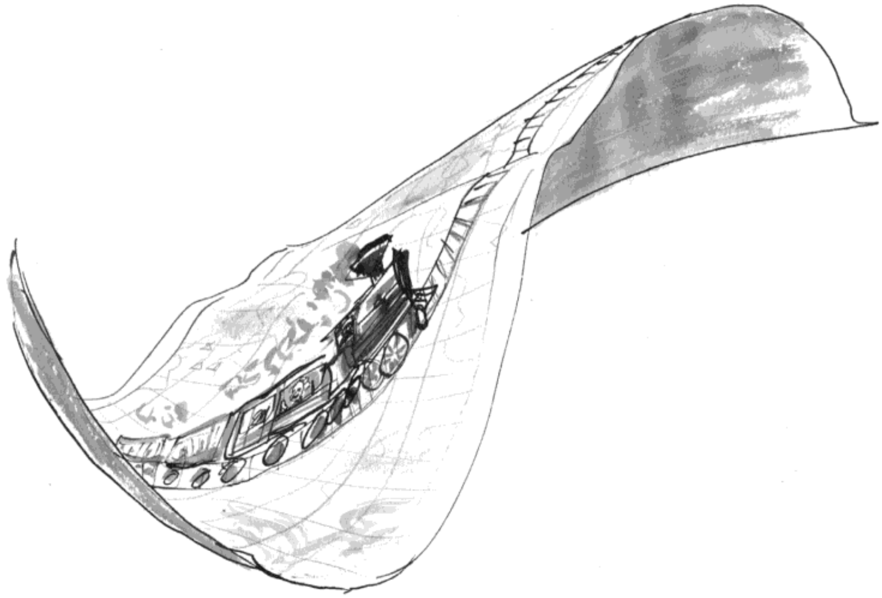
GEOMETRY ENTERS PHYSICS

The inclusion of rotating reference frames presented another conceptual challenge. Einstein and Max Born had encountered this challenge in 1909 in connection with the special theory of relativity. Paul Ehrenfest had also found independently that, according to special relativity, rods that are used to measure the circumference of a rotating disk should experience a so-called "Lorentz contraction." [23] Therefore, more rods are needed, and the circumference will appear longer than that of a disk at rest. However, rods used to measure the radius of the rotating disk will be unchanged, being perpendicular to the direction of motion. Therefore, the ratio of a rotating disk's circumference to its radius will have to be greater than the value determined in Euclidean geometry when both distances are measured in the frame of reference in which the disk is a rest. This difficulty became known as the "Ehrenfest paradox" and led to controversial discussions. Most participants in this debate considered this problem to be primarily a problem of the definition of a rigid body. However, Einstein identified the Ehrenfest paradox as a key issue to be addressed in seeking a generalization of the theory of relativity. In an article published in 1912, he argued that the ratio of circumference to diameter of a disk in a rotating laboratory is no longer given by π, indicating that general relativity implies a departure from Euclidean geometry.[24]



What if the world is intrinsically bent?

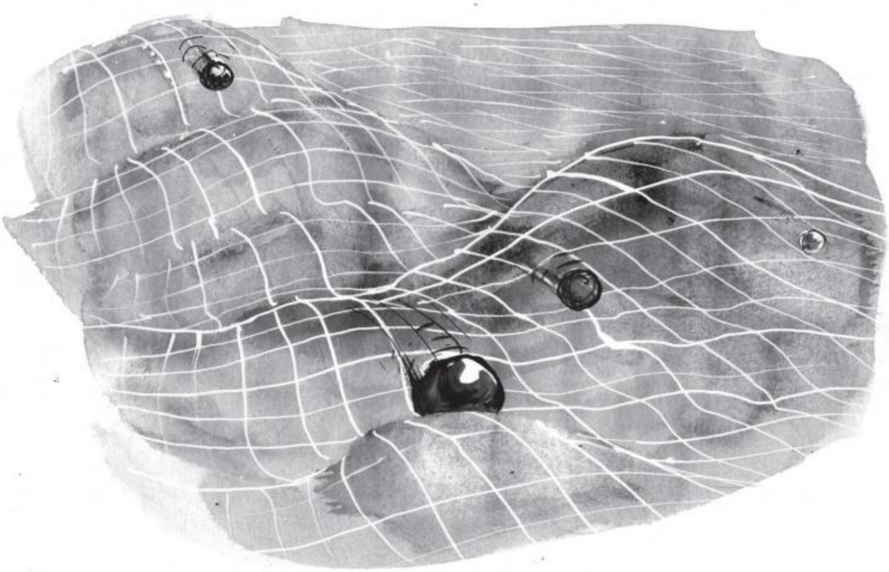
In Einstein's thought process, the equivalence principle and the use of accelerated laboratory models became subordinate to a newly formulated heuristic principle: the principle of general relativity. According to this principle, the new theory of gravitation should admit reference systems in arbitrary states of motion, and it should describe the inertial forces occurring therein as the action of a generalized dynamic gravitational field. This principle and the conceptual changes implied by the accelerated elevator and rotating bucket models played a crucial role in considering the kind of mathematics to be used in formulating the theory of gravitation. Einstein had realized that it would be necessary to go beyond Euclidean geometry. The desire to include arbitrary systems of reference gave him the idea in the summer of 1912 to construct the new theory of gravitation using a generalization of the Gaussian theory of curved surfaces, but he first had to generalize this theory to the four-dimensional world of the theory of relativity. Mathematicians like Bernhard Riemann, Elwin Christoffel, and Tullio Levi-Civita had provided the important background for this generalization, but Einstein was not familiar with their works and had to acquire this new mathematics gradually with help from his friend Marcel Grossmann.



This is how a straight path looks on a curved surface.

The mental model of motions along curved surfaces that was familiar from the world of classical physics also pointed directly to a solution to the problem of determining the equations of motion in an arbitrary gravitational field. An object that is constrained to move along a two-dimensional frictionless curved surface with no other forces than those exerted by the surface itself will always move along the shortest path, called a *geodesic*. This is the simplest generalization of a straight line. The idea could immediately be transferred to the case of motion observed from an arbitrarily accelerated system of reference, corresponding to motion in a gravitational field in the absence of any other forces. Such motion can also be represented as a four-dimensional spacetime geodesic in the curvilinear coordinates used to describe such a system of reference. (Curiously, however, the trajectory described by a freely moving object turns out to be the longest possible path between two given points in spacetime. This is a consequence of the peculiar mathematical properties of the spacetime metric.)

The revised description of the action of gravity meant that the gravitational field was no longer considered to be a force in the sense of Newtonian physics but as the embodiment



"Spacetime tells matter how to move; matter tells spacetime how to curve." (John Archibald Wheeler)

of geometric properties of a generalized spacetime continuum. The concept of a metric as a generalization of the concept of distance has already been introduced. Whereas a flat surface is characterized by a metric that behaves in the same way everywhere on the surface, the geometric properties of a curved surface must be described by a variable metric. Such a metric associates different actual distances with a given coordinate distance at different locations on the surface. This variable metric turned out to be a suitable representation of the gravitational potential.

EINSTEIN'S HEURISTICS: A PLAN OF ACTION

In his search for a relativistic theory of gravitation, Einstein could orient himself using a model very familiar to contemporary physicists, because it represented one of the great successes of nineteenth-century physics, namely, the unified theory of all electromagnetic interactions established by James Clerk Maxwell and Heinrich Hertz. It was in fact a remarkable feature of this theory that it did not describe electric and magnetic fields separately but as components of a unified electromagnetic field. This theory was developed into its definitive form by the Dutch physicist Hendrik Antoon Lorentz, who later became one of Einstein's mentors. The central concept of this theory was that of "field." In contrast with describing the interactions of particles due to forces acting at a distance, a field theory is not restricted to the interacting particles but extends to their complete surroundings. Field theory describes how the space-filling field is generated by charges and currents, considered to be the "source" of the field, and it also describes how this field in turn determines the motion of charged particles. A mathematical representation of the physical processes interpreted according to this "Lorentz model" therefore necessarily includes two parts:

Mathematics or physics—where to start? That is the question.

interpretation possible, and this became a necessary part of the strategy. In particular, a candidate field equation had to fulfill the demand that Newton's theory could be recovered for the special case of a weak static gravitational field and assure the conservation of energy and momentum. Furthermore, it had to fulfill the condition that even after it was modified to fulfill these demands, the group of admissible coordinate transformations still remained wide enough to include at least transformations to accelerated reference frames representing the special cases of uniform acceleration and uniform rotation.

The issue of the Newtonian limit of general relativity is complicated by the fact that there are actually two approaches: one going through the intermediate stage of special relativity, and the other via a generalization of gravitational fields within Newtonian physics, allowing slow-motion and quasi-static solutions to be treated. The latter, however, demands a reformulation of Newtonian theory—including the equivalence principle—in terms of mathematical concepts that were introduced only much later by the French mathematician Elie Cartan in reaction to the work of Levi-Civita and Weyl. Before this sophisticated mathematical approach was developed, Einstein was compelled to introduce assumptions about the Newtonian limit that later turned out to be problematic.