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# Contents

Prologue

# Listening to the Universe

> I hold it to be true that pure thought can grasp reality, as the Ancients dreamed.
>
> Albert Einstein, 'On the Method of Theoretical Physics', 1933

'Einstein is completely cuckoo.' That was how the cocky young Robert Oppenheimer described the world's most famous scientist in early 1935, after visiting him in Princeton.[1] Einstein had been trying for a decade to develop an ambitious new theory in ways that demonstrated, in the view of Oppenheimer and others, that the sage of Princeton had lost the plot. Einstein was virtually ignoring advances made in understanding matter on the smallest scale, using quantum theory. He was seeking an ambitious new theory, not in response to puzzling experimental discoveries, but as an intellectual exercise – using only his imagination, underpinned by mathematics. Although this approach was unpopular among his peers, he was pioneering a method similar to what some of his most distinguished successors are now using successfully at the frontiers of research.

Oppenheimer and many other physicists at that time can hardly be blamed for believing that Einstein's mathematical approach was doomed: for one thing, it seemed to contradict one of the principal lessons of the past 250 years of scientific research, namely that it is unwise to try to understand the workings of nature using pure thought, as Plato and other thinkers had believed. The conventional wisdom was that physicists should listen attentively to what the universe tells them about their theories, through the results of observations and experiments done in the real world. In that way, theorists can avoid deluding themselves into believing they know more about nature than they do.

Einstein knew what he was doing, of course. From the early 1920s, he often commented that experience had taught him that a mathematical strategy was the best hope of making progress in his principal aim: to discover the most fundamental laws of nature. He told the young student Esther Salaman in 1925, 'I want to know how God created this world. I'm not interested in this or that phenomenon, in the [properties] of this or that element. I want to know His thoughts, the rest are details.'[2] In his view, 'the supreme task of the physicist' was to comprehend the order that underlies the workings of the entire cosmos – from the behaviour of the tiny particles jiggling around inside atoms to the convulsions of galaxies in outer space.[3] The very fact that underneath the diversity and complexity of the universe is a relatively simple order was, in Einstein's view, nothing short of a 'miracle, or an eternal mystery'.[4]

Mathematics has furnished an incomparably precise way of expressing this underlying order. Physicists and their predecessors have been able to discover universal laws – set out in mathematical language – that apply not only here and now on Earth but to everything everywhere, from the beginning of time to the furthest future. Theorists, including Einstein, who pursue this programme may be accused quite reasonably of overweening hubris, though not of a lack of ambition.

The potential of mathematics to help discover new laws of nature became Einstein's obsession. He first set out his mathematical approach to physics research in the spring of 1933, when he delivered a special lecture to a public audience in Oxford. Speaking quietly and confidently, he urged theoreticians not to try to discover fundamental laws simply by responding to new experimental findings – the orthodox method – but to take their inspiration from mathematics. This approach was so radical that it probably startled the physicists in his audience, though understandably no one dared to contradict him. He told them that he was practising what he was preaching, using a mathematical approach to combine his theory of gravity with the theory of electricity and magnetism. That goal could be achieved, he believed, by trying to predict its mathematical structure – the mathematics of the two theories were the most potent clues to a theory that unified them.

As Einstein well knew, a mathematical strategy of this type would not work in most other scientific disciplines, because their theories are usually not framed in mathematical language. When Charles Darwin set out his theory of evolution by natural selection, for example, he used no mathematics at all. Similarly, in the first description of the theory of continental drift, Alfred Wegener used only words. One potential shortcoming of such theories is that words can be treacherous – vague and subject to misinterpretation – whereas mathematical concepts are precise, well defined and amenable to logical and creative development. Einstein believed that these qualities were a boon to theoretical physicists, who should take full advantage of them. Few of his colleagues agreed – even his most ardent admirers scoffed. His acid-tongued friend Wolfgang Pauli went so far as to accuse him of giving up physics: 'I should congratulate you (or should I say send condolences?) that you have switched to pure mathematics.... I will not provoke you to contradict me, in order not to delay the death of [your current] theory.'[5] Brushing such comments aside, Einstein continued on his lonely path, though he had little to show for his labours: he had become the Don Quixote of modern physics.[6] After he died in 1955, the consensus among leading physicists was that the abject failure of his approach had vindicated his critics, but this judgement has proved premature.

Although Einstein was wrong to gloss over advances in theories of matter at the subatomic level, he was in one respect more far-sighted than his many detractors. In the mid-1970s, twenty years after he died, several prominent physicists were following in his footsteps, trying to use pure thought – bolstered by mathematics – to build on well-established but flawed theories. At that time, I was a greenhorn graduate student, wary of this cerebral strategy and pretty much convinced that it was perverse and heading nowhere. It seemed obvious to me that the best way forward for theorists was to be guided by experimental findings. That was the orthodox method, and it had worked a treat for the theorists who developed the modern theory of subatomic forces. Later known as the Standard Model of particle physics, it was a thing of wonder: based on only a few simple principles, it quickly superseded all previous attempts to describe the behaviour of subatomic particles. It accounted handsomely for the inner workings of every atom. What I did not fully appreciate at the time was how fortunate I was to be sitting in the back row of the stalls, watching an epic contemporary drama unfold.

During those years, I remember attending dozens of seminars about exotic new theories that looked impressive but agreed only roughly with experiments. Yet their champions were obviously confident that they were on to something, partly because the theories featured interesting new mathematics. To me, this seemed a peculiar way of researching physics – I thought it much better to listen to what nature was telling us, not least because it never lies.

I sensed a new wind was blowing and, as far as I could tell, it was going in an unappealingly mathematical direction. Privately, I expected the trend to peter out, but once again I was wrong. In the early 1980s, the wind gathered momentum, as the flow of new information from experiments on subatomic particles and forces slowed from a gush to a drip. For this reason, more theoreticians turned to pure reasoning, supplemented by mathematics. This led to a new approach to fundamental physics – string theory, which aspires to give a unified account of nature at the finest level by assuming that the basic constituents of the universe are not particles but tiny pieces of string. Theorists made progress with the theory but, despite a huge effort, they could not make a single prediction that experimenters could check. Sceptics like me began to believe that the theory would prove to be no more than mathematical science fiction.

I found it striking, however, that many of the leading theoretical physicists were not discouraged by the glaring absence of direct experimental support. Time and again, they stressed the theory's potential and also the marvellous breadth and depth of its connections to mathematics, many of which were revelatory even to world-class mathematicians. This richness helped to shift collaborations between theoretical physicists and mathematicians into an even higher gear, and generated a welter of mind-blowing results, especially for mathematicians. It was clearer than ever not only that mathematics is indispensable to physics, but also that physics is indispensable to mathematics.

This intertwining of mathematics and physics seemed to exemplify the view expressed in the 1930s by the physicist Paul Dirac, sometimes described as 'the theorist's theorist'.[7] He believed that fundamental physics advances through theories of increasing mathematical beauty.[8] This trend convinced him – as 'a matter of faith rather than of logic' – that physicists should always seek out examples of beautiful mathematics.[9] It was easy to see why this credo had a special appeal for string experts: their theory had abundant mathematical beauty, so, according to Dirac's way of thinking, held commensurately huge promise.

The ascendancy of string theory did much to give modern fundamental physics a strong mathematical hue. Michael Atiyah, a brilliant mathematician who had switched his focus to theoretical physics, later wrote provocatively of the 'mathematical takeover of physics'.[10] Some physicists, however, were dismayed to see many of their most talented colleagues working on recondite mathematical theories that in many cases were impossible to test. In 2014, the American experimenter Burton Richter bluntly summarised his anxieties about this trend: 'It seems that theory may soon be based not on real experiments done in the real world, but on imaginary experiments, done inside the heads of theorists.'[11] The consequences could be disastrous, he feared: 'Theoreticians would have to draw their inspiration not from new observations but from mathematics. In my view, that would be the end of research into fundamental physics as we now know it.'

Disenchantment with the state of modern theoretical physics has even become a public talking point. Over the past decade or so, several influential commentators have taken aim at string theory, describing it as 'fairy tale physics' and 'not even wrong', while a generation of theoretical physicists stand accused of being 'lost in math'.[12] It is now common to hear some critics in the media, especially in the blogosphere, complain that modern physics should get back on the straight and narrow path of real science.

This view is misguided and unnecessarily pessimistic. In this book, I shall argue that today's theoretical physicists are indeed taking a path that is entirely reasonable and extremely promising. For one thing, their approach draws logically and creatively on centuries of achievements all the way back to Isaac Newton. By setting out mathematical laws that describe motion and gravity, he did more than anyone else to construct the first mathematically based and experimentally verifiable framework for describing the real world. As he made clear, the long-term aim is to understand more and more about the universe in terms of fewer and fewer concepts.[13] Leading theorists today pursue this agenda by standing squarely on the two granitic foundation stones of the twentieth century: Einstein's basic theory of relativity, a modification of Newton's view of space and time, and quantum mechanics, which describes the behaviour of matter on the smallest scale. No experiment has ever disproved either of the two theories, so they form an excellent basis for research.

As Einstein often pointed out, quantum mechanics and the basic theory of relativity are devilishly difficult to meld. Physicists were eventually able to combine them into a theory that made impressively successful predictions, in one case agreeing with the corresponding experimental measurement to eleven decimal places.[14] Nature seemed to be telling us loud and clear that it wanted both theories to be respected. Today's theoretical physicists are building on that success, insisting that every new theory that aspires to be universal must be consistent with both basic relativity and quantum mechanics. This insistence led to consequences that nobody had foreseen: not only to new developments in physics – including string theory – but also to a host of links with state-of-the-art mathematics. It had never been clearer that physics and mathematics are braided: new concepts in fundamental physics shed light on new concepts in mathematics, and vice versa. It is for this reason that many leading physicists believe that they can learn not only from experiments but also from the mathematics that emerges when relativity and quantum mechanics are combined.

The astonishing effectiveness of mathematics in physics has enthralled me since I was a schoolboy. I remember being surprised that the abstract techniques we learned in our mathematics lessons were perfectly suited to solving the problems we were tackling in physics classes. Most remarkable for me was that some of the mathematical equations that linked unknown quantities *x* and *y* also applied to observations that describe the real world, with *x* and *y* standing for quantities that experimenters could measure. It amazed me that a few simple principles, underpinned by mathematics we had only recently learned, could be used to predict accurately everything from the paths of golf balls to the trajectories of planets.

As far as I recall, none of my schoolteachers commented on the way abstract mathematics lends itself to physics so exquisitely, one might even say miraculously.[15] At university, I was even more impressed that theories that incorporated basic mathematics could describe so much about the real world – from the shapes of magnetic fields near current-carrying wires to the motion of particles inside atoms. It seemed something like a fact of scientific life that mathematics is utterly indispensable to physics. Only much later did I glimpse the other side of the story: that physics is indispensable to mathematics.

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One of my main aims in this book is to highlight how mathematics, as well as proving useful to physicists, has supplied invaluable clues about how the universe ticks. I begin with Newton's epoch-making use of mathematics to set out and apply the law of gravity, which he repeatedly tested against observations and careful measurements. Next, I explain how the mathematical laws of electricity and magnetism were discovered in the nineteenth century, using a mathematical framework that had huge implications for our understanding of nature.

I then move on to discuss two groundbreaking discoveries – first, basic relativity, and then quantum mechanics, the most revolutionary theory in physics for centuries. When Einstein used relativity to improve our understanding of gravity, he was forced to use mathematics that was new to him, and the success of this approach changed his view about the utility of advanced mathematics to physicists. Likewise, when physicists used quantum mechanics to understand matter, they were forced to use tranches of unfamiliar mathematics that changed their perspective on, for example, the behaviour of every one of nature's smallest particles.

Since the mid-1970s, many talented thinkers have been drawn to fertile common ground between mathematics and physics. Nonetheless, most physicists have steered clear of this territory, preferring the conventional and more prudent approach of waiting for nature to disclose more of its secrets through experiments and observations. Nima Arkani-Hamed, one of Einstein's successors on the faculty of the Institute for Advanced Study at Princeton, made his name by taking this orthodox approach. About a decade ago, however, after he began to study the collisions between subatomic particles, he and his colleagues repeatedly found themselves working on the same topics as some of the world's leading mathematicians. Arkani-Hamed quickly became a zealous promoter of the usefulness of advanced mathematics to fundamental physics.

He remains a physicist to his fingertips: 'My number one priority will always be physics – to help discover the laws underlying the Universe,' he says. 'We must listen to [nature] as attentively as we possibly can, making use of every observation and measurement that might have something to teach us. Ultimately, experiments will always be the judge of our theories.' But his mathematical work has radically changed the way he thinks about physics research – 'we can eavesdrop on nature not only by paying attention to experiments but also by trying to understand how their results can be explained by the deepest mathematics. You could say that the universe speaks to us in numbers.'[16]

**Notes**

1 Robert Oppenheimer to his brother Frank, 11 January 1935: Smith and Weiner (eds) (1980: 190). For interesting reflections on the meaning of 'fundamental physics', see Anderson (1972) and Weinberg (1993: 40–50)

2 Salaman (1955: 371). Note that the word in brackets, 'properties', is my rendition of the technical term she uses, 'spectrum'.

3 Einstein on 'Principles of Research' 1918: Einstein (1954: 226)

4 Einstein to Solovine, 30 March 1952: Solovine (ed.) (1986: 131) (I have amended Solovine's choice of the word 'world' to 'universe', which I believe is more accurate.) Einstein made a similar remark in 1936: 'The eternal mystery of the world is its comprehensibility': see Einstein on 'Physics and Reality' in Einstein (1954: 292)

5 Pauli to Einstein, 19 December 1929: Pais (2000: 216)

6 Schweber (2008: 282)

7 Farmelo (2009: 188)

8 Farmelo (2009: 300–301)

9 Dirac (1954: 268–269)

10 Atiyah (2005: 1081); interview with Atiyah, 15 April 2016

11 Interview with Burton Richter, 30 April 2015 (he later confirmed his comments in an e-mail). Richter died in July 2018. RIP.

12 'Fairytale physics' is a phrase favoured by the science writer Jim Baggott; *Lost in Math* is the title of a 2018 book by the theorist and prolific blogger Sabine Hossenfelder; 'Not Even Wrong' is the name of the popular blog by the physicist Peter Woit.

13 Iliffe (2007: 98)

14 Feynman (1985: 7)

15 Yang (2005: 74)

16 Interview with Arkani-Hamed, 10 May 2018

1

# Mathematics Drives Away the Cloud

> The things that so often vexed the minds of the ancient philosophers
> And fruitlessly disturb the schools with noisy debate
> We see right before our eyes, since mathematics drives away the cloud
>
> Edmond Halley,
> ode to Newton and his *Principia*, 1687

Einstein was modest about his achievements. He knew his place in the history of science, however, and was aware that he was standing on giants' shoulders, none broader than Isaac Newton's. Two centuries after the Englishman's death, Einstein wrote that 'this brilliant genius' had 'determined the course of western thought, research and practice, like no one else before or since'.[1] Among Newton's greatest achievements, Einstein later remarked, was that he was 'the first creator of a comprehensive, workable system of theoretical physics'.[2]

Newton never spoke of 'physicists' and 'scientists', terms that were coined more than a century after his death.[3] Rather, he regarded himself as primarily a man of God and only secondarily as a mathematician and natural philosopher, attempting to understand rationally the entirety of God's creation, using a combination of reasoning and experiment. He first publicly set out his mathematical approach to natural philosophy in 1687, when he published his *Principia*, a three-book volume soon to make him famous and help to establish him as one of the founders of the Enlightenment. In the preface to that edition, he made clear that he was proposing nothing less than 'a new mode of philosophising'.[4]

Newton rejected the way of working that virtually all his contemporaries regarded as the best way to proceed. They were making guesses about the mechanisms that can explain how nature works, as if it were a giant piece of clockwork that needed to be understood. Instead, Newton focused on the motion of matter, on Earth and in the cosmos – part of God's creation that he could describe precisely using *mathematics*. Most significantly, he insisted that a theory must be judged solely according to the accuracy of the account it gives of the most precise observations on the real world. If they do not agree within experimental uncertainties, the theory needs to be modified or replaced by a better one. Today, all this sounds obvious, but in Newton's day it was radical.[5]

When Newton published his *Principia*, he was a forty-four-year-old professor living a quiet bachelor life in Trinity College, Cambridge, in rooms that now overlook the row of stores that includes Heffers Bookshop.[6] Almost two decades before, the university had appointed him to its Lucasian Chair of Mathematics, although he had published nothing on the subject. Mathematics was only one of his interests – he was best known in Cambridge for designing and building a new type of telescope, which attested to his exceptional practical skills.

A devout and stony-faced Protestant, he believed he was born to understand God's role in creating the world, and he was determined to rid Christian teaching of corruptions by perverted priests and others who preyed on the tendency of many people to wallow in idolatry and superstition.[7] To this and all his other work, Newton brought a formidable energy and a concentration so intense that he would occasionally forget to eat.[8] For this prickly and suspicious scholar, life was anything but a joke – a smile would occasionally play across his face, but he was only rarely seen to laugh.[9]

Newton invited only a small number of acquaintances into his chambers, and relatively few experts appreciated the extent of his talent. He was not interested in sharing his new knowledge and once remarked that he had no wish to have his 'scribbles printed' – the relatively new print culture was not for him.[10] His circle of confidants included the chemist Francis Vigani, who was disappointed to find himself cut off after telling the great thinker a 'loose story' about a nun.[11]

Newton's new scheme for natural philosophy did not arrive out of the blue – it emerged after decades of gestation and close study. In the opening words of the *Principia*, he acknowledged his debts: first to the ancient Greeks, who had focused above all on the need to understand *motion*, and second to recent thinkers who had 'undertaken to reduce the phenomena of nature to mathematical laws'.[12] To understand the background to Newton's achievement, it is instructive to look briefly at these influences, beginning with the ancient Greeks, who had taught Europeans the art of thinking.

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The nearest the ancient Greeks came to doing science (from *scientia*, Latin for 'knowledge') in the modern sense was in the work of the philosopher Aristotle (384–322 BC). He believed that, underneath the messiness of the world around us, nature runs on principles that human beings can discover and that are not subject to external interference from, for example, meddlesome deities.[13] Of all the ancients' schools of philosophy, Aristotle's paid most attention to *physica* – a word derived from *physis*, meaning nature – which included studies that ranged from astronomy to psychology. The word 'physics' derives from this but didn't acquire its modern meaning until the early nineteenth century.

The sheer breadth of Aristotle's studies – from cosmology to zoology and from poetry to ethics – made him perhaps the most influential thinker about nature in our history. He believed that the natural world can be described by general principles that express the underlying reasons for all the types of change that can affect any matter, including changes in its shape, colour, size and motion. His writings on science, including his book *Physica*, seem strange to most modern readers partly because he attempted to understand the world using pure reason, albeit supported by careful observation.

One characteristic of his view of the world is that mathematics has no place in it. Aristotle declined, for example, to use the elements of arithmetic and geometry, whose rudiments were already thousands of years old when he began to think about science. Both branches of mathematics were grounded in human experience and had been developed by thinkers who had taken the crucial step of moving from observations of the real world to completely general abstraction. The most basic elements of arithmetic, for example, began when human beings first generalised the concept of two sticks, two wolves, two fingers and so on, to the existence of the abstract concept of the number 2, not associated with any one concrete object. This was a profound insight, though it is not easy to say when it was first made. The beginnings of geometry – the relationships between points, lines and angles in space – are easier to date: about 3000 BC, when people in ancient Babylonia and the ancient Indus Valley began to survey the land, sea and sky. In Aristotle's view, however, there was no place in science for mathematics, whose 'method is not that of natural science'.[14]

Aristotle's rejection of mathematical thinking was antithetical to the philosophy of his teacher Plato and of another of the most famous ancients, Pythagoras, who may never have existed (his putative teachings may have been the work of others). Pythagoreans studied arithmetic, geometry, music and astronomy and held the view that whole numbers were crucially important. Their remarkable ability to explain, for example, the relationship between musical harmonies and the properties of geometric objects led the Pythagorean school to believe that whole numbers were essential to a fundamental understanding of the way the universe works.

Plato had believed that mathematics was fundamental to philosophy and was convinced that geometry would lead to understanding the world. For Plato, the complicated realities around us are, in a sense, shadows of perfect mathematical objects that exist quite separately, in the abstract world of mathematics. In that world, shapes and other geometric objects are perfect – points are infinitely small, lines are perfectly straight, planes are perfectly flat and so on. So, for example, he would have regarded a roughly square table top as the 'shadow' of a perfect square, whose infinitely thin and perfectly straight lines all meet at precisely 90 degrees. Such a perfect mathematical object cannot exist in the real world, but it is a feature of what modern mathematicians often describe as the Platonic world, which can seem to them no less real than the world around us.

Within a quarter of a century of Aristotle's death, the Greek thinker Euclid introduced new standards of rigour to mathematical thinking. In his magnificent thirteen-book treatise *The Elements*, he set out the fundamentals of geometry clearly and comprehensively, setting new standards of logical reasoning in the subject. Although no one's idea of easy reading, *The Elements* became the most influential book in the history of mathematics and exerted a powerful influence on thinkers for centuries. One of the leading physicists who later fell under its spell was Einstein, who remarked, 'If Euclid failed to kindle your youthful enthusiasm, then you were not born to be a scientist.'[15]

Mathematics was becoming practically useful, too. Archimedes was especially adept at putting mathematical ideas to work in his inventions, for example, his water-raising screw and parabolic mirror. Several of his contemporaries in Greece used geometric reasoning to measure the distance of the Sun and the Moon from the Earth, the circumference of the Earth, and the tilt of the Earth's axis of spin, often to an impressively high degree of accuracy. The idea that regularities in the behaviour of objects that human beings observe around them on Earth could be described by mathematical laws was centuries away. However, mathematics was already enabling earth-bound human beings to transcend their senses and deploy their powers of imaginative reasoning way out into the heavens.

Simple mathematical concepts began to be useful to many of the thinkers who were advancing science. In the Middle Ages, many of the most notable mathematical innovations arose in Islamic territories, roughly in the areas now spanned by Iran and Iraq.[16] Scholars of this region made impressive mathematical advances, including the development of algebra, from the Arabic *al-jabr*, meaning 'reunion of broken parts'. These innovations formed the basis of modern algebra, which uses abstract symbols, say $x$ and $y$, to represent quantities that can take numerical values and be mathematically manipulated.

By the middle of the sixteenth century, when Shakespeare was born, mathematics featured prominently in almost every branch of physical science – including astronomy, optics and hydraulics – as well as in music. New ideas about the way mathematics relates to the world were gaining traction, calling into question the Aristotelian way of thinking that had dominated Christian and Islamic thinking for 2,000 years. One of the most important contributions was Nicolaus Copernicus's proposal in 1543 that the centre of the universe is not the Earth but the Sun, a radical notion that marked the beginning of what became known as the Scientific Revolution. Among its leading pioneers were two astronomers who were also mathematicians: the German Johannes Kepler and the Italian Galileo Galilei. They believed that the best way to understand the world was not to focus on the superficial appearances of things but to give precise descriptions of motion. To them, it was especially important to identify mathematical regularities in measurements made on moving objects. Among Kepler's achievements, he identified such regularities in the motion of the planets orbiting the Sun, while Galileo discovered regularities closer to home – in the paths of objects falling freely to the ground.

For the devout Kepler, God was the 'architect of the universe' and had created it according to a plan that human beings could understand using geometry, a subject that Kepler regarded as divine.[17] The disputatious Galileo often stressed the importance of comparing the predictions of scientific theories directly with observations made on the real world: this insistence made him 'the father of modern science' in Einstein's view, though Galileo was given to exaggerating the accuracy of his experimental data.[18] He was also no slouch as a mathematician and appreciated its importance to human understanding of the natural world, famously declaring in 1623 that the book of nature 'is written in the language of mathematics'.[19] His thinking was part of a cultural trend in many of the most prosperous European cities: mathematics was beginning to underpin commercial and artistic life, through new bookkeeping methods and the use of geometric perspective in art and architecture.[20]

Neither Kepler nor Galileo fully grasped an idea that was to become central to science – that the natural world appears to be described by laws that apply everywhere, perhaps for all time.[21] The idea, mooted by Aristotle, that there exist fundamental laws of nature emerged most clearly in the writings of the Frenchman René Descartes, whose work was to dominate European thinking about nature for several decades from the early 1640s, an era that saw both the death of Galileo and the birth of Newton. Descartes set aside Aristotelian science and tried to account for gravity, heat, electricity and other aspects of the real world using mechanisms that he described with impressive vividness, bearing in mind that neither he nor anyone else had any direct evidence that they were correct.[22]

Descartes published his ideas in the book *Principles of Philosophy*, which he recommended should be read straight through like a novel (he advised his readers that most of their difficulties with the text will have disappeared by their third reading). The book used very little mathematics and gave no indication of how experimenters could test his mechanical theories, such as his idea that huge swirling vortices of matter drive each planet around the Sun. London's most eminent experimenter, Robert Hooke, was a fulsome admirer of Descartes but was nonetheless becoming impatient with the prevailing cerebral approach to science: 'The truth is, the Science of Nature has already been too long made only a work of the Brain and the Fancy: it is now high time that it should return to the plainness and soundness of observations on material and obvious things.'[23]

When Hooke wrote those words, in 1665, the twenty-two-year-old Isaac Newton was doing breathtakingly creative work in both mathematics and natural philosophy. By then, he was familiar with the thinking of the ancient Greeks, and in one of his notebooks he had written an old scholastic tag: 'Plato is a friend, Aristotle is a friend, but truth is a greater friend.'[24] Newton was also well acquainted with the discoveries of Kepler, Galileo and Descartes and how these thinkers and others had overturned the Aristotelian consensus. The most decisive event in Newton's mathematical education was his reading of Descartes's *Geometry*: in the words of the eminent Newton scholar David Whiteside, from the first hundred or so pages of this book, Newton's 'mathematical spirit took fire'.[25] If he had published the mathematical discoveries he made in this period, he would have been recognised as one of the world's leading experts in the discipline, though hardly any of his peers knew what he had done. The world found out almost a quarter of a century later, when he began to move science towards more systematic studies of the natural world, grounded in mathematics and quantitative observations. He did this in his magnum opus, one of the most important volumes in the history of human thought.

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Newton may never have written the *Principia* had it not been for the initiative and perseverance of the astronomer Edmond Halley, now best remembered for the observations of the comet later named after him. One of Newton's few friends, Halley spent almost three years coaxing, assisting and cajoling the reluctant author to deliver his masterpiece. He even offered to pay the cost of publishing it. The *Principia*, about five hundred pages long, went on sale in London on Saturday 5 July 1687 – a red-letter day in the history of science, though at the time it was a non-event. The publishers printed about six hundred copies, but selling them all proved to be a struggle, even after an anonymous review praised the 'incomparable Author' for delivering 'a most notable instance of the powers of the Mind' (the words were Halley's).[26] Newton had presented his scheme in a forbiddingly austere style, partly to 'avoid being baited by little smatterers in mathematics', as he later put it.[27] As a result, the volume was virtually impenetrable for everyone except a handful of his peers: during his lifetime, fewer than a hundred people read the entire *Principia*, and it is certain that only a few of them had even a faint chance of understanding it.[28]

Newton had subtitled his volume *Mathematical Principles of Natural Philosophy*, in an unsubtle swipe at Descartes's *Principles of Philosophy*. The point of this was to signal that he was focussing entirely on *natural* philosophy – the *real* world – rather than on philosophy in general, and that his principles were essentially *mathematical*.[29] Newton had studied Descartes's error-strewn book closely and had become increasingly critical of its 'tapestry of assumptions'.[30] In a sense, the *Principia* was a narrowing and mathematical corrective to Descartes's narrative philosophy of nature: Newton focused entirely on the part of the real world that could be accounted for mathematically, with both generality and precision.

In the *Principia*, Newton assumes that time and space are the same for everyone, everywhere.[31] Time 'flows uniformly', and space exists 'without reference to anything external'. Adopting a relentlessly logical and austere style, similar to the one Euclid used in his *Elements*, Newton made perhaps the boldest unifying insights in the history of science: the force of gravity that pulls any object (an apple, say) towards the ground on Earth, he argued, is the same as the force that acts on the planets, the Moon and every other material object in the cosmos. He described this force using mathematics that later formed the basis of a simple formula that became familiar to all students of physics: any two particles separated by distance $d$, and with masses $m$ and $M$, are attracted directly towards each other with a force whose size is given by $GmM/d^2$, an 'inverse square law' ($G$ is a constant, with the same value everywhere and for all time, later known as Newton's gravitational constant). Newton's way of demonstrating that this is a valid law of nature has remained a template for science ever since.

To predict the effects of this force of gravity on planets, Newton used three new laws of motion, implemented using geometric methods that seem extremely obscure to modern scientists. They do these calculations using the technique known as

*Principia*'s author, the French were in the grip of *Anglomanie* – Anglomania.[39]

The most influential among the Newtonians in France was Pierre-Simon Laplace, sometimes known as the French Newton. A chilly rationalist, not given to philosophising, he sought every opportunity to apply his mathematical skills to describe the world around him and the cosmos, seizing every opportunity to use and test Newton's law of gravity. In the countryside of Normandy, he had begun to train for the priesthood before he ventured to Paris to begin a career as a mathematician, which flourished with impressive speed.[40] He swiftly ascended to the heights of the French scientific aristocracy.

To advance the Newtonian program, Laplace and his colleagues developed a wealth of new mathematics, much of it relating to calculus. However, they used the version of this technique set out by Newton's *bête noire*, Gottfried Leibniz, rather than the one introduced by the Englishman. The German's methods were much simpler to use and, more importantly, much easier to develop. By concentrating on the Leibniz framework, many of the techniques in calculus that are now part of every physicist's education were invented by several great mathematicians, including Laplace's first mentor, Jean le Rond d'Alembert, the Turin-born Joseph-Louis Lagrange, and Swiss adepts Leonhard Euler and Johann Bernoulli. Perhaps the most important of their achievements was their development of differential equations, which feature *rates of change* of quantities that relate to the real world (such as speed, temperature and magnetic field). A classic example of one such equation is the modern version of Newton's second law of motion, which says that the force acting on a mass is equal to the mass multiplied by the rate of change of its velocity with time.[41] This equation was a boon to natural philosophers: in principle, it enabled them to predict the motion of any mass whatsoever once they had a formula for the force acting on it.

Differential equations proved to be an essential tool for every theorist. These equations often generate surprising new connections between physical quantities, enable fresh perspectives on familiar ideas, and give unexpected insights into the way nature works. In a sense, the differential equations that describe the real world are akin to poetry: if 'poetry is language in orbit', as the writer Seamus Heaney later observed, then differential equations are mathematical language in orbit.[42]

Through the work of Laplace, his colleagues in Paris and other experts on the Continent, international leadership in natural philosophy passed across the English Channel. This went down badly in Britain, where Regency intellectuals whinged that Laplace and his followers relied too much on mathematical abstractions and not enough on concrete observations. London's most insightful natural philosopher, Thomas Young, complained that the Frenchman was leading natural philosophers astray: 'Mr Laplace may walk about and even dance ... in the flowery regions of algebra, without exciting our smiles, provided that he does no worse than return to the spot from which he set out.'[43] Unmoved, Laplace advanced a Newtonian agenda with formidable industry, dismissing the demands of previous generations of French philosophers that Newton's theory was unsatisfactory because it said nothing about a mechanical cause of gravity. The French mathematicians and philosophers never formally settled the dispute but stopped talking about it, and it eventually disappeared, along with Descartes's vortices.

Unlike Newton's approach to understanding the natural world, Laplace's was godless. After Newton discovered that his mathematical scheme incorrectly predicted that the cosmos was unstable, he explained the disparity by asserting that God occasionally tinkered with the motion of the planets to ensure their stability.[44] Laplace wanted nothing to do with that kind of reasoning. According to a widely believed anecdote, after Napoleon asked Laplace about the place of God in his view of the cosmos, the great *physicien* replied loftily, 'I have no need of that hypothesis.'[45] There was a crucial difference between Newton's view of the application of mathematics to nature and Laplace's. Whereas Newton was trying to give the most precise description of the universe through mathematics, to better appreciate God's work, his French successor believed that nature could be described using only mathematical laws. The essence of Laplace's faith was that these laws were in some sense out there, waiting to be discovered, rather as Plato regarded mathematics – the view most scientists take today.[46]

Laplace also took an uncompromising view about the ability of mathematical laws to tell us not only about the natural world but also about the past and future. He and his colleagues rendered obsolete vague talk about the role of chance in determining events, by setting up the first comprehensive mathematical theory of probability. Laplace believed that the universe is completely deterministic – the future can in principle be calculated entirely from complete information about the present.[47] He wrote, 'We ought to regard the present state of the universe as the effect of its antecedent state and as the cause of the state that is to follow.'[48] Laplace had a touching faith in the power of universal mathematical laws – although he knew of only a few, he was certain that others existed and that all the particles in the universe were dancing to mathematical tunes.

Laplace and his colleagues laid the foundations of the modern discipline of physics. It may be said to be a French invention, a consequence of the collective determination to systematise knowledge, to measure accurately, and to calculate using theories based on mathematics.[49] In 1765, Denis Diderot and Jean le Rond d'Alembert summed it up well in their classic compendium of Enlightenment thought, the *Encyclopédie*: 'The mathematical method belongs to all the sciences, is natural to the human mind, and leads to discoveries of truths of all kinds.'[50] New mathematics was playing a crucial role in physics research, which in turn was generating a great deal of new mathematics.

*

Physics as we know it today began to take shape around the turn of the nineteenth century, only a few years after the upheaval of the French Revolution. A group of experts, known as *physiciens*, began to focus their studies on heat, electricity, magnetism, pneumatics, hydrology and a few other subjects, with relatively little overlap with biology, chemistry and geology.[51] Laplace in particular, during the terrible Reign of Terror, which cost the lives of several of his peers, kept his head down and went about his work as if nothing untoward was happening. Napoleon was close to dozens of world-class astronomers, mathematicians and *physiciens*, and became the most influential and benevolent supporter of research into physics, especially electricity, the scientific craze of the eighteenth century.[52] All over Europe, lecturers made a good living by entertaining audiences with electrical science, drawing on stored supplies of electrical charge to make spectators' hair stand on end, to generate impressive bright sparks and loud bangs. It was science as theatre. At the same time, experimenters and engineers were making increasingly precise and accurate electrical measurements, which physicists sought to understand.

Laplace was not only a dedicated *physicien*, he was always ready to oblige his political masters. In 1799, he was delighted when Napoleon appointed him minister of the interior, though Laplace's tenure in the post lasted only six weeks – 'He looked for subtleties everywhere … and carried into administration the spirit of the infinitely small,' the French leader observed.[53] Without turning a hair, Laplace returned to his goal of completing Newton's programme to use the law of gravity to understand the cosmos. Laplace succeeded magnificently. He presented his findings in the five-volume masterpiece *Celestial Mechanics* and dedicated its third volume to Napoleon, 'the enlightened Protector of the Sciences … the Hero, the Pacificator of Europe, to whom France owes her prosperity, her greatness and the most brilliant epoch of her glory'.[54]

Supported by his friends in the Paris establishment, Laplace set up the world's first school of mathematical physics, based in his mansion, three miles south of Paris, in the village of Arcueil.[55] Most summer weekends between 1806 and 1822, he held court with dozens of able young protégés and scientific tourists on a wide variety of topics (at the same time, his next-door neighbour Claude-Louis Berthollet held an equally successful school in chemistry). The home Laplace shared with his wife and two children befitted his status, with liveried servants, sumptuous furnishings, Raphaels on the walls and a horse-drawn carriage always on hand to transport him and his guests.[56]

*

Laplace was building on Newton's vision of a world that ultimately consists of particles with central forces acting between them. The aim was to identify all those forces, describe them in mathematical theories, and compare their predictions with the most accurate observations.[57] Laplace's strategy was to use laws that were mathematically similar to Newton's law of gravity to describe the massless ('imponderable') fluids that he believed underlie the workings of virtually everything experimenters were investigating, including electricity, magnetism, heat, light and the flows of liquids through capillaries. This approach delivered some notable successes, including a much-lauded interpretation of Étienne-Louis Malus's discovery in 1808 that reflected light could have the special property of polarisation.[58] For Laplace and his coterie of disciples, there was little doubt that thinking about imponderable fluids was by far the best way of advancing physics.

By 1810, the Laplacian way of studying the natural world had become, as the historian John Heilbron later described it, the Napoleonic Standard Model of physics.[59] As a framework, it appeared to be so comprehensive that it promised to describe the whole of physics. The principal challenge, it seemed, was to work out all its details and ensure that its predictions agreed with all experimental results. For several years, Laplace was the toast of leading European scholars, who were convinced that he had discovered by far the best way to do physics research and had set a compelling agenda for the subject. But Laplace's influence and reputation began to fall precipitously by the summer of 1815, and it was no coincidence that this occurred a few months after the defeat of his most influential supporter at the Battle of Waterloo.[60]

Although Laplace's status was waning fast, he worked as hard as ever on the theory of imponderable fluids, trying to square its predictions with experimental observations, especially those relating to electricity and magnetism. Most investigators believed that the two phenomena were separate and unrelated but, as Hans Christian Ørsted demonstrated in 1820, this was incorrect. He discovered in his laboratory in Copenhagen that an electrical current flowing through a wire generates a magnetic field that encircles the wire, an observation that became the talk of his peers across Europe. This was the first evidence that electricity and magnetism are inextricably related and the first hint that they needed to be understood within a single framework – electromagnetism.

Laplace and his disciples found it difficult to explain Ørsted's observations within the scope of theories that incorporated forces that acted between the centres of particles. This was just one of the many problems that beset Laplace's once-invincible theory of imponderable fluids, and within five years it was a busted flush. The limitations of the Napoleonic Standard Model had gradually become more obvious, along with the dogmatism of its principal inventor.

A new generation gradually took over, with Laplace stripped of almost all his formidable authority and influence. For most of the brightest young physicists, he was yesterday's man – they preferred to work in the tradition of other leading thinkers, including Joseph Fourier, who wanted nothing to do with imponderable fluids but instead concentrated on describing the behaviour of matter on a large scale. Among his achievements, he described heat flow using an approach that did not refer to atoms and the forces between them but simply accounted for heat flow using differential equations, in ways that accounted well for observations. Fourier's work has endured – to this day, his equation and several of his other mathematical innovations are part of every physicist's education.

At about the same time as physics began to crystallise into an identifiable subject, the discipline of mathematics also changed shape, as Continental *philosophes* began to develop mathematics that aspired, above all, to be perfectly rigorous. An international leader in this field was Laplace's young neighbour Augustin-Louis Cauchy. Although he was interested in natural philosophy, Cauchy was at heart a mathematician, with zero tolerance for sloppiness, logical errors and loopholes.[61] Gradually, it became convenient to make a broad distinction between pure mathematics, done without regard to any practical uses it might have, and its applied sibling, mainly concerned with solving real-world problems.[62] Laplace had been a prince of applied mathematicians, while Cauchy was the prince of the purists.

Laplace died in March 1827, almost exactly a century after the death of Newton. Laplace's funeral was a major public event in Paris, if not quite on the scale of Beethoven's, which took place in Vienna about three weeks later.[63] Although Laplace was not a beloved figure among mourning colleagues, they lauded him as superhuman, a man who, in the words of one admirer in Britain, rose over 'all the Great Teachers of mankind', a compliment that even Newton would have relished.[64]

Within a few years, the writings of Mary Somerville, a science writer and the first to translate Laplace's *Celestial Mechanics* into English, had done much to increase the public appreciation of the value of mathematics in science and society.[65] By then, 150 years after Newton published his *Principia*, almost all experts agreed that mathematical laws underlie the workings of nature and that all proposed laws must be continually checked against observations. Laplace had played a crucial role in cementing the Newtonian approach, clarifying it, and bringing it to fruition. He and his colleagues had hugely improved our understanding of how gravity shapes the entire Solar System, and they brought the cosmos securely within the ambit of human imagination. For Laplace's successors, the main challenge was to bring the same degree of understanding to phenomena observed on Earth, especially electricity and magnetism. When would natural philosophers be able to explain the phenomena that lecturers were demonstrating so thrillingly? As we shall see, the explanation took longer than most experts expected. And, to the surprise of many, the enduring mathematical theory of electromagnetism was first presented not in French or German but in English, with a Scottish accent.

**Notes**

1 Einstein (1954: 253)
2 Einstein (1954: 273)
3 Ross (1962: 72)
4 Cohen and Whitman (trans.) (1999: 27–29)
5 Cohen and Whitman (trans.) (1999: 29). For the clearest statements on Newton's way of doing science, see his 'Four Rules of Scientific Reasoning' in his *Principia*: http://apex.ua.edu/uploads/2/8/7/3/28731065/four_rules_of_reasoning_apex_website.pdf
6 Newton's room at this time was E3 Great Court.
7 Iliffe (2017: 14–16)

55 Planck (1915: 6); see also Planck (1949: 13)

56 Google image search ‘Einstein 1896’: https://www.google.co.uk/search?q=bern+wiki&client=firefox-b-ab&biw=1363&bih=1243&source=lnms&tbm=isch&sa=X&sqi=2&ved=0ahUKEwigm5iIxJ7SAhWLAsAKHQcDDyUQ_AUICCgD#tbm=isch&q=Einstein+1896&img

57 Cahan, D., ‘The Young Einstein’s Physics Education’, in Howard and Stachel (eds) (2000); Pyenson (1980: 400); McCormmach (1976: xiv, xv, xviii, xix, xx, xxvii)

58 McCormmach (1976: xiv)

59 Schilpp (ed.) (1997: 33)

60 Stachel (ed.) (1998)

61 Einstein to Conrad Habicht, 18 or 25 May 1905: https://einsteinpapers.press.princeton.edu/vol5-trans/41

62 Einstein, A., ‘Ether and the Theory of Relativity (1920): http://www-history.mcs.st-andrews.ac.uk/Extras/Einstein_ether.html; Born (1956: 189)

63 Solovine (ed.) (1986: 7–8)

64 Einstein (1954: 270)

From the moment of Einstein's epiphany in his office chair, the process of discovering a new law of gravity took him eight years. The result delivered the pièce de résistance of what the German physicist Wilhelm Wien declared to be 'the now-mighty theoretical physics' and eventually made Einstein the world's most famous scientist.[5]

While developing his theory, Einstein changed his attitude to advanced mathematics. As we shall see, he discovered that the mathematics that he already knew was not enough to understand how gravity works. According to his later recollections, which scholars have since questioned, it was this mathematics that enabled him to complete the theory in a few frantic weeks, during which he was deeply concerned that his main competitor might well beat him to it. As a result, he completely changed his opinion about the usefulness of higher mathematics for theoretical physicists – it was not a luxury; it was essential. The experience also convinced him that theoreticians should not focus on the results of new experiments but instead use pure thought, guided by advanced mathematics.

By 1910, Einstein, then thirty-one, had caught the eye of most of the world's leading physicists. Most of them were impressed by his boldness and originality. That year, after a largely unsuccessful four-year struggle to understand quanta, he began to focus on developing a new understanding of gravity. He first made strong progress when he began to think afresh about the special theory of relativity using geometry rather than algebra. This new perspective was not Einstein's brainchild, but had been conceived by Hermann Minkowski, one of Einstein's mathematics professors at ETH. Minkowski had been the first to argue that because space and time are not separate, they should be treated as aspects of what he named 'space-time'. Separate space and time were, in Minkowski's view, 'doomed to fade away into mere shadows, and only a kind of union of the two will preserve independence'.[6] He suggested that, rather than connect observers' space and time measurements using algebraic formulae, it would be better to represent events on space-time diagrams, easily drawn on a flat piece of paper. This made it easier to visualise what was going on.

Einstein initially dismissed this innovation as 'superfluous learnedness', and it was not until several years later that he accepted Minkowski's space-time framework for a new theory of gravity.[7] At the same time, he contemplated how observations could test the theory, which must account for the fine detail of the motion of the planet Mercury around the Sun. By 1911, Einstein had concluded that the path of any beam of light – which has energy and an equivalent mass – should be affected if it passes close to a hugely massive object, such as the Sun, and that a respectable theory of gravity should be able to predict the deflection correctly.

Einstein was not the only physicist trying to reach a better understanding of gravity. In 1912, he was struck by the beauty and simplicity of the formulae produced by one of his competitors, the Göttingen theoretician Max Abraham. Within weeks, Einstein realised that he had been misled by the aesthetic appeal of his rival's ideas: Abraham had relied too much on formal mathematics, Einstein believed, and did not stay sufficiently close to reality. 'This is what happens when one operates formally, without thinking physically!' he harrumphed, adding that Abraham's theory was 'totally untenable' and 'totally unacceptable'.[8] Einstein was not going to make that mistake.

At around this time, it dawned on him that empty (flat) space-time is curved by matter, in much the same way a mattress is curved when someone lies on it.[9] The curvature of space-time determines the motion of matter: in concrete terms, any particle that feels no other net force follows the straightest possible path. This was a crucial insight, but Einstein did not know how to express it in mathematical terms: to do that, he needed the help of an expert mathematician. So, in the late summer of 1912, after he arrived at Zurich Polytechnic to take up a professorship, he sought out his old friend and undergraduate classmate Marcel Grossmann, now chair of the mathematics department. Einstein pleaded to him, 'You must help me, or else I'll go crazy.'[10]

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Having followed Grossmann's guidance, Einstein realised that, to understand gravity using curved space-time, he had no choice but to use advanced mathematics. It was not going to be possible to develop a new theory using the Newtonian picture in which the strength of the gravitational force in space is described by a smooth mathematical function that varies in space. Rather, he was going to have to use arrays of similar functions expressed in the form of a mathematical object known as a tensor. The concept of a 'tensor', in its modern sense, had been introduced fourteen years before by Göttingen theorist Woldemar Voigt, in connection with the stresses and strains in material objects.[11] It was in Göttingen, almost a century before, that the mathematician Carl Friedrich Gauss had pioneered the theory of curved surfaces, in which the sum of the angles of a triangle is not always 180 degrees. By studying curved spaces in familiar three-dimensional space, Gauss laid the foundations of differential geometry. His student Bernhard Riemann – later one of Gauss's successors in Göttingen – then extended the technique to apply to higher-dimensional spaces.

Einstein was thrilled to find this mathematics lying on the shelf. It seemed to be waiting to be used, precision-built to help physicists work out a theory of four-dimensional space-time.[12] This was a classic example of what he later described as the 'pre-established harmony' between mathematics and physics – a phrase coined by Newton's contemporary Leibniz and more recently favoured by Göttingen's mathematical cognoscenti to describe the relationship between pure mathematics and human understanding of the physical world.[13]

Soon, Einstein and Grossmann were collaborating – Einstein leading on the physics, Grossmann on the mathematics. The theory of gravity they were seeking would be a generalisation of the special theory of relativity, which applied not only to observers moving at constant speeds in straight lines, but also to observers in all states of motion. In other words, the general theory of relativity would itself be a new theory of gravity. Einstein intended to build it using a two-pronged approach, mathematical and physical.[14] On one hand, the mathematical strategy focused on setting up the theory in the most logical and elegant way; on the other, the physical strategy anchored the

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