THINKING THINGS THROUGH

An Introduction to Philosophical Issues and Achievements

second edition

CLARK GLYMOUR

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A Bradford Book
The MIT Press
Cambridge, Massachusetts
London, England

First edition 1992; second edition 2015

MIT Press books may be purchased at special quantity discounts for business or sales promotional use. For information, please e-mail special_sales@mitpress.mit.edu.

This book was set in ITC Stone Serif by Toppan Best-set Premedia Limited, Hong Kong. Printed and bound in the United States of America.

Library of Congress Cataloging-in-Publication Data

Glymour, Clark N.
Thinking things through: an introduction to philosophical issues and achievements / Clark Glymour.—Second edition.
pages cm
"A Bradford book."
Includes bibliographical references and index.
ISBN 978-0-262-52720-0 (pbk. : alk. paper)
1. Philosophy. 2. Evidence. 3. Knowledge, Theory of. 4. Philosophy of mind. I. Title.
B74.G59 2015
121—dc23
2014034221

10 9 8 7 6 5 4 3 2 1

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Preface to the Second Edition

A new edition of a book requires the author to think through again what has been written. *Thinking Things Through* is not a neutral history of philosophy; it is tinctured with viewpoints—mine—both about particular episodes and doctrines in the history of philosophy, and more generally about what counts as philosophy and what counts as important philosophy. For the most part, I have retained the tone and the particular assessments of the first edition, and I have retained entirely its conception of philosophy.

This book originally came about in a rather odd and perhaps unscrupulous way. When, many years ago, I agreed to form a philosophy department at Carnegie Mellon University, I found myself faced with teaching introductory philosophy to a class of 250 students each semester, without anyone to assist me in grading. My solution was to make careful lecture notes, sell them to the students, and with the funds hire a grader. It was my good fortune that the enterprise preceded the arrival of the Internet. The lecture notes became the first edition of this book, which I and several of my colleagues have used for many years in an advanced introductory course in philosophy.

I had originally intended that the book contain chapters on ethics, but I had not the energy. I am grateful to Philip Laughlin of The MIT Press for creating the opportunity to fill that gap in the first edition, which, on reading the second edition, some may think was much needed.

This edition differs from the original chiefly in the addition of four new chapters, one on decisions and causes, and three on ethics and political theory. None of them are comprehensive but I hope they will serve as critical introductions to their subjects. Other changes have been made to correct errors in the first edition, to update examples, and occasionally to insert further information. I am quite certain that errors remain, even errors that have been pointed out by colleagues and students but which I have since forgotten.

I am grateful to Jeremy Avigad who suggested additional material on Euclid, and especially to Alex London for help with a chapter on ethics. They bear no responsibility beyond their choice of colleagues. Alison Kost and Madelyn Glymour generously helped me with reading and correcting the manuscript. Judith Feldmann, the copy editor for the book, saved me from multiple embarrassments. Holly Glymour graciously helped with proofreading.

Clark Glymour
Pittsburgh, August 2014

Preface to the First Edition

An old story about a great teacher of philosophy, Morris Raphael Cohen, goes like this. One year, after the last lecture in Cohen's introductory philosophy course, a student approached and protested, "Professor Cohen, you have destroyed everything I believed in, but you have given me nothing to replace it." Cohen is said to have replied, "Sir, you will recall that one of the labors of Hercules was to clean the Augean stables. You will further recall that he was not required to refill them."

I'm with the student. People who are curious about the subject may want a historical view of philosophy, but they also may want to know what, other than that very history, philosophy has left them. In fact, the history of philosophy has informed and even helped to create broad areas of contemporary intellectual life; it seems a disservice both to students and to the subject to keep those contributions secret. The aim of this text is to provide an introduction to philosophy that shows both the historical development and modern fruition of a few central questions. The issues I consider are these:

- What is a demonstration, and why do proofs provide knowledge?
- How can we use experience to gain knowledge or to alter our beliefs in a rational way?
- What is the nature of minds and of mental events and mental states?

In our century the tradition of philosophical reflection on these questions has helped to create the subjects of cognitive psychology, computer science, artificial intelligence, mathematical logic, and the Bayesian branch of statistics. The aim of this book is to make these connections accessible to qualified students and to give enough detail to challenge the very best of them. I have selected the topics because the philosophical issues seem especially central and enduring and because many of the contemporary fields they have given rise to are open-ended and exciting. Other connections between the history of philosophy and contemporary subjects, for example the connection with modern physics, are treated much more briefly. Others are not treated at all for lack of space. I particularly regret the absence of chapters on ethics, economics, and law.

This book is meant to be used in conjunction with selections from the greats, and suggestions for both historical and contemporary readings accompany most chapters. The book is intended as an introduction, the whole of which can be read by a well-educated high school graduate who is willing to do some work. It is not, however, particularly easy. Philosophy is not easy. My experience is that much of this book can be read with profit by more advanced students interested in epistemology and metaphysics, and by those who come to philosophy after training in some other discipline. I have tried in every case to make the issues and views clear, simple, and coherent, even when that sometimes required ignoring real complexities in the philosophical tradition or ignoring alternative interpretations. I have avoided disingenuous defenses of arguments that I think unsound, even though this sometimes has the effect of slighting certain passages to which excellent scholars have devoted careers. A textbook is not the place to develop original views on contemporary issues. Nonetheless, parts of this book may interest professional philosophers for what those parts have to say about some contemporary topics. This is particularly true, I hope, of chapters 10, 11, and 13.

Especially challenging or difficult sections and chapters of the book are marked with an asterisk. They include material that I believe is essential to a real understanding of the problems, theories, and achievements that have issued from philosophical inquiry, but they require more tolerance for mathematical details than do the other parts of the book. Sometimes other chapters use concepts from sections with asterisks, and I leave it to instructors or readers to fill in any background that they omit. Each chapter is accompanied by a bibliography of suggested readings. The bibliographies are not meant to be exhaustive or even representative. Their purpose is only to provide the reader with a list of volumes that together offer an introduction to the literature on various topics.

I thank Kevin Kelly for a great deal of help in thinking about how to present the philosophical issues in historical context, for influencing my views about many topics, and for many of the illustrations. Andrea Woody read the entire manuscript in pieces and as a whole and suggested a great many improvements. She also helped to construct the bibliographies. Douglas Stalker gave me detailed and valuable comments on a first draft. Alison Kost read and commented on much of the manuscript. Martha Scheines read, revised, and proofread a preliminary draft. Alan Thwaits made especially valuable

I The Idea of Proof

1 Proofs

Introduction

Philosophy is concerned with very general questions about the structure of the world, with how we can best acquire knowledge about the world, and with how we should act in the world. The first topic, the structure of the world, is traditionally known as *metaphysics*. The second topic, how we can acquire knowledge of the world, is traditionally called *epistemology*. The third topic, what actions and dispositions are best, is the subject of *ethics*. These three topics inevitably go together. What one thinks about the structure of the world has a lot to do with how one thinks inquiry should proceed, and vice versa. These topics in turn involve issues about the nature of the mind, for it is the mind that knows. Considerations of ethics depend in part on our metaphysical conception of the world and ourselves, on our conception of mind, and on how we believe knowledge can be acquired.

These traditional branches of philosophy no doubt seem very abstract and vague. They may seem superfluous as well: Isn't the question of the structure of the world part of *physics*? Aren't questions about how we acquire knowledge and about our minds part of *psychology*? Indeed they are. What, then, are metaphysics and epistemology, and what are the methods by which these subjects are supposed to be pursued? How are they different from physics and psychology and other scientific subjects? Questions such as these are often evaded in introductions to philosophy, but let me try to answer them.

First, there are a lot of questions that are usually not addressed in physics or psychology or other scientific subjects but that still seem to have *something* to do with them. Consider the following examples:

- How can we know there are particles too small to observe?
- What constitutes a scientific explanation?

- How do we know that the process of science leads to the truth, whatever the truth may be?
- What is meant by "truth"?
- Does what is true depend on what is believed?
- How can anyone know there are other minds?
- What facts determine whether a person at one moment of time is the same person as a person at another moment of time?
- What are the limits of knowledge?
- How can anyone know whether she is following a rule?
- What is a proof?
- What does "impossible" mean?
- What is required for beliefs to be rational?
- What is the best way to conduct inquiry?
- What is a computation?
- How should people behave?
- How should social and political institutions be organized and ruled?

The questions have *something* to do with physics or psychology (or with mathematics or linguistics or political science), but they aren't questions you will commonly find addressed in textbooks on these subjects. The questions seem somehow too fundamental to be answered in the sciences; they seem to be the kind of questions that we just do not know how to answer by a planned program of observations or experiments. And yet the questions don't seem unimportant; how we answer them might lead us to conduct physics, psychology, mathematics, or other scientific disciplines very differently. These are the sorts of questions particular scientific disciplines usually either ignore or else presume to answer more or less without argument. And they are a sample, a small sample, of the questions that concern philosophy.

If these questions are so vague and so general that we have no idea how to conduct experiments or systematic observations to find their answers, what can philosophers possibly have done with them that is of any value? The philosophical tradition contains a wealth of proposed answers to fundamental questions about metaphysics and epistemology and ethics. Sometimes the answers are supported by arguments based on a variety of unsystematic observations, sometimes by reasons that ought to be quite unconvincing in themselves. The answers face the objections that they are either unclear or inconsistent, that the arguments produced for them are unsound, or that some other body of unsystematic observations conflicts with them. Occasionally an answer or system of answers is worked out precisely and fully enough that it can deservedly be called a theory, and a variety of consequences of the theory can be rigorously drawn, sometimes by mathematical methods. What is the use of this sort of philosophical speculation? On occasion the tradition of attempts at philosophical answers has led to theories that seem so forceful and so fruitful that they become the foundation for entire scientific disciplines or moral perspectives; they enter our culture, our science, our politics, and guide our lives. That is the case, for example, with the discipline of computer science, created by the results of more than 2,000 years of attempts to answer one apparently trivial question: What is a demonstration, a proof? An entire branch of modern statistics, often called Bayesian statistics, arose through philosophical efforts to answer the question: What is rational belief? The theory of rational decision making, at the heart of modern economics, has the same ancestry. Contemporary cognitive science, which tries to study the human mind through experiments aided by computer models of human behavior and thought, is the result of joining a philosophical tradition of speculation about the structure of mind with the fruits of philosophical inquiry into the nature of proof.

So one answer to why philosophy was worth doing is simply that it was the most creative subject: rigorous philosophical speculation formed the basis for much of contemporary science; it literally created new sciences. Moreover, the role of philosophy in forming computer science, Bayesian statistics, the theory of rational decision making, and cognitive science isn't ancient history. These subjects were all informed by developments in philosophy within the last 100 years or so.

But if that is why philosophy *was* worth doing, why is it still worth doing? Because not everything is settled and there may be fruitful alternatives even to what has been settled. In this chapter and those that follow we will see some of the history of speculation and argument that generated a number of contemporary scientific disciplines. We will also see that there can be reasonable doubts about the foundations of some of these disciplines. And we will see a vast space of further topics that require philosophical reflection, conjecture, and argument. In later chapters we will consider some philosophical contributions to science that have taken place over the last quarter century.

Forms of Reasoning and Some Fundamental Questions

Part of the process by which we acquire knowledge is the process of reasoning. There are many ways in which we reason, or *argue* for conclusions. Some ways seem more certain and convincing than others. Some forms of reasoning seem to show that

that *separate* valid deductive arguments from invalid deductive arguments. Any theory of deductive reasoning we construct should provide a way to distinguish the arguments that seem valid from the arguments that seem invalid. To get some practice for this part of the task of theory building, we will look at simple cases in which we want to form a theory that will include some examples and exclude a number of other examples. The cases we will consider first don't have to do with the idea of deductive reasoning, but they do illustrate many aspects of what a theory of deductive reasoning ought to provide: they separate the correct instances, the positive examples, of a concept from the incorrect instances, the negative examples.

Here is a very simple case. Suppose you are given this sequence of numbers: 1, 2, 5, 10, 17, 26, 37, 50, 65, 82. What is the general rule for continuing the sequence? In this case the numbers listed are positive examples to be included in a formula and all the whole numbers between 1 and 82 that are not listed are not in the sequence. (The listed sequence can be generated by the formula $n^2 + 1$ for $n = 0, 1, 2, 3$, and so on.)

Let's consider a very different kind of example, one where there are again a number of positive examples and a number of negative examples. Suppose you are given the positive and negative examples of arches shown in figure 1.1. How could you state conditions that include the positive examples but exclude the negative examples? You might try something like this: "*X* is an arch if and only if *X* consists of two series of blocks, and in each series each block except the first is supported by another block of that series, and no block in one series touches any block in the other series, and there is a block supported by a block in each series."

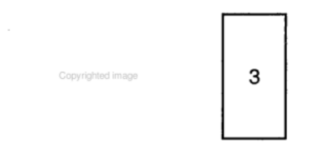
Here is still another kind of example. Artificial languages, such as programming languages or simple codes, are constructed out of vocabulary elements. A statement in such a language is a finite sequence of vocabulary elements. But not *every* sequence of vocabulary elements will make sense in the language. In BASIC or JAVA you can't just write down any sequence of symbols and have a well-formed statement or command. The same is true in natural languages, such as English. Not just any string of words in English makes an English sentence. Suppose you learned that the examples in table 1.1 are positive and negative examples of well-formed sequences in some unknown code, and suppose you also knew that there are an infinite number of other well-formed sequences in the code. What do you guess is the condition for a well-formed sequence in this code? Can you find a general condition that includes all of the positive examples and none of the negative examples?

For several reasons the philosophical problem with which we are concerned is more difficult than any of these examples. We want a theory that will separate valid

Block 1 and block 2 support block 3, and blocks 1 and 2 do not touch one another.

Block 1 supports block 2; block 3 supports block 4; blocks 2 and 4 support block 5; blocks 1 and 2 do not touch block 3 or block 4.

Negative examples



Block 1 supports block 2; blocks 1 and 2 do not touch block 3.

Blocks 1 and 2 support block 3; blocks 1 and 2 touch.

Figure 1.1
Positive and negative examples of arches.

deductive arguments from deductive arguments that are not valid. The problem is intrinsically difficult because the forms of deductive argument are very complex. It is also difficult because we are not always sure whether or not to count specific arguments as valid. And finally, this philosophical problem is intrinsically more difficult because we not only want a theory that will separate valid demonstrations from invalid ones, we also want to know *why* and *how* valid demonstrations ensure that if their premises are true then necessarily their conclusions are true.

In keeping with the Socratic method, the first thing to do in trying to understand the nature of demonstration is to collect a few examples. The histories of philosophy, science, mathematics, and religion are filled with arguments that claim to be proofs of their conclusions. Unfortunately, the arguments don't come labeled "valid" or "invalid," and we must decide for ourselves, after examination, whether an argument is good, bad, or good enough to be reformulated into a valid argument. We will next consider a series of examples of simple arguments from geometry, theology,

Table 1.1
Sequences in a code

Positive examples (well formed)	Negative examples (not well formed)
AA	AAAA
BB	BBBB
AABB	BBBBAA
AAABB	AABBBB
BBAA	AAAAAA
BBAAA	BBBBBB
BBAAABB	AAAAAAAAABB
BBAAABBB	BBAAAAAAAAA
AABBB	AAAAAAAAA
AABBAAA	BBBBBBBBB
AAABBAAA	
AAABBBAAA	
AAA	
BBB	
AAAAA	
BBBBB	
A	
B	

metaphysics, and set theory. The point of the examples is always to move toward an understanding of the three questions above.

Geometry

Euclid's geometry is still studied in secondary schools, although not always in the form in which he developed it. Euclid developed geometry as an *axiomatic system*. After a sequence of definitions, Euclid's *Elements* gives a sequence of assumptions. Some of these have nothing to do with geometry in particular. Euclid calls them "common notions." Others have specifically geometrical content. Euclid calls them "postulates." The theorems of geometry are deduced from the common notions and the postulates. Euclid's aim is that his assumptions will be sufficient to necessitate, or as we now say, *entail*, all the truths of geometry. We aspire for *completeness*. This means that every question about geometry expressible in Euclid's terms can be answered by his assumptions if only the proof of the answer can be found. Some of Euclid's definitions, common notions, postulates, and the first proposition he proves from them are given below:

Plato and Euclid

Plato, who died about 347 BC, is recognized as the first systematic Western philosopher. During the height of the Athenian empire Plato directed a school, the Academy, devoted to both mathematics and philosophy. No study of philosophy was possible in the Academy without a study of mathematics. The principal mathematical subject was geometry, although arithmetic and other mathematical subjects were also studied. It seems likely that textbooks on geometry were produced in Plato's Academy and that these texts attempted to systematize the subject and derive geometrical theorems from simpler assumptions (the Greeks called the simple parts of a thing its *elements*). Euclid studied in the Academy around 300 BC, and his book, *The Elements*, is thought to be derived from earlier texts of the school. Euclid later established his own mathematical school in Alexandria, Egypt.

Definitions

1. A point is that which has no part.
2. A line is breadthless length.
3. The extremities of a line are points.
4. A straight line is a line that lies evenly with the points on itself.
5. A surface is that which has length and breadth only.
6. The extremities of a surface are lines.
7. A plane surface is a surface that lies evenly with the straight lines on itself.
8. A plane angle is the inclination to one another of two lines in a plane that meet one another and do not lie in a straight line.
9. And when the lines containing the angle are straight, the angle is called rectilinear.
10. When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is right, and the straight line standing on the other is called a perpendicular to that on which it stands.
11. An obtuse angle is an angle greater than a right angle.
12. An acute angle is an angle less than a right angle.
13. A boundary is that which is an extremity of anything.
14. A figure is that which is contained by any boundary or boundaries.
15. A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another.

16. And the point is called the center of the circle.

⋮

19. Rectilinear figures are those contained by straight lines, trilateral figures being those contained by three.

20. Of trilateral figures, an equilateral triangle is that which has its three sides equal.

⋮

23. Parallel straight lines are straight lines that, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

Common Notions

1. Things that are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes are equal.
3. If equals be subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.

Postulates

1. It is possible to draw a straight line from any point to any point.
2. It is possible to produce a finite straight line continuously in a straight line.
3. It is possible to describe a circle with any center and distance.
4. All right angles are equal to one another.
5. If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on the side of the angles less than the two right angles.

Proposition 1 For every straight line segment, there exists an equilateral triangle having that line segment as one side.

Proof Let *AB* be the given finite straight line. Thus it is required to construct an equilateral triangle on the straight line *AB*. Let circle *BCD* be drawn with center *A* and distance (i.e., radius) *AB* (postulate 3). Again, let circle *ACE* be drawn with center *B* and distance *BA* (postulate 3). And from point *C*, at which the circles cut one another, to points *A*, *B*, let the straight lines *CA*, *CB* be joined (postulate 1). (See figure 1.2.) Now since point *A* is the center of the circle *CDB*, *AC* is equal to *AB*. Again, since point *B* is the center of circle *ACE*, *BC* is equal to *BA*. But *CA* was also proved equal

4. Are there contexts in which a proof consists of nothing more than a picture? Consider questions about whether or not a plane surface can be completely covered by tiles of a fixed shape, hexagons or pentagons, for example.
5. One of the aims of Euclid's formulation of geometry seems to have been to derive all of geometry from assumptions that are very simple and whose truth seems self-evident. Do any of Euclid's five postulates seem less simple and less self-evident than the others? Why?

God and Saint Anselm

From the first centuries after Christ until the seventeenth century, most Europeans believed in nothing so firmly as the existence of God or of gods. Despite the scarcity of doubters, for centuries (especially from the 11th century on) Christian intellectuals still sought proofs of God's existence and wrote arguments against real or imagined atheists. Some of these attempts at demonstrations of the existence of God are still presented in religious schools today, even though most logicians regard them as simple fallacies. However, at least one of the medieval proofs of the existence of God, Saint Anselm's (1033–1109), is still of some logical interest. Let's consider it.

Anselm gave his proof of the existence of God in several forms. Two versions of the argument are given in the following passage:

> And so, O Lord, since thou givest understanding to faith, give me to understand—as far as thou knowest it to be good for me—that thou dost exist, as we believe, and that thou art what we believe thee to be. Now we believe that thou art a being than which none greater can be thought. Or can it be that there is no such being since "the fool hath said in his heart, 'there is no God'" [Psalms 14:1; 53:1]. But when this same fool hears what I am saying—"A being than which none greater can be thought"—he understands what he hears, and what he understands is in his understanding, even if he does not understand that it exists. For it is one thing for an object to be in the understanding, and another thing to understand that it exists. When a painter considers beforehand what he is going to paint, he has it in his understanding, but he does not suppose that what he has not yet painted already exists. But when he has painted it, he both has it in his understanding and understands that what he has now produced exists. Even the fool, then, must be convinced that a being than which none greater can be thought exists at least in his understanding, since when he hears this he understands it, and whatever is understood is in the understanding. But clearly that than which a greater cannot be thought cannot exist in the understanding alone. For if it is actually in the understanding alone, it can be thought of as existing also in reality, and this is greater. Therefore, if that than which a greater cannot be thought is in the understanding alone, this same thing than which a greater cannot be thought is that than which a greater can be thought. But obviously this is impossible. Without doubt, therefore, there exists, both in the understanding and in reality, something than which a greater cannot be thought.

God cannot be thought of as nonexistent. And certainly it exists so truly that it cannot be thought of as nonexistent. For something can be thought of as existing, which cannot be thought of as not existing, and this is greater than that which can be thought of as not existing. Thus if that than which a greater cannot be thought can be thought of as not existing, this very thing than which a greater cannot be thought is *not* that than which a greater cannot be thought. But this is contradictory. So, then, there truly is a being than which a greater cannot be thought—so truly that it cannot even be thought of as not existing.[2]

Anselm's argument in the second paragraph just cited might be outlined in the following way:

Premise 1: A being that cannot be thought of as not existing is greater than a being that can be thought of as not existing.

Therefore, if God can be thought of as not existing, then a greater being that cannot be thought of as not existing can be thought of.

Premise 2: God is the being than which nothing greater can be thought of.

Conclusion: God cannot be thought of as not existing.

The sentence in the reconstruction beginning with "Therefore" does not really follow from premise 1. It requires the further assumption, which Anselm of course believed, that it is possible to think of a being than which nothing greater can be conceived or thought of.

The argument of the first paragraph seems slightly different, and more complicated. I outline it as follows:

Premise 1: We can conceive of a being than which none greater can be conceived.

Premise 2: Whatever is conceived exists in the understanding of the conceiver.

Premise 3: That which exists in the understanding of a conceiver and also exists in reality is greater than an otherwise similar thing that exists only in the understanding of a conceiver.

Therefore, a being conceived, than which none greater can be conceived, must exist in reality as well as in the understanding.

Premise 4: God is a being than which none greater can be conceived.

Conclusion: God exists in reality.

The arguments seem very different from Euclid's proof. Anselm's presentation is not axiomatic. There is no system of definitions and postulates. In some other respects, however, Anselm's arguments have similarities to Euclid's geometric proof. Note the following about Anselm's arguments:

• Anselm's arguments are meant to be demonstrations of their conclusions from perfectly uncontroversial premises. The arguments aim to show that the truth of the premises necessitates the truth of the conclusions.

• In the first argument, the discussion of the painter and the painting is not essential to the proof. Anselm includes the discussion of a painter and painting to help the reader understand what he, Anselm, means by distinguishing between an object existing in the understanding and understanding that an object exists. The painter discussion therefore plays a role in Anselm's proof much like the role apparently played by the drawing in Euclid's proof: it is there to help the reader see what is going on, but it is not essential to the argument.

• Like Euclid's proof, Anselm's arguments can be viewed as little essays in which, if we discount explanatory remarks and digressions, each claim is intended to follow either from previous claims or from claims that every reader will accept.

Study Questions

1. Suppose one grants Anselm's proof that a being exists none greater than which can be conceived. Does it follow, as Anselm surely believed, that there is only one such being?
2. One famous objection to Anselm's argument is this: If Anselm's argument were valid, then by the same form of reasoning, we could prove that a perfect island exists. But an island than which none greater can be conceived does not exist in reality. Therefore, something must be wrong with Anselm's proof of the existence of God. Give an explicit argument that follows the form of Anselm's and leads to the conclusion that there exists an island than which none greater can be conceived. Is the objection a good one? Has Anselm any plausible reply?
3. Try to explain specifically what is wrong with your proof that there exists a perfect island.

God and Saint Thomas

I will add another example to our collection of demonstrations. The most famous proofs of the existence of God are due to Saint Thomas Aquinas (ca. 1225–1274). Aquinas gave five proofs, which are sometimes referred to as the "five ways." They are presented in relatively concise form in his *Summa Theologica*. Four of the five arguments have essentially the same form, and the fifth is particularly obscure. I will consider only the first argument. In reading the argument, you must bear in mind that Aquinas had a very different picture of the physical universe than ours, and he assumed that his readers would fully share his picture. That picture derives from Aristotle. According to the picture Aquinas derived from Aristotelian physics, objects do not change unless acted on by another object. Further, Aristotle distinguished between the properties an object *actually* has and the properties it has the *potential* to have.

Aquinas and Aristotle

Aristotle was a student of Plato's. After Plato's death, Aristotle left Athens and subsequently became tutor to Alexander of Macedonia, later Alexander the Great. When Alexander conquered Greece, Aristotle returned to Athens and opened his own school. With the collapse of the Macedonian empire, Aristotle had to flee Athens, and he died a year later. During his life he wrote extensively on logic, scientific method and philosophy of science, metaphysics, physics, biology, cosmology, rhetoric, ethics, and other topics. Saint Thomas Aquinas helped to make Aristotle's philosophy acceptable to Christian Europe in the late Middle Ages. Writing in the thirteenth century, Aquinas gave Christianized versions of Aristotle's cosmology, physics, and metaphysics. The result of the efforts of Aquinas and others was to integrate Aristotelian thought into the doctrines of the Roman Catholic Church in the late Middle Ages. Aristotle's doctrines also became central in the teachings of the first universities, which began in Europe during the thirteenth century. The tradition of Christian Aristotelian thought that extends from the Middle Ages to the seventeenth century is known as *scholasticism*.

Any change in an object consists in the object coming actually to have properties that it previously had only potentially.

In translation Aquinas' argument is as follows:

> The existence of God can be proved in five ways.
>
> The first and most manifest way is the argument from motion. It is certain, and evident to our senses, that in the world some things are in motion. Now whatever is moved is moved by another, for nothing can be moved except it is in potentiality to that towards which it is moved; whereas a thing moves inasmuch as it is in actuality. For motion is nothing else than the reduction of something from potentiality to actuality. But nothing can be reduced from potentiality to actuality, except by something in a state of actuality. Thus that which is actually hot, as fire, makes wood, which is potentially hot, to be actually hot, and thereby moves and changes it. Now it is not possible that the same thing should be at once in actuality and potentiality in the same respect, but only in different respects. For what is actually hot cannot simultaneously be potentially hot; but it is simultaneously potentially cold. It is therefore impossible that in the same respect and in the same way a thing should be both mover and moved, i.e., that it should move itself. Therefore, whatever is moved must be moved by another. If that by which it is moved be itself moved, then this also must needs be moved by another, and that by another again. But this cannot go to infinity, because then there would be no first mover, and, consequently, no other mover, seeing that subsequent movers move only inasmuch as they are moved by the first mover; as the staff moves only because it is moved by the hand. Therefore it is necessary to arrive at a first mover, moved by no other; and this everyone understands to be God.[3]

Aquinas' attempted demonstration again shares many of the features of Euclid's and Anselm's arguments. From premises that are supposed, at the time, to be uncontroversial, a conclusion is intended to follow necessarily. The argument is again a little essay, with claims succeeding one another in a logical sequence. The example of heat is another illustration, like Anselm's painter and Euclid's diagram, intended to further the reader's understanding, but it is not an essential part of the argument.

Aquinas' argument illustrates that a proof (or attempted proof) may have another proof contained within it. Thus the remarks about potentiality and actuality are designed to serve as an argument for the conclusion that nothing moves itself, and that conclusion in turn serves as a premise in the argument for the existence of an unmoved mover.

Neglecting Aquinas' remarks about potentiality, which serve as a subargument for premise 2, we can outline the argument in the following way:

Premise 1: Some things move.

Premise 2: Anything that moves does so because of something else.

Therefore, if whatever moves something itself moves, it must be moved by a third thing.

Therefore, if there were an infinite sequence of movers, there would be no first mover, and hence no movers at all.

Therefore, there cannot be an infinite sequence of movers.

Conclusion: There is a first, unmoved mover.

One way to show that the premises of the argument do not necessitate Aquinas' conclusion is to imagine some way in which the premises of the argument could be true and the conclusion could at the same time be false. With this argument, that is easy to do. We can imagine that if object *A* moves object *B*, object *B* moves object *A*. In that case no third object would be required to explain the motion of *A*. We can also imagine an infinite chain of objects in which the first object is moved by the second, the second by the third, the third by the fourth, and so on forever. Neither of these imaginary circumstances is self-contradictory (although Aquinas would certainly have denied their possibility). So we can criticize Aquinas' argument on at least two counts:

- The first "therefore" doesn't follow. The two premises are consistent with the assumption that if one thing moves another, then the second, and not any third thing, moves the first.
- The second "therefore" doesn't follow. We can consistently imagine an infinite sequence of movers without there being an endpoint, a "first mover," just as we can

Spinoza and Euclid

Baruch Spinoza (1632–1677) was the child of Spanish Jews who had moved to Holland to avoid religious persecution. He himself was ostracized from the Jewish community for his opinions about God, and threatened with death for his biblical scholarship. (Spinoza noted that various stories in the Bible, for example the creation story, are repeated in contradictory ways in the Bible, which he took as evidence that the book had multiple authors.) Spinoza earned his living as a lens grinder, but he was well known to his intellectually prominent contemporaries and was offered university positions, which he refused.

Spinoza's major work, *The Ethics*, develops a view of nature in which there is a single substance, God. Most remarkable to a modern reader, Spinoza's *Ethics* is presented in the same format as Euclid's *Elements*. There are definitions, postulates, propositions, and proofs, or at least attempted proofs. In putting his theological views in this form, Spinoza exemplified the view, common among the great intellects of his time, that reasoning about metaphysical and epistemological questions should be rigorously scientific, and Euclid's geometry represented, even then, the ideal deductive science.

would say that the sequence of distances between Achilles and the tortoise converges to zero and the *sum* of the sequence of temporal intervals is some finite number. That sum, whatever it is, represents the time required for Achilles to catch the tortoise.

The concept of infinity also created problems for later philosophical writers interested in the properties of God. Baruch (or Benedict) Spinoza (1632–1677) was a seventeenth-century *pantheist*; he held that God consists of everything there is. Individual minds and bodies are, in Spinoza's terms, *modes* of God's existence.

Spinoza was troubled by the following objection to his view:

> We showed that apart from God no substance can be or can be conceived; and hence we deduced that extended substance is one of God's infinite attributes.
>
> However, for a fuller explanation, I will refute my opponents' arguments, which all come down to this. First, they think that corporeal substance, insofar as it is substance, is made up of parts, and therefore they deny that it can be infinite, and consequently that it can pertain to God. This they illustrate with many examples, of which I will take one or two. They say that if corporeal substance is infinite, suppose it to be divided into two parts. Each of these parts will be either finite or infinite. If the former, then the infinite is made up of two finite parts, which is absurd. If the latter, then there is an infinite twice as great as another infinite, which is also absurd.[5]

Spinoza was unsure whether or not this argument is valid. He responded, rather implausibly, that even though everything corporeal is an attribute of God, God does not have *parts*.

The argument Spinoza must address has a special form. It sets out to prove something, in this case that God is not corporeal. It proceeds by assuming the *denial* of what is to be proved. That is, it proceeds by assuming that God *is* corporeal. From that assumption, perhaps with the aid of other assumptions that are thought to be obvious, the argument then tries to establish something thought to be *false*. The idea is that if the denial of a claim necessitates something false, then the claim itself must be true. This form of argument is known as *reductio ad absurdum* (reduction to the absurd), or more briefly, as a *reductio* argument.

We can outline the argument of Spinoza's opponents in the following way:

Assumption: God is corporeal.

Premise: Whatever is corporeal can be divided into two parts.

Premise: God is infinite.

Hence, an infinity can be divided into parts.

Premise: Every part is either infinite or finite.

Premise: The whole is the union of its parts.

Hence, either an infinity is the union of two finite parts, which is impossible, or an infinity is the union of two lesser infinities, which is also impossible.

Conclusion: The assumption is false, i.e., God is not corporeal.

We can see that the argument is invalid, and for several different reasons, all having to do with the next to last sentence, beginning "Hence." First and most simply, the last step before the conclusion omits a possible case: the infinity might be divided into two parts, one of which is finite and the other infinite. Second, an infinite collection of objects *can* be divided into two subcollections, each of which is infinite. The integers, for example, consist of all negative integers together with all nonnegative integers. The set of all negative integers is infinite, and the set of all nonnegative integers is also infinite.

Infinity and Cardinality*

The argument Spinoza considers does raise an interesting and fundamental question about the infinite: Can one infinity be larger than another infinity? In the nineteenth century this question engendered a number of simple proofs that created a revolution in our understanding of infinity, and since the question touches on an issue that runs through the history of philosophy, it is worth considering some of the relevant ideas and arguments here.

What do we mean when we say that one set or collection is *larger* than another? Consider the two collections below:

{*A*, *B*, *C*, *D*}

{*X*, *Y*, *Z*, *U*, *V*}

Clearly, the second set is bigger that the first set, but what makes it so? One answer is this: If we try to match each member of the first set with a unique member of the second set, we can do so. For example, we can match *A* with *X*, *B* with *Y*, *C* with *Z*, and *D* with *U*. But if we try to match each member of the second set with a unique member of the first set, we run out of distinct things. For example, we can match *X* with *A*, *Y* with *B*, *Z* with *C*, and *U* with *D*. But then we still have *V* left over; whatever member of the first set we choose to match with *V*, that member will already have been matched with *X*, *Y*, *Z*, or *U*. We say that there is a *one-to-one mapping* from the first set into the second set, but there is no one-to-one mapping from the second set into the first.

I will take this as our definition of "larger than" for sets:

Definition Set *K* is *larger than* set *L* if and only if there is a one-to-one mapping relating each member of *L* to a distinct member of *K* but there is no one-to-one mapping relating each member of *K* to a distinct member of *L*.

Continuing with this idea, we can say what it means for two sets to be of the *same size*. Two sets are of the same size if the first is *not* larger than the second and also the second is *not* larger than the first. When neither of two sets is larger than the other in this sense, we say they have the same *cardinality*.

Definition Any two sets *K*, *L* have the same *cardinality* if and only if there is a one-to-one mapping relating each member of *K* to a distinct member of *L* and there is a one-to-one mapping relating each member of *L* to a distinct member of *K*.

For finite sets, the notion of cardinality is just our ordinary notion of the size of a set. All sets with 4 members have the same cardinality, all sets with 5 members have the same cardinality, sets with 5 members are larger than sets with 4 members, and so on.

An obvious property of finite sets is this: If *K* and *L* are finite sets and if *K* is a *proper subset* of *L* (that is, every member of *K* is a member of *L* but some member of *L* is not a member of *K*), then *L* is larger than *K*. The set {*X*, *Y*, *Z*}, for example, is larger than the set {*X*, *Y*}. Infinite sets behave differently. *An infinite set can have the same cardinality as one of its proper subsets*. Consider an example, the set of positive integers, and a proper subset of it, the set of even positive integers. There is a one-to-one correspondence that takes every positive integer to a distinct even positive integer, and the same correspondence viewed in the other direction takes every even positive integer to a distinct positive integer:

1	2	3	4	5	6	7	8	9	10	…
↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	…
2	4	6	8	10	12	14	16	18	20	…

The rule of correspondence is that each positive integer *n* is mapped to the even positive integer 2*n*. So the set of positive integers has the same cardinality as the set of even positive integers. You can also easily show that the set of positive integers has the same cardinality as the set of odd positive integers, and also the same cardinality as the set of all integers, whether positive, zero, or negative. All of these distinct infinite sets have the same size.

The property of having the same cardinality as a proper subset of itself neatly separates the finite from the infinite. Finite sets can't have that property, whereas every infinite set will have it for some of its proper subsets. The distinction is sometimes used to define the notion of an infinite set:

Definition A set is infinite if and only if it can be put into one-to-one correspondence with a proper subset of itself.

Now an obvious question raised by Spinoza's argument is this: Are some infinities larger than other infinities? In view of the considerations we have just discussed, we can understand that question in the following way: Are there two infinite sets that cannot be put into one-to-one correspondence with one another? In the nineteenth century, Georg Cantor (1845–1918) proved that there are. I will consider simple versions of two of his proofs. One concerns the number of subsets of any set. It is easy to see that any finite set has more distinct subsets that it has members. The set {*A*}, for example, has only one member, but it has two distinct subsets, namely itself and the empty set. The set {*A*, *B*} has two members, but it has four distinct subsets. Given any finite set *S* with *n* members, we can count the distinct subsets of *n* in the following way. Imagine forming an arbitrary subset *U* of *S*. For any member of *S* there are two choices: either the member is in *U* or it isn't in *U*. To determine *U*, we have to make that choice for each of the *n* members of *S*, so we have *n* choices, each with 2 options. Every distinct way of making the choices results in a distinct subset of *S*, so there are 2^n distinct subsets. And for all *n*, 2^n is greater than *n*. Cantor extends the conclusion to sets with infinite cardinality:

Cantor's first theorem For any set *K*, the set, denoted β(*K*), whose members are all subsets of *K* is larger than *K*.

Proof Suppose the theorem is false. Then there is some set *W* such that the set β(W) of all subsets of *W* is not larger than *W*. So β(*W*) can be put into a one-to-one correspondence with *W*, i.e., for every member of β(*W*) there will be a corresponding distinct member of *W*. Let *g* denote such a correspondence or mapping. So *g* maps the set of all subsets of *W*, β(*W*), one-to-one into *W*. Let g^{-1} denote the *inverse* of *g*. The inverse mapping g^{-1} maps members of *W* to subsets of *W*, and for all subsets *S* of *W*, $g^{-1}(g(s)) = S$. If *K* is any subset of *W*, then *K* is a member of β(*W*), and so *g* puts *K* into correspondence with some member of *W*, which I denote by *g*(*K*). Then the following subset of *W*, which I will call *R*, must exist: $R = \{x \text{ in } W \text{ such that } x \notin g^{-1}(x)\}$. Remember that because *g* is a one-to-one correspondence, for each *x* there can be only one set *S* such that $x = g(S)$.

Now consider *R* as defined. *R* is a subset of *W*, so *R* is a member of β(*W*). So *g*, which I have assumed to exist, puts *R* in correspondence with some member *g*(*R*) of *W*. Every member of *W* is either a member of *R* or not a member of *R*. Hence *g*(*R*) is either a member of *R* or not a member of *R*. Suppose that *g*(*R*) is a member of *R*. Then since *R* is the set of all members *x* of *W* such that *x* is not a member of $g^{-1}(x)$, it must be the case that *g*(*R*) is a member of *W*, which is not a member of *R*. So if *g*(*R*) is a member of *R*, then *g*(*R*) is not a member of *R*, which is a contradiction. Hence *g*(*R*) cannot be a member of *R*. But if *g*(*R*) is *not* a member of *R*, then since *R* is the set of all members *x* of *W* such that $x \notin g^{-1}(x)$, it follows that *g*(*R*) is a member of *R* (because *g*(*R*) satisfies the necessary and sufficient condition for being a member of *R*).

Hence the assumption entails that there exists a set whose existence implies a contradiction. Since a contradiction must be false, the assumption must be false. QED.

The proof of Cantor's first theorem is more complex than any of those we have considered previously. It is a reductio argument; that is, the theorem is proved by assuming its denial and deducing a contradiction. It has as an immediate corollary the result that there are infinite sets of different size.

Cantor gave a particular example of two infinite sets one of which is larger than the other. His example does not consider a set and the corresponding set of all subsets of that set. Instead, it concerns the natural numbers 0, 1, 2, 3, ... and the set of all functions defined on the natural numbers. Cantor proved that the set of all functions taking natural numbers as arguments and having natural numbers as values is larger than the set of all natural numbers itself. To understand his argument we need a few definitions.

Definition A *function of one argument* is any set of ordered pairs of objects such that for all *a*, *b*, *c*, if $\langle a, b\rangle$ and $\langle a, c\rangle$ are both in the set, then $b = c$. Equivalently, argument could be revised so that the premises do necessitate the conclusion. Thus Euclid's proof of his first proposition fails to show that the two circles he constructs intersect, and for that reason his postulates and common notions do not necessitate the first proposition. But it seems plausible that we could add axioms to Euclid's postulates so that the resulting system would permit us to deduce proposition 1. Modern reformulations of Euclid's theory do just that. On the other hand, some attempts at proof just seem to involve fundamental mistakes of reasoning. Other attempts at proof may leave us uncertain. Thus after reading and thinking about Anselm's proof of the existence of God, many people are left uncertain as to whether or not the proof is valid. (Of course, the proof could be valid—which means that *if* the premises of the argument are true, then necessarily the conclusion is true—even though the premises of the argument are in fact false.)

I have yet to formulate a theory that will agree with, and in some sense explain, our judgment about which demonstrations are valid and which are not. In the next chapter we will consider the first such theory ever formulated, Aristotle's theory of the syllogism.

Review Questions

1. Why is deductive reasoning often thought to be the first kind of reasoning that philosophy should try to understand?
2. What are three fundamental questions about deductive reasoning?
3. Explain what we want a theory of deductive reasoning to accomplish.
4. Why is finding a good theory of deductive reasoning more difficult than finding conditions that will include the positive and exclude the negative examples in the coding problem, the series problem, and the arch problem?
5. What is the Socratic method?
6. What features are common to the good deductive arguments considered in this chapter?
7. What is the role of the illustrations that accompany some of the arguments given in this chapter?
8. What was Aquinas' relation to Aristotle?

Further Reading

This bibliography contains some sources for further information on the people and issues discussed in this chapter. It is by no means complete. Many other relevant books and essays are available in any good college or university library.

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2 Aristotle's Theory of Demonstration and Proof

Aristotle and Greek Science

In the fourth century before Christ the entire human population consisted of perhaps 130 million people. Mediterranean civilization was spread around the coast of Greece and the Greek islands and in areas of modern Italy, Turkey, Syria, Lebanon, Israel, and northern Egypt. Most travel of any distance was by open boat with one or two square rigged sails and oarsmen. Such ships carried the products of one region to another; they also carried soldiers for the almost perpetual wars between groups and city-states.

What was known around 400 BC? A wide variety of practical arts, including metal production and metal working sophisticated enough to make good hand tools, weapons, and armor; carpentry sophisticated enough to make sea-going boats; the principles of navigation, architectural engineering, quarrying, and stone work; methods of manufacturing cloth and writing surfaces; methods of animal husbandry, fishing, and peasant agriculture.

And what about science? In mathematics, knowledge consisted principally of geometry and the theory of numbers. Many physical laws of mechanics and hydraulics were understood and used, but astronomy was the most developed subject in the physical sciences. Ancient astronomy was based on naked-eye observations of the positions of the stars, planets, and the sun using simple instruments. Astronomy developed because it was easy to make a large number of relevant observations, because the motions of the planets, moon, and sun could be studied as applications of geometry, and because astronomy was of practical use in navigation and ritual use in religion. Other scientific subjects like biology and medicine were also studied, and broad speculations about the structure of the universe and the structure of matter were common.

In this setting Aristotle (384–322 BCE) developed an ethical framework, a science of biology, a theory of cosmology, a theory of motion, and a theory of the of the internal angles of a triangle equal two right angles. But any physical triangle we try to construct will be imperfect. Lines in nature aren't perfectly straight; the sum of the internal angles of the figures we make or draw aren't exactly the sum of two right angles. Plato in effect argued as follows: Since geometry is known, it must be true. Accordingly, whatever geometry is about, it must be true of its subject. But since geometry is not true of the objects of the physical world, it is not about them. So it is about something else.

Plato called the objects of knowledge *forms*. In Plato's conception, the forms aren't in the world, and they certainty are not parts, aspects, or properties of things in the world. They are quite literally not of the world. Of course, the objects and properties of this world have some relation to the forms, but the relation is obscure. Plato says that worldly things *participate* in forms. The idea, very roughly, is that earthly things are crude models of forms, the way a chalk drawing of a triangle is a crude model of a Euclidean triangle.

Aristotle shared with Plato the view that knowledge requires certainty, and also the view that what we seek to know are combinations of properties or features that make a thing an *x*—a man or a triangle or whatever may be the topic of inquiry. But Aristotle brought the forms down to Earth, and the result was a conception of nature and of scientific inquiry that is rather different from Plato's.

Aristotle's Conception of Nature

In Aristotle's conception, if a thing changes, it acquires some new property or loses some old property. For change to be possible, there must exist something that can be identified as one and the same thing before and after the change. So what is the same before and after the change must itself be unchanged. Aristotle calls *substance* whatever endures through change. *Attributes* or *properties* are features that can attach to a substance at one time and not attach to it at other times. Substance that has no properties and is completely unformed, Aristotle calls *prime matter*. Aristotle's conception of the fundamental stuff of the universe can be very roughly pictured as gobs of stuff enduring through time but having various attributes stuck to it at any moment. Of course, Aristotle didn't think of properties as literally stuck to substance, like notices on a bulletin board or clothes on a clothesline.

It is tempting to think that the world is put together in the same way that our descriptions of it are assembled. In English, as in Greek, we assemble sentences from noun phrases and verb phrases. Noun phrases typically occur as subjects in sentences.

They include common nouns such as "cat," "dog," "noon," "eclipse," "tree." Verb phrases typically occur as predicates that are applied to subjects; they include verbs and verbs together with adjectives or adverbs, such as "is black," "is mean," "occurs rarely," "is deciduous." If we put subject terms together with predicate terms in the appropriate way and introduce extra grammatical words (such as "the") in the appropriate places, we get sentences:

The cat is black.

The dog is mean.

Eclipses occur rarely.

Vines are deciduous.

Aristotle thought that the fundamental distinctions in the world are indeed reflected in fundamental distinctions in language, and he described them in some detail in *The Categories*. He held, for example, that the particular objects, such as a mean dog, are constituted by *matter* and by *form*. A mean dog is matter *formed into* a dog that is mean.

We have devices in our language for turning a sentence into a new subject for a new predicate. We can say, for example:

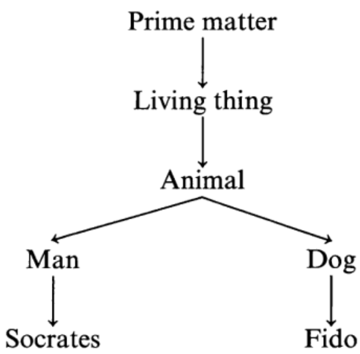
The animal that is a cat is trained.

The mean dog is four legged.

The deciduous vines are broad leafed.

According to Aristotle, the world has the same feature. When form is applied to matter, the combination becomes the matter or substance for the application of still other forms. When we have a black cat, for example, we have a particular object constituted by matter and form. That object can then be the matter that we cause another form to obtain. So if we train the black cat, the black cat is caused to acquire a further form; in other words, the black cat is formed into a trained black cat.

Aristotle thought of nature in terms of *hierarchies*. In particular, he thought of complex entities as built up by the application of a sequence of forms to bare, unformed matter. Suppose that there is bare matter with no form of any sort. If that bare matter is formed into something living, the result is living matter. If living matter is formed into an animal, the result is something animate. If animate matter is given canine form, the result is a dog. If instead animate matter is formed into something with a rational soul, the result is a human. We can picture the process by means of a kind of diagram.



(Diagrams consisting of nodes connected by directed lines are now called *directed graphs*. If the connections are just lines and not arrows, so that the order does not matter, the diagram is simply called a *graph* or an *undirected graph*.)

Another fundamental idea in Aristotle's conception of nature is the distinction between the properties that a thing has *accidentally* and the properties that a thing has *essentially*. A thing has a property accidentally if it could possibly not have had that property. It is an accidental property of a dog that it is a *trained* dog. Fido, the trained dog, would still be Fido if it had not been trained. In Aristotle's terms, it is an accidental feature of Ronald Reagan that he was elected president. Ronald Reagan would still have been Ronald Reagan if he had not been elected president. Essential attributes of a thing are those features without which the thing would lose its identity. Fido is essentially a dog. Anything that is not a dog could not be Fido. Any creature that is *by nature* not furred, not four legged, or not born of a bitch is not a dog, and hence is not Fido. These are essential properties of Fido. Similarly, Ronald Reagan was essentially a man, and anything that could not have been a man could not have been Ronald Reagan. (Of course, anything could have been *named* "Ronald Reagan" but that would not have made the thing the very person *we* denote by "Ronald Reagan.")

For each part of nature, there is a hierarchy that includes only the essential attributes or forms of objects and ignores accidental attributes. According to Aristotle, the goal of science is to find the structure of the appropriate hierarchy for any subject, whether it is astronomy, biology, or cosmology.

Aristotle thought that natural processes have natural ends or purposes. An acorn does not have leaves or roots or bark, but it has the *potential* to acquire leaves and roots and bark, and in the natural course of things, it will come to be an oak tree that *actually* has those features. A human infant does not have language or reason, but it has the potential to acquire both, and in the natural course of things, it will do so. Aristotle thought of all natural processes in the same way; each has an end, and in the natural course of things, that end will be achieved.

Aristotle's conception of nature involves a conception of causality different from our own. Consider questions such as "Why does the sun give warmth?" or "Why does water boil when heated?" or "Why do stars twinkle?" or "Why are vines deciduous?" These questions are requests for causal explanations. Often causal questions are about how something came to be or how it came to be a certain way. In Aristotle's view, there is not just one sort of answer to be given to these questions; there are four different sorts, corresponding to four different senses of "cause." Each question asks about an object or kind of object and about an attribute of that object or objects of that kind.

An object has a specific attribute just in case the object is obtained by imposing a specific form on an appropriate substance. So one sort of cause is the form of the object responsible for the attribute, and another sort of cause is the matter on which the form is imposed. The first is called the *formal cause*, and the second is called the *material cause*. Aristotle tended to think of formal causes as internal principles of development in natural objects, as whatever it is that determines that acorns grow up to be oak trees rather than hemlock trees, for example.

For an attribute to be acquired by a thing, some action must take place to impose a further form on matter. An acorn doesn't become an oak tree unless it is covered with earth in a place where light and water fall. A block of marble does not become a statue of Venus without the action of a sculptor. For Aristotle, the *efficient cause* of a thing possessing a certain attribute is the process by which the matter of the thing acquires the appropriate form. Efficient causes are the kinds of events or processes that we nowadays think of as causes.

According to Aristotle, natural processes have purposes or ends, just as human activities have purposes. The qualities and attributes that things take on in the normal course of events are attributes they have *so that* these purposes or ends will be achieved. One aspect of the explanation of why the sun gives warmth, for example, is the purpose or goal of that state of affairs. One might hold, for example, that the sun gives warmth *so that* life can endure on Earth. Aristotle did not mean, of course, that the sun deliberately intends or plans to make life prosper on Earth. The plan is nature's, not the sun's. Whatever it is *for the purpose of which* an object has an attribute, Aristotle calls the *final cause* of the thing's having the attribute.

The doctrine of four causes forms one of the centerpieces of Aristotle's conception of science. Scientific inquiry is an attempt to answer "why" questions. When such questions are about why something comes to be, they are ambiguous, according to Aristotle: their meaning depends on whether one is asking for the material, efficient, final, or formal cause.

Aristotle's conception of causality and his conception of scientific explanation as the statement and demonstration of causes formed a framework for understanding scientific inquiry that lasted until the eighteenth century. Together with his theory of proof, these conceptions make up an important part of the background against which modern philosophy was formed. I will return to them again in the next chapter when I describe seventeenth-century approaches to the idea of a proof, and I will consider them yet again in later chapters when I take up the subject of inductive inference, and still again when I turn to ethics.

Study Questions

1. Does the sentence "Sam and Suzy love one another" consist of a predicate applied to a subject? What about the sentence "Equals added to equals are equal"?
2. Biological taxonomy describes hierarchies of species, genera, and so on. Do such classifications exemplify Aristotle's conception of nature?
3. Use your own judgment to determine which of the following attributes of water are essential properties of water and which are accidental properties of water.
   - It covers most of the surface of the Earth.
   - It is composed of molecules having two atoms of hydrogen and one atom of oxygen.
   - It can be obtained from wells.
   - It boils at 100 degrees centigrade at one atmosphere pressure.
   - It is sold in bottles by Perrier.
   - It is sometimes drunk with scotch.
   - It is of two kinds, salt and fresh.
4. What do you suppose are the four Aristotelian causes that explain why mammals give milk?

Aristotle's Conception of Science

Aristotle thought that the science of any subject should constitute a system of knowledge claims. Fundamental claims, or axioms, could be used to deduce less fundamental claims. The scientific explanation of a general fact about the world consists in a valid deductive argument that has a description of that general fact as its conclusion and has true, fundamental claims as its premises. Different sciences might have quite different axiomatic systems; there is one theory for biology, another for the constitution of matter, another for astronomy, and so on. These diverse theories may share certain fundamental assumptions, but they will also have postulates that are peculiar to their

Syllogism 1

All humans are animals.

All animals are mortal.

Therefore, all humans are mortal.

This is a *valid syllogism*. *What makes it valid is that if the premises are true, then it follows necessarily that the conclusion is also true*. If the premises happen to be false in a valid syllogism, then the conclusion may be either true or false. What matters is that in every conceivable case in which the premises could be true, the conclusion would also be true.

You can see why this syllogism counts as valid by drawing some circles. (This is not a device that Aristotle used. It was first developed during the Renaissance, and is known today as Venn diagrams, after the 19th-century logician and philosopher, John Venn [1834–1923]). Suppose you introduce a circle *H* to represent the set of all humans, another circle *A* to represent the set of all animals, and a third circle *M* to represent the set of all mortal things. The first premise says that the set of all men is contained in the set of all animals. So put circle *H inside circle A* to represent the state of affairs required for the first premise to be true (figure 2.1). The second premise says that the set of all animals is contained in the set of all mortal things. So put circle *M* around circle *A* to represent the state of affairs required for the second premise to be true (figure 2.2). Now consider the figure drawn (2.2). To represent the state of affairs required to make both premises true, you *had* to put *H* inside *A* and *A* inside *M*. So *H*

Figure 2.1
Premise 1 of syllogism 1.

Figure 2.2
Syllogism 1.

Figure 2.3
Two possibilities for syllogism 2.

is inside *M*, which is what the conclusion asserts. What makes a syllogism valid is that in any way you represent circumstances so that both of the premises are true, the conclusion is true as well.

Here is another valid syllogism:

Syllogism 2

All humans are animals.

Some humans are quiet.

Therefore, some quiet things are animals.

Represent the class of all humans by the circle *H*, and the class of all animals by the circle *A*, and the class of all quiet things by the circle *Q*. The first premise, as before, says that *H* is contained in *A*. The second premise is different. It says that there are things that are both human and quiet. This can only be represented by having circle *Q*, representing the set of all quiet things, *intersect* circle *H*, representing the set of all humans. So every representation that makes the first two premises of the syllogism both true has *Q* intersecting *H* and *H* contained in *A* (figure 2.3). But then *Q* must necessarily intersect *A*, which is what the conclusion asserts.

By contrast, the following syllogism is *not valid*, even though all its premises and its conclusion are true:

Syllogism 3

All humans are animals.

Some animals are mortal.

Therefore, all humans are mortal.

To see that the syllogism is not valid, remember that for validity there must be *no possible way* of arranging the circles representing the sets of things that are human, *H*, animals, *A*, and mortal, *M*, so that in that representation of possible circumstances the premises are both true but the conclusion of the syllogism is false. The first premise says, as before, that *H* is included in *A*. The second premise says that circles *A* and *M*

Figure 2.4
A counterexample to syllogism 3.

intersect. One way in which the two premises *could imaginably* be true is given in figure 2.4. In this figure *M* intersects *A*, and *H* is included in *A*, but *M* does not intersect any of *H*. The figure represents an imaginable circumstance in which all humans are animals, some animals are mortal, but some humans (in fact, all humans) are *immortal*. The circumstances represented are not those that obtain in our world, where in fact all humans are mortal, but they are consistently imaginable circumstances, and they show that *the truth of the premises of the syllogism does not by itself necessitate the truth of the conclusion of the syllogism*.

That a syllogism is *valid* does not imply that its premises are true or that its conclusion is true. A valid syllogism may have false premises and a true conclusion, false premises and a false conclusion, or true premises and a true conclusion. What it may *not* have is true premises and a false conclusion. What it means for a syllogism to be valid is that if its premises were true, its conclusion would of necessity be true. So if the premises are actually true and the syllogism is valid, then the conclusion must actually be true.

Here is an example of a valid syllogism in which the premises are in fact false but the conclusion is true:

All humans are apes.

All apes have opposing thumbs.

Therefore, all humans have opposing thumbs.

Here is an example of a valid syllogism in which the premises are false and the conclusion is false:

All humans are apes.

All apes are stockbrokers.

Therefore, all humans are stockbrokers.

Aristotle realized that the validity of a syllogism has nothing to do with what the predicate terms and the subject terms *mean*, but has everything to do with what quantifiers occur in the premises and the conclusion and with where one and the same term occurs in both the premises and the conclusion. The first syllogism we considered has the following form:

All *A* are *B*.

All *B* are *C*.

Therefore, all *A* are *C*.

Any syllogism of this form will be valid, no matter what classes *A*, *B*, and *C* denote. *A* could be stars, *B* olives, *C* dragons. The following syllogism is silly, but valid.

All stars are dragons.

All dragons are olives.

Therefore, all stars are olives.

By contrast, the following form is not valid.

All *A* are *B*.

Some *B* are C.

Therefore, some *A* are *C*.

It is easy to see that this form of syllogistic argument is not valid by considering an example of that form in which the premises are true but the conclusion is false:

All humans are mammals.

Some mammals are chimpanzees.

Therefore, some humans are chimpanzees.

Study Questions

1. Give new examples of *valid* syllogisms with the following properties: (a) The premises are false and the conclusion is true. (b) The premises are false and the conclusion is false. (c) One premise is false, one premise is true, and the conclusion is false.
2. Give examples of *invalid* syllogisms with the following properties: (a) The premises are true and the conclusion is true. (b) The premises are false and the conclusion is true.

The Theory of the Syllogism

Aristotle described fourteen valid forms of syllogistic argument. Medieval logicians gave each of them names, such as *Barbara* and *Celarent*. In Aristotle's logical theory there are four expressions, now called *quantifiers*, that can be prefixed to a subject-predicate phrase. The quantifiers are "all," "no," "some," and "not all." The traditional abbreviations for these quantifiers are respectively A, E, I, and O. By prefixing one of the quantifiers to a subject-predicate phrase, we obtain a sentence. An Aristotelian syllogism consists of three such sentences: two premises and a conclusion. (The names of the syllogisms contain a code for the quantifiers in the sentences in syllogisms of that form. The vowels in the names indicate the kind of quantifier in the second premise, the first premise, and the conclusion. Thus Darapti is a syllogism with two premises having "all" as their quantifier and a conclusion having "some" as its quantifier.)

These syllogisms are written so that the conclusion is always "(Quantifier) *A* are *C*." The term that occurs in the subject place in the conclusion (*A* in the examples below) is called the *minor* term. The term that occurs in the predicate place in the conclusion (*C* in the examples below) is called the *major* term. The term that occurs in the premises but not in the conclusion (*B* in the examples) is called the *middle term*.

The *form of a syllogistic argument* is determined entirely by the quantifiers attached to each sentence and by the positions of the terms in the premises. If we ignore the quantifiers for the moment, it is easy to see that there are four different patterns or *figures* (as they are called) in which the major, middle, and minor terms can be distributed (table 2.1). The valid Aristotelian syllogisms, with their medieval names, are listed in table 2.2. You may notice that the table of valid syllogisms contains no syllogisms having the pattern of figure 4. Aristotle did not include a study of syllogisms of this figure.

There are four possible quantifiers, any of which can attach to any sentence in a syllogism of any figure. Each syllogism has three sentences, and there are four choices of quantifier for the first sentence, four choices for the second sentence, and four choices for the third sentence, and thus there are $4 \times 4 \times 4 = 64$ distinct syllogistic forms in each figure. And since there are four figures, there are 256 distinct forms of syllogistic arguments altogether. Of the 192 syllogistic forms in the first three figures, Aristotle held that only the 14 illustrated are valid. All others are invalid. How did Aristotle come to this conclusion?

Table 2.1
The four figures of syllogistic arguments

Figure 1	Figure 2	Figure 3	Figure 4
A are *B*	*A* are *B*	*B* are *A*	*B* are *A*
B are *C*	*C* are *B*	*B* are *C*	*C* are *B*
A are *C*	*A* are *C*	*A* are *C*	*A* are *C*

No *A* are *B*	No *A* are *B*	No *A* are *B*
All *B* are *C*	All *B* are *C*	All *B* are *C*
No *A* are *C*	Some *A* are *C*	No all *A* are *C*

2. Use the valid syllogistic forms of the first figure and the rules of conversion to show the validity of the form Camestres and the form Felapton.
3. Do the rules of conversion given in the text suffice to show the validity of the forms Baroco and Bocardo? Why or why not?
4. Find the valid syllogistic forms in the fourth figure.*

Limitations of Aristotle's Syllogistic Theory of Deductive Argument

Although the theory of the syllogism is an interesting and impressive theory of deductive inference, it is not comprehensive. It does not include arguments that we recognize as valid. In other respects it is *too* comprehensive: Aristotle counts as valid some arguments that we would not count as valid.

Aristotle developed his theory of the syllogism as part of a theory of scientific demonstration. One of the great ironies of intellectual history is that while geometry was the paradigmatic Greek science and Euclid lived only a generation after Aristotle, the theory of the syllogism cannot account for even the simplest demonstrations in Euclid's *Elements*. There are several reasons why.

First, the propositions of geometry are not all of a simple subject-predicate form. In fact, rather few of them are. Instead, geometrical propositions deal with *relations* among objects. Second, the propositions of geometry do not all have *just* one quantifier; they may essentially involve repeated uses of "all" and "there exists." Third, proofs require devices for referring to the same object in different ways within the same sentence. Recall from chapter 1 the content of Euclid's first proposition:

Proposition 1 For every straight line segment, there exists an equilateral triangle having that line segment as one side.

To treat this claim as the conclusion of a syllogism, Aristotle would have to treat this sentence as having a single quantifier, "all"; a subject, "straight line segment"; and a predicate, "thing for which there exists an equilateral triangle having that thing as one side." Aristotle would therefore have to interpret the conclusion of Euclid's first proof as of the form

All *A* are *C*.

That is,

All straight line segments are things for which there exists an equilateral triangle having that thing as one side.

If we look at the table of valid syllogistic forms, we see that a conclusion of this form can only be obtained from a syllogism of the form Barbara. So for Aristotle's theory of deductive argument to apply, Euclid's proof would have to provide some middle term *B* and axioms or subconclusions of the following forms:

All *A* are *B*.

All *B* are *C*.

Or more concretely,

All straight line segments are *B*.

All *B* are things for which there exists an equilateral triangle having that thing as one side.

But that is not how Euclid's proof works. Recall that if the line segment has endpoints *P* and *Q*, Euclid constructs a circle centered on *P* and another circle centered on *Q*, each having the line segment as a radius. One of his postulates says that for every point and every length, a circle centered on that point having that length as radius exists (or can be constructed). Then Euclid assumes that there is a point at which the circle centered on *Q* and the circle centered on *P* intersect one another. This point, call it *S*, must be the same distance from *P* as *P* is from *Q*, and also the same distance from *Q* as *Q* is from *P*. By the construction and the definition of circle, the distance from *Q* to *P* is the same as the distance from *P* to *Q*, so point *S* must be the same distance from *Q* as *P* is from *Q*. Then Euclid uses the axiom that things equal to the same thing are equal to one another to infer that the distance from *S* to *P* is the distance from *P* to *Q*. So the distances *PQ*, *PS*, and *QS* are all equal. Another axiom guarantees that for all pairs of points there is a line segment connecting the points, and the definition of a triangle shows that the figure thus shown to exist is a triangle.

Aristotle might let *B* stand for "thing with endpoints that are the centers of circles with radii equal to the distance between the points." Then Aristotle would need to show that Euclid's proof contains a syllogistic demonstration of each of the following:

All straight line segments are things with endpoints that are centers of circles with radii equal to the distance between the points.

All things for which there exists an equilateral triangle having that thing as one side are things with endpoints that are the centers of circles with radii equal to the distance between the points.

Each of these will again have to be established by means of a syllogism of the form Barbara. But however many times we compound syllogisms of the Barbara form, we will never obtain a proof that looks at all like the argument that Euclid provided.

Aristotle's theory also fails to cover several other types of arguments. Recall that Aristotle *proves* that the syllogistic forms of the second and third figures shown in table 2.2 are valid forms. What is the form of those proofs? The proof I illustrated has the following form:

If Celarent is valid, then Cesare is valid.
Celarent is valid.
Therefore, Cesare is valid.

This is a perfectly valid deductive argument. It has the following form:

If *P* then *Q*
P
Therefore *Q*

Here *P* and *Q* stand for any complete sentences that are either true or false. *This argument is not one of Aristotle's valid syllogistic forms*. So Aristotle's own proof of the properties of his logical system uses logical principles that his system can neither represent nor account for. The argument just sketched depends on the logical properties of "If ... then ___," where the ellipsis and the blank are filled by *sentences*. This form of argument is sometimes called a *hypothetical syllogism*.

There is a third difficulty with Aristotle's theory of the syllogism. Look at the first four valid syllogisms of the third figure: Darapti, Felapton, Disamis, and Datisi. Each of them has an existential conclusion; that is, in each case the conclusion says that something exists having specified properties. So, for example, in Darapti we have the following inference:

All *B* are *A*
All *B* are *C*
Some *A* are *C*

Aristotle meant "Some *A* are *C*" to be read as "There exist some things that are *A* and *C*." So understood, it is not clear that Darapti is a valid form of inference. Consider the following example:

All unicorns are animals with hoofs.
All unicorns are horses with one horn.
Therefore, some animals with hoofs are horses with one horn.

This looks like an argument in which the premises are true but the conclusion is false. The problem is with the second rule of conversion:

From "All *X* are *Y*," infer "Some *Y* are *X*."

We don't think it is legitimate to infer "Some little people are leprechauns" from "All leprechauns are little people." We don't think it is legitimate to infer "Some numbers that are divisible by two are both even and odd" from "All numbers that are both even and odd are divisible by two." We reason all the time (both in fairy tales and in mathematics) about *all* things of a certain kind, even when we don't believe or mean to imply that things of that kind exist. In fact, in mathematics we often reason about such things just to prove that they don't exist! Presumably, Aristotle would have agreed with our practice, but his *theory* seems not to agree.

After Aristotle

Aristotle's theory of deductive reasoning may have had many flaws. Yet despite minor improvements in the theory of syllogistic reasoning and some other developments in logical theory, no fundamental advances appeared for the next 2,400 years. Aristotle's successors at the Lyceum, and after them the Stoic philosophers, developed some of the principles of the logic of propositions that were also understood by medieval logicians. For example, it was recognized that for any propositions *P* and *Q*, one could infer *Q* from premises consisting of the assertion of *P* and the assertion of "If *P* then *Q*." Medieval logicians even gave this form of inference a name, *modus ponens*:

Modus ponens From "*P*" and "If *P* then *Q*," infer "*Q*."

Other related logical principles were also understood, for example, the principle *modus tollens*:

Modus tollens From "Not *Q*" and "If *P* then *Q*," infer "Not *P*."

Theophrastus (c. 371–c. 287 BC), who succeeded Aristotle as the head of the Lyceum, gave conditions for the truth of sentences compounded of simpler sentences. He proposed that any sentence of the form "If *P* then *Q*" is false only when *P* is true and *Q* is false. In any other circumstance, "If *P* then *Q*" is true. So in Theophrastus' view, "If *P* then *Q*" is true if *P* and *Q* are both false, if *P* is false and *Q* is true, and if both *P* and *Q* are true. In Theophrastus' conception, therefore, the truth or falsity of "If *P* then *Q*" is a function of the truth-values (true or false) of *P* and *Q*. In other words, the truth-value (true or false) of "If *P* then *Q*" is uniquely determined by the truth values of *P* and *Q*, just as the numerical value of the sum $X + Y$ is uniquely determined by the numerical values of *X* and *Y*. Sentences of the form "If ... then ___" are now known as *conditional sentences* or simply *conditionals*. The account of conditionals as truth functions of the simpler sentences from which they are composed was not widely accepted by logicians of the Middle Ages. They held instead that "If *P* then *Q*" is true only if the truth of *P* necessitates the truth of *Q*. With that understanding, the truth-value of "If *P* then *Q*" is not a function only of the truth-values of *P* and *Q*. It isn't the truth or falsity of *P* and *Q* alone that determines the truth or falsity of "If *P* then *Q*," but whether the truth of *P* would necessitate the truth of *Q*. These ideas were axiomatized in various ways as "modal logics" by Clarence Irving Lewis (1883–1964) in the twentieth century.

Further principles about inference with quantifiers were also recognized by Aristotle's successors. For example, they recognized the principle that from a universal claim one may infer any instance of it. From "Everything is such that if it is human, then it is mortal" one may infer "If Socrates is human, then Socrates is mortal."

Logic was extensively studied in the late Middle Ages from the twelfth through the fourteenth centuries. The theory of the syllogism was understood and extended in minor ways, and tracts were written on various sorts of quantifiers. Medieval logicians were especially interested in what we call *modal logic*, which is the study of deductive inferences that involve notions of necessity, possibility, and ability. Aristotle himself had written on the subject. Aristotle had maintained the following logical principles (which he did not clearly distinguish):

For any proposition *P*, "Necessarily *P*" is true if and only if "Not possibly not *P*" is true.

"*A* is necessarily *B*" is true if and only if "*A* is not possibly not *B*" is true.

Modal reasoning was of special concern to logicians of the Middle Ages because the motivation for their studies of logic was as much religious as it was scientific. They were concerned with features of God and with humanity's relations with God. These subjects involved complicated uses of claims about necessity and possibility. For example, Saint Anselm's proof of God's existence seems to turn on the idea that God is an entity that could not possibly not exist, an entity that necessarily exists. Notions of possibility and necessity can easily lead to paradoxes, which require a logical theory to untangle.

These and other logical investigations amounted to some limited progress in understanding valid reasoning. But at the end of the fourteenth century, Western civilization was not substantially closer to understanding deductive inference than it had been in the fourth century BC. It was still not possible, for example, to give a systematic theory *image not available*

from "Dumbo" to "elephant," to "mammal," to "animal." Upon reaching "animal," it would answer that yes, Dumbo is an animal. In this sort of procedure the computer is carrying out the simplest sort of syllogistic inference. (The key problem is how to program the computer so that it can efficiently find the path if it exists, or efficiently and correctly conclude that no such path exists.)

One interesting thing about trying to simulate human reasoning using a computer is that we are forced to consider logical features that might otherwise be ignored. Suppose that instead of reasoning about elephants, the computer is to reason about birds. From the information that Tweety is a bird, the computer should be able to answer such questions as "Does Tweety have feathers?" "Does Tweety have wings?" "Can Tweety fly?" "Is Tweety a mammal?" "Is Tweety an animal?" The relevant information about birds needed to answer these questions can be represented by a graph, just as the information about elephants is represented. The graph would encode such information as that birds have feathers, that birds have wings, that winged things can fly, and so on. The computer can then carry out simple syllogistic inferences to answer these questions. Given the information that Tweety is a bird, a person will generally answer the question, Can Tweety fly? with a yes. The computer will answer in the same way. But if you give a person a further piece of information about Tweety, you get a different answer. If a person is given the further information that Tweety is an ostrich, the person will not infer that Tweety can fly. People, in other words, make the following inference:

Tweety is a bird.

Birds can fly.

Therefore, Tweety can fly.

And they also make this inference:

Tweety is a bird.

Birds can fly.

Tweety is an ostrich.

Ostriches cannot fly.

Therefore, Tweety cannot fly.

"Tweety can fly" may look at first like the conclusion of a syllogistic inference, but actually something much more complicated is going on. Syllogistic inference, as Aristotle and his successors understood it, is *monotonic*, meaning that if a conclusion *C* can be validly inferred from a set of premises, then it can also he validly inferred from *any* set of premises that include the original premises. The Tweety example shows that the kind of reasoning humans do is sometimes (in fact quite often) *nonmonotonic*: adding information to the premises prevents us from drawing conclusions we would otherwise draw. One mark of the difference is that we are inclined to agree that *birds can fly*, but not that *all birds can fly*. In the same way, we are inclined to agree that whales give milk, but not that all whales give milk (male whales don't). Sentences such as "Birds can fly" are sometimes said to be *generalized*, whereas sentences such as "All birds can fly" are said to be *universal*. While universal and generalized sentences are sometimes synonymous, they aren't always. When they aren't, reasoning that looks syllogistic may actually be nonmonotonic. To make a computer reason as humans do in contexts where knowledge consists of generalized but not universal sentences, the computer must make inferences according to principles of nonmonotonic logic. The principles of nonmonotonic logic and their efficient implementation in computer programs are areas of contemporary research.

Study Questions

1. Write out a graph for reasoning about birds that is like the graph shown for elephants.
2. Suppose that someone reconstructed a particular deduction as a syllogism and you wished to show that the inference principles used actually involved nonmonotonic reasoning. How could you argue for your view? Give an example.
3. Knowledge of causes is often used to reason nonmonotonically. Give an example.
4. What is the name of the syllogistic form used in the reasoning that Dumbo is an animal?
5. Explain why the theory of the syllogism cannot fully account for everyday reasoning about properties of things.
6. How would you answer the question of whether an omnipotent God can make a rock He cannot lift—without contradiction?

Review Questions

1. What questions should a theory of deductive argument address? How well does Aristotle's theory of deductive arguments succeed in answering these questions?
2. What are three major difficulties with Aristotle's theory of deductive argument?
3. What are the four senses of "cause" in Aristotle's philosophy?
4. Do you think that syllogistic reasoning could be used to account for proofs in arithmetic or the theory of numbers?
5. Explain Aristotle's strategy for justifying his theory of syllogisms.
6. Which of Aristotle's syllogistic forms of the second and third figures can be converted into a first-figure form without using Aristotle's second rule of conversion?
7. What role does the theory of syllogisms play in Aristotle's understanding of scientific demonstration?
8. Plato, and many subsequent philosophers, sought theories that had the logical form of definitions, e.g. "X is an equilateral triangle if and only if X is a plane figure composed of three line