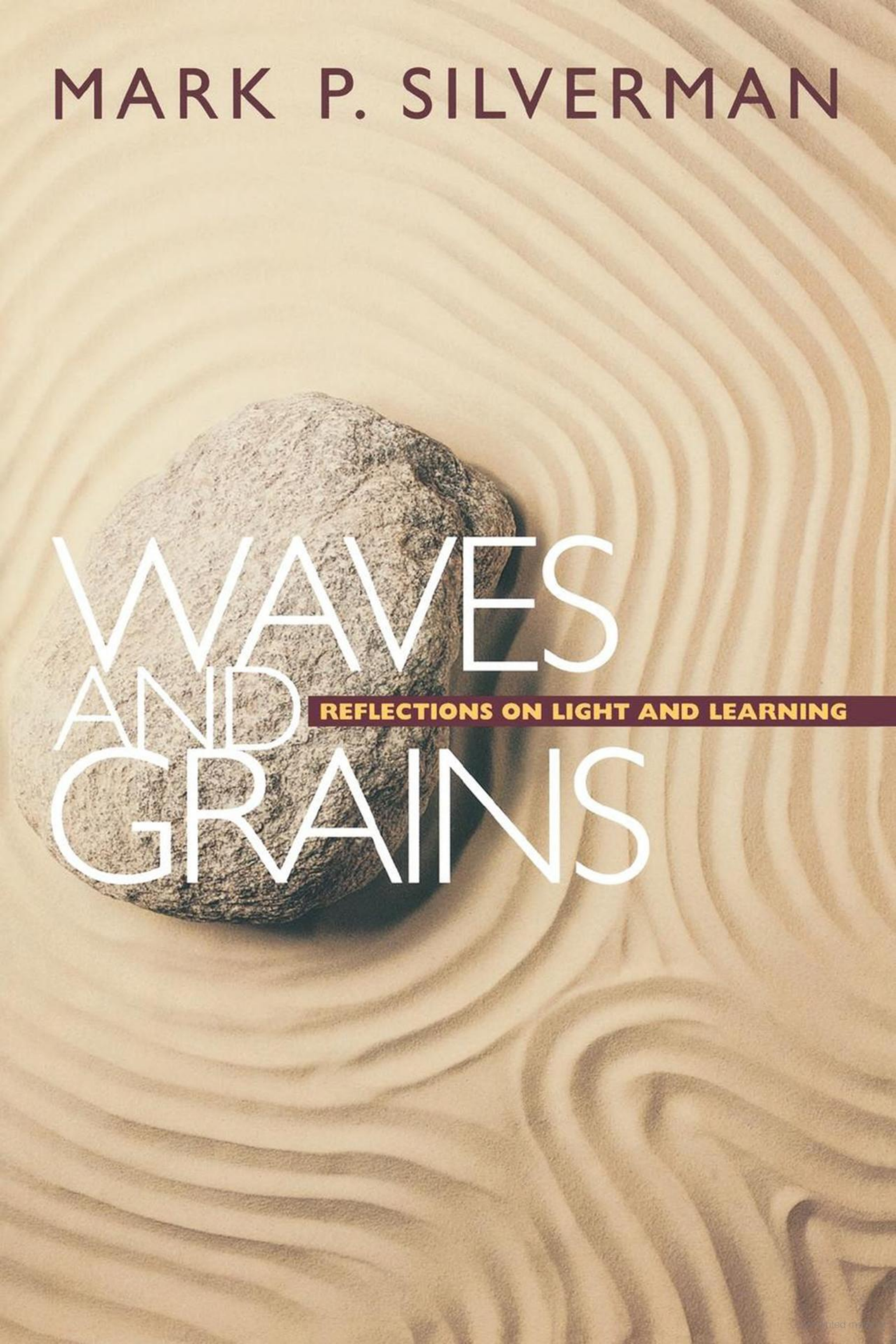
MARK P. SILVERMAN
WAVES
AND
GRAINS
REFLECTIONS ON LIGHT AND LEARNING

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Mark P. Silverman

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Preface

A FEW years ago I wrote a popular book of essays, *And Yet It Moves: Strange Systems and Subtle Questions in Physics.*[1] Designed for anyone with scientific curiosity, the book described some of my experiences as a scientist, drawing on projects that ranged widely over the subdisciplines of physics. The parts of those undertakings specifically concerned with fundamental processes of quantum mechanics have been treated more comprehensively in a sequel, *More Than One Mystery: Explorations of Quantum Interference.*[2] In an analogous way, the present book expands in scope and mathematical and experimental detail those of my investigations devoted primarily to optics, except for spectroscopy, which will be published separately.[3]

This is a book of technical essays based upon studies in classical optics that I have made over the past twenty years. In keeping with the activities of a scientist who throughout his career has had little inclination to become a "specialist," the thematic content of these essays is broad; among them are theoretical, experimental, and historical investigations representing virtually all the major elements of physical optics: propagation (in various media), reflection, refraction, diffraction, interference, polarization, and scattering.

The essays explore unusual—indeed, fascinating—questions linking diverse and far-reaching threads throughout the fabric of physics. What, for example, are the explanation and significance of the strange hyperbolic diffraction pattern first reported by Isaac Newton? What is one to make of interference fringes emerging one photon or one electron at a time? How can one magnify small structures a millionfold or more without a single lens or mirror? Can more light reflect from a surface than is incident upon it? What do all living things have in common that might one day render a Star Trek "life scanner" a possibility? How is it possible to look into densely turbid media and see embedded objects ordinarily hidden from view by multiple scattering?

Besides testing or applying important physical concepts, I have long been fascinated with the unfolding of scientific discovery. The field of optics is particularly rich in this regard, and I have included essays (comprising chapters 2, 4, 8, and 10) based primarily upon my own readings of the seminal papers and books of natural philosophers like Newton, Fresnel, Maxwell, and others whose names figure prominently in the exploration of light. These essays not only help give the scientific investigations a historical perspective, but also provide insight into the social dimensions of a life in science.

Finally, I have always been deeply involved in education, as well as in scientific research. There are general lessons that the study of physics, and particularly optics, presents regarding teaching, learning, and the attributes required of a scientist. These, too, I discuss throughout the text and especially in the concluding essays.

To be a physicist is a difficult undertaking, but its intellectual and spiritual rewards are almost beyond imagination to those who have never experienced them. The adventure of discovery is all the more enjoyable when shared with enthusiastic colleagues, and I gratefully acknowledge Professor Jacques Badoz (Ecole Supérieure de Physique et Chimie), Professor Ismo Lindell (Helsinki University of Technology), Professor John Spence (Arizona State University), Mr. Wayne Strange (Trinity College), and Dr. Akira Tonomura (Hitachi Advanced Research Laboratory) for their companionship in some of the investigations described in this book.

I am also thankful to Dr. Trevor Lipscombe, Mathematical and Physical Sciences Editor at Princeton University Press, for his invitation to write this book and for his vigorous support and encouragement, and to my copyeditor, Mr. David Anderson, for his careful attention to detail and his helpful suggestions.

NOTES

1. Mark P. Silverman, *And Yet It Moves: Strange Systems and Subtle Questions in Physics* (New York: Cambridge University Press, 1993).
2. Mark P. Silverman, *More Than One Mystery: Explorations in Quantum Interference* (New York: Springer-Verlag, 1995).
3. Mark P. Silverman, *Probing the Atom: A Study of Fast Beams, Loose Electrons, and Coupled States* (Princeton: Princeton University Press) (forthcoming).

WAVES AND GRAINS

CHAPTER 1

Introduction: Setting the Agenda

When asked by practical men of affairs for reasons which would justify the investment of large sums of money in researches in pure science, he was quite able to grasp their point of view and cite cogent reasons and examples whereby industry and humanity could be seen to have direct benefits from such work. But his own motive he expressed time and again . . . in five short words, "It is such good fun."
(H. B. Lemon, on A. A. Michelson)[1]

FOR MILLENNIA light has exerted an enchantment over the human imagination. What is light? How does it move? How fast does it move? How does it affect matter? Questions like these are, for the most part, no longer mysteries—except, ironically, the first. No one, I suspect, can say what light *is*, any more than what an electron is, or what a quark is. To reply "an electromagnetic wave" or a "massless spin-1 boson" is merely to append labels evocative of two irreducible elements of a dual nature.

Resolving the enigma of light is a dramatic episode in the history of great ideas. As an experimental and theoretical physicist deeply immersed in the interactions and applications of light, I have long been fascinated by the people of this drama—people alive well before I was born and whose acquaintance I would dearly have wished to make. They appeal to me because of their creative genius, experimental skill, articulate expression, incisive wit, personal courage, and, in a few cases at least, their compassion and simple humanity. They have made empirical and conceptual discoveries that I encounter daily in the course of being a physicist and that, in one way or another, have influenced my own scientific pathways and far more modest contributions.

In the inner sanctum of my office/library my eyes have glanced innumerable times at the wall before me to see the grave, yet gentle countenance of Maxwell—a smile, I surmise, beneath the hirsute exterior. Behind me looking over my shoulder is the youthful visage of Fresnel—wistful, distant, perhaps mindful of his fragile health and impending death, his lusterless eyes staring vacantly into the ether whose essence he was unable to penetrate. These two men—major actors in the drama and principal stimuli, in a certain sense, of this book—both died intolerably young: Maxwell at age forty-nine in 1879 and

Fresnel at age thirty-nine in 1827. One can only wonder what further discoveries they might have made had they lived to the venerable age of Newton who took his leave at eighty-five in 1727. The severe and pensive features of Newton, his "mind forever voyaging through strange seas of thought, alone" (in the memorable words of Wordsworth), stare down at me from my right wall. And close beside him is the middle-aged Einstein, a broad, mischievous smile on his face as he steers his bicycle somewhat precariously toward the photographer. I have lived with these "friends" for many years; indeed, quite literally, much of the physics I know I learned directly from them through their writings.

Newton's self-deprecating remark about seeing farther from the shoulders of giants may have been intended sarcastically in his ongoing difficulties with his illustrious contemporary Robert Hooke, but there is surely a deep truth in it. Scientists not only learn from those who have built the foundations of their science; they define themselves in large measure by the company they keep—and for over three decades these colleagues of a distant era have seemed as real to me as those with whom I am in daily discourse by means of electronic mail, fax, and telephone. I thank them, in part, that I am a physicist.

It is with a certain maturity both in age and experience that a scientist comes to see more clearly how his or her work relates to the historical scheme of things. Assuredly at no time in my career have I ever consciously designed a particular project to complement or generalize an investigation by Fresnel or Maxwell. Motivating influences were always more direct. There was the allure of a controversial issue, the practical applications of a new experimental method, the challenge of a difficult mathematical analysis, the delectation of some striking physical phenomenon. And yet, with hindsight, out of this random diversity emerged a pattern. That pattern winds through this volume like a three-stranded rope with intertwining and inseparable strands of theory, experiment, and chronicle.

For a number of years now I have wanted to write a book of essays on different aspects of my studies of light. In one way or another, over the course of a long and varied scientific career, light was either my subject or my tool. To the young medical researcher studying autoimmune disease at a university hospital it was the means of examining fluids essential to life. To the slightly older researcher in a microbiology laboratory, it was the agent that revealed through the microscope the delicate structures of living cells. To the organic chemist attempting to create odd new molecules, it carried the spectroscopic fingerprints of chemical bonds and molecular arrangements. To the atomic physicist probing the limits of the Dirac equation and quantum electrodynamics, the arrival of individual photons spoke tellingly of the interactions that held an atom together. To the materials scientist it signaled new ways to observe the electromagnetic properties of matter. To the quantum physicist it suggested novel experiments for imaging and interferometry with electrons. And to the perpetual student—despite his gray hair—who has throughout his life marveled at the wonders of the natural world, it brought an abundance of colors, patterns, and curious problems to enjoy. Exploring light *is* such good fun, as Michelson said.

Although colored by the many occupational threads that make up the fabric of my experiences, this is nevertheless a book of *physics* essays, addressing virtually all the major themes of physical optics: light propagation, polarization, reflection, refraction, diffraction, interference, and scattering.

I begin in "Following the Straight and Narrow" with an account of light propagation and the various ways of understanding the law of refraction. The accompanying essay, "How Deep Is the Ocean?/How High Is The Sky?" then takes up the theme of geometric optics in the special context of stratified media, in particular, common media compressed by gravity. Here, surprisingly, are examples of the influence of gravity on light in familiar systems close to home rather than more exotic ones reflecting the agency of remote black holes in the deep recesses of interstellar space.

"Dark Spots—Bright Spots" addresses critical experiments on interference and diffraction leading to the recognition of light as a wavelike phenomenon. The essay that follows, "Newton's Two-Knife Experiment," investigates the theoretical underpinnings and experimental details of an unusual example of Fresnel diffraction—a study first performed by Newton and reproduced with modern technology in my laboratory—with an ironical twist of history. In the next essay, "Young's Two-Slit Experiment with Electrons," I look at the striking implications of a modern rendition of an experiment first performed with candlelight by a young Englishman at the turn of the century. Captured on video, the creation of an electron interference pattern one electron at a time illustrates how inadequate "common sense" may be when it confronts fundamental quantum events. The last essay of the second part, "Pursuing the Invisible," discusses the practical applications of interference and diffraction in creating focused, magnified images without the use of lenses. It examines imaging as an interferometric process and the potential advantages of one of the newest types of microscopes based on one of the most ancient forms of image making. Moreover, as the world's simplest electron interferometer employing what is perhaps the brightest beam in science, the device has a special role to play in the exploration of quantum mechanical phenomena.

"Poles Apart" completes the narrative of the wave nature of light by tracing the experimental path to understanding light polarization. In the associated essay, "The State of Light," I give a detailed description of light polarization from the enlightening, yet uncommonly encountered, perspective of a quantum physicist. Besides illustrating a theoretical framework—considerably simpler in my opinion than the prevailing use of four-dimensional matrices—for analyzing optical systems, I discuss as well the operation and application of one of the most versatile, yet "unsung," devices in optics for quantitatively characterizing polarization: the photoelastic modulator.

"The Grand Synthesis" carries the story of light beyond the point of scalar waves to Maxwell's awe-inspiring creation of electromagnetism. Every physics student learns (or at least is taught) electromagnetism—but almost no one, of course, learns it from Maxwell's original papers or *Treatise*. I did, however, and I try to convey in this essay elements of Maxwell's deep insight that have been lost through generations of modern textbooks. The electromagnetic foundation of light provides the basis for understanding light reflection and scattering, the subject matter of the three accompanying essays of this section. "New Twists on Reflection" introduces the theme of chiral asymmetry in nature and discusses the conceptual intricacies and pioneering experiments related to light reflection from materials whose molecules are like a spiral staircase. "Through a Glass Brightly" tells of the theoretical subtleties and ultimate experimental confirmation of an innovative approach, quite different from that of the laser, to amplifying light by reflection. Finally, "A Penetrating Look at Scattered Light" provides a unifying perspective on all the optical processes of the book by regarding them as special cases of light scattering. It also discusses the notoriously difficult problem of light scattering in turbid media (air, fog, clouds, blood, etc.), showing how both chemical and visual information can be extracted by measurements of light polarization.

Although this book is primarily concerned with light, I have included as the last technical essay "Voice of the Dragon," which is actually about sound. The ostensible justification is that the device under study—a child's toy—resembles superficially an unusual and fascinating light source. Another motivation, however—as in Michelson's case—is that the entire undertaking was pleasurable and instructive, and therefore worth relating to others.

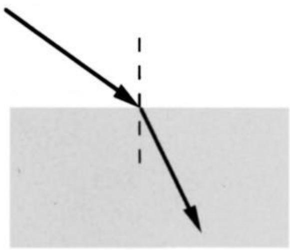
The title of this book (aside from evoking an American patriotic song that I often sang to my children when they were very young) has a twofold significance. On the one hand is the obvious allusion to the dual nature of light, which constitutes a central theme throughout the book. But on the other hand, waves also signify in a more general context a disturbance, the "shaking up" of an otherwise unperturbed medium. And this, too, is intended. As one who has long been seriously concerned about contemporary attitudes toward teaching and learning, who believes that there are more appropriate, more effective, more humane ways to do both, I included the three essays "A Heretical Experiment," "Why Brazil Nuts Are on Top," and "What Does It Take . . . ?" as gentle waves in the status quo containing grains of truths culled from a long career of helping students achieve their potential.

NOTES

1. Harvey B. Lemon, from the foreword to A. A. Michelson, *Studies in Optics* (Chicago: University of Chicago Press, 1962), xxi.

Part One

REFRACTION



CHAPTER 2

Following the Straight and Narrow

> When a hypothesis is true it must lead to the discovery of numerical relationships that tie together the most distant facts. When, on the contrary, it is false, it can represent at best the phenomena for which it has been imagined . . . but it is incapable of revealing the secret bonds that unite these phenomena to those of another class.
>
> (*Augustin Fresnel*)[1]

ANYONE WHO has observed a laser beam passing through a cloudy liquid or smoky room has seen one of the most salient properties of light rays: They propagate rectilinearly (i.e., in straight lines) through a homogeneous medium. This observation is so familiar as to seem almost beneath consideration—and yet from Newton's day to the time of Fresnel, it served as one of the most cogent arguments for a corpuscular or particle description of light. Waves, after all, spread out, as one can readily see by throwing a stone into a pond and observing the ripples.

When light passes from one medium to another, however, the direction of light changes; the rays are "broken" or refracted (from the Latin *frangere*, to break). Newton (1642–1727), having achieved prodigious success in deducing the existence, mathematical properties, and manifold applications of the law of gravity, saw in the phenomenon of refraction another instance of the effects of forces between particles. As a working hypothesis Newton envisaged each color of light to be composed of particles of a particular mass and, possibly, shape. The program of research that he set for himself and "as a farther search to be made by others"—as outlined in a series of some thirty-one queries at the end of his comprehensive and personal treatise, *Opticks*[2]—is in effect to discover the nature of the forces that matter exerted upon the particles of light.

It is worth pointing out that Newton's treatise is remarkably interesting to read. Scientific classics often sit on one's shelf, venerable, sometimes mentioned or referenced, rarely opened and read except by historians. There are reasons for that. The language is antiquated, if not virtually incomprehensible; the mathematical notation or analytical methods obscure; the factual content quite possibly out of date by centuries; the conclusions often incorrect. Try reading Newton's earlier masterpiece, Principia,[3] first published in Latin in

1686. Translations in English are of course available, but one must still be a first-rate geometer to follow Newton's arguments competently and facilely. I suspect that few scientists today are likely to consult the *Principia* to learn mechanics.

By contrast, *Opticks*, which first appeared in 1704, was written in English (although a Latin edition soon followed). Far from being a dry mathematical tome weighted with theorems and proofs, it delineated with obvious loving care Newton's optical experiments—investigations that even today are instructive and enjoyable to read. Moreover, the ordinarily wary Newton expressed himself with greater freedom; he was not averse (as in his study of gravity) to making hypotheses, although these were couched in the less committal form of questions. Nonetheless, these questions permit a penetrating glimpse into what he was thinking, for, as I. B. Cohen pointed out in his preface to *Opticks*, "every one of the Queries is phrased in the negative! Thus Newton does not ask in a truly interrogatory way ... 'Do Bodies act upon Light at a distance ...?'—as if he did not know the answer. Rather, he puts it: 'Do not Bodies act upon Light at a distance ...?'—as if he knew the answer well—'Why, of course they do!'" Whereas the austere and daunting *Principia* concluded an ancient line of inquiry, *Opticks* exposed the exciting phenomena of a new direction of research. Little wonder, then, that experimenters of Newton's day and well over a century later avidly read the book and were much influenced by it.

There is some irony in the fact that Newton's towering reputation was to earn him reproaches—although by that time he was dead and (one would hope) less sensitive to criticism. So deeply had Newton impressed his stamp upon the fabric of science that by the beginning of the nineteenth century most natural philosophers were, like him, partisans of a corpuscular theory of light. When, by the end of that century, a wave theory was firmly established, historians or textbook writers would lament that it was Newton's great authority that held back the progress of a wave theory of light for more than a century.

Such criticism seems to me of dubious validity. That the direction Newton set may have dominated natural philosophy for as long as it did does not necessarily indicate slavish adherence to authority; in fact, the conception of light as a stream of particles acted upon by matter through short-range color-correlated forces actually accounted reasonably well for the phenomena it attempted to explain. Moreover, although a corpuscular theory of light was to become inextricably associated with his name, Newton actually did not find it inconsistent, when necessary, to attribute both particle and wavelike attributes to optical processes. For example, in attempting to understand the pattern of colored fringes in a thin glass plate—a phenomenon eventually to be attributable to wave interference—he wrote:

> As Stones, by falling upon Water put the Water into an undulating Motion and all Bodies by percussion excite vibrations in the Air; so the Rays of Light, by impinging on any refracting or reflecting Surface, excite vibrations in the ...

> Medium . . . and, by exciting them agitate the . . . Body; . . . the vibrations thus excited are propagated in the . . . Medium . . . and move faster than the Rays so as to overtake them.[4]

with the consequence that "every Ray is successively disposed to be easily reflected, or easily transmitted" depending upon whether the motion of the ray is aided or hindered by the overtaking wave. Not a bad try. Indeed, the model bears an uncanny qualitative similarity to a modern picture of photon scattering!

In the opening years of the 1800s when Thomas Young presented his wave theory of interference to a harshly unaccepting community of British scientists, he tried to exploit the ambiguous state of Newton's attitude toward light by pleading that it was indeed the great Sir Isaac himself who first introduced an undulatory theory into optics. The ruse was not very successful, and Young's contributions were actually to find greater appreciation across the Channel among the French—in particular, the remarkably versatile Augustin Fresnel—rather than among the British. But this anticipates matters to come.

By historical accord, it is Christian Huygens (1629–1695) who is usually regarded as the first to propose a wave theory of light, as detailed in his *Traité de la Lumiére* (1690) which predated Newton's own treatise by some fourteen years. Committed Cartesian, Huygens conceived of light as a kind of pressure vibration spreading spherically from each point source throughout an all-pervasive medium, the ether. At each fixed interval of time, the surface tangent to these component wavefronts defined the instantaneous total wavefront—and by outward progression of this envelope Huygens could locate the wavefront at any subsequent time.

It was in extending this mode of reasoning to the more complicated and challenging case of a homogeneous, yet optically anisotropic, medium like calcite that Huygens displayed the full novelty and power of his conception. Here he envisioned the elementary wave surfaces as ellipsoidal, rather than spherical, expanding with different characteristic speeds in the directions of the principal axes (figure 2.1).

As a wavelike description of light, however, Huygens's construction left much to be desired. For one thing, the wave model contained no element of periodicity. Without periodicity, Huygens's waves could not give rise to the attribute of color or to interference phenomena such as the circular fringes Newton saw reflected from a convex lens pressed against a glass slab. For another, it did not even account convincingly for the property of rectilinear propagation. Outwardly spreading spherical waves generate both backward- and foreward-moving wavefronts, and the former are not observed. (This was, in fact, a difficulty with which Fresnel, too, would have to contend in developing a comprehensive mathematical theory of diffraction based on Huygens's conception of light; eventually Kirchhoff would provide the final element of the solution, the "obliquity factor.") More damaging still, Newton found a glaring inconsistency in Huygens's argument for constructing anisotropic wavefronts: There

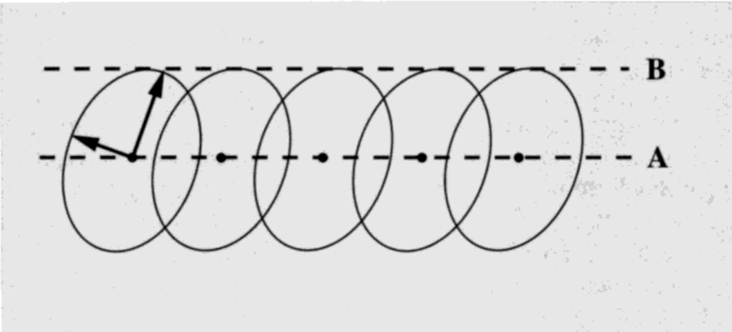


Figure 2.1. Huygens's construction for advance of a plane wavefront in an optically anisotropic medium like calcite with two principal axes. Each luminous point on the initial wavefront *A* produces an ellipsoidal wavelet that propagates with characteristic velocities (arrows) along the principal axes. The new wavefront at *B* is the tangent envelope to all such wavelets.

can be *no* directional dependence of wave speed for "Pressions"—longitudinal vibrations like sound waves—propagating through a uniform medium. Neither Newton nor Huygens apparently entertained the possibility that light could be a transverse vibration.

Although the triumph of the wave theory of light over the particle theory was eventually to be established by Fresnel's sweeping investigations of interference and diffraction, there was, in fact, a far simpler and more direct way in which the two models led to an irreconcilable predictive difference: the speed of light in materials. In this regard, the phenomenon of refraction could have played a key role in the quest to understand the nature of light. History, however, took a different course. Nevertheless, the pertinent physics is instructive.

That light actually moves with a finite speed—no trivial matter to ascertain by direct time-of-flight measurement—had only recently been demonstrated in Huygens's and Newton's day. Indeed, in his treatise Huygens refers appreciatively to "the ingenious proof of Mr. Römer" who determined ($\sim$1675) the passage of light to be "more than six hundred thousand times greater than that of sound" by observing the eclipses of Jovian moons at different locations of the Earth about the Sun. Actually, content to have demonstrated that the speed of light was not infinite, Römer, to my knowledge, never gave a concrete numerical value for it; the foregoing figure was deduced by Huygens.

One can regard the eclipses of a moon as a uniform series of time signals sent out by the planet. As the Earth moves away from conjunction (closest approach) with Jupiter, the time required for light to reach the Earth increases. According to Römer the signals were delayed 996 seconds when the two planets were in opposition (farthest approach). Dividing the diameter of the Earth's orbit, known approximately from measurements of solar parallax, by the delay time furnished by Römer, one would obtain a highly respectable value of ~192,000 miles/second (~3.2×10^8 m/s).

It is a much more difficult undertaking, however, to ascertain the speed of light in matter. Although the law of reflection was already known to the Greeks of classical antiquity, the law of refraction was discovered empirically by Willebrord Snel (or sometimes Snell) around 1621—and is consequently referred to as Snel's law in English-speaking countries. Snel died before publishing his discovery, and the law first entered the public record, as far as I know, in the *Dioptrique* of Descartes who is believed to have seen Snel's manuscript, yet not to have acknowledged it. Not everyone believes it; in a French-speaking country—as my Parisian colleagues have gently reminded me—one refers to *la loi de Descartes*.

Without knowing why it was so, Snel observed that the sine of the angle of refraction bears a constant ratio to the sine of the angle of incidence (figure 2.2)—the constant of proportionality depending on the color of the light and nature of the two media. In modern notation the law would read

$$\frac{\sin\phi}{\sin\theta} = \frac{n_1}{n_2} = \text{constant} \tag{1}$$

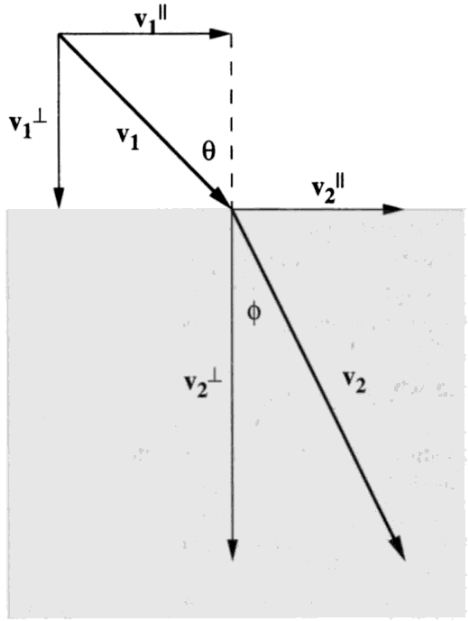
where n_1 and n_2 are, respectively, the indices of refraction of the incident and refracting media, and θ and ϕ are the angles made by the incident and refracted rays with respect to the normal to the surface (figure 2.2a) and are equivalent to the corresponding angles made with respect to the surface by the wavefronts (figure 2.2b).

Newton knew of relation (1) and made extensive measurements, which he reported in Book 2 of *Opticks*, of the "refractive Power" of substances of different densities. It was Pierre-Simon de Laplace, however, who first derived the law from Newton's corpuscular hypothesis. Published in Book 10 of his *Traité de Mécanique Céleste*, the relation formed the underpinnings of Laplace's treatment of the effects of atmospheric refraction on astronomical observations.

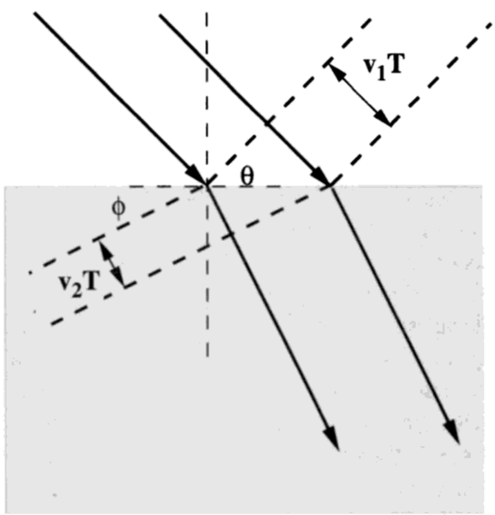
Consider a beam of light originating in air and incident upon the surface of a transparent material such as water or glass. Upon penetrating the surface, the beam bends or refracts toward the normal. This is explicable, according to Newton's ideas, because the molecules of the bulk medium exert a net vertical attractive force on the "molecules" of light. By symmetry there should be no net lateral force if the surface is sufficiently—actually infinitely—extensive. Thus, light moves faster in the refracting medium although the parallel component of velocity remains unchanged. From figure 2.2a, the precise relationship between initial (v_1) and final (v_2) speeds and angles of incidence and refraction is clearly seen to be

$$\frac{\sin\phi}{\sin\theta} = \frac{v_1}{v_2} \quad \text{[particles]}. \tag{2}$$

In a particle theory such as Newton's, the rays of light are effectively identified with a stream of real objects. In Huygens's theory, however, it is the wavefronts



(a)



(b)

Figure 2.2. Refraction of light according to (a) Newton's corpuscular theory and (b) Huygens's wave theory. In (a) the ray is equivalent to the velocity vector of particles of light. Upon moving from medium 1 (air) to medium 2 (water), the particles increase in speed. In (b) a ray is simply the normal to a wavefront. In moving from air to water, the speed of the wave decreases.

that are accorded a physical existence, the rays being merely geometrical signposts of beam direction. Figure 2.2b depicts the process of refraction according to Huygens's wave model. Wavefronts (let us say in air) are incident upon the surface, penetrate it, and propagate through the denser medium (for example, water) at a slower speed. In a fixed interval of time T, the distance $v_1 T$ traveled by a wavefront in air is greater than the corresponding distance $v_2 T$ covered by the portion of the same wavefront in water. Sequential wavefronts in water are therefore closer together than in air, and the normal to the wavefront (the ray) must bend toward the normal to the surface.

Although Huygens did not explicitly consider periodic waves, his model readily accommodates them. If T is the period of one oscillation, then the wavelength $\lambda = vT$ represents the distance between points of equal phase on two sequential wavefronts (e.g., the spatial interval between two crests or two troughs). The essential point to recognize is that the oscillation period (or its reciprocal, the frequency $\nu = 1/T$) depends on the light source and is independent of the various media through which the light propagates. Thus, if $v_2 < v_1$, the wavelength λ_2 in the refracting medium must be *shorter* than the wavelength λ_1 in air, as illustrated in figure 2.2b. From the geometry of the figure, the ratio of the wavelengths is

$$\frac{\sin\phi}{\sin\theta} = \frac{\lambda_2}{\lambda_1} \quad \text{[waves]} \tag{3a}$$

which, upon substitution of corresponding speeds, leads to the reciprocal of Eq. (2)

$$\frac{\sin\phi}{\sin\theta} = \frac{v_2}{v_1} \quad \text{[waves].} \tag{3b}$$

Equations (2) and (3a, b) are both compatible with Snel's empirical law. To ascertain, however, that the second relation, and not the first, correctly represents the behavior of light requires that one measure directly either the speed or the wavelength of light in transparent bulk matter relative to that in air. Unfortunately, the former could not be done during Huygens's or Newton's lifetimes, or even during the lifetimes of Laplace, Fresnel, and Young. Indeed, by the time the experiment was actually performed—around 1850 by Léon Foucault who compared the speed of light in water to that in air—the issue of light wave versus light particle had been essentially resolved (until the appearance of quantum difficulties after 1900).

Interestingly, the second measurement could well have been performed during Newton's lifetime, for it entailed the use of apparatus already familiar to him, especially in his study of the annular fringes produced by reflection from an air gap of variable width ("Newton's rings"). To my knowledge, however, it was Thomas Young who first reported ($\sim$1802) the shortening of wavelengths of visible light in water and inferred from this effect the reduction in light speed.

Young's memoir had little impact. To appreciate its significance one must have already recognized as fundamental the concept of wavelength. But in 1802 the particle theory reigned supreme—and within this framework wavelength had no relevance. Velocity, by contrast, would have been understood by all.

Although the quantum theory of light lies well outside the time frame of the present discussion, I think it is nonetheless useful to relate that the correct law of refraction of light "particles" follows from the conception of light as a stream of photons. Assuming, as did Laplace, that the medium does not affect the lateral motion of the particles, and directing attention to photon linear momentum **p**—a more significant quantity in quantum theory than velocity—figure 2.1a leads to an expression perfectly analogous to Eq. (2):

$$\frac{\sin\phi}{\sin\theta} = \frac{p_1}{p_2} \quad \text{[photons]}. \tag{4}$$

The momentum of a photon, however, is not (as in the mechanics of massive particles) proportional to speed, but reciprocally related to wavelength by de Broglie's famous formula

$$p = \frac{h}{\lambda} \tag{5}$$

in which the constant of proportionality is Planck's consant $h = 6.627 \times 10^{-27}$ erg-sec. Substitution of Eq. (5) into the particle relation (4) leads precisely to the wave result (3a, b) deduced from Huygens's principle. This is but one of many ways in which quantum theory weaves its unifying thread through the seemingly disparate realm of waves and particles.

NOTES

1. Cited (in French) in *Les Cahiers de Science & Vie*, no. 5 (October 1991), p. 1.
2. Sir Isaac Newton, *Opticks: Or a Treatise of the Reflections, Refractions, Inflections & Colour of Light* (New York: Dover, 1952; based on the 4th edition, London, 1730).
3. Sir Isaac Newton, *Principia*, 2 vols. (Berkeley: University of California Press, 1966; based on the 1729 translation by Andrew Motte of *Philosophiae Naturalis Principia Mathematica* [London: S. Pepys, 1686]).
4. Newton, *Opticks*, p. 280.

CHAPTER 3

How Deep Is the Ocean? How High Is the Sky? Imaging in Stratified Media

We saw an ingenious use of this mathematical fact
during our visit to ... China.... [Our friend] placed
the optics of the instrument inside an enclosure and
removed the air. The instrument looks out at the stars
through a horizontal glass window. Because there is
no air inside ... theoretically there will be no effect
of the atmosphere above the glass window, and the
stars will appear in exactly their correct positions.
And, in fact, they do.
(*Aden and Majorie Meinel*)[1]

A STACK of transparent parallel plates produces a virtual image whose location depends sensitively on the angle of viewing. Far from being an idle curiosity confined to a laboratory bench, these "stacks" are everywhere. We breathe in one and swim in another, for gravitational compression of gases and liquids creates stratified media with analogous optical properties. The difference between the actual and apparent locations of objects viewed through the atmosphere and ocean can be significant—and provides a dramatic example of the indirect effect of gravity on light propagation,[2] an influence ordinarily associated with massive stars. Moreover, in a stratified medium of appropriate refractive index gradient, light can move in a closed circular path as if in orbit about a black hole.

3.1 VIEWING THROUGH FLAT LENSES

Although it is doubtless considered ill-mannered to poke a finger in one's tea, this breach of etiquette reveals (with the simplest of apparatus) a striking phenomenon of geometrical optics. The effect is especially marked if the immersed finger is viewed obliquely. The apparent foreshortening of distance that occurs with viewing through a thick refractive medium is a perfect example of optical imaging by a parallel plate or layer.

Most discussions of geometrical optics ordinarily focus on lenses with spherical surfaces. There is, of course, good reason for this: Such lenses are important for historical and practical reasons. Nevertheless, there are interesting things to be learned from the optics of lenses with plane-parallel surfaces.

To the extent that the subject is broached at all in the more than two dozen physics and introductory optics books I have examined, it appears as a standard problem.[3] A coin lies at the bottom of a pond (or a puddle or bathtub of water, etc.) whose depth is x and refractive index is n; what is the apparent depth of the coin as seen from above? The problem is as uninspiring as it is old, for its solution effectively requires only simple substitution into the familiar "lensmaker's" formula applied to lenses of infinite radii of curvature. Phrased as it is, the problem does not stimulate one to discover the enhanced effect of oblique viewing or to explore the implications for stratified continuous media.

Two important aspects of parallel-plate optics are worth mentioning explicitly. First, the location of an image point of a single plate can be determined simply and exactly for arbitrary plate thickness. Second, the analysis of stacked plates with different indices of refraction is not much more complicated than that of one plate. By contrast, the lensmaker's formula for spherical surfaces that one ordinarily encounters in elementary physics is valid only for thin lenses in a paraxial approximation (i.e., light rays close to the optical axis), and analysis of combinations of thick lenses can become algebraically messy indeed.

Perhaps the most prominent feature of the optics of a parallel plate immersed in an ambient medium (e.g., air) is that the incident and transmitted rays are parallel, a consequence readily deduced from Snel's law. A secondary feature, usually negligible for thin plates, is that the refracted ray is laterally shifted upon leaving the plate and reentering the surrounding medium. This has its consequences, however. Insertion of a parallel plate of glass into a converging light beam focuses the beam farther from the converging lens. One might have surmised at first that the plate, being a "lens" with infinite focal length, would have no effect. Moreover, if a parallel plate is sufficiently thick, it can give rise to a surprising effect. Look at both a distant and a nearby object simultaneously through a thick slab of glass or calcite—and turn the slab clockwise and counterclockwise about the vertical axis. The far object appears unmoved, but the apparent position of the near object is markedly altered. The lateral shift does not affect the angular location of an object at infinity but will displace one located at a finite distance.

That the incident and transmitted rays are parallel in the case of multiple layers was well known to Isaac Newton, who drew from that fact a not-so-obvious conclusion regarding imaging by the Earth's atmosphere:

> if Light pass through many refracting Substances or Mediums gradually denser and denser, and terminated with parallel Surfaces, the Sum of all the Refractions will be equal to the single Refraction which it would have suffer'd in passing immediately out of the first Medium into the last. ... And therefore the whole Refraction of Light in passing through the Atmosphere from the highest and rarest Part thereof down to the lowest and densest Part, must be equal to the Refraction

viewing. A detector with large depth of field, such as the human eye, would be largely insensitive to the effect for arbitrary angle of viewing—and so we will not consider parallel-plate astigmatism further.

From the geometry delineated above, the image distance d is given by

$$d = y \cot \theta_0 \tag{2}$$

where

$$y = y_0 + y_1 + y_2 = d_0 \tan \theta_0 + d_1 \tan \theta_0 + d_2 \tan \theta_2 \tag{3}$$

is the vertical distance between the tip of the arrow and the point of emergence of ray R_1 at the rear surface of plate 2 (interface between medium 2 and the air). By using Snel's law one can express the tangent functions in Eq. (3) in terms of the incident (and directly measurable) angle θ_0. Doing so, and generalizing the resulting relation to cover an arbitrary number m of plates, leads to the desired virtual image distance

$$d = d_0 + \sum_{i=1}^{m} \frac{d_i \cos \theta_0}{\sqrt{(n_i/n_0)^2 - \sin^2 \theta_0}}. \tag{4}$$

It is worth pointing out explicitly here that the dependence of the image location on the angle of ray incidence is a true refractive effect and not merely a geometrical consequence of oblique viewing, for the apparent distance d reduces to the actual distance $d = d_0 + d_1 + \cdots + d_m$ irrespective of the angle θ_0 when all refractive indices are the same (n_0).

In the frequently treated example of normal incidence $\theta_0 = 0$, Eq. (4) reduces to

$$d = d_0 + n_0 \sum_{i=1}^{m} \frac{d_i}{n_i} \tag{5}$$

and reproduces the familiar result $d = d_1(n_0/n_1)$ in the simplest case of an object lying in the left interface of a single refracting plate.

The virtual image of an extended object, as a result of its dependence on the angle of incidence, is not simply a projection of the object onto a displaced parallel plane. Look again at figure 3.1; a ray R_1' emitted from the base of the arrow must be incident upon the first surface at a larger angle θ_0' in order to intersect ray R_1 at the aperture of the detector, here taken to be the pupil of the eye. The apparent location d' of the virtual image of the base, determined by the intersection of the normally emitted ray R_0' and the backward extension of R_1', is less than d for $n_i > n_0$ ($i = 1, 2, \ldots, m$).

From Eqs. (2) and (3) and the comparable geometric relation

$$d' + d_e = (y + a + d_e \tan \theta_0) \cot \theta_0' \tag{6}$$

one can derive an implicit relation between θ_0' and θ_0 that, for objects of small extension, leads to the following displacement δ between the virtual images of the tip and base:

$$\delta = d' - d = \frac{aD\cos^2\theta_0}{d + d_e - D\sin\theta_0\cos\theta_0} \tag{7}$$

where D is the rate at which the apparent tip location varies with viewing angle

$$D = \left|\frac{\partial d(\theta)}{\partial\theta}\right|_{\theta=\theta_0} = \sum_{i=1}^{m} \frac{d_i \sin\theta_0[(n_i/n_0)^2 - 1]}{[(n_i/n_0)^2 - \sin^2\theta_0]^{3/2}}. \tag{8}$$

For the configuration of figure 3.1, where the top of the arrow lies closest to the eye and $n_2 > n_1$, the base of the arrow appears closest to the front surface ($d' < d$). This distortion vanishes for normal viewing since $D(\theta_0 = 0) = 0$.

3.3 IMAGING IN A STRATIFIED GAS

Let us consider an isothermal ideal gas in a uniform gravitational field as an analytically tractable model of an atmosphere without thermally induced refractive index gradients. (The real atmosphere is of course more complicated—but this book is not a treatise on meteorology!) The gas compresses under its own weight, thereby leading to variations in pressure, density, and refractive index that depend on height. From the perspective of geometric optics, the atmosphere is an infinite stack of infinitesimally thin plates.

From the ideal gas equation of state and the barometric pressure relation[5]

$$\frac{dp}{dz} = \rho(z)g, \tag{9}$$

where g is the acceleration of gravity near the Earth's surface, one can derive the dependence of mass density $\rho(z)$ on height z (measured from ground level $z = 0$)

$$\rho(z) = \rho(0)\exp\left(\frac{-z}{h_c}\right). \tag{10}$$

The characteristic height

$$h_c = \frac{RT}{Mg}, \tag{11}$$

determined from the universal gas constant $R = 8.31$ J/K-mol, absolute temperature T, and formula weight (actually mass) M of gas, is a rough measure of the vertical extension of the atmosphere. (At a height $z = h_c$ the density falls to $1/e = 0.37$ its value at ground level.) For the Earth's atmosphere—comprising about 75% N_2 and 25% O_2—h_c is approximately 8.8 km, which is roughly the cruising height of a passenger jet. The validity of relation (10) requires that the acceleration of gravity a_g remain close to g over the range of heights of interest. This is a reasonable assumption here. From Newton's law of gravity it is not difficult to show that at the height h_c the fractional departure from g is $(g - a_g)/g \cong 2h_c/R_E \sim 3 \times 10^{-3}$ where $R_E \sim 6400$ km is the radius of the Earth.

The index of refraction n of a gas is simply related to density by an expression of the form

$$n - 1 = K\rho \tag{12}$$

where K is a constant depending on the light frequency and atomic structure. Later, in chapter 13, I will discuss the physics underlying this expression from the perspective of light scattering. (As a historical aside, it is of interest to note at this point that François Arago and Jean-Baptiste Biot—two French physicists of whom I shall say more soon—jointly investigated, at the suggestion of Laplace, the refractive index of the components of air with the intention of using refractive power as a means of chemical identification. The two collaborators were to become bitter enemies—the first a partisan of the wave theory, and the second a defender of the corpuscular theory.) Substitution of the mass density, Eq. (10), into Eq. (12) leads to the refractive index

$$n(z) = 1 + \beta \exp\left(\frac{-z}{h_c}\right) \tag{13}$$

where $\beta = K_\rho(0) = n(0) - 1 \ll 1$. For air at sea level one has $n(0) - 1 \sim 3 \times 10^{-4}$.

To someone at ground level looking up at angle θ_0 (to the vertical) at an object at height h, the apparent height, deduced from Eq. (4) applied to a continuous medium (with $d_0 = 0$), is

$$d = \int_0^h \frac{\cos\theta_0 \, dz}{\sqrt{(n(z)/n_0)^2 - \sin^2\theta_0}}. \tag{14}$$

Changing the integration variable from z to $v = \exp(-z/h_c)$ leads to a form found in integral tables. Then, following a little algebraic rearrangement, one obtains the expression

$$d(h) = \frac{n(0)\cos\theta_0}{\sqrt{1 - n(0)^2 \sin^2\theta_0}} \left[1 - \frac{h_c}{h} \ln\left(\frac{Q_1}{Q_2}\right)\right] \tag{15}$$

in which

$$Q_1 = n(0)(1 - n(0)\sin^2\theta_0) + n(0)\cos\theta_0(1 - n(0)^2\sin^2\theta_0)^{1/2}, \tag{16a}$$

$$\begin{aligned} Q_2 = {} & n(h) - n(0)^2 \sin^2\theta_0 \\ & + \{(n(h)^2 - n(0)^2\sin^2\theta_0)(1 - n(0)^2\sin^2\theta_0)\}^{1/2}. \end{aligned} \tag{16b}$$

For Eq. (15) to be meaningful the denominator must not vanish. The viewing angle is therefore limited by the inequality

$$\sin\theta_0 < \frac{1}{n(0)}. \tag{17}$$

This is not particularly restrictive, for air at sea level θ_0 must be less than 88.6°. Thus, the expression for $d(h)$ is applicable to nearly the full range of viewing.

There is a subtle, but physically important, point relating to the form of Eq. (15), which can be written as the sum of two terms. With Eq. (4) in mind, it is tempting to interpret the first term—which does not depend on h_c or, equivalently, on the gravitational acceleration g—as the effect of a uniform layer of gas with index $n(0)$ on the refraction of light originating in a vacuum. The second term, in which h_c appears, would then represent the refractive influence of gravity. One might think that the first term is always by far the larger, and that the location of the virtual image effectively results from refraction at a "boundary" between the vacuum and the atmosphere at ground level. This picture calls to mind the remark of Newton cited earlier (which concerned the net angular deviation of a light ray in air). This interpretation, however, is not strictly valid.

The gas is not a uniform layer, but has an effective thickness, as indicated in relation (13), of the order of h_c (or, at most, a few tens of h_c) that can be much smaller than the object distance h. Even for purposes of analogy there is no refractive boundary separating the object and the gas; both object and observer are immersed in the gas, which is a continuous stratified layer. Had the gas actually been an optically homogeneous layer, no refractive effect would have resulted at all. That is, in the limit that g approaches zero, the image must be located at the object ($d = h$), and *not* at the position given by the first term of Eq. (15).

Examination of the integral in Eq. (14) shows immediately that d reduces to h when g vanishes. This is not so apparent, however, in the integrated expression Eq. (15) in which h_c becomes infinitely great and the argument of the logarithm approaches unity in the limit of vanishing g. Careful evaluation of this limit nevertheless yields, as it must, the expected result. The physical significance of these remarks is that the gravitational compression of the gas is responsible for the entire difference between object and image locations.

With regard to the title of this chapter, let us examine the case $h \gg h_c$ for which Eq. (15) reduces to

$$d(h \gg h_c) = \frac{n(0)\cos\theta_0}{\sqrt{1 - n(0)^2\sin^2\theta_0}} \times \left\{1 - \left(\frac{h_c}{h}\right)\ln\left[\frac{n_0}{2}\left(1 + \frac{\cos\theta_0}{\sqrt{1 - n(0)^2\sin^2\theta_0}}\right)\right]\right\}. \tag{18}$$

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named Schmidt that the Sun was such a body—and that the bright disc we see is an optical illusion![11]

In Schmidt's model the Sun is a ball of gas whose density decreases from the center outward attaining at a certain radius—which defined what he called the "critical sphere"—a value such that light rays within that spherical surface had the same radius of curvature. The rays that reached our eyes, and that therefore defined the size of the Sun's disc, were just those escaping in a narrow range beyond tangency. Rays leaving the surface of the critical sphere more obliquely would pass into space without reaching us.

Much has been learned about the Sun since then, including the origin of its luminosity, but the idea of light constrained to move in circular paths by virtue of simple geometric optics, rather than the esoteric laws of general relativity, is an intriguing one well worth examining. How must the refractive index vary if orbiting light rays are to result? The answer is in fact quite simple.

The fundamental equation that locates a point $\mathbf{r}$ on a light ray in a medium of refractive index $n(\mathbf{r})$ is

$$\frac{d}{ds}\left(n\frac{d\mathbf{r}}{ds}\right) = \nabla n \tag{30}$$

where s is the length of the ray as measured from a fixed point through which it passes. This equation, whose derivation will be left to the appendix, follows straightforwardly from the definition of a light ray as a trajectory orthogonal to the wavefront.[12] In particular, the gradient of the wavefront yields a vector in the direction $d\mathbf{r}/ds$ (a unit vector since $d\mathbf{r}\cdot d\mathbf{r} = ds^2$) with magnitude n.

As a quick test of the plausibility of Eq. (30), consider the simplest case of an optically uniform medium, $n =$ constant. Then $d^2\mathbf{r}/ds^2 = 0$, and the solution for the trajectories is a straight line (with two constants of integration to be specified).

The case of a medium with spherical symmetry is also simple. The refractive index $n(r)$ depends only on the radial distance $r = |\mathbf{r}|$; all rays are plane curves, and it is not difficult to show that along each ray

$$nr\sin\phi = \text{constant}. \tag{31}$$

In the above relation ϕ is the angle between the position vector $\mathbf{r}$ of a point on the ray and the tangent to the ray at the same point (as shown in figure 3.2). If the ray is to trace out a circular path, ϕ must be 90°, whereupon it follows from Eq. (31) that the index of refraction must take the form

$$n(r) = \frac{\text{constant}}{r}. \tag{32}$$

One can verify that an inverse radial dependence of the refractive index leads to circular rays of light by solving directly the fundamental equation (30), which,

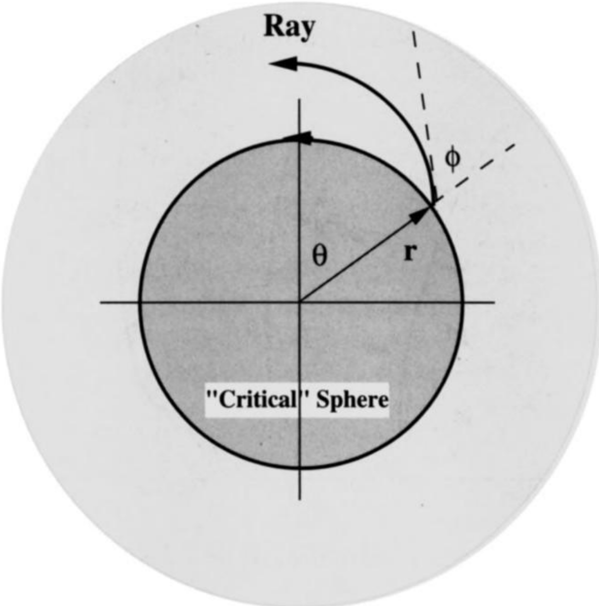

Figure 3.2. Rays in a medium of spherical symmetry.

for spherical symmetry, reduces to

$$\frac{dr}{d\theta} = \frac{r}{a}\sqrt{n^2r^2 - a^2}. \tag{33}$$

In Eq. (33) θ is the polar angle of the ray (whose vertex is at the origin $r = 0$), and a is the constant in relation (32). Since r does not vary with θ for a circular orbit, the left-hand side of Eq. (33) vanishes, and one arrives again at Eq. (32).

Suppose, then, that one had a huge ball of gas extending at radius R to the vacuum of space where $n(R) = 1$. Then circular orbits of light should occur at the apparent solar radius $R_S < R$ if the refractive index at that radius were $n(R_S) = R/R_S$.

The foregoing example raises a simple question that, at first thought, may seem perplexing. Since the circular ray is always moving in the direction in which the refractive index does *not* vary, why does the ray bend? In fact, the same question could be raised with respect to any ray moving in a stratified medium perpendicular to the optical gradient. Yet refraction occurs.

In directing attention to rays of light, it is sometimes easy to forget their correct significance—that rays are merely normals to wavefronts. The answer to the question is that refraction occurs because the wavefront, not the ray, extends into the medium and experiences the variation in refractive index. The different parts of the wavefront move at different speeds; the wavefront changes

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Part Two

DIFFRACTION AND INTERFERENCE

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a simplification of the actual state of affairs. Nevertheless, Fresnel's binary ray theory was neither consistent nor unified and could not in general be applied to situations—like the illuminated region behind a diffracting aperture—where many rays clearly superposed at each point.

Directing his attention to the three-dimensional spatial distribution of fringe maxima and minima, Fresnel deduced what he considered one of the most remarkable consequences of the wave theory of diffraction. Indeed, it is a striking result that I have rarely seen mentioned in the vast number of general physics and optics textbooks in which diffraction patterns are usually shown only in a single plane a fixed distance from the diffractor. What, however, is the locus of points traced out by a fringe as the distance of the viewing screen from the diffractor is varied? One might think—based on familiar treatments of diffraction within (what we now call) the far-field or Fraunhofer approximation—that the variation is linear. Fresnel recognized that the fringes do not propagate in a straight line, but along a hyperbola whose foci, in the case of fringes outside the geometric shadow of a diffracting wire, are the point source and one of the edges. Furthermore, he devised a simple, yet precise, micrometer to demonstrate this fact.

Why a hyperbola? Recall that within a binary ray model of interference there are always just two point sources. They may be, for example, two pinholes in Young's experiment, or a point source and an edge point in Fresnel's treatment of diffraction from a wire. The locus of all points equidistant from two given points (the sources of light) is a perpendicular plane bisecting the line joining the two points. Thus, the principal (zeroth-order) maximum in the fringe pattern is spatially distributed in a plane lying midway between the two sources. The first-order maximum is the locus of all points in space so situated that their distances to the two sources differ by one wavelength. This, however, is the condition that defines a hyperboloid of revolution, a surface generated by the movement of a point whose distance from two fixed points (the foci) is a constant. Likewise, the second-order maximum will be spatially distributed on the surface of another hyperboloid for which the constant difference is twice the wavelength. In this way the interference fringes form in space a family of confocal hyperboloids—and the intersection of these surfaces with a plane passing through the two foci will be a system of hyperbolas.[5]

Under conditions characteristic of most interference experiments, where the source points are very close and the viewing screen far removed, the hyperbolic variation is practically linear. To discern the actual fringe variation with the apparatus available to Fresnel required great patience and experimental skill. Fresnel, like Newton, was not only a mathematically adept theorist, but a superb experimentalist.

The hyperbolic fringe variation with distance from the source carries a major conceptual implication. If light, as Newton construed it, were truly a beam of particles deviating from rectilinear propagation as it passed by the edges of

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