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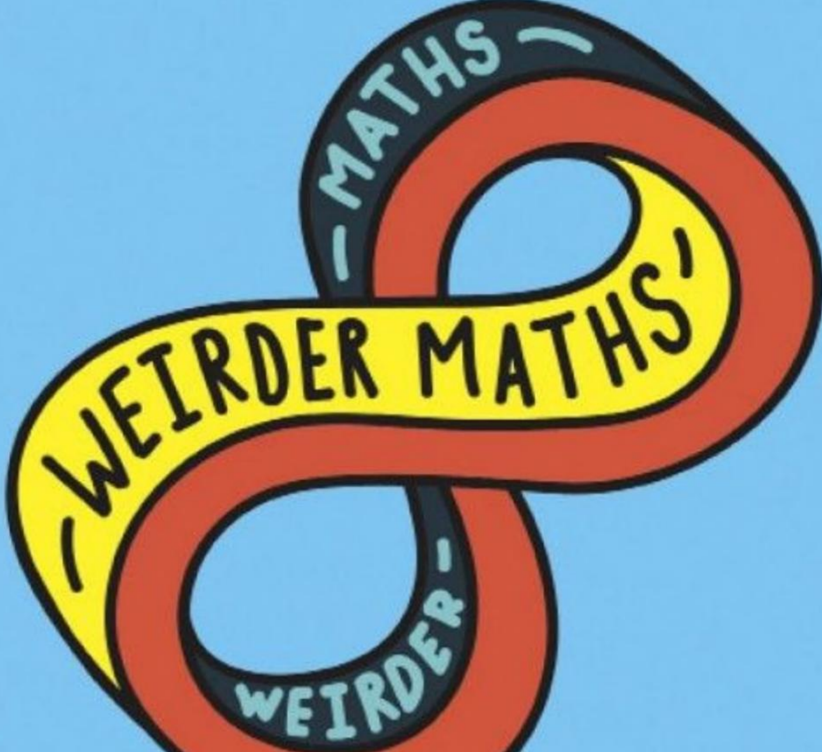

AT THE EDGE OF THE POSSIBLE

Contents

Introduction

This is our second foray into some of the most outrageous, fascinating, and downright peculiar parts of mathematics, following our earlier adventures in *Weird Maths*. We'll be venturing into a land of bizarre shapes and numbers, exploring, like Gulliver, realms of the ultra small and the fantastically large, wandering down passageways with many twists and turns, and, along the way, coming face to face with some of the greatest challenges the human mind has ever confronted.

Maths is a subject vaster than most of us realise – so scarily vast that perhaps we're lucky to have minds limited in how far they can see. Maths penetrates every aspect of our lives and underpins not just science and technology, but also music and the arts, the forms, patterns, and movements that surround us, and even the games we play. It can be as hard as solving a page full of convoluted equations in a graduate class at Princeton or as easy as blowing bubbles for a child. We do maths every minute of every day because maths infuses all aspects of the universe around us, part of the infrastructure of reality. Some of it's as familiar as 1, 2, 3, or the symmetry of a circle. But much of maths is extraordinary and dazzling, beautiful, diverse, and strange. Its wonder and weirdness literally have no bounds.

As a writing team we're a little unusual. One of us (David) is a physicist and astronomer by training, who's spent the past 35 years writing books on everything from cosmology to consciousness. The other (Agnijo) is a teenage maths prodigy who started coming for private lessons with David several years ago and in 2018 took joint first place in the International Mathematical Olympiad with a perfect score of 42 points out of 42. He recently headed off to Cambridge University to continue his mathematical explorations there. About three years ago we began work on *Weird Maths*, dividing the chapters between us, and then cross checking each other's work – Agnijo focusing on the maths itself while David concentrated on making the writing clear and adding historical and biographical detail. The collaboration proved so successful and relatively stress-free that here we are again with the sequel!

So much is happening in this field – the pace of discovery is so dizzying – that we've added more chapters to *Weirder Maths*. Our goals, however, remain the same. We want to bring the most unusual, interesting, and important ideas in maths within reach of the general reader, not avoiding topics simply because they may seem hard to explain. Our mantra continues to be that anyone can grasp maths if it's framed in the right language. We've also tried, whenever possible, to show how the maths is relevant in everyday life or how it's proved useful to science and other fields.

We hope that some of the enthusiasm we feel for this most amazing and often misunderstood subject spills over onto these pages. Maths truly can be weird but, more than anything, it's a very human endeavour, full of the fun and foibles that mark us out as a species.

CHAPTER 1

Get Out of That

> Ts'ui Pên must have said once: *I am withdrawing to write a book*. And another time: *I am withdrawing to construct a labyrinth*. Every one imagined two works; to no one did it occur that the book and the maze were one and the same thing.
>
> – Jorge Luis Borges

The most famous maze of all probably never existed and, even if it had, would have been a doddle to solve – if depictions of it on Cretan coins are anything to go by. The architect Daedalus, the story goes, built a winding series of passageways called the Labyrinth for King Minos of Crete as a place to contain the Minotaur. This monstrosity, with a bovine head, a human body, and an understandably bad temper, was the offspring of Minos's wife and a white bull given to the king by Poseidon, god of the sea. As a punishment to the Athenians, whom he had defeated in battle, Minos demanded that a number of their young men and women be periodically sacrificed to the creature lurking at the Labyrinth's heart. One year, the hero Theseus of Athens took the place of one of the youths to be sacrificed, entered the dreaded system of chambers, unravelled a ball of thread given to him by Ariadne, Minos's daughter, as he went, slew the Minotaur, and then escaped by following the thread trail back to the entrance.

We don't know how the Labyrinth of Minos was laid out. In any case, it's just a legend – more bull than the actual product of human ingenuity. What we do have are coins from the island of Crete dating back to between 300 and 100 BCE that bear designs presumed to represent the layout of the famous lair of the Minotaur. Most of these depict a rather simple yet ingenious pattern, typically in the form of a seven- or eight-level unicursal maze. The number of levels is how many times you cross the path to the centre if you draw a line from the outside to the ultimate goal. 'Unicursal' means that there's only one way in and out. As for the distinction between 'maze' and 'labyrinth', that's a matter of what definition you choose.

Some languages have only one word for maze or labyrinth; the Spanish *laberinto*, for instance, translates as either. 'Maze' is Old English for 'confuse' or 'confound', while 'labyrinth' comes from the Greek *labyrinthos*, the etymology of which is controversial. Some scholars have linked the Greek term with an old Lydian word, *labrys*, for 'double-edged axe', a symbol of royal power. So, the theory goes, the Labyrinth was part of the palace of the double axe – the home of the Minoan kings. It's a tentative and questionable link. In any event, we're left with a choice of definition and how, if at all, we want to distinguish between a labyrinth and a maze.

Our purposes being mainly mathematical, we'll assume that a labyrinth is a special type of maze – a unicursal maze. A labyrinth, then, is just a winding passage with no choices of which way to go or leave (except back the way we came). A maze, on the other hand, we'll take to be the general case of a system of paths that may have multiple branches and a layout as confusing and convoluted as the maze-designer cares to imagine. A maze may also have multiple entrances, exits, and dead ends, whereas in the form of a labyrinth, although it may be ingeniously long to traverse given its total area, it consists of nothing more than an unbranching path, which leads to the centre and then back out the same way, with only one point of entry and exit.

Labyrinths aren't so much an intellectual challenge as they are a place to spend time in an unusual environment. As such they tend to have been employed as a form of meditation, a point nicely captured in the phrase 'you enter a maze to lose yourself and a labyrinth to find yourself.' Not surprisingly, designs for labyrinths are found in places of spiritual reflection. A well-known one is set into the floor of the nave of Chartres Cathedral, the border made of blue-black marble and the path itself of 276 slabs of white limestone. With a diameter of just under 13 metres (about 42 feet) it's large enough for a person to walk around the snaking track, as pilgrims have done since its construction sometime in the early thirteenth century. Rumour has it that there was once a depiction of the Minotaur at the centre of the eleven concentric rings of the pattern, but the primary symbolism, for obvious reasons, is Christian. The design features four arms standing for the branches of the cross and a winding path intended to symbolise the road to Jerusalem. Those not able, or willing, to make an actual trek to the Holy City could thus simulate the journey, more conveniently, by walking around this handy representation or, for the genuinely pious, shuffling around it on their knees. Although not the most ornate or embellished of labyrinths found in ecclesiastical buildings around the world, the one at Chartres is considered archetypal and others like it are known as 'Chartres mazes'.

Many other labyrinthine patterns are to be found in other parts of the world from all periods of history, from the Neolithic and Bronze Ages to recent times. As we've seen, they're intended, not as puzzles to solve, but as devices for religious or spiritual practice, ritual, or ceremony. It's thought that, long ago, Nordic fishermen would walk labyrinths before heading out to sea as a way of ensuring a plentiful haul and safe return, and that, in Germany, young men did the same as a rite of passage to adulthood. But these motives for their creation and design don't detract from the mathematical interest of labyrinths. The ingenuity and variety of techniques used to pack such a long path into a comparatively small space are fascinating in themselves. There's the study, too, of all the different ways of producing unicursal paths from 'seeds' – the starting shapes in the form of short sections of curves in a symmetrical pattern, that determine the initial course of the path of the labyrinth, from the centre working out. Labyrinths can be left- or right-handed depending on the direction of the first turn after entering, have different numbers of circuits, and take any of a couple of dozen or more distinct forms (largely determined by the choice of seeds) known to specialists in the subject.

A Chartres-style labyrinth in the Abbey of Our Lady of Saint-Remy, Rochefort, Wallonia, Belgium.

The first mathematician to do a thorough analysis of unicursal mazes was the prolific Swiss theoretician Leonhard Euler (pronounced 'oiler') in the mid-eighteenth century. His interest in the subject stemmed from an answer he presented to the St Petersburg Academy in 1736 to the problem of the Bridges of Königsberg. The question was whether it was possible to walk from anywhere in the East Prussian city of Königsberg (present-day Kaliningrad in Russia) and cross every bridge there exactly once before returning to the starting point. Six of the bridges connected the banks of the river (three on either side) with two islands in the middle, while a seventh joined the islands. Euler reduced the problem to its mathematical essentials and, in this way, made it much easier to solve. He realised that the only information of relevance had to do with the connections: each land mass could be thought of as a point and each bridge a line joining two points. Euler was able to prove that for *any* arrangement of points and connecting lines, it would be possible to arrive back at the starting point, having traversed every connecting line exactly once, if and only if a certain condition was satisfied. This condition was that either no point along the way had an odd number of connecting lines or only two points did. Since the Königsberg layout of bridges broke the rule, there was no way to solve the original problem of crossing all the bridges just once and returning to the place you began.

The beauty of Euler's approach to the famous problem was that it could be generalised. His Königsberg analysis provided the first clear mathematical definition of a unicursal figure, as one that obeys the rule of connectivity just mentioned. But, more importantly, his work on this puzzle gave birth to a whole new field of maths known as graph theory and was important, too, in the rise of another major nascent subject – topology.

Both graph theory and topology are among the tools that mathematicians can bring to bear when tackling the thornier issue of multicursal mazes. Not only are such mazes designed to pose a mental challenge but also they can be fiendishly hard to solve, exist in two, three, or more dimensions, and take a form that doesn't, at first sight, even look like a maze.

Outside of legend, the first maze of which there's any historical record is that referred to by the Greek historian Herodotus who lived in the fifth century BCE. He describes a maze in Egypt so grand that 'all the works and buildings of the Greeks put together would certainly be inferior to this labyrinth as regards labour and expense.' Whether it was actually a labyrinth in the sense of being unicursal, we don't know. But, if Herodotus is to be believed, it would certainly have been impressive, with 12 courts, 3,000 chambers, and one side consisting of a pyramid 243 feet high.

Among more recent puzzle mazes are those that European royalty had built on their properties to amuse guests or provide places for secret meetings and trysts. The one at Hampton Court Palace, on the banks of the Thames, commissioned in the 1690s, is the best known and has now become a popular tourist attraction. The oldest surviving hedge maze in Britain, with walls tall enough to block any view of the way ahead, it covers a third of an acre but isn't hard to solve. Though not unicursal, it has only a few places where the path forks so that no one can get lost for long. Daniel Defoe mentions it in *From London to Land's End*, as does Jerome K. Jerome in *Three Men in a Boat*:

> We'll just go in here, so that you can say you've been, but it's very simple. It's absurd to call it a maze. You keep on taking the first turning to the right. We'll just walk round for ten minutes, and then go and get some lunch.

Far more convoluted is Il Labirinto Stra (the labyrinth of Stra). Located just outside the city limits of Venice, in the grounds of the Villa Pisani, and created in 1720, it's reputed to be one of the hardest public mazes in the world to solve. Even Napoleon, a smart guy and no mean mathematician, is said to have been baffled by it. Anyone, however, who manages to navigate their way through the nine concentric rings of the maze, with their multiple openings and branches, can then climb the spiral staircase of the turret at the centre to get a bird's-eye view of the whole affair.

The octagonal Jubilee Maze at Symonds Yat, Herefordshire.

Two record-breaking mazes are in the United States. The Dole Plantation's giant Pineapple Garden Maze in Hawaii, made up of 14,000 tropical plants bordering two and a half miles of paths, was declared the world's longest in 2008. Meanwhile, not to be outdone, Cool Patch Pumpkins in Dixon, California, grew a corn maze that earned an entry in the *Guinness Book of Records* as the largest such temporary maze. So confounded by it were some visitors that, fearing they wouldn't escape before closing time, they dialled 911 to be rescued!

Let's suppose, then, that you've just entered a maze, about which you know nothing, for the first time. You've no idea how big or complicated it is, the walls are too high to see over, and there's no one else around with whom to compare notes. All you've been told is that there's a goal – a place in the middle that you need to reach in order to solve the puzzle – and definitely at least one route that leads there. A classic and straightforward approach is the 'wall following' method, in which you keep your hand in contact with one side of the maze and just keep walking. This will work in many cases, in the sense that it'll eventually lead you to your goal. But it has two drawbacks. First, it may take you a very long time, and second, it may fail completely if the maze has loops in it as well as dead ends that aren't connected to the outer wall. The key to solving mazes in a systematic way that won't let you down is to turn to maths.

Following Euler's example, the first step towards successfully traversing a maze is to transform it into an abstract plan. We can do this by using ideas from a subject known as network topology. In negotiating a maze, all that matters is what we do at points where there's a choice available – the so-called decision points. The first decision point comes at the entrance because we can choose to go in or not! Dead ends are decision points, too, although they offer only the option of stopping or turning around. More interesting are where the path splits and we can opt to go down one of two or more branches. If a maze is shown as a network, in other words as a series of points connected by lines, it's easy to see the solution – the best way to get from the entrance to the centre. Complicated subway systems, such as the London Underground, are maze-like and confusing to those unfamiliar with them, but maps in the form of network diagrams, on station walls and in every carriage, make it clear how to travel from any station to the desired destination.

We're supposing, though, that you've entered a maze without the benefit of such a map. This is where a bag of popcorn and a bag of peanuts come in handy – and not as snacks to be eaten if you end up getting lost! The popcorn and peanuts are to lay down trails so that you can take advantage of what Euler discovered from his work on the Königsberg problem. The trick is to ensure that, whatever choices you make at decision points, you never go along any stretch of path more than twice. So here's the method: leave a trail of popcorn as you go and a piece of popcorn at every decision point. That way you'll know if you've been down that path and to that point before. If you choose to go down a path a second time, then leave a trail of peanuts. The rule is, if you come to a path that you've already marked with peanuts, don't go down it again. Now, for a bit of nomenclature. If you arrive at a decision point that's popcorn-less, call this a *new node* and put down a piece of popcorn, thereby turning it into an *old node*. In the same way, if you come to a path that has no popcorn on it this is a *new path*. If you walk down it, drop popcorn as you go. The next time you walk down it, trail peanuts so that it becomes an *old path*.

With all this in mind, here's how you crack the maze. At the entrance, choose any path. When you come to a new node go down any new path. If you're on a new path and you come to an old node or a dead end, turn around and go back along the path. If you're on an old path and you come to an old node, take a new path if there is one or an old path otherwise. Don't go down a path twice. Follow these steps, make sure you're well stocked with popcorn and peanuts, and you're bound to reach the centre. You can then turn around and follow the route that's marked only by popcorn to find your way back out again.

A series of well-defined instructions that gives a guaranteed solution to a certain class of problems is known as an algorithm. This one for solving mazes is called Trémaux's algorithm, after the nineteenth-century French author Charles

Trémaux who first described it. It's now recognised as a version of what's called depth-first search (DFS) – a method that can be used to search data structures known in maths as trees or graphs. Both of these structures consist of points, or nodes, which are linked by connecting lines, or 'edges'. Graph theory, in particular, which, as we mentioned, sprang from Euler's work on the Königsberg problem, is the source of a number of algorithms useful in tackling mazes. It's also a powerful tool for representing as mazes problems that don't superficially look like mazes at all – like Rubik's Cube.

Astonishingly, a standard 3 × 3 × 3 Rubik's Cube has 43,252,003,274,489,856,000 possible arrangements. Each of these positions corresponds to a decision point in a maze of fiendish complexity. Simply spinning a cube at random would be as likely to succeed as a drunk staggering around a planet-sized maze in the hope of reaching the centre. The key to solving the puzzle in a reasonable amount of time lies in applying algorithms so that more pieces are brought into position without disturbing those already in place.

In graph theory there's a concept known as graph diameter. This is the largest possible number of nodes that have to be passed through in order to travel from one node to another when paths that backtrack, detour, or loop around are ignored. In the case of Rubik's Cube this equates to the maximum number of moves needed to solve the puzzle from *any* starting position (including the most randomised, worst-case scenario). Although the Cube was invented in 1974 it took until 2010 for its graph diameter, sometimes referred to as God's Number, to be calculated. Eventually, a team of researchers at Google, having burned through 35 CPU-years of computer time, found the answer: just 20. This surprisingly low number explains how top 'speedcubers' can solve the puzzle in under five seconds (the world record is 4.22 seconds, from a random starting position, set by a 22-year-old Australian in 2018). At least, it explains how it's *physically* possible. The real key to such extraordinary proficiency is endless hours of practice and memorising the steps involved in various efficient algorithmic strategies. To these demands must be added exceptional memory in the case of cubers who are able to solve the puzzle blindfold.

Complex mazes sometimes arise naturally on Earth, providing plenty of opportunity for people to get lost. In South Florida, large stands of mangrove forming impenetrably thick walls and rising to a height of 70 feet line the sides of twisting channels. Although the waterways may not be long, a kayaker who enters without a guide or a map is liable to end up going round in circles for hours. Geological formations too can form natural mazes, which often become popular tourist attractions. Rock Maze, near Rapid City, in the Black Hills of South Dakota, includes an area of massive granite boulders that have separated and cracked to create a network of narrow, sinuous passageways.

When mazes form underground, as twisting interconnected systems of caves, they can have the added complication of being three-dimensional. Among the most extraordinary examples is the Optymistychna Cave near the Ukrainian village of Korolivka. Discovered as recently as 1966 the cave is confined to a layer of gypsum less than 30 metres (98 feet) thick and consists mostly of small passages that are no more than 3 metres (10 feet) wide and 1.5 metres (5 feet) high, although they can rise taller at intersections. To date, more than 265 kilometres (165 miles) have been mapped, making it the fifth longest known cave in the world. Longest of all – by a wide margin – is Mammoth Cave in central Kentucky, with passageways stretching for 663 kilometres (412 miles) through limestone that dates back more than 300 million years.

Rotunda Room, Mammoth Cave, Kentucky. USGS photo.

One of the amateur cavers who, in the early 1970s, helped create survey maps of Mammoth Cave, was Will Crowther, a programmer with the R&D company Bolt, Beranek and Newman. Crowther was part of the original, small team that developed ARPANET (a forerunner of the Internet). A fan of the tabletop role-playing game Dungeons & Dragons, he hit upon the idea of combining a computer simulation of his caving explorations with elements of fantasy role-playing. The result, developed in 1975 and 1976, was *Colossal Cave Adventure*, which became popularly known as *Adventure* or simply ‘Advent’ (after the name of its executable file). Crowther’s original 700 lines of FORTRAN code were expanded by Don Woods, a graduate student at Stanford University, who added more fantasy ideas and settings based on his love of Tolkien's writings. By 1977, the canon version of *Adventure* was complete and soon became widely distributed among programmers in the US and elsewhere. Its 3,000 lines of code were supplemented by 1,800 lines of data, which included 140 map locations, 293 vocabulary words, 53 objects (15 of them treasure objects), travel tables, and various messages, the most famous of which has become: 'You are in a maze of twisty little passages, all alike.' Part of the fun of the game was in figuring out how to map this maze with pen and paper. A useful approach was to drop objects in the rooms you encountered as you went along to serve as landmarks.

Our look at cave mazes wouldn't be complete without mention of the Labyrinthos Caves, located beneath a quarry at Gortyn in the south of Crete, just 20 miles or so from the Minoan palace at Knossos. This series of chambers and tunnels, some researchers claim, may be the true source of the legend of the Minotaur. Visitors can explore up to two and a half miles of intertwining passageways, which occasionally open into large rooms such as the Altar Chamber. Whether this natural maze inspired the famous legend we'll probably never know, but the Labyrinthos Caves aren't short of fascinating historical episodes as it is, including a time when the spies of Louis XVI carried out covert operations from there and the Nazis employed them as a secret ammunitions dump in World War II.

Psychologists use mazes for experiments in animal cognition, while researchers in artificial intelligence challenge their robotic inventions to navigate mazes in the most efficient way. The maze that is the Internet is one of the most elaborate creations of the human mind, which, in turn, inhabits the maze of neurons and their interconnections, which make up our brain. Curiously, James Knierim of Johns Hopkins University and his colleagues found that, in some situations, such as when we try to recall if we've seen a person's face before, our brains work in a way that's similar to how a rat navigates its way out of a maze. Different areas of the hippocampus arrive at two different conclusions – either the face is familiar or it isn't – which are then voted on by other parts of the brain in order to come to a decision. The researchers found that a similar decision-making process takes place in a rat's brain if the animal is taught how to recognise a certain maze but then, later, some landmarks in the maze are slightly altered.

In creating mazes as challenges for the mind, or labyrinths for the purpose of contemplation, we're in a sense externalising the nature of our own brains and how they operate. The Argentinian writer Jorge Luis Borges repeatedly used the labyrinth as a metaphor for some of the great mysteries of the world, including time, mind, and physical reality. The epigraph for this chapter comes from his short story 'The Garden of Forking Paths' (1941), while in 'Ibn-Hakam Al-Bokhari, Murdered in His Labyrinth' (1951) one of the characters, Unwin the mathematician, remarks: 'There's no need to build a labyrinth when the entire universe is one.'

CHAPTER 2

At the Vanishing Point

I love to talk about nothing. It's the only thing I know anything about.

– Oscar Wilde

Zero: there isn't much to it. Certainly not if it happens to be your bank balance, how many birthday cards you received, or a good approximation to your chances of winning the rollover lottery jackpot. At the same time, what zero is seems obvious. We all know what it means and take its existence for granted. It's hard to imagine that there was a time when mathematicians got by without it and that it actually had to be discovered – or invented, depending on how you look at it.

Intuitively, of course, the idea of zero goes back before the dawn of history. Early humans – even animals for that matter – know what it's like to have zero food or zero shelter. Having nothing, or the threat of having nothing, is what gives us the impetus to survive.

For philosophers, zero, and the concept of nothing, has long been a subject of fascination. The notion of the void plays an especially important role in many Eastern philosophies. In some forms of Buddhism, for instance, Śūnyatā (emptiness) is considered to be a state of mind in which all conscious thoughts, including awareness of self, are released leaving just pure consciousness of the moment. It's a state that, for

It was as a place-holder, to make clear the value of a multi-digit number, that the idea of zero first appeared in maths. Place-value number systems, in which the position of a digit indicates its value, go back at least 4,000 years to when the Babylonians started using them. But there's no evidence that these people also felt the need to have an empty place indicator, at least not for a very long time. Original texts from around 1700 BCE survive, in the form of cuneiform writing pressed into clay tablets with a stylus to leave wedge-shaped marks. These tablets reveal how the Babylonians represented numbers and did arithmetic with them. Their notation was quite different from ours and their number system was based on 60 rather than 10. But it's clear that the early Babylonians didn't distinguish between what we would write as, say, 1036 and 136, except by context. It wasn't until around 700 BCE that they started to include symbols in the same way that we do for the idea of zero as a place-marker. Various notations were used, according to the city and the era, but one, two, or three wedge-shaped symbols can be seen on Babylonian and Mesopotamian tablets in place of where we would put a '0'. The same idea occurred later to other civilisations, including the Chinese, who left an empty space as the equivalent of zero in their counting-rod system, and the Mayans.

Clay tablets, such as this one from southern Iraq, dating to 3100–3000 BCE, contain the earliest known symbols for zero.

The drawbacks of not having a place-value number system quickly become obvious if we try doing maths using the alternative – where each symbol stands for a value that's fixed and can't be changed. The Romans were lumbered with such an approach, which is perhaps why we hear a lot about Roman generals, politicians, conquests, and methods of government and town planning, but not so much about Roman breakthroughs in mathematics. Roman numerals use seven letters as symbols for writing numbers: I for 1, V for 5, X for 10, L for 50, C for 100, D for 500, and M for 1,000. Their use gets cumbersome very quickly. For example, 1,999 in Roman numerals is MCMXCIX and any number much larger than 5,000 can be ridiculously hard to represent. The other big issue is doing arithmetic Roman style. For us, figuring out that 47 + 72 = 119 is pretty straightforward. But try adding XLVII and LXXII. The easiest way is to convert the Roman numerals into our decimal (base-10) system and then convert back to give the answer, CXIX. Doing it the Roman way is tortuous, and as for multiplication ...

Zero as a number in itself, rather than as a place-holder, is a much more recent invention (or discovery). Its introduction in this guise is attributed to Pingala, an Indian scholar who lived during the third and second centuries BCE. Pingala used a place-value notation, based on binary rather than decimal because a binary notation allowed numbers to be encoded into Sanskrit verse. But he also employed the word 'sunya', Sanskrit for empty, to refer to the number zero. The earliest appearance of the symbol in its modern form is in the Bakhshali manuscript, a text written on birch bark and found in the summer of 1881 near the village of Bakhshali in what at the time was British-ruled India but is now Pakistan. A large part of the manuscript had been destroyed and only about 70 leaves of bark, of which a few were mere scraps, survived to the time of its discovery. From what we can gather it seems to be a commentary on an earlier mathematical work, setting out rules and techniques for solving problems, mostly in arithmetic and algebra, but also to a lesser extent in geometry and mensuration (the maths of measurements). Now kept at the Bodleian Library in Oxford, the manuscript has been recently carbon-dated to the third or fourth century CE making it several centuries older than previously thought.

Later, in the seventh century CE, the Indian mathematician Brahmagupta put the concept of zero as a number on a firm footing. He laid down various rules for doing arithmetic when zero and negative numbers (another

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to the result in one quick step, because we end up with 0/0. Instead, we have to let x creep up on the value 1, bit by bit: when $x = 0.5$, $(x^2 - 1)/(x - 1) = 1.5$; when $x = 0.9$, $(x^2 - 1)/(x - 1) = 1.9$; when $x = 0.999$, $(x^2 - 1)/(x - 1) = 1.999$, and so on. The endpoint is clearly 2, even though we can't, in this case, put in the value for x that transports us instantly to this final answer. It's simply the limit of the process.

In some ways, approaching zero closer and closer in maths is analogous to the efforts of physicists to produce an ever more perfect vacuum – a space devoid of all matter. Those efforts began in earnest when seventeenth-century Italian physicist and mathematician Evangelista Torricelli learned of the fact that no matter how strong a team of workers were, they couldn't use their water pumps to draw water more than 10 metres vertically. In 1643, Torricelli decided to try this experiment using mercury instead of water, as mercury, being denser, would involve a much smaller height. He found in this case the limit to be around 76 centimetres. He then took a tube that was slightly longer than 76 centimetres, sealed one end, filled it with mercury, and placed the upturned tube in a bowl that also contained mercury. Whenever he did this, the mercury level in the column would always drop to 76 centimetres. As air couldn't enter the space at the top of the column – the open end of the column being submerged in mercury – Torricelli deduced that he'd created a vacuum. To be sure, it wasn't a perfect vacuum (for one thing, it would contain traces of mercury vapour) but it was good enough to give the lie to the ancient philosophical claim that Nature abhorred a vacuum.

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