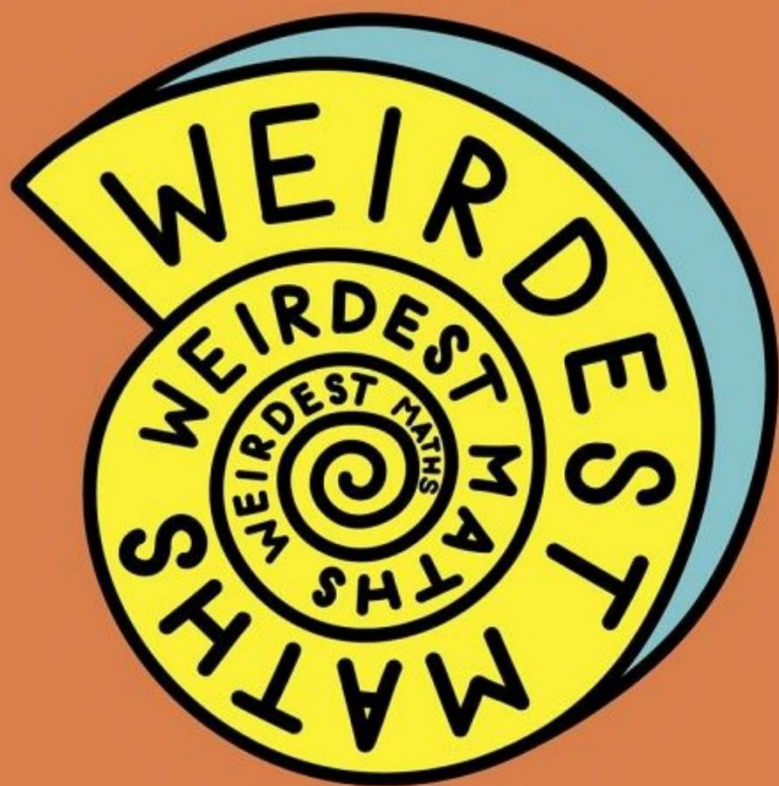


DAVID DARLING
AGNIJO BANERJEE



AT THE FRONTIERS
OF REASON

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Introduction

We began writing our first book together when one of us was a 61-year-old science writer and the other a 15-year-old schoolboy. It seemed an unlikely combination but Agnijo was no ordinary teenager. His father, who'd heard that I (David) do some tutoring in maths and science, asked if I'd be available to work with Agnijo to help develop his wider knowledge of maths, modern physics, and areas where science and philosophy overlap. His school was running out of things to teach him and he was taking exams four years ahead of his age group (nearly always scoring full marks).

From the outset, it was clear that Agnijo had outstanding abilities. He could perform extraordinary feats of mental arithmetic and had a phenomenal memory. The first time he came to me I lent him a copy of a book I'd written about ten years before called *The Universal Encyclopedia of Mathematics*, a fairly dense 400-page A-to-Z of everything under the mathematical sun. A week later he brought the book back – not only had he read it and memorised large chunks but he'd also found several minor mistakes! From then on, our sessions weren't so much conventional tutorials as they were conversations at a graduate level, on everything from the nature of dark energy to ways of naming numbers vast beyond imagination.

At some point in 2015 I suggested to Agnijo that we write a book together, dividing up the chapters according to our special interests and cross-checking each other's work. Fortunately, my publisher, Oneworld, saw the merit in our

unusual partnership. Our proposal was accepted and the start of 2018 saw the fruits of our collaboration – *Weird Maths* – appear in print.

If any seventeen-year-old mathematicians have had a bigger year than Agnijo did in 2018, they must be few and far between. Shortly after becoming a published author, Agnijo was selected for the UK team at the prestigious Balkan Mathematical Olympiad and then to represent Britain at the 2018 International Mathematical Olympiad. His one and only appearance at an IMO saw him take joint first place with a perfect score – the best result by a UK competitor in twenty-four years. No sooner had he returned to Britain than he was on his travels again, this time to India with his family, to help promote a new edition of *Weird Maths* there. As if that were not enough, he had only a few weeks to recover after coming back to British soil before starting his maths degree at Trinity College, Cambridge, the alma mater of so many of his mathematical heroes, including Srinivasa Ramanujan, G. H. Hardy, and Charles Babbage.

In *Weirdest Maths*, the final book of the *Weird Maths* trilogy, we trek far and wide across the landscape of this strangest and most wonderful subject. We explore the maths of sport, life, and the universe as a whole. We ask what makes a mathematical genius and follow the adventures of Agnijo from elementary school prodigy to first place in the world's greatest maths competition. We look at maths in fiction, examine the claim that beauty is a reliable guide to truth, and ask what mathematicians may accomplish over the decades ahead.

To many people maths may seem a dull, hard subject – one of the necessary evils of school to be avoided later on as much as possible. But maths is infinitely greater than a system of calculation. It permeates everything around and within us –

music, art, the natural world, and the human mind – forming an invisible infrastructure of reality. Mathematics, far from being dry and difficult, is vibrant and fascinating, as wonderful as it is weird.

Genius

Talent is a flame. Genius is a fire.

– Bernard Williams

When he was six, John von Neumann could multiply and divide two 8-digit numbers in his head. A couple of years later he'd progressed to solving tough problems in differential and integral calculus. He'd also amuse his parents' friends by chatting in ancient Greek or reciting whole pages of a telephone directory that he'd memorised at a glance – early signs of the startling ability that would blossom in adulthood. Economist Paul Samuelson said of von Neumann that he had 'the fastest mind' of anyone he'd met. Polish-born British mathematician Jacob Bronowski, in the 1973 documentary series *The Ascent of Man*, considered him 'the cleverest man I ever knew, without exception' (the second cleverest he judged to be Italian-American physicist Enrico Fermi).

These days 'genius' is a much overused label. It's also, like beauty, an imprecise one that's partly in the eye of the beholder. The claim to be able to measure the threshold of genius with a single number – an 'intelligence quotient' or IQ – just doesn't bear scrutiny. A commonly applied type of IQ test, based on the work of French psychologist Alfred Binet in the early twentieth century, puts a figure of 100 or thereabouts on average intelligence and rates genius as anything above

about 160. But like the cryptic crossword puzzles in a newspaper, IQ tests are things you get better at with practice and age (up to a point). They also favour certain types of thinker. Who'd feel happy about ranking the relative genius of Beethoven, Picasso, and Einstein based, say, on their scores in the entrance test to Mensa? Richard Feynman, by common consent one of the brightest theoretical physicists of the twentieth century, and co-winner of the 1965 Nobel Prize for Physics (with Julian Schwinger and Sin-Itiro Tomonaga), managed just 125 in a high school IQ test. My co-author, Agnijo, took the Mensa entrance test at age twelve and achieved a maximum possible score of 162, putting him ahead of Stephen Hawking at the same age. But he's modest enough to recognise that such comparisons are meaningless unless a great intellect is turned to good use later in life.

What is genius – and, in particular, *mathematical* genius? Does the potential for it lie within each of us if only we knew how to tap it? Or does the spark of genius have to be there from the start, in the makeup of an individual's brain? There are no easy answers because the forms in which exceptional ability and achievements come are so varied.

John von Neumann, for instance, though unquestionably a genius by any definition, had a privileged upbringing. Born to Jewish parents in Budapest in 1903, he was given every advantage money could buy including, at the age of eight, entrance to the Fasori Evangélikus Gimnázium, one of three prep schools at the pinnacle of a superb education system in the Hungarian capital – for those who could afford it. Between the late 1890s and the 1930s, this elite system turned out a generation of mega-intellecets who played important roles on the world stage of science and maths: von Neumann himself, mathematician and space engineer Theodore von Kármán,

radiochemist George de Hevesy, physicists Leó Szilárd, Eugene Wigner, and Edward Teller, and extraordinarily prolific mathematician Paul Erdős, among them. Most of these talented Jewish Hungarians ended up in the United States in the first half of the twentieth century and soon gained a reputation for their almost superhuman abilities. On one occasion, Szilárd was asked why no intelligent extraterrestrials had been found despite the seeming likelihood that they existed – the so-called Fermi Paradox. He replied: ‘They are already here among us – they just call themselves Hungarians.’

Copyrighted image

John von Neumann, widely regarded as one of the greatest mathematical geniuses of the twentieth century, shown here in 1956.

No one would claim that eastern Europeans are inherently

smarter than anyone else. All the Hungarian example shows is that the right upbringing and education can help foster intellectual attainment, but there's surely more to genius than that. Some of the greatest mathematicians the world has ever known came from humble backgrounds.

Take Carl Friedrich Gauss, born in 1777 in what's now Lower Saxony, Germany. Today, he stands shoulder to shoulder with the likes of Euclid, Isaac Newton, and Leonhard Euler as a colossus of mathematics. Yet his origins were humble. His father turned his hand to gardening, bricklaying, butchering, and anything else that would help make ends meet. His mother could neither read nor write and never recorded the date of Carl's birth. She did remember, though, that it was on a Wednesday, eight days before the Feast of Ascension, which, in turn, is the fortieth day of Easter. In time, Gauss came up with a formula that gave not only his own birthdate but also the date of Easter in any year, past or future.

Even as a toddler, Carl's talent with numbers stood out: he could do sums when he could barely speak. Just after he'd turned three, he spotted a mistake in his father's tax calculations. As a seven-year-old, he solved a problem in seconds that his teacher had expected would keep his class busy for ages. The problem was to add together all the numbers from one to a hundred. Gauss quickly spotted that the sum could be broken down into fifty pairs – $(1 + 100) + (2 + 99) + \dots + (50 + 51)$ – each of which added up to 101, giving a total of 101 times 50, or 5050. By the age of ten he'd discovered an important result in maths called the binomial theorem that, unbeknown to him, had been derived earlier by Newton. Word spread of the young prodigy's achievements and he found a sponsor in the Duke of Brunswick who offered to fund his further education.

Thanks to his benefactor, Gauss was able to attend the Collegium Carolinum – where he earned a degree in maths at the age of eighteen – and then move on to the prestigious University of Göttingen for his graduate studies. In 1796, a year after receiving his doctorate, he cracked a major problem in geometry by showing that a regular polygon with 17 sides could be constructed using just an unmarked ruler and a compass. The Greeks had known how to construct polygons with 3, 5, and 15 sides with ruler and compass alone but making the heptadecagon in the same way had resisted all their efforts. It was this breakthrough that persuaded Gauss to focus on maths instead of languages, at which he also excelled. Later in the same year, he discovered that every number is the sum of at most three triangular numbers (numbers of the form $1 + 2 + \dots + n$, for example 1, 3, 6, or 5050).

One of Gauss's most spectacular achievements was to track down a heavenly body that had gone missing. In 1801, Italian astronomer Giuseppe Piazzi discovered a faint object, which he called Ceres, that didn't appear in catalogues of known stars at the time. Piazzi followed Ceres over a period of several weeks and found that it wasn't a star at all but something in orbit around the Sun. Then he fell ill and lost track of the newcomer. Fortunately, Gauss was able to figure out its orbit, along with its whereabouts, using just the handful of observations that Piazzi had already made. As we now know, Ceres is the largest object in the asteroid belt, so large, in fact, that it's been reclassified as a dwarf planet.

Gauss showed an exceptional talent for maths even before he started school. His genius, it seems, was innate – although who knows if it would have flourished later on had not his ability been recognised and allowed to develop. In some ways his story parallels that of another genius who lived more

recently.

The most extraordinary mathematician of the twentieth century was born, like Gauss, to working-class parents and had, early on, a very modest education. Yet by the age of eleven, when he encountered formal maths in school for the first time, it was already clear that Srinivasa Ramanujan was operating on a different plane. In his early teens he tutored other pupils, mastered new concepts with ease, and won a string of academic awards. In 1903, as a sixteen-year-old, he got hold of a library copy of a book with the disarmingly simple title *A Synopsis of Elementary Results*, which, in fact, was a dense collection of about five thousand results in pure maths based on the notoriously challenging Mathematical Tripos at Cambridge. Not satisfied merely to absorb the book's contents, Ramanujan set out to derive all of its results himself with no outside help. In the process he came up with a wealth of other extraordinary conclusions that seemed to spring from nowhere.

This almost manic creativity, with no obvious point of origin, became a hallmark of Ramanujan's work. To the end of his life, he attributed all of his major insights and discoveries to a singular source beyond logic – the Goddess of Namagiri (his hometown) who, he said, appeared to him in visions and revealed formulae, which, upon waking, he'd seek to verify. Ramanujan's proofs, however, were often incomplete, making it hard to check them or sometimes even to make sense of his propositions. They were also, sometimes, just plain wrong.

It's possible that Ramanujan would have remained in relative obscurity had he not, in his twenties, written a series of letters to distinguished British mathematicians. Only one of them took him seriously. Fortunately, that one happened to be G. H. Hardy, famed Cambridge scholar and distinguished

number theorist who had himself been precocious as a child. While still a toddler he wrote down numbers into the millions and, later, when taken to church on the Sabbath he'd pass the time factorising the numbers of hymns. In Ramanujan's writings, Hardy recognised something very special indeed. Some of the Indian's results corresponded to known maths, but of a very advanced nature and arrived at by unfamiliar means. Other results seemed utterly new but, in Hardy's opinion, probably true 'because, if they were not true, no one would have the imagination to invent them'. In Ramanujan's obituary, which, sadly, Hardy would pen just seven years later, he wrote that Ramanujan was 'a mathematician of the highest quality, a man of altogether exceptional originality and power'. On his personal scale of maths ability, Hardy scored himself a modest 25, another close colleague at Cambridge, John Littlewood, 30, and David Hilbert, the most renowned mathematician of the time, 80. Ramanujan he rated at 100.

Hardy invited Ramanujan to join him at Cambridge and for a few years the two formed a formidable team, at the very college, Trinity, where Agnijo now studies. Hardy taught the younger man how to set down proofs in an orthodox way so that they could be published in academic journals and checked by other mathematicians. At the same time, he was aware that it was neither possible nor desirable to give the Indian a conventional education in all the areas of maths that he'd missed. Hardy understood well the danger of such an education: that it can stifle the kind of extreme creativity that's so often the sign and greatest product of true genius.

Knowing too much about a subject can make us overly cautious. Having a lot of conventional wisdom may make us doubt our own hunches and intuition because we're more likely to think that any seemingly good ideas that pop into our heads

are wrong if they don't square with what we've previously learned. Had Ramanujan received an expensive but traditional education, would his genius have burned so brightly and uniquely? For sure, mathematical genius needs some foundation on which to build, but what's the optimal amount of formal schooling to nurture genius but not at the same time crush it with conformity?

The strangest aspect of Ramanujan's genius was his conviction that it had a supernatural source. Artists and musicians have often, especially in the past, expressed the view that their work was divinely inspired. Kahlil Gibran, the Lebanese-American mystic poet and artist, wondered: 'Am I a harp that the hand of the almighty may touch me or a flute that his breath may pass through me?' It's more unusual for mathematicians or scientists to regard themselves as agents of a higher power, but it's common for them to talk about the importance of sudden inspiration. Famously, the German chemist August Kekulé ascribed his discovery of the benzene ring to a dream in which he saw a serpent with a tail in its mouth – an ancient symbol, known as the *ouroboros* (Greek for 'tail-devourer'), representing an endless cycle or loop. French philosopher and mathematician René Descartes and mathematician Henri Poincaré also accounted for some of their important discoveries in terms of pictorial revelations from the unconscious. In a chapter titled 'Mathematical Creation' from his 1904 book *The Foundations of Science*, Poincaré put it this way:

The subliminal self is in no way inferior to the conscious self; it is not purely automatic; it is capable of discernment; it has tact, delicacy; it knows how to choose, to divine.

Poincaré is another of those widely regarded as being among the greatest of mathematical geniuses. He's been described as 'the last universalist' because after his time (he died in 1912) maths spread so far and wide that even the most penetrating minds on Earth could no longer master all aspects of it.

Swiss psychiatrist and psychoanalyst Carl Jung spoke of the ancient tradition of knowledge being imparted through dreams and visions and how twentieth-century rationality tended to devalue its significance. Yet Albert Einstein, the towering figure in physics from the past hundred years, was quick to acknowledge that his biggest breakthroughs came out of the blue. He recalled that, one night in the spring of 1905, 'a storm had broken loose in my mind' and in the morning it was as if he had the master plan for the universe in his grasp. The days and weeks that followed saw Einstein working feverishly, non-stop, filling thirty-one pages of notes that formed the basis for his special theory of relativity – a new physics of space and time.

When insights come in this way, in dreams or daydreams, or in waking from a deep sleep, it isn't hard to see how they might be attributed to a deity or some mysterious cosmic influence. Ramanujan was steeped in the religion of his family and birthplace, which habitually gave credit for happy outcomes to Namagiri Thayar, the local form of Lakshmi, the Hindu goddess of wealth and success. It would be natural for visions of her, constructed in his mind, to overlap with subconscious mathematical musings – his two great obsessions fused into one. Even Einstein spoke sometimes of his ideas coming from God, although he professed to believe not in a personal god but instead something more along the lines of Spinoza's god – 'a superior mind that reveals itself in the world of experience'.

We're all familiar with occasions when we go to bed with some problem niggling away at us only to wake to find the solution there crystal clear, arrived at effortlessly sometime in the night. Our brains are just naturally good at sorting things out without conscious intervention – in fact, in many cases, thinking just seems to get in the way. But no amount of R&R can help if we're trying to do something difficult without first having the necessary knowledge or skill. Great mathematicians may have great insights but they already know a lot about their subject and spend much of their waking time involved with it. The same is true in every walk of life. A top tennis player will speak of being 'in the zone', when shots flow effortlessly with fluid grace and the player is effectively on autopilot. But that experience can only come after years of practice and dedication.

Argentine-American mathematician and computer scientist Gregory Chaitin, who's made important contributions to information theory, has described the intellectual equivalent of being in the zone:

I can only look at my own experience creating a new mathematical theory and say I don't know where it comes from... I seem to be in some kind of energised or more perceptive state and it's a wonderful state to be in. It doesn't last long. It feels wonderful.

Whenever the subject of extreme ability crops up, whether it's intelligence, artistic skill of some kind, or athletic prowess, there's the age-old issue of nature versus nurture. Both, on their own, have limits. With the best will – and training regime – in the world, the reader, even in his or her prime, would never be able to run as fast as Usain Bolt. Nurturing will only get you so far if the basic physical ingredients aren't in place.

At the same time, a potential genius might go to waste without the right encouragement and support. In some cases, it seems, nature takes the lead and nurture follows, as in the case of Gauss and Ramanujan. In other instances, exceptional mathematicians and scientists have emerged after not-very-promising starts to their careers.

Jacques Hadamard was a French mathematician who rose to fame at the end of the nineteenth century with his proof (arrived at independently by Belgian contemporary Charles de la Vallée Poussin) of the prime number theorem. This theorem has to do with how prime numbers are distributed on the number line and bears upon the Riemann hypothesis – the biggest unsolved conundrum in maths. Yet, as Hadamard pointed out, he was a late bloomer: ‘in arithmetic, until the fifth grade [ten or eleven years old], I was last or nearly last.’

The same was true of Hermann Grassmann, one of the founders of linear algebra – a subject that now has all kinds of applications in science, from quantum mechanics to machine learning. A couple of centuries ago, the young Grassmann was studying at Stettin Gymnasium, in what was then Prussia, where his father taught maths and physics. Grassmann senior wasn’t optimistic about his offspring’s mathematical potential and recommended that perhaps he become a gardener instead. Luckily for the world, Hermann persevered with his academic studies, although at university, in Berlin, he took courses not in maths but theology, classical languages, and philosophy. He came back to maths in a modest way, taking a year out to prep himself in the basics before sitting exams so that, like his dad, he could teach the subject in secondary schools. It was while writing an essay for one of these exams, on the theory of tides, that Grassmann introduced an entirely new mathematical approach – what became known as linear algebra – together

with the concept of vector spaces.

Grassmann got little or no credit for his innovations at the time. He qualified to become a schoolteacher and, in 1852, at the age of forty-three, rose to his late father's position at the Stettin Gymnasium with the title of 'professor'. But his greatest ambition, to teach at a university, was stymied by a report to the Prussian Ministry of Education by another mathematician, Ernst Kummer, in which he described Grassmann's ground-breaking essay on tides as 'commendably good material expressed in a deficient form'.

Such criticism became a recurrent theme in Grassmann's life. Over and over again, with a handful of exceptions, his peers attacked his methods and failed to grasp the underlying importance of his ideas. His first great published work, *Ausdehnungslehre* ('Theory of Extension'), in 1844, was almost totally ignored, as was a new version of the book eighteen years later. Eminent mathematicians such as August Ferdinand Möbius, Augustin-Louis Cauchy, and Giuseppe Peano were aware of what Grassmann was doing, and Peano was generous enough to acknowledge the part that Grassmann's concepts played in his own treatment on the foundations of natural numbers. But the fact is that Grassmann was too far ahead of his time to be properly appreciated. He also lacked the language and mathematical tools, such as those of set theory, which hadn't yet been developed, to be able to express his ideas rigorously. In retrospect, he's seen as one of the few people ever to invent a whole new branch of mathematics pretty much single-handed. But it wasn't until the first quarter of the twentieth century, when Hermann Weyl and others came up with formal definitions of the key concepts at the heart of linear algebra, notably that of a vector space, that Grassmann's genius and the extent of his achievement were recognised.

of relativity, and the equivalence of mass and energy, any one of which might have won him a Nobel Prize (although, in fact, only the first did). Einstein was now twenty-six and at the peak of his powers. He'd remain at that summit for perhaps another decade during which he hatched a radically new theory of gravity – the general theory of relativity. But after 1915 his creativity fell away and he pioneered no more ground-breaking science for the rest of his life.

In his 1940 memoir, *A Mathematician's Apology*, G. H. Hardy wrote: 'No mathematician should ever allow himself to forget that mathematics, more than any other art or science, is a young man's game.' Hardy's sentiment is commonly extended to include physics, especially theoretical physics, which is highly mathematical. Certainly there are many examples to bolster the argument – Einstein being a case in point. His genius burned intensely for a dozen or so years after the turn of the century, but then flickered out.

By the time Einstein arrived at his final academic home, the Institute for Advanced Study (IAS), in Princeton, New Jersey, in 1933, he'd begun a long and futile quest for a unified theory of gravity and electromagnetism. At the IAS he found himself in the company of two other intellectual giants, both unusual in their own ways: the Austrian Kurt Gödel, a logician, who became Einstein's closest friend, and John von Neumann.

As premier geniuses of the twentieth century, Einstein and von Neumann make an interesting contrast. Einstein today is overwhelmingly the better known, but von Neumann's achievements span a broader range and began earlier in his career. By the age of nineteen, von Neumann had published two major mathematical papers, the second of which gave the modern definition of so-called ordinal numbers – numbers that can be used to generalise the concept of natural numbers. He

later pioneered game theory and early electronic computers and played a prominent role in the Manhattan Project, the top-secret US programme to develop an atomic bomb.

One of von Neumann's colleagues on the Manhattan Project was fellow high-IQ Hungarian Eugene Wigner. The two had been a year apart in the same elite school in Budapest. When asked why the Hungary of his generation had spawned so many geniuses, Wigner, who won the Nobel Prize in Physics in 1963, replied that von Neumann was the only genius. Perhaps being a close friend from childhood, Wigner was biased but he said of von Neumann 'only he was fully awake'. Comparing him with Einstein, however, he commented:

Einstein's understanding was deeper even than von Neumann's. His mind was both more penetrating and more original than von Neumann's.

Various factors, it seems, contribute to what we call genius and the forms it may take: speed of thought (at which von Neumann, by all accounts, was exceptional), depth of understanding (at which, according to Wigner, Einstein excelled), originality, creativity, and so forth. Sometimes, too, genius may be narrow in its focus – as in the case of Einstein or Ramanujan – while at other times, as illustrated by von Neumann, and to an even greater extent by some Renaissance figures such as Leonardo da Vinci, it can range over many subjects. For all his recognition during life, von Neumann doesn't have the celebrity status today of Einstein whose office at the IAS was just down the hall. Yet while Einstein essentially stagnated after his arrival at the Institute, von Neumann continued to flourish, taking on one massively difficult challenge after another right up until the end of his relatively short life. From the maths of quantum mechanics he'd pivot to

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A Oneworld Book

First published by Oneworld Publications in 2020

This ebook edition published 2020

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ISBN 978-1-78607-805-6

eISBN 978-1-78607-806-3

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Typesetting and ebook by Tetragon, London

Oneworld Publications

10 Bloomsbury Street

London WC1B 3SR

England